Superconductivity in the Pseudogap State due to Fluctuations of Short – Range Order.

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Abstract

We analyze the anomalies of superconducting state (s and d–wave pairing) in a simple model of pseudogap state, induced by fluctuations of short – range order (e.g. antiferromagnetic), based on the model Fermi surface with "hot patches". We derive a system of recursion relations for Gorkov's equations which take into account all diagrams of perturbation theory for electron interaction with fluctuations of short – range order. Then we find superconducting transition temperature and gap behavior for different values of the pseudogap width and correlation lengths of short – range order fluctuations. In a similar approximation we derive the Ginzburg – Landau expansion and study the main physical characteristics of a superconductor close to the transition temperature, both as functions of the pseudogap width and correlation length of fluctuations. Results obtained are in qualitative agreement with a number of experiments on underdoped HTSC – cuprates.

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I. INTRODUCTION.

The pseudogap state, observed mainly in the underdoped region of the phase diagram of high – temperature superconducting (HTSC) cuprates, leads to a wide range of anomalies of physical properties of these system both in normal and superconducting states [1]. There are two main theoretical scenarios to explain these anomalies. The first is based upon the model of Cooper pairs formation already above the temperature of superconducting transition [2–4] with phase coherence appearing only for $T < T_c$. The second assumes that the pseudogap state is induced by fluctuations of antiferromagnetic (AFM) short – range order which exist in the underdoped region of the phase diagram [5–7]. Recently a number of experiments appeared which rather convincingly favor the second scenario [8,9].

Most theoretical papers in the field are dedicated to the study of the models of the pseudogap state in normal phase for $T > T_c$. In Refs. [10,11] we proposed a very simple (toy) exactly solvable model of the pseudogap state, which assumes the existence of certain "hot" (flat) patches on the Fermi surface. Within this model we derived the Ginzburg – Landau expansion for different types of Cooper pairing [10] and studied the anomalies of superconducting state for $T < T_c$ [11] induced by fluctuations of AFM short – range order. In these papers we dealt with (over) simplified model of static Gaussian fluctuations of short – range order with infinite correlation length, which allowed us to obtain an exact analytical solution for the pseudogap state. In real systems the correlation length of AFM fluctuations is finite and relatively short [6]. The main aim of the current paper is to generalize the main results of Refs. [10,11] to the case of finite correlation lengths of fluctuations of AFM short – range order and to analyze the dependence of the main physical characteristics of the superconducting state on this correlation length, as well as on the effective width of the pseudogap.

II. MODEL OF THE PSEUDOGAP STATE.

We consider the simplified model of the pseudogap [10,11] based upon the picture of well – developed fluctuations of AFM short – range order and is in some respects close to the "hot – spot" model of Ref. [6]. We assume the Fermi surface of two – dimensional electronic system as shown in Fig.1. In fact such a Fermi surface was observed in a number of ARPES – experiments on HTSC – cuprates [12,13]. Note that the assumption of the flatness of these patches is non crucial for our model, but significantly simplify calculations, which, in principle, can be done also in more realistic "hot-spot" approach with apparently similar results. The model Fermi surface as in Fig.1 has already been applied to HTSC – cuprates in Refs. [14–16], where the details of microscopic criteria for the existence of antiferromagnetic and superconducting phases were analyzed. Here we just assume a kind of phenomenological model with static Gaussian fluctuations of short – range order with correlation function (structure factor) of the following form [5]:

$$S(\mathbf{q}) = \frac{1}{\pi^2}\frac{\xi^{-1}}{(q_x - Q_x)^2 + \xi^{-2}}\frac{\xi^{-1}}{(q_y - Q_y)^2 + \xi^{-2}} \tag{1}$$

where ξ – is correlation length of fluctuations, while the scattering vector is taken to be $Q_x = \pm 2k_F$, $Q_y = 0$ or $Q_y = \pm 2k_F$, $Q_x = 0$, anticipating that these fluctuations are incommensurate. The factorized form of (1) was first introduced in Ref. [5] and leads to great simplification of all calculations. At the same time we stress that in practice it practically coincides with the usual isotropic Lorentzian in the most important region of $|\mathbf{q}-\mathbf{Q}|<\xi^{-1}$ [7].

Less justified physically is the assumption of the static nature of fluctuations which can be reasonable only at high enough temperatures [6,7]. At low temperatures, including the superconducting phase, spin dynamics can be very important, e.g. for the microscopics of Cooper pairing according to the picture of "nearly – antiferromagnetic" Fermi liquid [17,18]. However, we assume that our static approximation can be sufficient for qualitative analysis of the influence of superconductivity upon superconductivity, which will be described below within the standard BCS – like approximation.

Let us write down the interaction of electrons with AFM fluctuations in the following form:

$$V_{eff}=(2\pi)^2W^2S(\mathbf{q}) \tag{2}$$

where W is determining the energy scale (width) of the pseudogap. We assume that only electrons from the flat ("hot") of the Fermi surface interact with AFM fluctuations, so that W is effectively non zero only for these electrons [10,11]. Note that we completely neglect the spin structure of interaction which can be rather easily taken into account [6], but makes calculations more cumbersome. In this sense, strictly speaking our discussion is more appropriate for the case of electrons interacting with fluctuations short – range order of CDW rather than SDW (AFM) type. We also assume that this simplification is relatively unimportant for the analysis of qualitative effects of pseudogap on superconductivity.

The factorized form of the correlator (1) and of interaction (2) leads to the one – dimensional nature of scattering on fluctuations. In the limit of infinite correlation length $\xi\to\infty$ this model acquires an exact solution [10,11,19]. For finite ξ we can construct "nearly exact" solution [7], directly generalizing the one – dimensional approach proposed in Ref. [20]. In this case we can (approximately) sum the whole diagram series for one – particle Green's function of electrons from the flat parts of the Fermi surface (where the "nesting" condition for the electronic spectrum $\xi_{\mathbf{p}\pm\mathbf{Q}}=-\xi_{\mathbf{p}}$ is satisfied).

For the contribution of an arbitrary diagram for the self – energy of an electron of the N-th order in (2) we use the following *Ansatz* [7,20]:

$$\Sigma^{(N)}(\varepsilon_n\mathbf{p})=W^{2N}\prod_{j=1}^{2N-1}G_{0k_j}(\varepsilon\mathbf{p}),$$
$$G_{0k_j}(\varepsilon_n\mathbf{p})=\frac{1}{i\varepsilon_n-(-1)^j\xi_{\mathbf{p}}+ik_j\kappa} \tag{3}$$

where $\kappa=v_F\xi^{-1}$ (v_F – Fermi velocity), k_j – is the number of interaction lines surrounding the j-th (from the beginning) electronic line in diagram, $\varepsilon_n=2\pi T(n+1/2)$ (in the following we write expressions for the case of $\varepsilon_n>0$). Thus the contribution of an arbitrary diagram is determined, in fact, only by some set of integer numbers k_j. An arbitrary diagram with intersections of interaction lines is seen to be equal to some diagram without intersections and the contribution of all diagrams with intersection can be taken into account with the help of some combinatorial factors $v(k_j)$, attributed to interaction lines on diagrams with no intersections [20,7,6]. In the model of incommensurate fluctuations which we only consider:

$$v(k)=\begin{cases}\frac{k+1}{2} & \text{for odd } k\\ \frac{k}{2} & \text{for even } k\end{cases} \tag{4}$$

As a result we obtain the following recursion procedure (continuous fraction representation) to calculate the one – particle Green's function $G(\varepsilon_n\mathbf{p})$ of electrons from "hot patches" [20,7,6]:

$$G_k(\varepsilon_n\mathbf{p})=\frac{1}{i\varepsilon_n-(-1)^k\xi_{\mathbf{p}}+ik\kappa-W^2v(k+1)G_{k+1}(\varepsilon_n\mathbf{p})};\quad G(\varepsilon_n\mathbf{p})\equiv G_0(\varepsilon_n\mathbf{p}) \tag{5}$$

Diagrammatically this procedure is shown in Fig.2.

Ansatz (3) for the contribution of an arbitrary diagram of the N-th order, in general case, is not exact [7,21]. However, in two – dimensional case we can explicitly demonstrate the topologies of the Fermi surface when (3) becomes exact [7], for the general case we can show [7] that this representation in some sense overestimates the role of finiteness of correlation length ξ in the given order of perturbation theory. For one – dimensional case, when this problem is particularly serious [7,21], it appears that for incommensurate fluctuations calculations of the density of states using (3) quantitatively almost ideally reproduce [22] the results of an exact numerical solution given in Refs. [23,24] [1]. In the limit of $\xi\to\infty$ *Ansatz* (3) reduces to an exact solution of Ref. [19], while in the limit of $\xi\to 0$ for the fixed value of W it describes the physically correct limit of free electrons.

Outside "hot patches" electrons in our model do not interact with fluctuations and the Green's function remains a free one:

$$G(\varepsilon_n\mathbf{p})=G_{00}(\varepsilon_n\mathbf{p})=\frac{1}{i\varepsilon_n-\xi_{\mathbf{p}}} \tag{6}$$

This model leads to non Fermi – liquid like behavior of the spectral density on "hot patches" and to the smeared pseudogap in the density of states (cf. similar results in "hot spots" model [6,7]). On "cold" parts of the Fermi surface we just get the usual Fermi – liquid (free electron) behavior.

III. GORKOV EQUATIONS FOR A SUPERCONDUCTOR WITH PSEUDOGAP.

In Refs. [10,11] we have analyzed the anomalies of superconducting state in an exactly solvable model of the pseudogap state induced by AFM short – range order fluctuations with infinite correlation length ($\xi\to\infty$). In particular, in Ref. [11] it was shown that AFM fluctuations may lead to strong fluctuations of superconducting order parameter (energy gap Δ) breaking the standard assumption of self – averaging gap [25–27] which allows independent

[1] In the case of one – dimensional problem with commensurate fluctuations *Ansatz* (3) does not describe a certain weak Dyson – type singularity of the density of states near the center of the pseudogap [23,24], while outside the region of this singularity it also gives rather good quantitative description of exact results. Note that in two -dimensional case Dyson – singularity, most probably, is just absent.

averaging (over configurations of the random field of static short – range order fluctuations) of the order parameter Δ and different combinations of electronic Green's functions, entering the basic equations of the theory. Usually, the possibility of such independent averaging is supported by the following argument [25,27]: the value of Δ changes on characteristic scale of the order of superconducting coherence length $\xi_0 \sim v_F/\Delta_0$ of BCS – theory, while Green's functions typically vary on much shorter lengths of the order of interatomic distances. Naturally, the last assumption becomes wrong if a new length scale $\xi \to \infty$ appears in electronic system. However, in case of AFM correlation length $\xi \ll \xi_0$ (i.e. when AFM fluctuations correlate on distances much smaller than the size of Cooper pairs) the self – averaging property of Δ remains, being broken only for $\xi > \xi_0$. For this reason all our analysis below will be done assuming the self – averaging nature of the energy gap, allowing us to use the standard approach of the theory of disordered superconductors (mean – field approach in terms of Ref. [11]). Thus, we temporarily put aside the very interesting problem of superconductivity in the absence of self – averageness of the order parameter. Note that in real HTSC – systems we, in fact, usually have $\xi \sim \xi_0$, so that these materials are in the region of parameters most difficult for the theory.

Similarly to Refs. [10,11] we assume that superconducting pairing is due to the following simplest attraction potential:

$$V(\mathbf{p},\mathbf{p}') = V(\phi,\phi') = -Ve(\phi)e(\phi'), \tag{7}$$

where ϕ is the polar angle determining the direction of electronic momentum $\mathbf{p}$ in highly conducting plane, while for $e(\phi)$ we take the simple model dependence:

$$e(\phi) = \begin{cases} 1 & (\ s\text{-wave pairing}) \\ \sqrt{2}\cos(2\phi) & (\ d\text{-wave pairing}) \end{cases}. \tag{8}$$

The attraction constant V is, as usual, non zero in some shell of the width of $2\omega_c$ around the Fermi level (ω_c – is characteristic frequency of the quanta responsible for electron attraction). In this case superconducting gap takes the form:

$$\Delta(\mathbf{p}) \equiv \Delta(\phi) = \Delta e(\phi). \tag{9}$$

In the following for brevity we shall denote gap simply as Δ instead of $\Delta(\phi)$ writing down explicit angular dependence only where it is necessary.

In the superconducting state perturbation theory over the interaction with AFM fluctuations (1) has to be built on "free" normal and anomalous Green's functions of a superconductor:

$$G_{00}(\varepsilon_n\mathbf{p}) = -\frac{i\varepsilon_n + \xi_{\mathbf{p}}}{\varepsilon_n^2 + \xi_{\mathbf{p}}^2 + |\Delta|^2}; \quad F_{00}^{+}(\varepsilon_n\mathbf{p}) = \frac{\Delta^*}{\varepsilon_n^2 + \xi_{\mathbf{p}}^2 + |\Delta|^2} \tag{10}$$

In the model with flat patches on the Fermi surface the electronic spectrum on the patches orthogonal to p_x is: $\xi_{\mathbf{p}} = v_F(|p_x| - p_F)$ as electron velocity $\mathbf{v}$ is perpendicular to p_y (everything is symmetric for patches orthogonal to p_y). Thus, in the case of s-wave pairing, when Δ is independent of the direction of electronic momentum, for the model interaction (1), (2) the problem becomes one – dimensional. In case of d-wave pairing situation is more difficult as even on the flat patches orthogonal to p_x the value of $\Delta(\phi)$ depends on p_y (and symmetrically on patches orthogonal to p_y). For this reason, in the analysis of d-wave pairing it is convenient to introduce instead of (1) the fluctuation correlator of the form

$$S(\mathbf{q}) = \frac{1}{\pi}\left\{\frac{\xi^{-1}}{(q_x \mp 2p_F)^2 + \xi^{-2}}\delta(q_y) + \frac{\xi^{-1}}{(q_y \mp 2p_F)^2 + \xi^{-2}}\delta(q_x)\right\} \tag{11}$$

In this case interaction does not change p_y or p_x on flat patches orthogonal to p_x or p_y and the problem again reduces to one – dimensional.

Now we can formulate an analogue of approximation (3) also for superconducting state. Some details of derivation are given in the Appendix A. The contribution of an arbitrary diagram of N-th order over interaction (2) to normal or anomalous Green's function is given by the product of $N+1$ "free" normal G_{0k_j} or anomalous $F^+_{0k_j}$ Green's functions with renormalized frequencies and gaps (cf. below). Here k_j – is the number of interaction lines surrounding the j-th (from the beginning of the diagram) electronic line. As in normal phase, the contribution of an arbitrary diagram is defined by the set of integers k_j and each diagram with intersection of interacting lines is equal to a certain diagram of the same order without intersections. Thus again we can use only diagrams without intersections of interaction lines accounting diagrams with intersections by the same combinatorial factors $v(k)$ (attributed to interaction lines) as in the normal phase. As a result we obtain diagrammatic analogue of Gorkov equations [28] shown in Fig.3. Accordingly we get a system of two recursion relations for normal and anomalous Green's functions:

$$\begin{aligned} G_k &= G_{0k} + G_{0k}\tilde{G}G_k - G_{0k}\tilde{F}F^+_k - F_{0k}\tilde{G}^*F^+_k - F_{0k}\tilde{F}^+G_k \\ F^+_k &= F^+_{0k} + F^+_{0k}\tilde{G}G_k - F^+_{0k}\tilde{F}F^+_k + G^*_{0k}\tilde{G}^*F^+_k + G^*_{0k}\tilde{F}^+G_k \end{aligned} \tag{12}$$

where

$$\tilde{G} = W^2 v(k+1)G_{k+1}; \quad \tilde{F}^+ = W^2 v(k+1)F^+_{k+1} \tag{13}$$

$$G_{0k}(\varepsilon_n\mathbf{p}) = -\frac{i\varepsilon_n + (-1)^k\xi_{\mathbf{p}}}{\tilde{\varepsilon}_n^2 + \xi_{\mathbf{p}}^2 + |\tilde{\Delta}|^2}; \quad F^+_{0k}(\varepsilon_n\mathbf{p}) = \frac{\tilde{\Delta}^*}{\tilde{\varepsilon}_n^2 + \xi_{\mathbf{p}}^2 + |\tilde{\Delta}|^2} \tag{14}$$

and we introduced renormalized frequency $\tilde{\varepsilon}$ and gap $\tilde{\Delta}$ as:

$$\tilde{\varepsilon}_n = \eta_k\varepsilon_n; \quad \tilde{\Delta} = \eta_k\Delta; \quad \eta_k = 1 + \frac{k\kappa}{\sqrt{\varepsilon_n^2 + |\Delta|^2}} \tag{15}$$

similar to those appearing for superconductors with impurities [28].

From (12)-(15) it is easy to obtain the system of recursion relations for real and imaginary parts of normal Green's function, as well as for anomalous Green's function:

$$\begin{aligned} ImG_k &= \frac{\tilde{\varepsilon} - Im\tilde{G}}{(\tilde{\varepsilon} - Im\tilde{G})^2 + ((-1)^k\xi_{\mathbf{p}} + Re\tilde{G})^2 + |\tilde{\Delta} + \tilde{F}|^2} \\ ReG_k &= \frac{(-1)^k\xi_{\mathbf{p}} + Re\tilde{G}}{(\tilde{\varepsilon} - Im\tilde{G})^2 + ((-1)^k\xi_{\mathbf{p}} + Re\tilde{G})^2 + |\tilde{\Delta} + \tilde{F}|^2} \\ F^+_k &= \frac{\tilde{\Delta}^* + \tilde{F}^+}{(\tilde{\varepsilon} - Im\tilde{G})^2 + ((-1)^k\xi_{\mathbf{p}} + Re\tilde{G})^2 + |\tilde{\Delta} + \tilde{F}|^2} \end{aligned} \tag{16}$$

Let us introduce the following notations:

$$ImG_k = -\varepsilon_n J_k; \quad ReG_k = -(-1)^k \xi_{\mathbf{p}} R_k; \quad F_k^+ = \Delta^* f_k \tag{17}$$

Then we can see that recursion relations for J_k and f_k just coincide, so that $J_k = f_k$. Finally we obtain the following system of recursion relations for J_k and R_k:

$$\begin{aligned} J_k &= \frac{\eta_k + W^2 v(k+1) J_{k+1}}{(\varepsilon_n^2 + \Delta^2)(\eta_k + W^2 v(k+1) J_{k+1})^2 + \xi_{\mathbf{p}}^2 (1 + W^2 v(k+1) R_{k+1})^2} \\ R_k &= \frac{1 + W^2 v(k+1) R_{k+1}}{(\varepsilon_n^2 + \Delta^2)(\eta_k + W^2 v(k+1) J_{k+1})^2 + \xi_{\mathbf{p}}^2 (1 + W^2 v(k+1) R_{k+1})^2} \end{aligned} \tag{18}$$

Then the normal and anomalous Green's functions of a superconductor are determined through R_0 and J_0:

$$ImG = -\varepsilon_n J_0; \quad ReG = -\xi_{\mathbf{p}} R_0; \quad F^+ = \Delta^* J_0 \tag{19}$$

and represent the complete sum of perturbation series for electron in a superconductor interacting with AFM short – range order fluctuations.

IV. CRITICAL TEMPERATURE AND TEMPERATURE DEPENDENCE OF THE GAP.

Energy gap of a superconductor is determined by the equation:

$$\Delta(\mathbf{p}) = -T \sum_{\mathbf{p}'} \sum_{\varepsilon_n} V_{sc}(\mathbf{p}, \mathbf{p}') F(\varepsilon_n \mathbf{p}') \tag{20}$$

On the flat parts of the Fermi surface the anomalous Green's function is defined by (19) and recursion relations (18). On the rest ("cold" part) of the Fermi surface the scattering on AFM fluctuations is absent (in our model), so that the anomalous Green's there function is given by (10). As a result, for the case of s-wave pairing, with the account of (8), the gap equation (20) takes the form:

$$1 = \lambda \left\{ \tilde{\alpha} T \sum_{\varepsilon_n} \int_{-\omega_c}^{\omega_c} d\xi J_0(\varepsilon_n \xi) + (1 - \tilde{\alpha}) \int_0^{\omega_c} d\xi \frac{th \frac{\sqrt{\xi^2 + \Delta^2}}{2T}}{\sqrt{\xi^2 + \Delta^2}} \right\} \tag{21}$$

where $\lambda = VN_0(0)$ is dimensionless coupling constant of pairing interaction ($N_0(0)$ – free – electron density of states at the Fermi level), $\tilde{\alpha} = 4\alpha/\pi$, where α is the angular size of a flat patch on the Fermi surface (cf. Fig.1). In numerical calculations below we, rather arbitrarily, choose $\tilde{\alpha} = 2/3$, i.e. $\alpha = \pi/6$, which is close e.g. to the data of Ref. [12].

In case of d-wave pairing we have to take into account the angular dependence of the gap (9), so that Eq. (20) becomes:

$$1 = \lambda \frac{4}{\pi} \left\{ T \int_0^{\alpha} d\phi e^2(\phi) \sum_{\varepsilon_n} \int_{-\omega_c}^{\omega_c} d\xi J_0(\varepsilon_n \xi) + \int_{\alpha}^{\pi/4} d\phi e^2(\phi) \int_0^{\omega_c} d\xi \frac{th \frac{\sqrt{\xi^2 + \Delta^2 e^2(\phi)}}{2T}}{\sqrt{\xi^2 + \Delta^2 e^2(\phi)}} \right\} \tag{22}$$

In Fig.4 we show the calculated (from Eq. (21)) temperature dependences of the gap for the case of s-wave pairing and for different values of correlation length (parameter $\kappa = v_F\xi^{-1}$) of fluctuations. For the case of d-wave pairing results are qualitatively similar.

Equation for superconducting critical temperature T_c immediately follows from (21), (22) as $\Delta \to 0$. In this case $J_0(\Delta \to 0)$ is independent of ϕ and is the same both for s and d-wave pairing. Then T_c – equation takes the following form:

$$1 = \lambda\left\{\alpha_{eff}T_c\sum_{\varepsilon_n}\int_{-\omega_c}^{\omega_c}d\xi J_0(\varepsilon_n\xi;\Delta \to 0) + (1-\alpha_{eff})\int_0^{\omega_c}d\xi\frac{th\frac{\xi}{2T_c}}{\xi}\right\} \tag{23}$$

where an "effective" size of flat patches on the Fermi surface is defined as:

$$\alpha_{eff} = \begin{cases} \tilde{\alpha} & (\ s\text{-wave pairing}) \\ \tilde{\alpha} + \frac{1}{\pi}\sin(\pi\tilde{\alpha}) & (\ d\text{-wave pairing}) \end{cases}. \tag{24}$$

Calculated dependences of T_c on the width of the pseudogap W and correlation length (parameter $\kappa = v_F\xi^{-1}$) are shown in Fig.5 (T_{c0} – transition temperature in the absence of pseudogap).

The general qualitative conclusion is the same as in Refs. [10,11]: pseudogap suppresses superconductivity due to a partial "dielectrization" of electronic spectrum on "hot" parts of the Fermi surface. This suppression effect is maximal for $\kappa = 0$ (infinite correlation length of AFM fluctuations) [10,11] and weakens as correlation length becomes shorter. These results are in full accordance with experimental phase diagram of HTSC – cuprates.

Let us stress once again that all our results are valid in case of self – averaging superconducting order parameter (mean – field approach of Ref. [11]), which is valid for not very large correlation lengths $\xi < \xi_0$, where ξ_0 – is superconducting coherence length (the size of Cooper pairs at $T = 0$). For $\xi \gg \xi_0$ important effects due to non self – averaging gap fluctuations appear, leading e.g to characteristic "tails" in temperature dependence of the average gap for $T_c < T < T_{co}$ [11].

V. COOPER INSTABILITY. RECURRENCE PROCEDURE FOR THE VERTEX PART.

It is well known that the critical temperature can also be determined from Cooper instability of the normal phase:

$$1 - V\chi(0,0) = 0 \tag{25}$$

where the generalized Cooper susceptibility is defined by the graph shown in Fig.6. Here we have to calculate the "triangular" vertex part accounting for interaction with AFM fluctuations. For the similar one – dimensional problem (and for real frequencies $T = 0$) the appropriate recurrence procedure was formulated in Ref. [29]. For our two – dimensional model this procedure was used in Ref. [30] to calculate optical conductivity. Generalization to Matsubara frequencies is rather straightforward. Below, for definiteness, we assume $\varepsilon_n > 0$. Then we obtain:

$$\Gamma_{k-1}(\varepsilon_n, -\varepsilon_n, \mathbf{q}) = 1 +$$

$$+W^2 v(k) G_k \bar{G}_k \left\{ 1 + \frac{2ik\kappa}{2i\varepsilon_n - (-1)^k v_F q - W^2 v(k+1)(G_{k+1} - \bar{G}_{k+1})} \right\} \Gamma_k(\varepsilon_n, -\varepsilon_n, \mathbf{q})$$

$$\Gamma(\varepsilon_n, -\varepsilon_n, \mathbf{q}) \equiv \Gamma_0(\varepsilon_n, -\varepsilon_n, \mathbf{q}) \qquad (26)$$

where $G_k = G_k(\varepsilon_n \mathbf{p} + \mathbf{q})$ and $\bar{G}_k = G_k(-\varepsilon_n, \mathbf{p})$ are calculated from (5).

To find T_c we have to know the vertex part at $\mathbf{q} = 0$. Then $\bar{G}_k = G_k^*$ and the vertex Γ_k becomes real, which considerably simplifies (26). Using notations similar to (17), from (5) and (26) we get:

$$\Gamma_{k-1} = 1 + W^2 v(k) \frac{J_k}{1 + W^2 v(k+1) J_{k+1}} \Gamma_k \qquad (27)$$

while for R_k and J_k we have recursion relations given by (18) with $\Delta = 0$.

There exists the following exact relation similar to the Ward identity (the proof of this relation will be given below):

$$G(\varepsilon_n \mathbf{p}) G(-\varepsilon_n \mathbf{p}) \Gamma(\varepsilon_n, -\varepsilon_n, 0) = (\xi_{\mathbf{p}}^2 R_0^2(\varepsilon_n \xi_{\mathbf{p}}) + \varepsilon_n^2 J_0^2(\varepsilon_n \xi_{\mathbf{p}})) \Gamma_0(\varepsilon_n, -\varepsilon_n, 0) \equiv J_0(\varepsilon_n \xi_{\mathbf{p}}) = -\frac{Im G(\varepsilon_n \mathbf{p})}{\varepsilon_n} \qquad (28)$$

Numerical analysis fully confirms this relation, demonstrating the self – consistency of our recursion relations for one – particle Green's function and vertex part [2]. As $J_0(\Delta \to 0)$ coincides with J_0 in normal phase, the relation (28) leads to T_c – equation obtained from Cooper instability (25):

$$1 = \lambda \left\{ \alpha_{eff} T_c \sum_{\varepsilon_n} \int_{-\omega_c}^{\omega_c} d\xi (\xi_{\mathbf{p}}^2 R_0^2(\varepsilon_n \xi_{\mathbf{p}}) + \varepsilon_n^2 J_0^2(\varepsilon_n \xi_{\mathbf{p}})) \Gamma_0(\varepsilon_n, -\varepsilon_n, 0) + (1 - \alpha_{eff}) \int_0^{\omega_c} d\xi \frac{th \frac{\xi}{2T_c}}{\xi} \right\} \qquad (29)$$

being the same as Eq. (23), obtained by linearization of the gap equation, despite seemingly different recursion procedures used to obtain these equations.

VI. GINZBURG – LANDAU EXPANSION.

In Ref. [10] we derived the Ginzburg – Landau expansion in exactly solvable model of the pseudogap with infinite correlation length of AFM fluctuations. Here we generalize these results to the case of finite correlation lengths.

Let us write the Ginzburg – Landau expansion for the difference of free energies of superconducting and normal state in the following form:

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[2] Note that an analytic proof of this relation from direct comparison of recursion procedures for Green's function and vertex part is non obvious, to say the least.

$$F_s - F_n = A|\Delta_{\mathbf{q}}|^2 + q^2 C|\Delta_{\mathbf{q}}|^2 + \frac{B}{2}|\Delta_{\mathbf{q}}|^4, \tag{30}$$

where $\Delta_{\mathbf{q}}$ – is the amplitude of the Fourier – component of the order parameter

$$\Delta(\phi, \mathbf{q}) = \Delta_q e(\phi). \tag{31}$$

Now (30) is defined by diagrams of loop – expansion for the free energy of an electron in the field of fluctuations of the order parameter with small wave – vector $\mathbf{q}$ [10].

We express the coefficients of Ginzburg – Landau expansion as:

$$A = A_0 K_A; \qquad C = C_0 K_C; \qquad B = B_0 K_B, \tag{32}$$

where A_0, C_0 and B_0 denote the standard expressions for these coefficients in the case of isotropic s-wave pairing:

$$A_0 = N_0(0)\frac{T - T_c}{T_c}; \qquad C_0 = N_0(0)\frac{7\zeta(3)}{32\pi^2}\frac{v_F^2}{T_c^2}; \qquad B_0 = N_0(0)\frac{7\zeta(3)}{8\pi^2 T_c^2}, \tag{33}$$

Then all the anomalies of the model under consideration, connected with the appearance of the pseudogap, are contained in dimensionless coefficients K_A, K_C and K_B. In the absence of pseudogap all these coefficients are equal to 1, only in case of d-wave pairing we have $K_B = 3/2$. Thus for d- wave pairing we shall appropriately normalize K_B, giving numerical results for $\tilde{K}_B = 2/3 K_B$.

Consider the generalized Cooper susceptibility shown in Fig.6.

$$\chi(\mathbf{q}0; T) = -T \sum_{\varepsilon_n} \sum_{\mathbf{p}} G(\varepsilon_n \mathbf{p} + \mathbf{q}) G(-\varepsilon_n \mathbf{p}) e^2(\phi) \Gamma(\varepsilon_n, -\varepsilon_n, \mathbf{q}) \tag{34}$$

Using (28) we can express the coefficients K_A and K_C as:

$$K_A = \frac{\chi(\mathbf{q}0; T) - \chi(00; T_c)}{A_0} =$$

$$= \alpha_{eff} \frac{T_c}{T - T_c} \left\{ T \sum_{\varepsilon_n = \pi T(2n+1)} \int_{-\omega_c}^{\omega_c} d\xi J_0(\varepsilon_n \xi) - T_c \sum_{\varepsilon = \pi T_c(2n+1)} \int_{-\omega_c}^{\omega_c} d\xi J_0(\varepsilon_n \xi) \right\} + 1 - \alpha_{eff} \tag{35}$$

$$K_C = \lim_{q \to 0} \frac{\chi(\mathbf{q}0; T_c) - \chi(00; T_c)}{q^2 C_0} =$$

$$= \frac{32\pi^2 T_c^3}{7\zeta(3) v_F q^2} \alpha_{eff} \left\{ \sum_{\varepsilon_n = \pi T_c(2n+1)} \int_{-\omega_c}^{\omega_c} d\xi J_0(\varepsilon_n \xi) - \right.$$

$$\left. - \sum_{\varepsilon = \pi T_c(2n+1)} \int_{-\omega_c}^{\omega_c} d\xi G(\varepsilon_n, \xi + \frac{1}{2} v_F q) \Gamma(\varepsilon_n, -\varepsilon_n, q) G(-\varepsilon_n, \xi - \frac{1}{2} v_F q) \right\} + 1 - \alpha_{eff} \tag{36}$$

Situation with coefficient B, in general case, is more complicated. Important simplifications appear if we limit ourselves in the order of $|\Delta_q|^4$, as is usually done, by considering only the case of $q = 0$. Then the coefficient B can be determined directly from the anomalous Green's function F, for which we already have the recurrence procedure (18), (19). Let us consider the diagrammatic expansion of the anomalous Green's function, shown in Fig.7(a). From this it becomes clear that:

$$\lim_{\Delta\to 0}\frac{F(\varepsilon_n\mathbf{p})}{\Delta}=G(\varepsilon_n\mathbf{p})G(-\varepsilon_n\mathbf{p})+...=G(\varepsilon_n\mathbf{p})G(-\varepsilon_n\mathbf{p})\Gamma(\varepsilon_n,-\varepsilon_n,0) \tag{37}$$

which, by the way, immediately proves (28) (use also (19)). Thus, for the two – particle loop $\chi(0,0)$ we get:

$$\chi(0,0)=T\sum_{\mathbf{p}}\sum_{\varepsilon_n}\lim_{\Delta\to 0}\frac{F(\varepsilon_n\mathbf{p})}{\Delta}=T\sum_{\mathbf{p}}\sum_{\varepsilon_n}J_0(\Delta=0) \tag{38}$$

For the "four – tail" diagram of Fig.7(b), determining the coefficient B, in the same way we obtain:

$$-T\sum_{\mathbf{p}}\sum_{\varepsilon_n}\lim_{\Delta\to 0}\frac{\frac{F(\varepsilon_n\mathbf{p})}{\Delta}-\lim_{\Delta\to 0}\frac{F(\varepsilon_n\mathbf{p})}{\Delta}}{|\Delta|^2}=-T\sum_{\mathbf{p}}\sum_{\varepsilon_n}\lim_{\Delta\to 0}\frac{J_0(\Delta)-J_0(\Delta=0)}{|\Delta|^2} \tag{39}$$

where $J_0(\Delta)$ is defined by the recursion procedure (18). Finally, for the dimensionless coefficient K_B we have:

$$K_B=\alpha_B\frac{8\pi^2T_c^3}{7\zeta(3)}\sum_{\varepsilon_n}\int_{-\omega_c}^{\omega_c}d\xi\lim_{\Delta\to 0}\frac{J_0(\Delta=0)-J_0(\Delta)}{|\Delta|^2}+1-\alpha_B \tag{40}$$

where

$$\alpha_B=\begin{cases}\tilde{\alpha} & (\ s\text{-wave pairing})\\ \tilde{\alpha}+\frac{4}{3\pi}\sin(\pi\tilde{\alpha})+\frac{1}{6\pi}\sin(2\pi\tilde{\alpha}) & (\ d\text{-wave pairing})\end{cases}. \tag{41}$$

These expressions allow direct numerical calculations of the coefficients K_A, K_C, K_B. In Fig.8, for example, we present the calculated dependence of K_C on the width of the pseudogap W and correlation length of AFM fluctuations (parameter $\kappa=v_F/xi^{-1}$). The appropriate dependences of K_A and K_B are qualitatively quite similar. In particular, for $\kappa=0$ we have just $K_B=K_C$ [10].

VII. PHYSICAL PROPERTIES OF SUPERCONDUCTORS WITH PSEUDOGAP.

Ginzburg – Landau expansion defines two characteristic lengths of superconductors: the coherence length and penetration depth of magnetic field.

The coherence length for given temperature $\xi(T)$ determines the characteristic scale of inhomogeneities of the order parameter Δ:

$$\xi^2(T)=-\frac{C}{A}. \tag{42}$$

In the absence of the pseudogap:

$$\xi^2_{BCS}(T)=-\frac{C_0}{A_0}, \tag{43}$$

$$\xi_{BCS}(T)\approx 0.74\frac{\xi_0}{\sqrt{1-T/T_c}}, \tag{44}$$

where $\xi_0 = 0.18 v_F/T_c$. In our model:

$$\frac{\xi^2(T)}{\xi^2_{BCS}(T)} = \frac{K_C}{K_A}. \tag{45}$$

The dependences of $\xi^2(T)/\xi^2_{BCS}(T)$ on the width of the pseudogap W and correlation length of fluctuations (parameter κ) for the case d-wave pairing are shown in Fig.9. Note that the changes of coherence length are relatively small.

For the penetration depth of a superconductor without the pseudogap we have:

$$\lambda_{BCS}(T) = \frac{1}{\sqrt{2}}\frac{\lambda_0}{\sqrt{1-T/T_c}}, \tag{46}$$

where $\lambda_0^2 = \frac{mc^2}{4\pi ne^2}$ is penetration depth at $T=0$. In general case:

$$\lambda^2(T) = -\frac{c^2}{32\pi e^2}\frac{B}{AC}. \tag{47}$$

Then for our model:

$$\frac{\lambda(T)}{\lambda_{BCS}(T)} = \left(\frac{K_B}{K_A K_C}\right)^{1/2}. \tag{48}$$

Graphical dependences of penetration depth for the case of d-wave pairing are shown in Fig.10.

Near T_c the upper critical magnetic field H_{c2} is defined via Ginzburg – Landau coefficients as:

$$H_{c2} = \frac{\phi_0}{2\pi\xi^2(T)} = -\frac{\phi_0}{2\pi}\frac{A}{C}, \tag{49}$$

where $\phi_0 = c\pi/e$ is magnetic flux quantum. Then the slope of the upper critical field close to T_c is defined as:

$$\left|\frac{dH_{c2}}{dT}\right|_{T_c} = \frac{24\pi\phi_0}{7\zeta(3)v_F^2}T_c\frac{K_A}{K_C}. \tag{50}$$

Graphic dependences of the slope of the upper critical field $\left|\frac{dH_{c2}}{dT}\right|_{T_c}$, normalized to the slope at T_{c0}, on the effective width of the pseudogap W and parameter of correlation length κ for the case of d-wave pairing are shown in Fig.11. It is seen that the slope for large enough correlation lengths rapidly drops with the width of the pseudogap. However, for short enough correlation lengths we can observe even some weak growth of this parameter for small values of the pseudogap width. For fixed pseudogap width the slope of H_{c2} significantly grows as correlation length becomes smaller.

Finally, let us consider the discontinuity of specific heat at the transition point:

$$\frac{C_s - C_n}{\Omega} = \frac{T_c}{B}\left(\frac{A}{T-T_c}\right)^2, \tag{51}$$

where C_s, C_n are specific heats of superconducting and normal states, Ω – sample volume. For $T = T_{c0}$ (in the absence of pseudogap, $W = 0$):

$$\left(\frac{C_s - C_n}{\Omega}\right)_{T_{c0}} = N(0)\frac{8\pi^2 T_{c0}}{7\zeta(3)}. \tag{52}$$

Then the normalized discontinuity of specific heat in our model can be expressed as:

$$\frac{(C_s - C_n)_{T_c}}{(C_s - C_n)_{T_{c0}}} = \frac{T_c}{T_{c0}}\frac{K_A^2}{K_B}. \tag{53}$$

Appropriate dependences on effective width of the pseudogap W and parameter of correlation length κ for the case of d-wave pairing are shown in Fig.12. It is seen that specific heat discontinuity rapidly drops with the growth of the pseudogap width and grows as correlation length of AFM fluctuations becomes smaller.

For s-wave superconductor the dependences of physical properties are quite similar, the only change is in larger scale of W for which the appropriate changes appear, corresponding to larger stability of isotropic superconductors to partial "dielectrization" of electronic spectrum due to pseudogap formation on "hot patches" of the Fermi surface [10,11].

Among the physical characteristics, analyzed above, relatively detailed experimental data are available for specific heat discontinuity [8]. In complete agreement with our conclusions, specific heat discontinuity for $Bi-2212$ rapidly drops as the system moves to the underdoped region, where the width of the pseudogap grows. According to Ref. [8] the width of the pseudogap (our parameter $2W$) changes from the values of the order of $700K$ for hole concentration $p = 0.05$ to the values of the order of $T_c \sim 100K$ near optimal concentration $p = 0.16$ and drops to zero for $p = 0.19$. A relation between the drop of specific heat discontinuity and the growth of the pseudogap width is clearly observed. Unfortunately, we do not know detailed enough data on concentration dependence of correlation length of fluctuations and appropriate dependences of physical characteristics of superconductors in pseudogap state. Qualitatively it is obvious that correlation length grows as system moves to the underdoped region, so that the drop of specific heat discontinuity is natural also from this point of view.

VIII. CONCLUSION.

In this paper we continued the study of anomalies of superconducting state in the framework of rather crude model of the pseudogap state of two – dimensional electronic system [10,11], which is, however, in qualitative agreement with a number of observed anomalies of electronic structure of underdoped HTSC – cuprates. In Refs. [10,11] we considered rather unrealistic limit of infinite correlation length of AFM short – range order fluctuations, which allowed us to obtain an exact analytic solution. Here we generalized our model to realistic case of finite correlation lengths, taking into account, as in Refs. [10,11], all diagrams of perturbation theory on electron interaction with fluctuations of short – range order. Our analysis was performed in a standard (mean – field in terms of Ref. [11]) approach, assuming self – averaging property of superconducting order – parameter over fluctuations of the random field of AFM fluctuations. In Ref. [11] we have shown that this assumption is not justified in the limit of $\xi \to \infty$. At the same time this assumption is apparently well justified for the case of $\xi \ll \xi_0$ (where ξ_0 is the coherence length of a superconductor at $T = 0$, i.e. the size of Cooper pairs). Thus, we are left with rather complicated task of the accounting of non self – averaging effects for $\xi > \xi_0$. We have already mentioned that in real HTSC systems in most cases $\xi \sim \xi_0$, so that effects of non self – averaging superconducting gap, similar to those considered in Ref. [11], may be very important, leading e.g. to characteristic "tails" in the temperature dependence of the average gap for $T > T_c$ and the physical picture of superconducting "drops" of Ref. [11].

Another serious simplification of our model is the assumption of static (and Gaussian) nature of short – range order of fluctuations. This assumption may be justified only for high enough temperatures $T \gg \omega_{sf}$ (where ω_{sf} – is characteristic scale of spin fluctuations) [6,7]. Thus, the use of static approximation in superconducting state for $T < T_c$ is rather doubtful. However, we think that our simplified treatment allows us to describe most important effects of the changes of the electronic spectrum (due to pseudogap formation on "hot patches" of the Fermi surface) upon superconductivity. The account of spin dynamics inevitably requires to drop the simple phenomenology of BCS model and consider the microscopic nature of pairing interaction. It is doubtful that such a program can be realized in near future. In particular, the problem of summation of all perturbation theory diagrams for the interaction with dynamical spin fluctuations seems absolutely hopeless.

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APPENDIX A: COORDINATE REPRESENTATION. NORMAL AND ANOMALOUS GREEN'S FUNCTIONS.

Let us consider some technical details of derivation of recursion relations for Gorkov equations (12) – (15). It is sufficient to limit the analysis to consideration of two flat parts of the Fermi surface, orthogonal to p_x – axis, which are connected by the scattering vector $\mathbf{Q} = (\pm 2p_F, 0)$. Then the problem becomes purely one – dimensional as the velocity projection $v_y = 0$ and the linearized electronic spectrum $\xi_{p_x \mp p_F} = \pm v_F p_x$ does not depend on y-component of electronic momentum. For brevity in the following we just put $v_F = 1$.

Calculations simplify in coordinate representation [21], considering the electron propagation if the field of Gaussian AFM fluctuations $W(x) \neq W^*(x)$ (incommensurate case) with correlator:

$$< W^*(x)W(x') >= W^2 e^{-\kappa|x-x'|} \tag{A1}$$

Then electron propagators corresponding to normal and anomalous Green's functions of a superconductor (10), take the form:

$$G_{00}(x) = \int_{-\infty}^{\infty} \frac{dp_x}{2\pi} e^{ip_x x} G_{00}(p_x) = -\frac{i}{2}\left(\frac{\varepsilon_n}{\sqrt{\varepsilon_n^2 + |\Delta|^2}} + \sigma_3 sign(x)\right) e^{-\sqrt{\varepsilon_n^2+|\Delta|^2}|x|}$$
$$F_{00}(x) = \int_{-\infty}^{\infty} \frac{dp_x}{2\pi} e^{ip_x x} F^+_{00}(p_x) = \frac{\Delta^*}{\sqrt{\varepsilon_n^2 + |\Delta|^2}} e^{-\sqrt{\varepsilon_n^2+|\Delta|^2}|x|} \tag{A2}$$

where $\sigma_3 = 1$ for right moving particles, and $\sigma_3 = -1$ for left moving particles. Scattering by fluctuations transforms "right" particles into "left" and vice versa. From (A2) it is seen that the particle moving along the path of the length l produces the factor $e^{-\sqrt{\varepsilon_n^2+|\Delta|^2}l}$.

During calculation of specific diagrams it is convenient [21] to change integration variables from coordinates of interaction vertices x_k to lengths of the paths l_k traversed by the particle between separate scatterings, fixing the total displacement $x - x'$. With interaction line connecting the vertices m and n on electronic line we have to associate the factor:

$$W^2 exp(-\kappa|x_m - x_n|) = W^2 exp(-\kappa|\sum_{k=m}^{n-1}(-1)^k l_k|) \tag{A3}$$

Integration over all l_k is performed from 0 to ∞.

It is seen that the finite correlation length of fluctuations in a given diagram leads to some "damping" of the appropriate transition amplitude with distance traversed by an electron. The exact treatment of this effect is difficult, but in Ref. [7] we used an obvious inequality:

$$exp\Big(-\kappa|\sum_{k=m}^{n-1}(-1)^k l_k|\Big) > exp\Big(-\kappa\sum_{k=m}^{n-1} l_k\Big) \tag{A4}$$

and replaced the exponential of (A3) by the exponential from the r.h.s. of (A4). This is equivalent to the replacement of correlator of random fields (A1) by similar expression, where in the exponent we just replace the distance $|x - x'|$ by the total length of the path traversed by the particle between scattering acts at x and x'. According to (A4) this procedure somehow overestimates the effect of damping κ in each diagram of perturbation theory. After such a replacement the diagrams of all orders are easily calculated and precisely reproduce the *Ansatz* of (3) for the normal phase [21]. We have already mentioned that results obtained in this way, e.g. for the density of states, are in good quantitative agreement with exact numerical simulation of the one – dimensional problem [23,24], which gives additional support for our approximation strengthening qualitative estimates of Ref. [7].

Let us use the same approximation during the analysis of diagrams of perturbation theory in superconducting phase, which are built upon propagators (A2). In this case the role of interaction reduces just to the appearance of additional factor of $e^{-\kappa l_k}$ in each normal or anomalous Green's function (A2), surrounded by the given interaction line, or (which is the same) to the addition of κ to $\sqrt{\varepsilon_n + |\Delta|^2}$ in the exponential of each Green's function. Making transformation back to the momentum representation it is easily seen that the contribution of an arbitrary diagram of the higher order of perturbation theory is determined by the product of the appropriate number of normal and anomalous Green's functions of the form:

$$G_{0k}(p) = -\frac{i\varepsilon_n \frac{\varepsilon_k}{\sqrt{\varepsilon_n + |\Delta|^2}} + (-1)^k \xi_p}{\varepsilon_k^2 + \xi_p^2}; \quad F^+_{0k}(p) = \frac{\Delta^* \frac{\varepsilon_k}{\sqrt{\varepsilon_n + |\Delta|^2}}}{\varepsilon_k^2 + \xi_p^2}; \tag{A5}$$

where $\varepsilon_k = \sqrt{\varepsilon_n + |\Delta|^2} + k\kappa$, while k – is the number of interaction lines, surrounding the given Green's function. The factor of $(-1)^k$ is due to scattering transforming "right" particles into "left" and vice versa. Introducing the renormalized frequency and gap as in (15), we can see that (A5) reduces to the standard form (14), which completes the justification of our recursion procedure (12), (15).

Figure Captions:

Fig.1. Fermi surface of two – dimensional system. "Hot patches" are shown by thick lines of the width $\sim \xi^{-1}$.

Fig.2. Diagrammatic representation of recursion relation for one – particle Green's function.

Fig.3. Diagrammatic representation of recursion relations for Gorkov equations.

Fig.4. Temperature dependence of superconducting energy gap for the case of s-wave pairing and different values of correlation length (parameter $\kappa = v_F\xi^{-1}$) of AFM fluctuations, calculated for $\lambda = 0.4$, $\frac{\omega_c}{W} = 3$:

$\frac{\kappa}{W}$=0 (1); 1.0 (2); 10.0 (3).

Dashed line — $\Delta(T)$ in the absence of the pseudogap.

Fig.5. Dependence of superconducting transition temperature on the width of the pseudogap W and correlation length of AFM fluctuations (parameter $\kappa = v_F\xi^{-1}$):

$\frac{\kappa}{W}$=0.1 (1); 1.0 (2); 10.0 (3).

Dashed line — $\kappa = 0$ [10].

At the insert: dependence of T_c on κ for $\frac{W}{T_{c0}} = 5$.

Fig.6. Diagram for the generalized Cooper susceptibility.

Fig.7. (a) – Diagram series for anomalous Green's function, dashed lines – AFM fluctuations. (b) – Diagram determining K_B.

Fig.8. Dependence of K_C on the width of the pseudogap W and correlation length of AFM fluctuations (parameter $\kappa = v_F\xi^{-1}$):

$\frac{\kappa}{W}$=0.1 (1); 1.0 (2); 10.0 (3).

Dashed line — $\kappa = 0$ [10].

At the insert: dependence of K_C on κ for $\frac{W}{T_{c0}} = 5$.

Fig.9. Dependence of supercoducting coherence length on the width of the pseudogap W and correlation length of AFM fluctuations (parameter $\kappa = v_F\xi^{-1}$):

$\frac{\kappa}{W}$=0.1 (1); 1.0 (2); 10.0 (3).

Dashed line — $\kappa = 0$ [10].

At the insert: dependence of coherence length on κ for $\frac{W}{T_{c0}} = 5$.

Fig.10. Dependence of penetration depth on the width of the pseudogap W and correlation length of AFM fluctuations (parameter $\kappa = v_F\xi^{-1}$):

$\frac{\kappa}{W}$=0.1 (1); 1.0 (2); 10.0 (3).

At the insert: dependence of penetration length on κ for $\frac{W}{T_{c0}} = 5$.

Fig.11. Dependence of the slope of the upper critical field on the width of the pseudogap W and correlation length of AFM fluctuations (parameter $\kappa = v_F\xi^{-1}$):

$\frac{\kappa}{W}$=0.1 (1); 1.0 (2); 10.0 (3).

Dashed line — $\kappa = 0$ [10].

At the insert: dependence of the slope of H_{c2} on κ for $\frac{W}{T_{c0}} = 5$.

Fig.12. Dependence of specific heat discontinuity on the width of the pseudogap W and correlation length of AFM fluctuations (parameter $\kappa = v_F\xi^{-1}$):

$\frac{\kappa}{W}$=0.1 (1); 1.0 (2); 10.0 (3).

Dashed line — $\kappa = 0$ [10].

At the insert: dependence of specific heat discontinuity on κ for $\frac{W}{T_{c0}} = 5$.

REFERENCES

[1] T.Timusk, B.Statt. Rep.Progr.Phys. **62**, 61 (1999)

[2] V.B.Geshkenbein,L.B.Ioffe,A.I.Larkin. Phys.Rev. **B55**,3173(1997)

[3] V.Emery, S.A.Kivelson, O.Zachar. Phys.Rev. **B56**,6120(1997)

[4] V.P.Gusynin, V.M.Loktev, S.G.Sharapov. Zh.Eksp.Teor.Fiz. (JETP) **115**, 1243 (1999)

[5] A.P.Kampf, J.R.Schrieffer. Phys.Rev. **B41**,6399(1990), **B42**,7967(1990)

[6] J.Schmalian, D.Pines, B.Stojkovic. Phys.Rev.Lett. **80**, 3839(1998); Phys.Rev. **B60**, 667 (1999)

[7] E.Z.Kuchinskii, M.V.Sadovskii, Zh. Eksp. Teor. Fiz. **115**, 1765 (1999); JETP **88**, 968 (1999); Preprint cond-mat/9808321

[8] J.L.Tallon, J.W.Loram. Preprint cond-mat/0005063; Physica C (2000) - in press

[9] V.M.Krasnov, A.Yurgens, D.Winkler, P.Delsing, T.Claeson. Phys.Rev.Lett. **84**, 5860 (2000); Preprint cond-mat/0006479

[10] A.I.Posazhennikova, M.V.Sadovskii. Zh. Eksp. Teor. Fiz. **115**, 632 (1999); JETP **88**, 347 (1999); Preprint cond-mat/9806199

[11] E.Z.Kuchinskii, M.V.Sadovskii. Zh. Eksp. Teor. Fiz. **117**, 613 (2000); JETP **90**, 535 (2000); Preprint cond-mat/9910261

[12] R.Gatt, S.Christensen, B.Frazer, Y.Hirai, T.Schmauder, R.J.Kelley, M.Onellion, I.Vybornik, L.Perfetti, G.Margaritondo, A.Morawski, T.Lada, A.Paszewin, C.Kendziora. Preprint cond-mat/9906070

[13] D.L.Feng, W.J.Zheng, K.M.Shen, D.H.Lu, F.Ronning, J.Shimoyama, K.Kishio, G.Gu, D.Van der Marel, Z.X.Shen. Preprint cond-mat/9908056

[14] A.Virosztek, J.Ruvalds. Phys.Rev. **B42**, 4064 (1990)

[15] J.Ruvalds, C.T.Rieck, S.Tewari, J.Thoma, A.Virosztek. Phys.Rev. **B51**, 3797 (1995)

[16] A.T.Zheleznyak, V.M.Yakovenko, I.E.Dzyaloshinskii. Phys.Rev. **55**, 3200 (1997)

[17] P.Monthoux, A.V.Balatsky, D.Pines. Phys.Rev. **B46**, 14803 (1992)

[18] P.Monthoux, D.Pines. Phys.Rev. **B47**, 6069 (1993); **B48**, 4261 (1994)

[19] M.V.Sadovskii. Zh. Eksp. Teor. Fiz. **66**, 1720 (1974); JETP **39**, 845 (1974); Fiz. Tverd. Tela **16**, 2504 (1974); Sov.Phys. Solid State **16**, 1632 (1974)

[20] M.V.Sadovskii. Zh.Eksp.Teor.Fiz. **77**, 2070(1979); JETP 50, 989 (1979)

[21] O.Tchernyshyov. Phys.Rev. **B59**, 1358 (1999)

[22] M.V.Sadovskii. Preprint cond-mat/9912318; Physica C (2000) - in press

[23] L.Bartosch, P.Kopietz. Phys.Rev. **B60**, 15488 (1999)

[24] A.Millis, H.Monien. Phys.Rev. **B61**, 12496 (2000)

[25] L.P.Gorkov. Zh.Eksp.Teor.Fiz **37**, 1407 (1959)

[26] P.De Gennes. Superconductivity of Metals and Alloys, W.A.Benjamin, NY, 1966

[27] M.V.Sadovskii. Superconductivity and Localization. World Scientific, Singapore 2000; Phys.Reports **282**, 225 (1997)

[28] A.A.Abrikosov, L.P.Gorkov, I.E.Dzyaloshinskii. Methods of the Quantum Field Theory in Statistical Physics. Prentice-Hall, NY, 1963

[29] M.V.Sadovskii, A.A.Timofeev. Superconductivity, Phys. Chem. Technol. **4**, 9 (1991); J.Moscow Phys.Soc. **1**,391(1991)

[30] M.V.Sadovskii. Pis'ma Zh.Eksp.Teor.Fiz. **69**, 447 (1999); JETP Letters **69**, 483 (1999); Preprint cond-mat/9902192

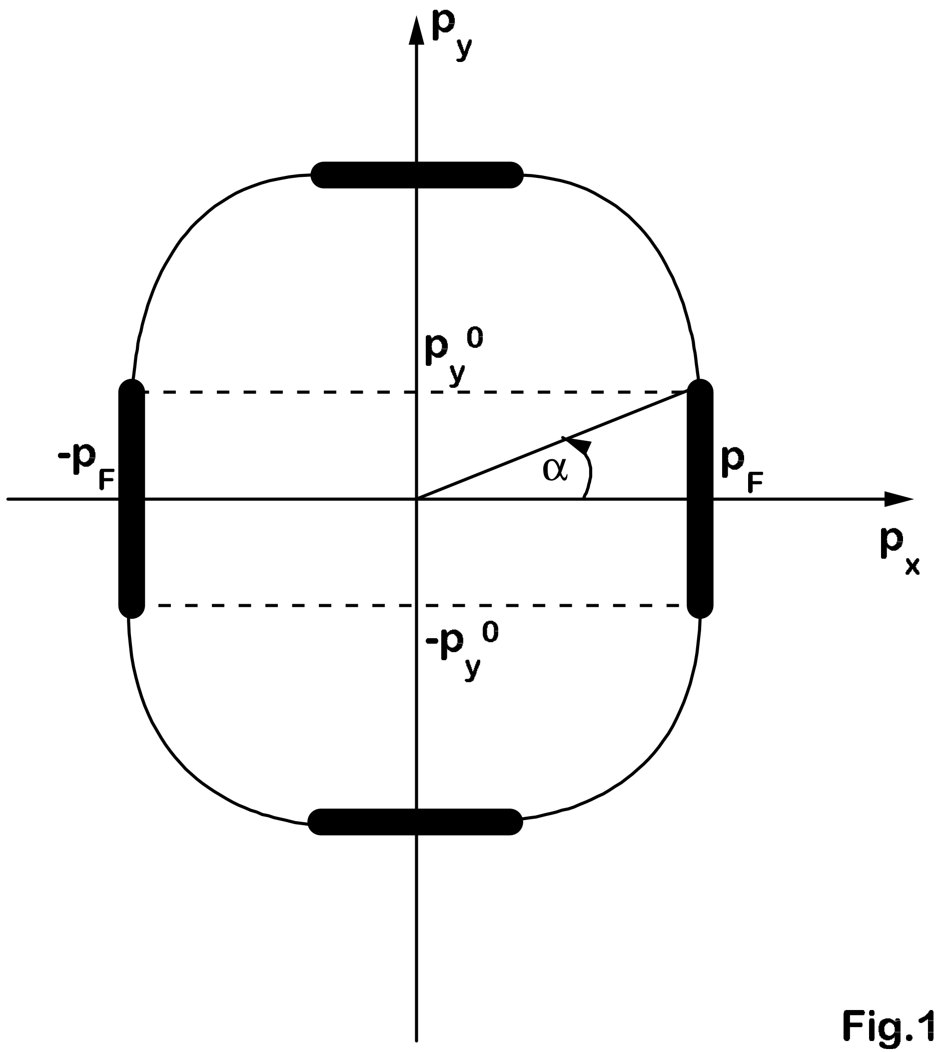
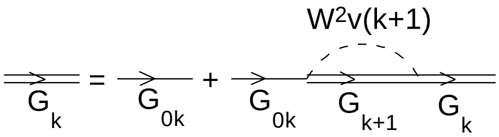


Fig.1



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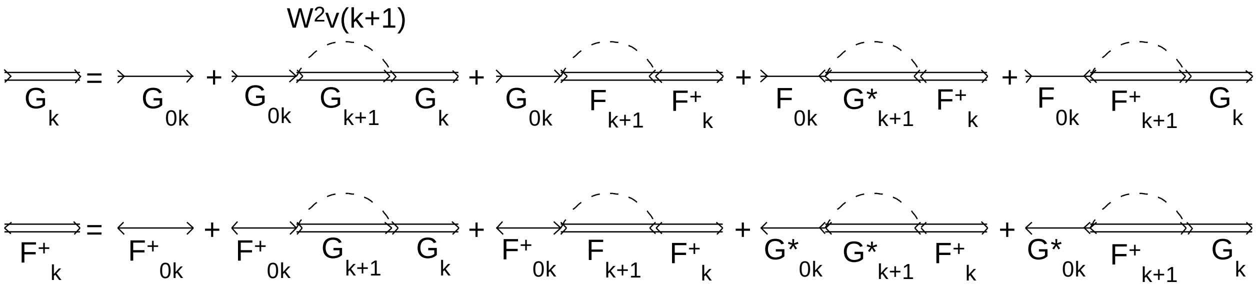


W2v(k+1)
Gk = G0k + G0k Gk+1 Gk + G0k Fk+1 F+k + F0k G*k+1 F+k + F0k F+k+1 Gk
F+k = F+0k + F+0k Gk+1 Gk + F+0k Fk+1 F+k + G*0k G*k+1 F+k + G*0k F+k+1 Gk

Fig.3

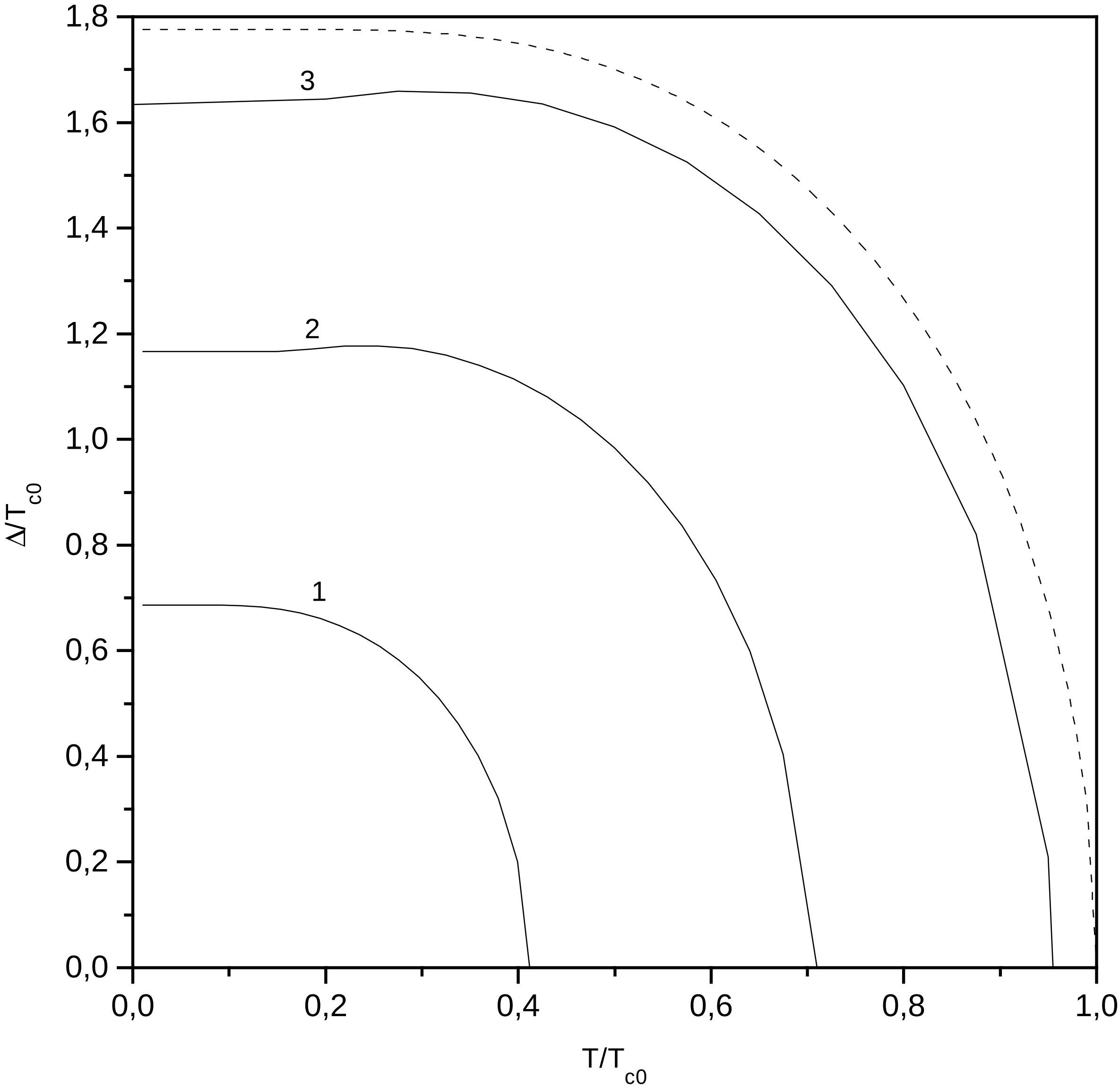


Fig.4

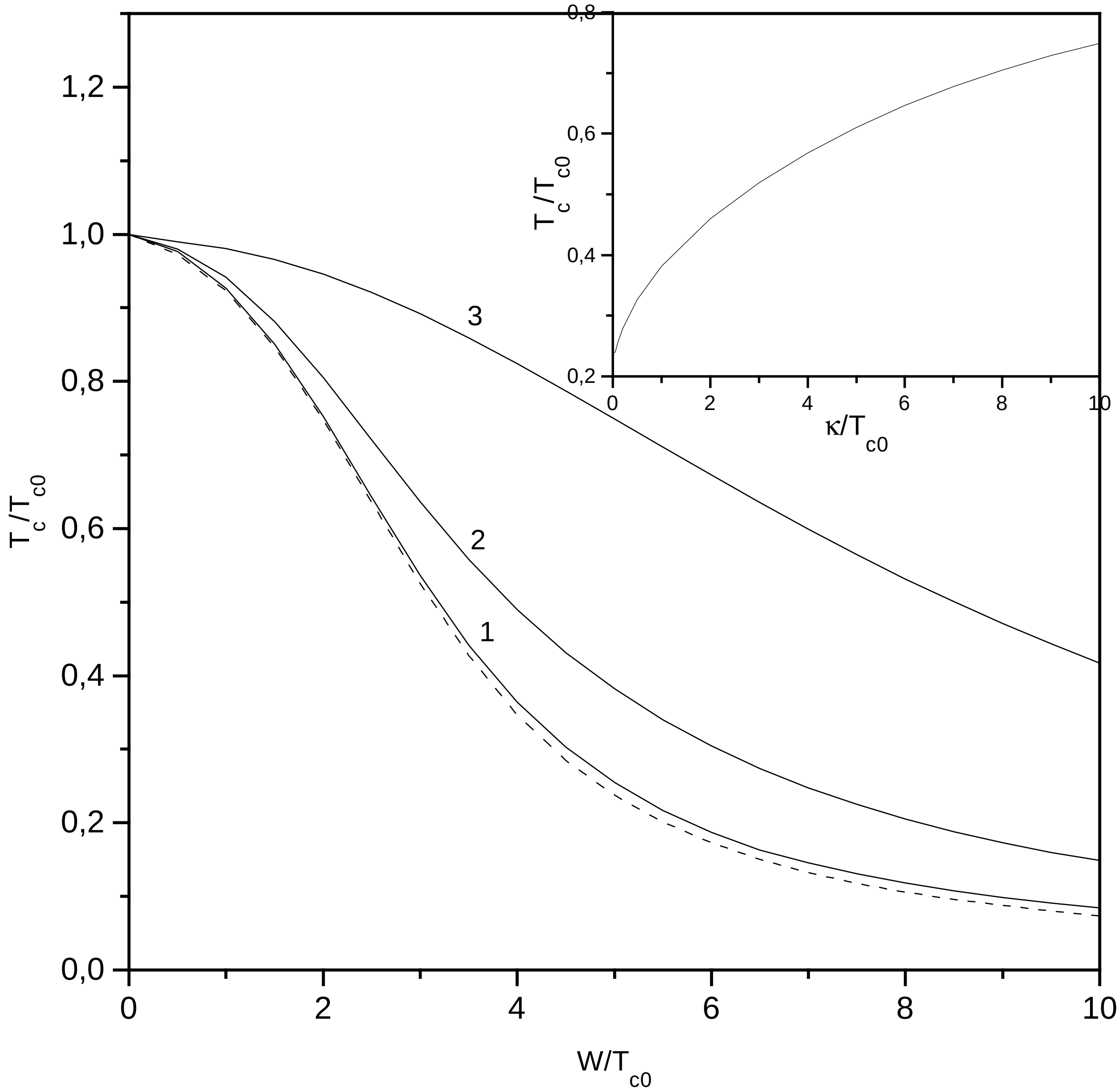
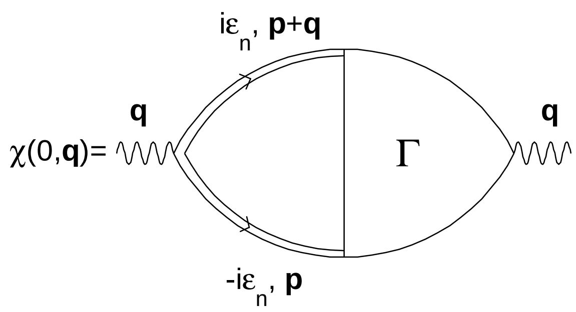


Fig.5



$i\varepsilon_n$, $\mathbf{p}+\mathbf{q}$
$\mathbf{q}$
$\chi(0,\mathbf{q})=$
Γ
$\mathbf{q}$
$-i\varepsilon_n$, $\mathbf{p}$

Fig.6

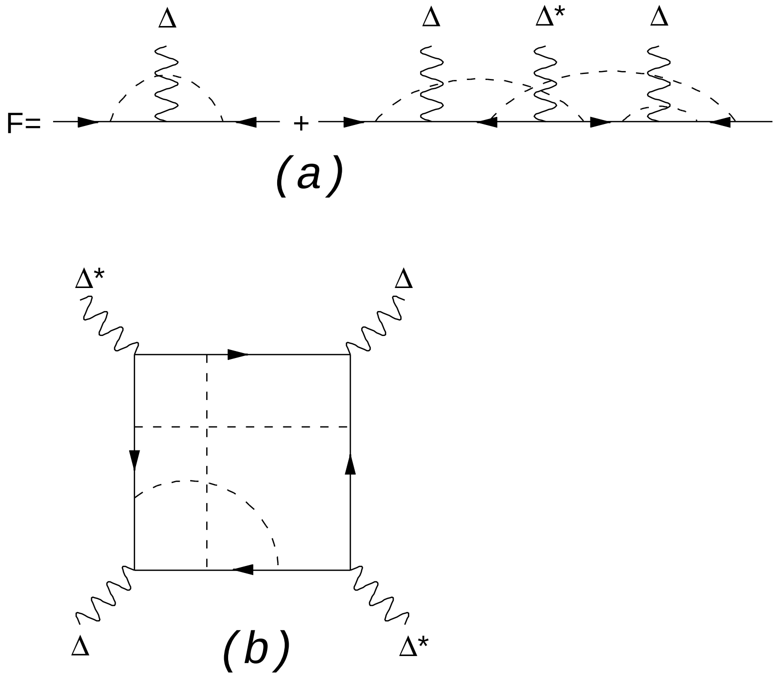


Fig.7

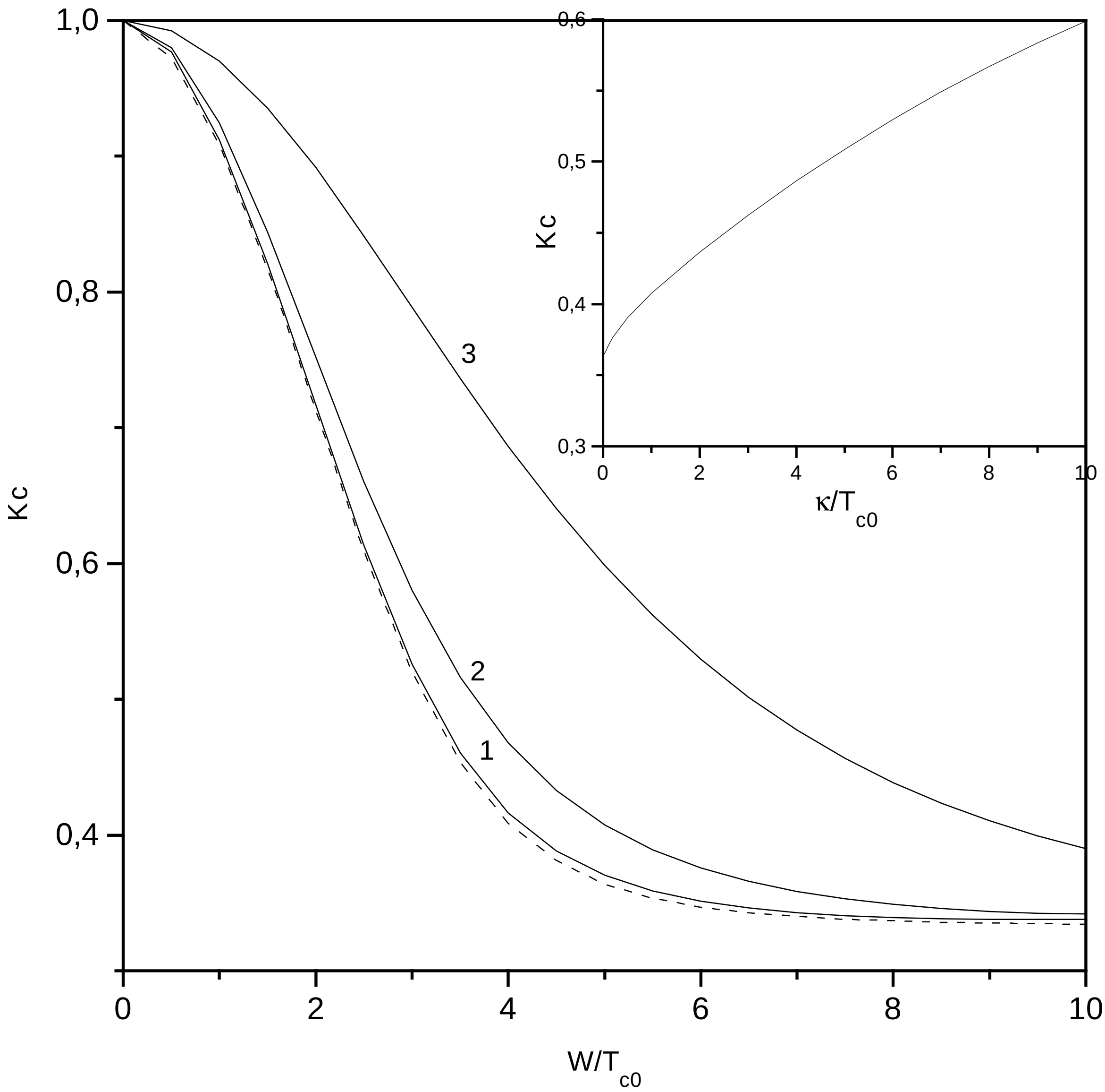
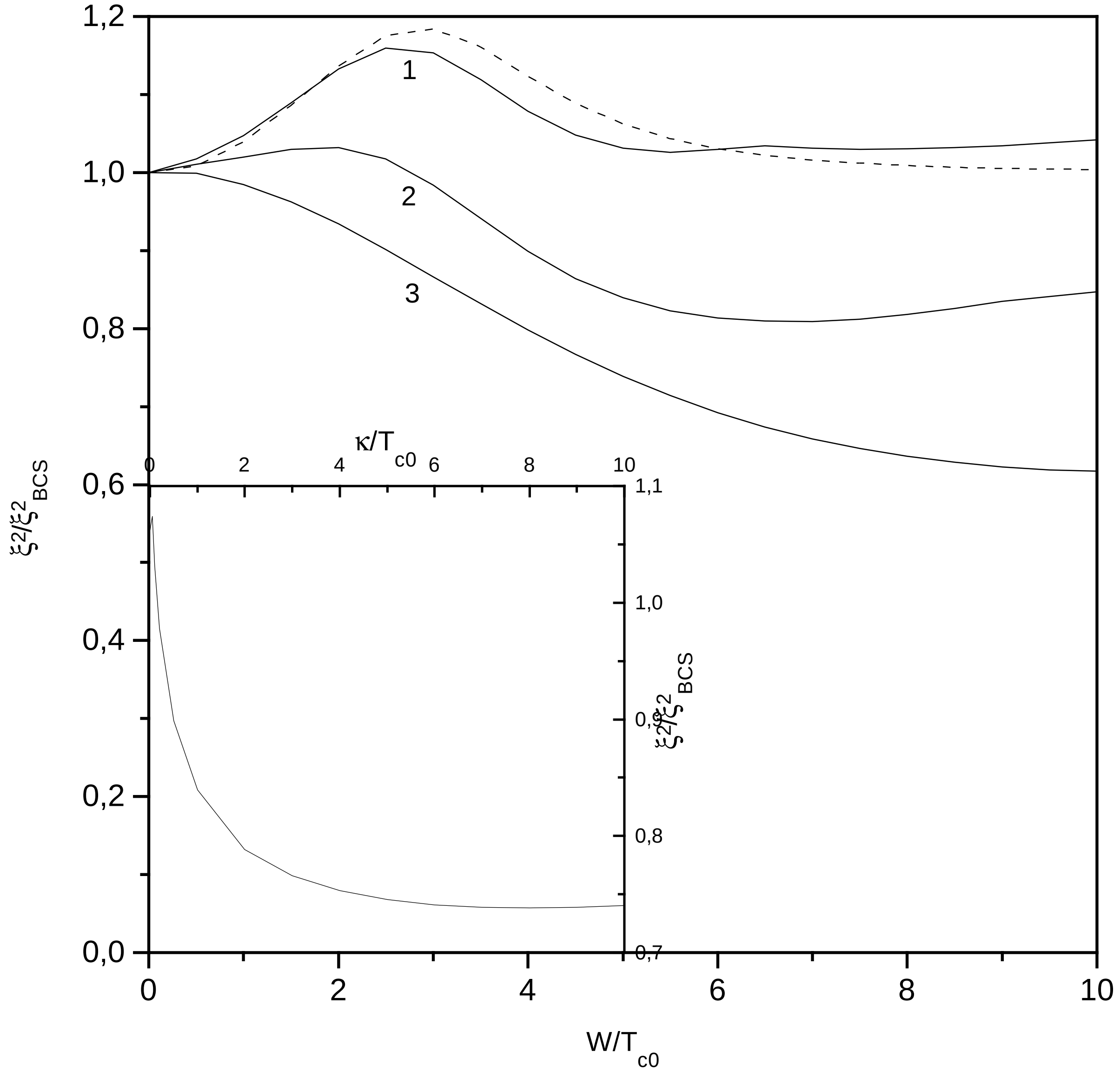


Fig.8



1,2
1,0
0,8
0,6
0,4
0,2
0,0
ξ^2/ξ^2_{BCS}
1
2
3
κ/T_{c0}
0
2
4
6
8
10
1,1
1,0
0,9
0,8
0,7
ξ^2/ξ^2_{BCS}
0
2
4
6
8
10
W/T_{c0}

Fig.9

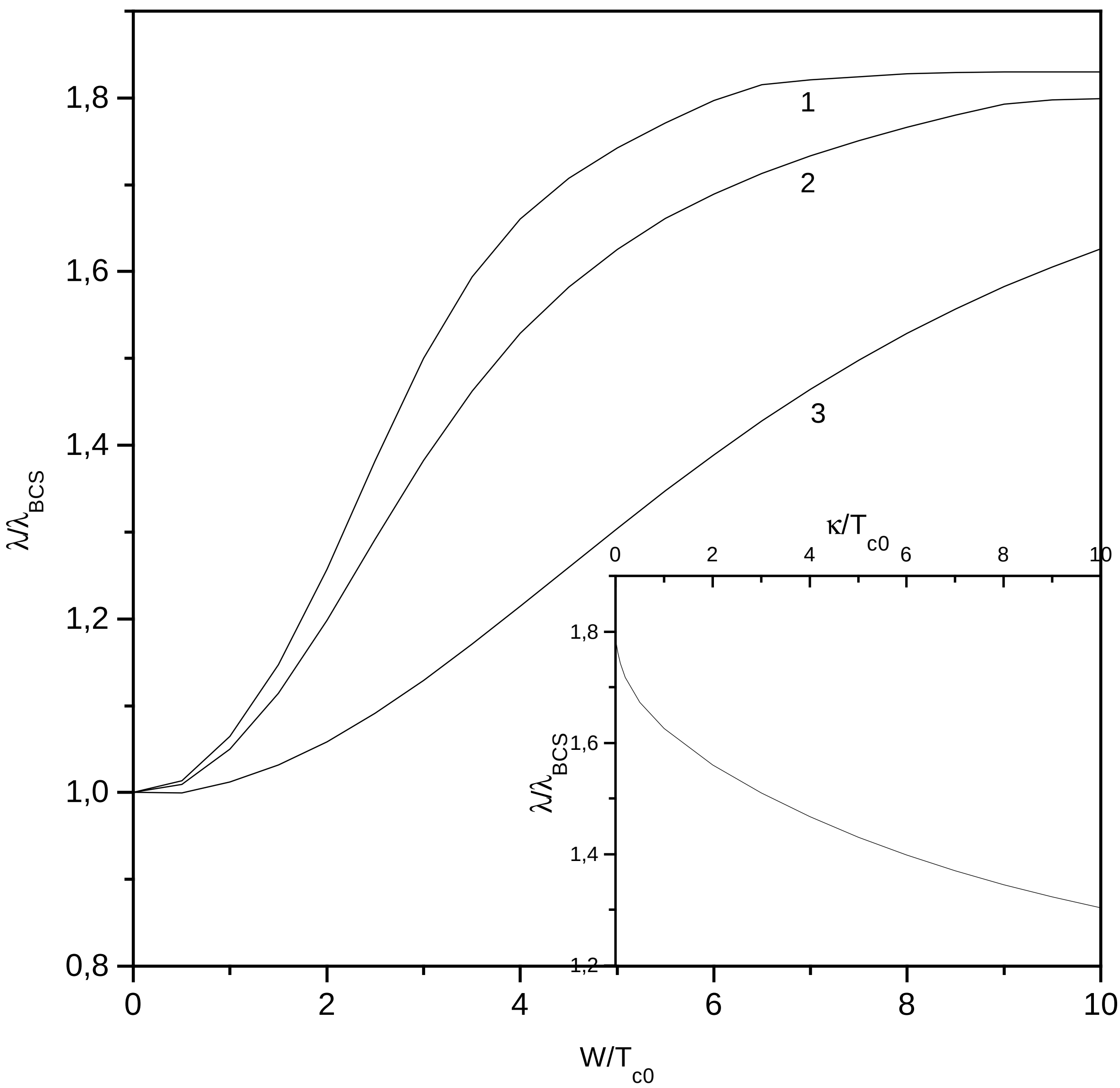

1,8
1,6
1,4
1,2
1,0
0,8
0
2
4
6
8
10
λ/λBCS
W/Tc0
1
2
3
κ/Tc0
0
2
4
6
8
10
1,8
1,6
1,4
1,2
λ/λBCS

Fig.10

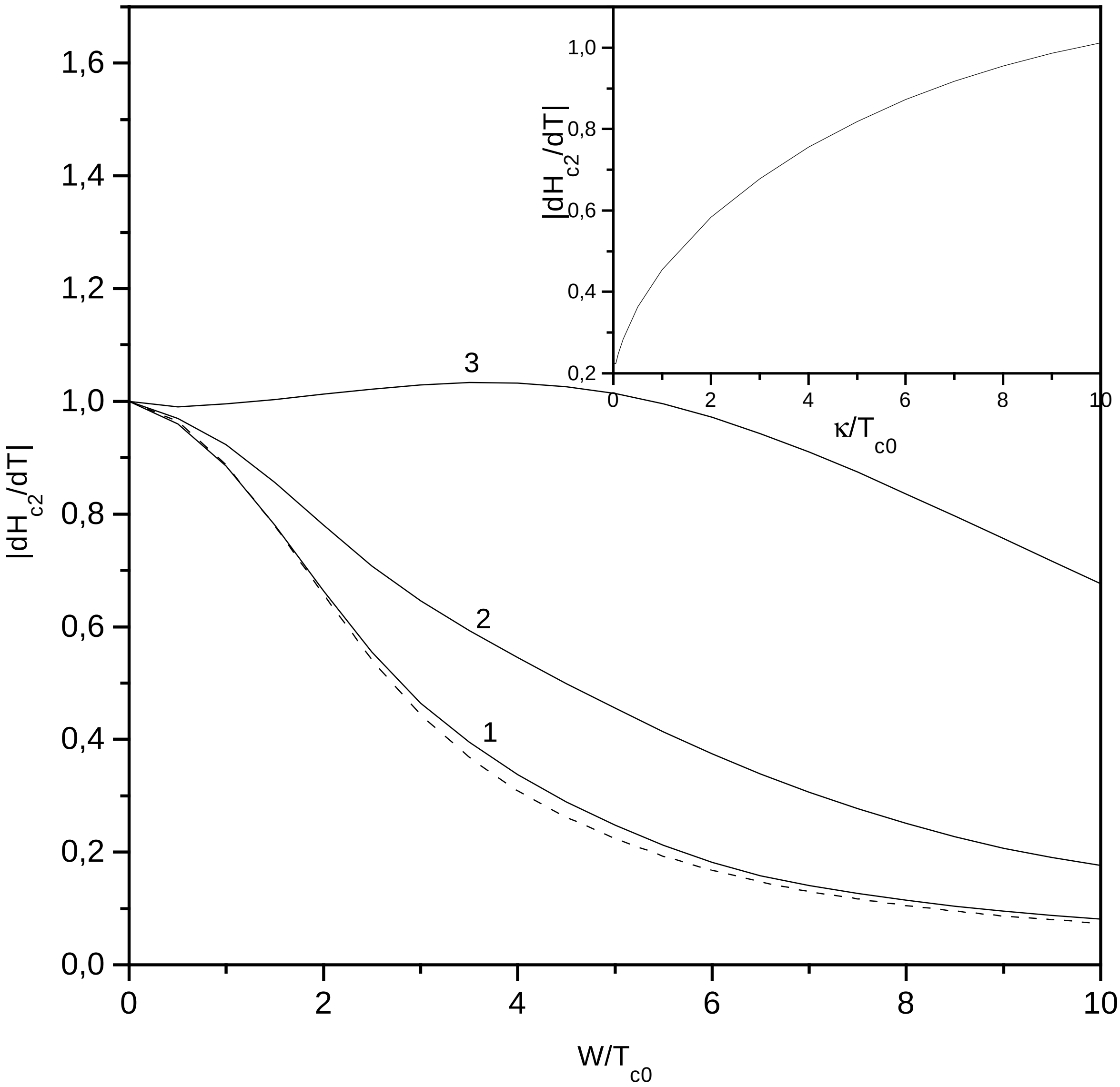


Fig.11

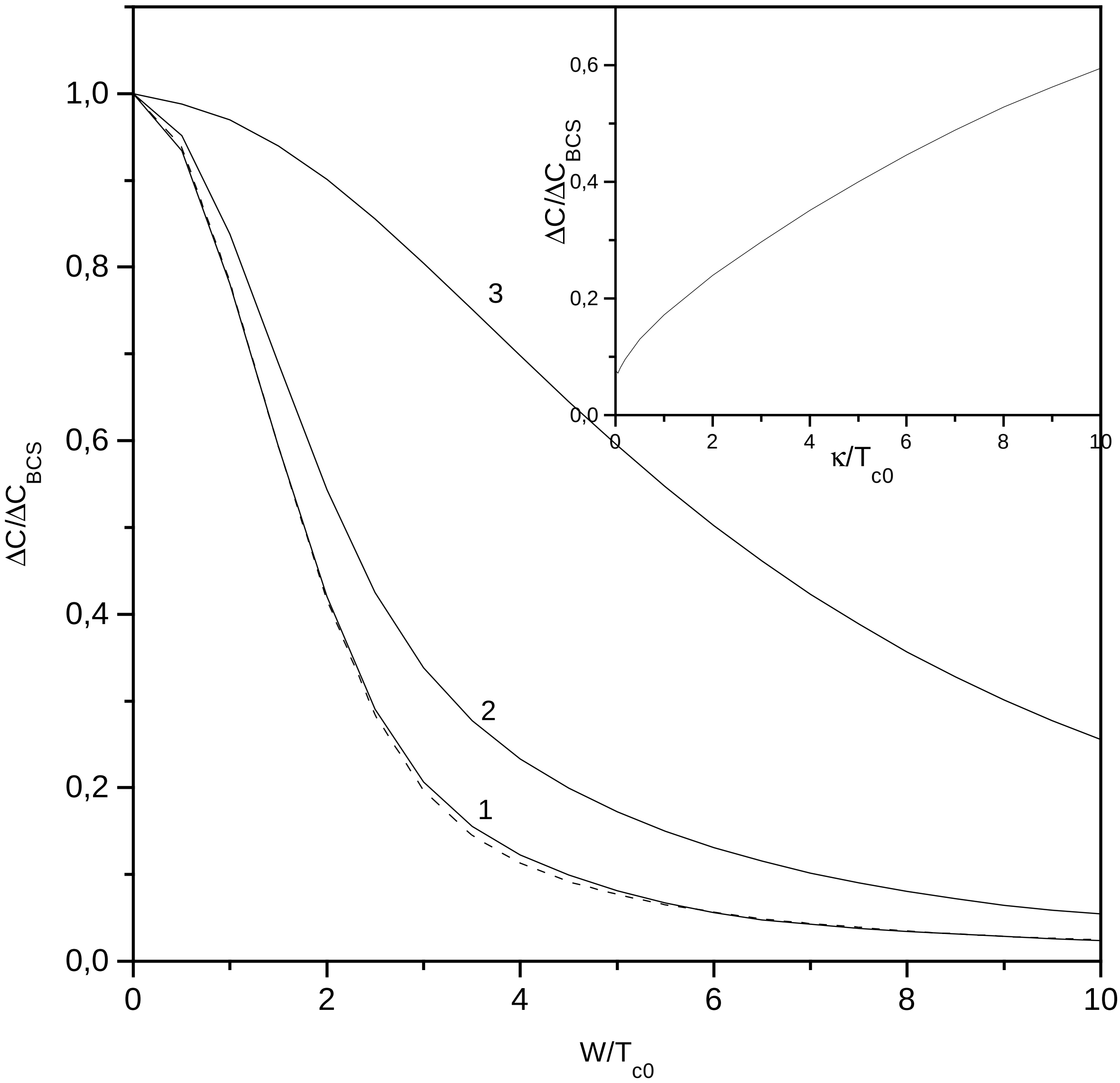


Fig.12