

Superconductivity in the Pseudogap State in “Hot – Spots” model: Ginzburg – Landau Expansion

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Abstract

We analyze properties of superconducting state (for both s -wave and d -wave pairing), appearing on the “background” of the pseudogap state, induced by fluctuations of “dielectric” (AFM(SDW) or CDW) short – range order in the model of the Fermi surface with “hot spots”. We present microscopic derivation of Ginzburg – Landau expansion, taking into account all Feynman diagrams of perturbation theory over electron interaction with this short – range order fluctuations, leading to strong electronic scattering in the vicinity of “hot spots”. We determine the dependence of superconducting critical temperature on the effective width of the pseudogap and on correlation length of short – range order fluctuations. We also find similar dependences of the main characteristics of such superconductor close to transition temperature. It is shown particularly, that specific heat discontinuity at the transition temperature is significantly decreased in the pseudogap region of the phase diagram.

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I. INTRODUCTION

Pseudogap state observed in a wide region of the phase diagram of high – temperature superconducting cuprates is characterized by numerous anomalies of their properties, both in normal and superconducting states [1,2]. Apparently, the most probable scenario of pseudogap state formation in HTSC – oxides can be based [2] on the picture of strong scattering of current carriers by fluctuations of short – range order of “dielectric” type (e.g. antiferromagnetic (AFM(SDW)) or charge density wave (CDW)) existing in this region of the phase diagram. In momentum space this scattering takes place in the vicinity of characteristic scattering vector $\mathbf{Q} = (\frac{\pi}{a}, \frac{\pi}{a})$ (a – lattice constant), corresponding to doubling of lattice period (e.g. vector of antiferromagnetism), being a “precursor” of spectrum transformation, appearing after the establishment of AFM(SDW) long – range order. Correspondingly, there appears an essentially non Fermi – liquid like renormalization of electronic spectrum in the vicinity of the so called “hot spots” on the Fermi surface [2]. Recently there appeared a number of experiments giving rather convincing evidence for precisely this scenario of pseudogap formation [3–5]. Within this picture it is possible to formulate simplified “nearly exactly” solvable model of the pseudogap state, describing the main properties of this state [2], and taking into account all Feynman diagrams of perturbation theory for scattering by (Gaussian) fluctuations of (pseudogap) short – range order with characteristic scattering vectors from the area of \mathbf{Q} , with the width of this area defined by the appropriate correlation length ξ [6,7].

Up to now the majority of theoretical papers is devoted to the studies of models of the pseudogap state in the normal phase for $T > T_c$. In Refs. [8–11] we have analyzed superconductivity in the simplified model of the pseudogap state, based on the assumption of existence of “hot” (flat) patches on the Fermi surface. Within this model we have constructed Ginzburg – Landau expansion for different types (symmetries) of Cooper pairing [8,10] and also studied the main properties of superconducting state for $T < T_c$, solving the appropriate Gor’kov’s equations [9–11]. At first stage, we have considered greatly simplified (“toy”)

model of Gaussian fluctuations of short – range order with an infinite correlation length, where it is possible to obtain an exact (analytic) solution for the pseudogap state [8,9], further analysis of more realistic case of finite correlation lengths was performed both for “nearly exactly” solvable model of Ref. [10] (assuming the self – averaging nature of superconducting order parameter over pseudogap fluctuations) and for a simplified exactly solvable model [11], where we have been able to analyze the effects due to the absence of self – averaging [9,11,12].

The aim of the present paper is to analyze the main properties of superconducting state (for different types of pairing), appearing on the “background” of the pseudogap of “dielectric” nature in more realistic model of “hot spots” on the Fermi surface. Here we shall limit ourselves to the region close to superconducting temperature T_c and perform an analysis based on microscopic derivation of Ginzburg – Landau expansion, assuming the self – averaging nature of superconducting order parameter and generalizing similar approach used earlier for “hot patches” model in Ref. [10].

II. “HOT – SPOTS” MODEL AND PAIRING INTERACTION.

In the model of “nearly antiferromagnetic” Fermi – liquid, which is actively used to describe the microscopic nature of high – temperature superconductivity [13,14], it is usually assumed that the effective interaction of electrons with spin fluctuations of antiferromagnetic (AFM(SDW)) short – range order is of the following form:

$$V_{eff}(\mathbf{q}, \omega) = \frac{g^2 \xi^2}{1 + \xi^2(\mathbf{q} - \mathbf{Q})^2 - i \frac{\omega}{\omega_{sf}}} \quad (1)$$

where g – is some interaction constant, ξ – correlation length of spin fluctuations, $\mathbf{Q} = (\pi/a, \pi/a)$ – vector of antiferromagnetic ordering in dielectric phase, ω_{sf} – characteristic frequency of spin fluctuations. Both dynamic spin susceptibility and effective interaction (1) are peaked in the region of $\mathbf{q} \sim \mathbf{Q}$, which leads to the appearance of “two types” of quasiparticles – “hot” one, with momenta in the vicinity of “hot spots” on the Fermi surface

(Fig. 1), and “cold” one, with momenta close to the parts of the Fermi surface, surrounding diagonals of the Brillouin zone [6]. This is due to the fact, that quasiparticles from the vicinity of the “hot spots” are strongly scattered with the momentum transfer of the order of \mathbf{Q} , due to interaction with spin fluctuations (1), while for quasiparticles with momenta far from these “hot spots” this interaction is relatively weak.

For high enough temperatures $\pi T \gg \omega_{sf}$, we can neglect spin dynamics [6], limiting ourselves to static approximation in (1). Considerable simplification, allowing to analyze higher – order contributions, can be achieved by substitution of (1) by model – like static interaction of the following form [7]:

$$V_{eff}(\mathbf{q}) = W^2 \frac{2\xi^{-1}}{\xi^{-2} + (q_x - Q_x)^2} \frac{2\xi^{-1}}{\xi^{-2} + (q_y - Q_y)^2} \quad (2)$$

where W is an effective parameter with dimension of energy. In the following, as in Refs. [6,7], we consider parameters W as ξ phenomenological (to be determined from the experiment). Anyhow, Eq. (2) is qualitatively quite similar to the static limit of (1) and almost indistinguishable from it in most interesting region of $|\mathbf{q} - \mathbf{Q}| < \xi^{-1}$, determining scattering in the vicinity of “hot spots”.

The spectrum of “bare” (free) quasiparticles can be taken as [6]:

$$\xi_{\mathbf{p}} = -2t(\cos p_x a + \cos p_y a) - 4t' \cos p_x a \cos p_y a - \mu \quad (3)$$

where t is the transfer integral between nearest neighbors, while t' is the transfer integral between second nearest neighbors on the square lattice, a is the lattice constant, μ – chemical potential. This expression gives rather good approximation to the results of band structure calculations of real HTSC – system, e.g. for $YBa_2Cu_3O_{6+\delta}$ we have $t = 0.25eV$, $t' = -0.45t$ [6]. Chemical potential μ is fixed by concentration of carriers.

The least justified is an assumption of the static nature of fluctuations, which can be valid only for rather high temperatures [6,7]. For low temperatures, particularly in superconducting phase, spin dynamics can become quite important, e.g. for microscopics of Cooper pairing in the model of “nearly antiferromagnetic” Fermi liquid [13,14]. However,

we assume here that the static approximation is sufficient for qualitative understanding of the influence of pseudogap formation upon superconductivity, which will be modelled within phenomenological BCS – like approach.

In the limit of infinite correlation length $\xi \rightarrow \infty$ this model acquires an exact solution [15]. For finite ξ we can construct “nearly exact” solution [7], generalizing the one – dimensional approach, proposed in Ref. [16]. Then we can (approximately) sum the whole diagrammatic series for the one – particle electronic Green’s function.

For the contribution of an arbitrary diagram for electronic self – energy, in the N -th order over the interaction (2), we write down the following *Ansatz* [7,16]:

$$\begin{aligned}\Sigma^{(N)}(\varepsilon_n \mathbf{p}) &= W^{2N} \prod_{j=1}^{2N-1} G_{0k_j}(\varepsilon_n \mathbf{p}), \\ G_{0k_j}(\varepsilon_n \mathbf{p}) &= \frac{1}{i\varepsilon_n - \xi_{k_j}(\mathbf{p}) + ik_j v_{k_j} \kappa}\end{aligned}\quad (4)$$

where $\kappa = \xi^{-1}$, k_j – is the number of interaction lines, surrounding the j -th (from the beginning) electronic line in a given diagram, $\varepsilon_n = 2\pi T(n + 1/2)$ (assuming $\varepsilon_n > 0$).

$$\xi_k(\mathbf{p}) = \begin{cases} \xi_{\mathbf{p}+\mathbf{Q}} & \text{for odd } k \\ \xi_{\mathbf{p}} & \text{for even } k \end{cases}\quad (5)$$

$$v_k = \begin{cases} |v_x(\mathbf{p} + \mathbf{Q})| + |v_y(\mathbf{p} + \mathbf{Q})| & \text{for odd } k \\ |v_x(\mathbf{p})| + |v_y(\mathbf{p})| & \text{for even } k \end{cases}\quad (6)$$

where $\mathbf{v}(\mathbf{p}) = \frac{\partial \xi_{\mathbf{p}}}{\partial \mathbf{p}}$ – velocity of a “bare” (free) quasiparticle.

In this approximation the contribution of an arbitrary diagram is determined, in fact, by the set of integers k_j . Any diagram with intersection of interaction lines is actually equal to some diagram of the same order without intersections and the contribution of all diagrams with intersections can be accounted with the help of combinatorial factors $s(k_j)$ attributed to interaction lines on diagrams without intersections [16,7,6].

Combinatorial factor:

$$s(k) = k \quad (7)$$

in the case of commensurate fluctuations with $\mathbf{Q} = (\pi/a, \pi/a)$ [16], if we neglect their spin structure [6]) (i.e. limiting ourselves to CDW – type fluctuations). Taking into account the spin structure of interaction within the model of “nearly antiferromagnetic” Fermi – liquid (spin – fermion model of Ref. [6]), we obtain more complicated combinatorics of diagrams. In particular, spin – conserving scattering leads to the formally commensurate combinatorics, while the spin – flip scattering is described by diagrams of incommensurate case (“charged” random field in terms of Ref. [6]). As a result the *Ansatz* (4) for one – particle Green’s function is conserved, but the combinatorial factor $s(k)$ takes the form [6]:

$$s(k) = \begin{cases} \frac{k+2}{3} & \text{for odd } k \\ \frac{k}{3} & \text{for even } k \end{cases} \quad (8)$$

However, in the case of two – particle processes and in the analysis of the pseudogap influence on superconductivity, the use of spin – fermion model leads to significant complications. In particular, in this model there is a significant difference between vertexes, corresponding to spin dependent and charge interactions, and combinatorics of diagrams for spin dependent vertex is different from that of (8) [6].

In spin – fermion model [6] the spin dependent part of interaction is usually described by isotropic Heisenberg model. If for this interaction we assume the Ising like form, only spin – conserving processes remain and commensurate combinatorics of diagrams (7) remains valid both for one – particle Green’s function and for spin and charge vertexes. For this reason, in the present work we shall limit ourselves to the analysis of commensurate (7) “Ising like” spin – fluctuations only (AFM, SDW), as well as commensurate charge fluctuations (CDW). Details concerning incommensurate fluctuations of CDW – type can be found in Refs. [7,15,16].

As a result we obtain the following recurrence procedure for the one – particle Green’s function $G(\varepsilon_n \mathbf{p})$ (continuous fraction representation) [16,7,6]:

$$G_k(\varepsilon_n \mathbf{p}) = \frac{1}{i\varepsilon_n - \xi_k(\mathbf{p}) + ikv_k\kappa - W^2s(k+1)G_{k+1}(\varepsilon_n \mathbf{p})} \quad (9)$$

and “physical” Green’s function is determined as $G(\varepsilon_n \mathbf{p}) \equiv G_0(\varepsilon_n \mathbf{p})$.

Ansatz (4) for the contribution of an arbitrary diagram of N -th order is not exact in general case [7]. However, for two – dimensional system we were able to show that for certain topologies of the Fermi surface (4) is actually an exact representation [7], in other cases it can be shown [7], that this approximation overestimates (in some sense) the effects of the finite values of correlation length ξ for the given order of perturbation theory. For one – dimensional case, when this problem is most serious [7], the values of the density of states found with the help of (4) in the case of incommensurate fluctuations are in almost ideal quantitative correspondence [17] with the results of an exact numerical modelling of this problem in Refs. [18,19]. In the limit of $\xi \rightarrow \infty$ *Ansatz* (4) reduces to an exact solution of Ref. [15], while in the limit of $\xi \rightarrow 0$ and fixed value of W it reduces to the trivial case of free electrons.

This model describes non Fermi – liquid like spectral density in the vicinity of “hot spots” on the Fermi surface and “smooth” pseudogap in the density of states [6,7].

To analyze superconductivity in such a system with well developed fluctuations of short – range order we shall assume that superconducting pairing is determined by attractive BCS – like interaction (between electrons with opposite spins) of the following simplest possible form:

$$V_{sc}(\mathbf{p}, \mathbf{p}') = -V e(\mathbf{p}) e(\mathbf{p}'), \quad (10)$$

where $e(\mathbf{p})$ is given by:

$$e(\mathbf{p}) = \begin{cases} 1 & (s\text{-wave pairing}) \\ \cos(p_x a) - \cos(p_y a) & (d_{x^2-y^2}\text{-wave pairing}) \\ \sin(p_x a) \sin(p_y a) & (d_{xy}\text{-wave pairing}) \\ \cos(p_x a) + \cos(p_y a) & (\text{anisotropic } s\text{-wave pairing}) \end{cases}. \quad (11)$$

Interaction constant V is assumed, as usual, to be non zero in some interval of the width of $2\omega_c$ around the Fermi level (ω_c – characteristic frequency of quanta responsible for attractive interaction of electrons). Then in general case the superconducting gap is anisotropic and given by: $\Delta(\mathbf{p}) = \Delta e(\mathbf{p})$.

In Refs. [8,9] we have analyzed the peculiarities of superconducting state in an exactly solvable model of the pseudogap state, induced by short – range order fluctuations with infinite correlation length ($\xi \rightarrow \infty$). In particular, in Ref. [9] it was shown that these fluctuations may lead to strong fluctuations of superconducting order parameter (energy gap Δ), which break the standard assumption of the self – averaging nature of the gap [20–22], which allows to perform independent averaging (over the random configurations of static fluctuations of short – range order) of superconducting order parameter Δ and different combinations of electronic Green’s functions, entering the main equations. Usual argument for the possibility of such an independent averaging goes as follows [20,22]: the value of Δ significantly changes on the length scale of the order of coherence length $\xi_0 \sim v_F/\Delta_0$ of BCS theory, while the Green’s functions oscillate on much shorter length scales of the order of interatomic spacing. Naturally, this assumption becomes invalid with the appearance (in electronic system) of a new characteristic length scale $\xi \rightarrow \infty$. However, when this correlation length of short – range order $\xi \ll \xi_0$ (i.e. when fluctuations are correlated on the length scale shorter than the characteristic size of Cooper pairs), the assumption of self – averaging of Δ is apparently still valid, being broken only in case of $\xi > \xi_0$ ¹. Thus, below we perform all the analysis under the usual assumption of self – averaging energy gap over the fluctuations of short – range order, which allows us to use standard methods of the theory of disordered superconductors (mean field approximation in terms of Ref. [9]).

III. COOPER INSTABILITY. RECURRENCE PROCEDURE FOR THE VERTEX PART.

It is well known that critical temperature of superconducting transition can be determined from the equation for Cooper instability of normal phase:

¹The absence of self – averaging of superconducting gap even for the case of $\xi < \xi_0$, obtained in Ref. [11], is apparently due to rather special model of short – range order used in this paper.

$$1 - V\chi(0; T) = 0 \quad (12)$$

where the generalized Cooper susceptibility is determined by diagram shown in Fig. 2 and equal to:

$$\chi(\mathbf{q}; T) = -T \sum_{\varepsilon_n} \sum_{\mathbf{p}, \mathbf{p}'} e(\mathbf{p})e(\mathbf{p}')\Phi_{\mathbf{p}, \mathbf{p}'}(\varepsilon_n, -\varepsilon_n, \mathbf{q}) \quad (13)$$

where $\Phi_{\mathbf{p}, \mathbf{p}'}(\varepsilon_n, -\varepsilon_n, \mathbf{q})$ is two - particle Green's function in Cooper channel, taking into account scattering by fluctuations of short - range order.

Let us consider first the case of charge (CDW) fluctuations, when electron - fluctuation interaction is spin independent. In case of isotropic s and d_{xy} -wave pairing superconducting gap does not change after scattering by \mathbf{Q} , i.e. $e(\mathbf{p} + \mathbf{Q}) = e(\mathbf{p})$ and $e(\mathbf{p}') \approx e(\mathbf{p})$. In case of anisotropic s and $d_{x^2-y^2}$ pairings superconducting gap changes sign after scattering by \mathbf{Q} , i.e. $e(\mathbf{p} + \mathbf{Q}) = -e(\mathbf{p})$, thus $e(\mathbf{p}') \approx e(\mathbf{p})$ for $\mathbf{p}' \approx \mathbf{p}$ and $e(\mathbf{p}') \approx -e(\mathbf{p})$ for $\mathbf{p}' \approx \mathbf{p} + \mathbf{Q}$. Thus, for diagrams with even number of interaction lines, connecting the upper (ε_n) and lower ($-\varepsilon_n$) electronic lines in Fig. 2, we have $\mathbf{p}' \approx \mathbf{p}$ and the appropriate contribution to susceptibility is the same for both isotropic s and d_{xy} -wave pairing. However, for diagrams with odd number of such interaction lines we obtain contributions differing by sign. This sign change can be accounted by changing the sign of interaction line, connecting the upper and lower lines in the loop in Fig. 2. Then for the generalized susceptibility we obtain:

$$\chi(\mathbf{q}; T) = -T \sum_{\varepsilon_n} \sum_{\mathbf{p}} G(\varepsilon_n \mathbf{p} + \mathbf{q})G(-\varepsilon_n, -\mathbf{p})e^2(\mathbf{p})\Gamma^\pm(\varepsilon_n, -\varepsilon_n, \mathbf{q}) \quad (14)$$

where $\Gamma^\pm(\varepsilon_n, -\varepsilon_n, \mathbf{q})$ is “triangular” vertex, describing electron interaction with fluctuations of short - range order, while the superscript \pm denotes the abovementioned change of signs for interaction lines connecting the upper and lower electronic lines.

Consider now the case of scattering by spin (AFM(SDW)) fluctuations. In this case interaction line, describing the longitudinal S^z part of interaction, surrounding the vertex changing spin direction, should be attributed an extra factor of (-1) [6]. Because of this factor, in case of electron interaction with spin fluctuations contributions to generalized

susceptibility for different types of pairing considered above just “change places”² and susceptibility for the case of isotropic s and d_{xy} -wave pairing is determined by “triangular” vertex Γ^- , while for anisotropic s and $d_{x^2-y^2}$ -wave case by “triangular” vertex Γ^+ .

Thus we come to the problem of calculation of “triangular” vertex parts, describing interaction with “dielectric” (pseudogap) fluctuations. For one – dimensional case and a similar problem (for real frequencies, $T = 0$) the appropriate recursion procedure was formulated, for the first time, in Ref. [23]. For a two – dimensional model of the pseudogap with “hot spots” on the Fermi surface the generalization of this procedure was performed in Ref. [24] and used to calculate optical conductivity. All the details of appropriate derivation can also be found there. The generalization to the case of Matsubara frequencies, of interest to us here, can be made directly. Below, for definiteness we assume $\varepsilon_n > 0$. Finally, for “triangular” vertex we obtain the recurrence procedure, described by diagrams of Fig. 3 (where the wavy line denotes interaction with pseudogap fluctuations), and having the following analytic form:

$$\begin{aligned} & \Gamma_{k-1}^{\pm}(\varepsilon_n, -\varepsilon_n, \mathbf{q}) = \\ & = 1 \pm W^2 s(k) G_k \bar{G}_k \left\{ 1 + \frac{2ikv_k \kappa}{2i\varepsilon_n - \mathbf{v}_k \mathbf{q} - W^2 s(k+1)(G_{k+1} - \bar{G}_{k+1})} \right\} \Gamma_k^{\pm}(\varepsilon_n, -\varepsilon_n, \mathbf{q}) \end{aligned} \quad (15)$$

where $G_k = G_k(\varepsilon_n \mathbf{p} + \mathbf{q})$ and $\bar{G}_k = G_k(-\varepsilon_n, -\mathbf{p})$ are calculated according to (9), v_k is defined by (6), and \mathbf{v}_k are given by:

$$\mathbf{v}_k = \begin{cases} \mathbf{v}(\mathbf{p} + \mathbf{Q}) & \text{for odd } k \\ \mathbf{v}(\mathbf{p}) & \text{for even } k \end{cases} \quad (16)$$

“Physical” vertex is defined as $\Gamma^{\pm}(\varepsilon_n, -\varepsilon_n, \mathbf{q}) \equiv \Gamma_0^{\pm}(\varepsilon_n, -\varepsilon_n, \mathbf{q})$.

To determine T_c we need vertices at $\mathbf{q} = 0$. Then $\bar{G}_k = G_k^*$ and vertex parts Γ_k^+ and Γ_k^- become real significantly simplifying our procedures (15). For ImG_k and ReG_k we obtain the following system of recurrence equations:

²This is due to the fact of spin projections change in the vertex, describing “interaction” with superconducting gap (restricting to the case of singlet pairing).

$$\begin{aligned}
ImG_k &= -\frac{\varepsilon_n + kv_k\kappa - W^2s(k+1)ImG_{k+1}}{D_k} \\
ReG_k &= -\frac{\xi_k(\mathbf{p}) + W^2s(k+1)ReG_{k+1}}{D_k}
\end{aligned} \tag{17}$$

where $D_k = (\xi_k(\mathbf{p}) + W^2s(k+1)ReG_{k+1})^2 + (\varepsilon_n + kv_k\kappa - W^2s(k+1)ImG_{k+1})^2$, and vertex parts at $\mathbf{q} = 0$ are determined by:

$$\Gamma_{k-1}^\pm = 1 \mp W^2s(k) \frac{ImG_k}{\varepsilon_n - W^2s(k+1)ImG_{k+1}} \Gamma_k^\pm \tag{18}$$

Going to numerical calculations we have to define characteristic energy (temperature) scale associated with superconductivity in the absence of pseudogap fluctuations ($W = 0$). In this case the equation for superconducting critical temperature T_{c0} becomes standard BCS – like (for the general case of anisotropic pairing):

$$1 = \frac{2VT}{\pi^2} \sum_{n=0}^{\bar{m}} \int_0^\pi dp_x \int_0^\pi dp_y \frac{e^2(\mathbf{p})}{\xi_{\mathbf{p}}^2 + \varepsilon_n^2} \tag{19}$$

where $\bar{m} = \lfloor \frac{\omega_c}{2\pi T_{c0}} \rfloor$ is the dimensionless cutoff for the sum over Matsubara frequencies. All calculations were performed for typical quasiparticle spectrum in HTSC given by (3) with $\mu = -1.3t$ and $t'/t = -0.4$. Choosing, rather arbitrarily, the value of $\omega_c = 0.4t$ and $T_{c0} = 0.01t$ we can easily determine the appropriate values of pairing constant V in (19), giving this value of T_{c0} for different types of pairing listed in (11). In particular, for the usual isotropic s -wave pairing we obtain $\frac{V}{ta^2} = 1$, while for $d_{x^2-y^2}$ pairing we get $\frac{V}{ta^2} = 0.55$. For other types of pairing from (11) the values of pairing constant for this choice of parameters are found to be unrealistically large and we shall not quote the the results of numerical calculations for these symmetries ³.

³Of course, our description, based on BCS equations of weak coupling theory, is not pretending to be realistic also for the cases of s -wave and $d_{x^2-y^2}$ -wave pairing. We only need to define characteristic scale of T_{c0} and express all temperatures below in units of this temperature, assuming certain universality of all dependences with respect to this scale.

Typical results of numerical calculations of superconducting transition temperature T_c for the system with pseudogap obtained using our recursion relations directly from (12) are shown in Figs. 4,5. We can see that in all cases pseudogap (“dielectric”) fluctuations lead to significant reduction of superconducting transition temperature. This reduction stronger in case of $d_{x^2-y^2}$ -wave pairing than for isotropic s -wave case. At the same time, reduction of the correlation length ξ (growth of κ) of pseudogap fluctuations leads to the growth of T_c . These results are similar to those obtained earlier in the “hot patches” model [8,10]. However, significant qualitative differences also appear. From Fig. 4 it is seen, that for the case of isotropic s -wave pairing and scattering by charge (CDW) fluctuations, as well as for $d_{x^2-y^2}$ pairing and scattering by spin (AFM(SDW)) fluctuations ⁴ (i.e. in cases when the upper sign is “operational” in Eqs. (15) and (18), leading to recursion procedure for the vertex with same signs) there appears characteristic plateau in the dependence of T_c on the width of the pseudogap W in the region of $W < 10T_{c0}$, while significant suppression of T_c takes place on the scale of $W \sim 50T_{c0}$. Qualitative differences appear also for the case of s -wave pairing and scattering by spin (AFM(CDW)) fluctuations, as well as for the case of $d_{x^2-y^2}$ pairing and scattering by charge fluctuations. From Fig. 5 we can see that in this case (when lower sign is “operational” in Eqs. (15) and (18), i.e. when we have recursion procedure for the vertex with alternating signs) reduction of T_c is an order of magnitude faster. In the case of $d_{x^2-y^2}$ pairing, for the values of W/T_{c0} corresponding to almost complete suppression of superconductivity, our numerical procedures become unreliable. In particular, we can observe here characteristic non single valued dependence of T_c on W , which may signify the existence of a narrow region of phase diagram with “reentrant” superconductivity⁵. This behavior of T_c resembles similar dependences, appearing in

⁴This last case is apparently realized in copper oxides.

⁵Our calculations show that manifestations of such behavior of T_c become stronger for the case of scattering by incommensurate pseudogap fluctuations.

superconductors with Kondo impurities [25]. Alternative possibility is the appearance here of the region, where superconducting transition becomes first order, similarly to situation in superconductors with strong paramagnetic effect in external magnetic field [26]. However, it should be stressed that our calculations mostly show the probable appearance of some critical value of W/T_{c0} , corresponding to the complete suppression of superconductivity. In any case, the observed behavior requires special studies in the future and below we only give the results, corresponding to the region of single – valued dependences of T_c .

IV. GINZBURG – LANDAU EXPANSION.

In Ref. [8] Ginzburg – Landau expansion was derived for an exactly solvable model of the pseudogap with infinite correlation length of short – range order fluctuations. In Ref. [10] these results were extended for the case of finite correlation lengths. In these papers the analysis was, in fact, done only for the case of charge fluctuations and simplified model of pseudogap state, based on the picture of “hot” (flat) parts (patches) on the Fermi surface. Also in this model the signs of superconducting gap after the transfer by vector \mathbf{Q} were assumed to be the same, both for s -wave and d -wave pairings [10]. Here we shall make appropriate generalization for the present more realistic model of “hot spots” on the Fermi surface.

Ginzburg – Landau expansion for the difference of free energy density of superconducting and normal states can be written in the usual form:

$$F_s - F_n = A|\Delta_{\mathbf{q}}|^2 + q^2 C|\Delta_{\mathbf{q}}|^2 + \frac{B}{2}|\Delta_{\mathbf{q}}|^4, \quad (20)$$

where $\Delta_{\mathbf{q}}$ is the amplitude of the Fourier component of the order parameter, which can be written for different types of pairing as: $\Delta(\mathbf{p}, \mathbf{q}) = \Delta_q \epsilon(\mathbf{p})$. In fact (refGiLa) is determined by diagrams of loop expansion of free energy in the field of random fluctuation of the order parameter (denoted by dashed lines) with small wave vector \mathbf{q} [8], shown in Fig. 6.

Coefficients of Ginzburg – Landau expansion can be conveniently expressed as:

$$A = A_0 K_A; \quad C = C_0 K_C; \quad B = B_0 K_B, \quad (21)$$

where A_0 , C_0 and B_0 denote expressions of these coefficients (derived in the Appendix) in the absence of pseudogap fluctuations ($W = 0$) for the case of an arbitrary quasiparticle spectrum ξ_p and different types of pairing:

$$\begin{aligned} A_0 &= N_0(0) \frac{T - T_c}{T_c} \langle e^2(\mathbf{p}) \rangle; & C_0 &= N_0(0) \frac{7\zeta(3)}{32\pi^2 T_c^2} \langle |\mathbf{v}(\mathbf{p})|^2 e^2(\mathbf{p}) \rangle; \\ B_0 &= N_0(0) \frac{7\zeta(3)}{8\pi^2 T_c^2} \langle e^4(\mathbf{p}) \rangle, \end{aligned} \quad (22)$$

where angular brackets denote the usual averaging over the Fermi surface: $\langle \dots \rangle = \frac{1}{N_0(0)} \sum_p \delta(\xi_p) \dots$, and $N_0(0)$ is the density of states at the Fermi level for free electrons.

Now all peculiarities of the model under consideration, due to the appearance of the pseudogap, are contained within dimensionless coefficients K_A , K_C and K_B . In the absence of pseudogap fluctuations all these coefficients are equal to 1.

Coefficients K_A and K_C , according to Fig. 6(a) are completely determined by the generalized Cooper susceptibility [8,10] $\chi(\mathbf{q}; T)$, shown in Fig. 2:

$$K_A = \frac{\chi(0; T) - \chi(0; T_c)}{A_0} \quad (23)$$

$$K_C = \lim_{q \rightarrow 0} \frac{\chi(\mathbf{q}; T_c) - \chi(0; T_c)}{q^2 C_0} \quad (24)$$

Generalized susceptibility, as was shown above, can be found from (14), where ‘‘triangular’’ vertices are determined by recurrence procedures (15), allowing to perform direct numerical calculations of the coefficients K_A and K_C .

Situation with coefficient B is, in general case, more complicated. Significant simplifications arise if we limit ourselves (in the order $|\Delta_q|^4$), as usual, to the case of $q = 0$, and define the coefficient B by diagram shown in Fig. 6(b). Then for coefficient K_B we get:

$$K_B = \frac{T_c}{B_0} \sum_{\varepsilon_n} \sum_{\mathbf{p}} e^4(\mathbf{p}) (G(\varepsilon_n \mathbf{p}) G(-\varepsilon_n, -\mathbf{p}))^2 (\Gamma^\pm(\varepsilon_n, -\varepsilon_n, 0))^4 \quad (25)$$

It should be noted that Eq. (25) immediately leads to positively defined coefficient B . This is clear from $G(-\varepsilon_n, -\mathbf{p}) = G^*(\varepsilon_n \mathbf{p})$, so that $G(\varepsilon_n \mathbf{p}) G(-\varepsilon_n, -\mathbf{p})$ is real and accordingly $\Gamma^\pm(\varepsilon_n, -\varepsilon_n, 0)$, defined by recurrence procedure (18), is also real.

V. PHYSICAL PROPERTIES OF SUPERCONDUCTORS IN THE PSEUDOGAP STATE.

It is well known that Ginzburg – Landau equations define two characteristic lengths — coherence length and penetration depth. Coherence length at a given temperature $\xi(T)$ determines characteristic length scale of inhomogeneities of the order parameter Δ :

$$\xi^2(T) = -\frac{C}{A}. \quad (26)$$

In the absence of the pseudogap:

$$\xi_{BCS}^2(T) = -\frac{C_0}{A_0} \quad (27)$$

Thus, in our model:

$$\frac{\xi^2(T)}{\xi_{BCS}^2(T)} = \frac{K_C}{K_A}. \quad (28)$$

For the penetration depth of magnetic field we have:

$$\lambda^2(T) = -\frac{c^2}{32\pi e^2} \frac{B}{AC} \quad (29)$$

Then, analogously to (28), we obtain:

$$\frac{\lambda(T)}{\lambda_{BCS}(T)} = \left(\frac{K_B}{K_A K_C} \right)^{1/2}. \quad (30)$$

Close to T_c the upper critical field H_{c2} is defined via Ginzburg – Landau coefficients as:

$$H_{c2} = \frac{\phi_0}{2\pi \xi^2(T)} = -\frac{\phi_0}{2\pi} \frac{A}{C}, \quad (31)$$

where $\phi_0 = c\pi/|e|$ is the magnetic flux quantum. Then the derivative (slope) of the upper critical field close to T_c is given by:

$$\left| \frac{dH_{c2}}{dT} \right|_{T_c} = \frac{16\pi\phi_0 \langle e^2(\mathbf{p}) \rangle}{7\zeta(3) \langle |\mathbf{v}(\mathbf{p})|^2 e^2(\mathbf{p}) \rangle} T_c \frac{K_A}{K_C}. \quad (32)$$

Specific heat discontinuity at the transition point is defined as:

$$(C_s - C_n)_{T_c} = \frac{T_c}{B} \left(\frac{A}{T - T_c} \right)^2, \quad (33)$$

where C_s and C_n are specific heats of superconducting and normal states. At temperature T_{c0} (in the absence of the pseudogap, $W = 0$):

$$(C_s - C_n)_{T_{c0}} = N(0) \frac{8\pi^2 T_{c0} \langle e^2(\mathbf{p}) \rangle^2}{7\zeta(3) \langle e^4(\mathbf{p}) \rangle}. \quad (34)$$

Then the relative specific heat discontinuity in our model can be written as:

$$\Delta C \equiv \frac{(C_s - C_n)_{T_c}}{(C_s - C_n)_{T_{c0}}} = \frac{T_c}{T_{c0}} \frac{K_A^2}{K_B}. \quad (35)$$

Numerical calculations of K_A , K_B , K_C were performed for the same typical parameters of the model as calculation of T_c described above. The values of these coefficients, are not of great interest and we drop these data⁶ presenting in Figs. 7–12 appropriate W/T_{c0} – dependences of physical characteristics, defined by Eqs. (26) – (35). In accord with typical behavior of T_c described above, here we can also observe two qualitatively different types of behavior due to either constant or alternating signs in recurrence equations for the vertex part (upper or lower signs in (15), charge or spin fluctuations). Results of our calculations of physical characteristics for the first type of behavior (s -wave pairing and scattering by charge (CDW) fluctuations, $d_{x^2-y^2}$ -wave pairing and scattering by spin (AFM(SDW)) fluctuations) are shown in Figs. 7 – 10. We can see that with the growth of pseudogap width W coherence length $\xi(T)$ is reduced, while penetration depth $\lambda(T)$ grows, as compared to the appropriate values of BCS theory. Dependence of both characteristic lengths on parameter κa is rather weak and in Figs. 7, 8 we present data for only one value of $\kappa a = 0.2$. The slope (derivative) of the upper critical field at $T = T_c$ initially grows, then starts to diminish. Most characteristic is the suppression of specific heat discontinuity (as compared to BCS value) shown in Fig. 10, which is in direct qualitative accord with experimental data [27].

⁶Typical dependences of these coefficients on the parameter W/T_{c0} are functions rather rapidly decreasing from 1 in the region of existence of superconducting state.

Note that for specific heat discontinuity we also have noticeable plateau in the region of $W/T_{c0} < 10$, analogous to that observed above in similar dependence of T_c .

Behavior of physical characteristics of superconductor for the case of s -wave pairing and scattering by spin (AFM(CDW)) fluctuations as well as for $d_{x^2-y^2}$ pairing and scattering by charge (CDW) fluctuations are shown in Figs. 11, 12. Data on characteristic lengths are not given, as both coherence length $\xi(T)$, and penetration depth $\lambda(T)$ are practically the same as in BCS theory in almost all region of existence of superconducting state (except very narrow region where T_c can be non single valued and disappear, where both lengths are rapidly growing). As to the slope of an upper critical field and specific heat discontinuity at superconducting transition, both are rapidly suppressed with the growth of W/T_{c0} , up to the region of non single valued T_c (the region of possible “reentrant” behavior or type I transition) (cf. Fig. 11, 12), where both are characterized by non monotonous behavior.

VI. CONCLUSION

In this paper we have analyzed peculiarities of superconducting state forming on the “background” of the pseudogap induced by electron scattering by fluctuations of “dielectric” short – range order in the model of “hot spots” on the Fermi surface. Our analysis is based on microscopic derivation of Ginzburg – Landau expansion and takes into account all high order contributions of perturbation theory for scattering by pseudogap fluctuations. “Condensed” phase of such superconductor should be described by appropriate Gor’kov’s equations for a superconductor with the pseudogap [10], which will be the task for the future work.

Our main conclusion is that superconductivity is suppressed by pseudogap fluctuations of CDW, AFM(SDW) type with two qualitatively different model of such suppression, related to either constant or alternating signs in recurrence equations for the vertex part (upper or lower signs in (15, charge or spin fluctuations). While the case of scattering by spin fluctuations and pairing with $d_{x^2-y^2}$ – symmetry is apparently realized in high – temperature superconducting copper oxides, we are not aware of systems with unusual behavior

obtained above for the case of s -wave pairing and scattering by spin (AFM(CDW)) fluctuations, as well as for the case of $d_{x^2-y^2}$ pairing and scattering by charge (CDW) fluctuations. Experimental search of systems with this behavior may be of significant interest.

The most important problem in the description of the pseudogap state in real HTSC – cuprates is an explanation of doping dependences of the main physical characteristics. In our model, these dependences should be expressed via corresponding doping dependences of the effective pseudogap width W and correlation length ξ . Unfortunately, these dependences are determined from rather indirect experimental data and are poorly defined [1,2]⁷. As a very crude statement, we can claim that correlation length ξ does not change significantly in rather wide region of doping concentrations, while the pseudogap width W drops linearly with concentration growth from the values of the order of $10^3 K$ close to the insulating phase, to the values of the order of T_c at optimal doping, becoming zero at slightly higher concentrations of current carriers (Cf. Fig.6 in the review paper [2], which is based of Fig.4 of Ref. [3], with some compilation of appropriate data for $YBCO$). Using such doping dependence we can immediately reexpress W – dependences given above as dependences on concentration of doping impurity. In the oversimplified version of our model with an infinite correlation length and Fermi surface with complete nesting, assuming also linear doping dependence for T_{c0} , such calculations were given in a recent paper [28]. It was shown that even within this simplest approach the qualitative form of the phase diagram of cuprates can be nicely reproduced. However, the obvious crudeness of the model, as well as the absence of reliable data on concentration dependences of W , ξ and T_{c0} , do not allow any serious “improvement” of these qualitative conclusions.

Among shortcomings of the present model, besides many times mentioned neglect of dynamics of short – range order fluctuations and limitation to Gaussian statistics of these

⁷Concentration dependence of T_{c0} may also be quite important, but in fact it is completely unknown.

fluctuations, we also note the simplified analysis of spin structure of interaction, which assumed its Ising like nature. It will be of great interest to perform similar analysis for the general case of Heisenberg interaction.

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**APPENDIX A: GINZBURG – LANDAU COEFFICIENTS FOR ANISOTROPIC
SUPERCONDUCTOR IN THE ABSENCE OF THE PSEUDOGAP.**

In the absence of pseudogap fluctuations ($W = 0$) the generalized Cooper susceptibility define by diagram of Fig. 2 takes the following form:

$$\chi_0(\mathbf{q}; T) = -T \sum_{\varepsilon_n} \sum_{\mathbf{p}} e^2(\mathbf{p}) \frac{1}{i\varepsilon_n - \xi_{\mathbf{p}+\mathbf{q}}} \frac{1}{-i\varepsilon_n - \xi_{\mathbf{p}}} \quad (\text{A1})$$

Then for susceptibility at $\mathbf{q} = 0$, defining the coefficient A_0 , we get:

$$\begin{aligned} \chi_0(0; T) &= -T \sum_{\varepsilon_n} \sum_{\mathbf{p}} e^2(\mathbf{p}) \frac{1}{\varepsilon_n^2 + \xi_{\mathbf{p}}^2} = -T \sum_{\varepsilon_n} \int_{-\infty}^{\infty} d\xi \frac{1}{\varepsilon_n^2 + \xi^2} \sum_{\mathbf{p}} \delta(\xi - \xi_{\mathbf{p}}) e^2(\mathbf{p}) \approx \\ &\approx -N_0(0)T \sum_{\varepsilon_n} \int_{-\infty}^{\infty} d\xi \frac{1}{\varepsilon_n^2 + \xi^2} \frac{\sum_{\mathbf{p}} \delta(\xi_{\mathbf{p}}) e^2(\mathbf{p})}{N_0(0)} = \chi_{BCS}(0; T) \langle e^2(\mathbf{p}) \rangle \end{aligned} \quad (\text{A2})$$

where angular brackets denote averaging over the Fermi surface and we introduced the standard susceptibility of BCS model for isotropic s -wave pairing $\chi_{BCS}(0; T)$.

As a result, we obtain coefficient A_0 as:

$$A_0 = \chi_0(0; T) - \chi_0(0; T_c) = A_{BCS} \langle e^2(\mathbf{p}) \rangle \quad (\text{A3})$$

where

$$A_{BCS} = \chi_{BCS}(0; T) - \chi_{BCS}(0; T_c) = N_0(0) \frac{T - T_c}{T_c} \quad (\text{A4})$$

is the standard expression for coefficient A for the case of isotropic s -wave pairing.

Coefficient C_0 of Ginzburg – Landau expansion is defined by susceptibility (A1) at small \mathbf{q} :

$$C_0 = \lim_{q \rightarrow 0} \frac{\chi_0(\mathbf{q}; T_c) - \chi_0(0; T_c)}{q^2} \quad (\text{A5})$$

Expanding expression (A1) for $\chi_0(\mathbf{q}; T_c)$ in powers of q we get:

$$\chi_0(\mathbf{q}; T_c) = \chi_0(0; T_c) + T_c \sum_{\varepsilon_n} \sum_{\mathbf{p}} \frac{3\varepsilon_n^2 - \xi_{\mathbf{p}}^2}{4(\varepsilon_n^2 + \xi_{\mathbf{p}}^2)^3} e^2(\mathbf{p}) (\mathbf{v}(\mathbf{p})\mathbf{q})^2 \quad (\text{A6})$$

so that coefficient C_0 is found to be:

$$\begin{aligned}
C_0 &= T_c \sum_{\varepsilon_n} \int_{-\infty}^{\infty} d\xi \frac{3\varepsilon_n^2 - \xi^2}{4(\varepsilon_n^2 + \xi^2)^3} \sum_{\mathbf{p}} \delta(\xi - \xi_{\mathbf{p}}) e^2(\mathbf{p}) |\mathbf{v}(\mathbf{p})|^2 \cos^2(\phi) \approx \\
&\approx T_c \sum_{\varepsilon_n} \int_{-\infty}^{\infty} d\xi \frac{3\varepsilon_n^2 - \xi^2}{4(\varepsilon_n^2 + \xi^2)^3} \sum_{\mathbf{p}} \delta(\xi_{\mathbf{p}}) e^2(\mathbf{p}) |\mathbf{v}(\mathbf{p})|^2 \cos^2(\phi) = \\
&= N_0(0) \frac{7\zeta(3)}{16\pi^2 T_c^2} \langle e^2(\mathbf{p}) |\mathbf{v}(\mathbf{p})|^2 \cos^2(\phi) \rangle \quad (\text{A7})
\end{aligned}$$

where ϕ is an angle between vectors $\mathbf{v}(\mathbf{p})$ and \mathbf{q} , $\zeta(3) = \sum_{n=1}^{\infty} \frac{1}{n^3} \approx 1,202$

For square lattice both the Fermi surface and $|\mathbf{v}(\mathbf{p})|$ are symmetric with respect to rotation over an angle $\frac{\pi}{2}$, the same symmetry is valid for $e(\mathbf{p})$ (for all types of pairing, mentioned above). Thus we easily find:

$$\langle e^2(\mathbf{p}) |\mathbf{v}(\mathbf{p})|^2 \cos^2(\phi) \rangle = \frac{1}{2} \langle e^2(\mathbf{p}) |\mathbf{v}(\mathbf{p})|^2 (1 + \cos(2\phi)) \rangle = \frac{1}{2} \langle e^2(\mathbf{p}) |\mathbf{v}(\mathbf{p})|^2 \rangle \quad (\text{A8})$$

because $\cos(2\phi)$ changes sign after the rotation of \mathbf{p} by an angle $\frac{\pi}{2}$. In fact, the direction of the velocity $\mathbf{v}(\mathbf{p})$ after this rotation changes to a perpendicular one, accordingly $\cos(2\phi) \rightarrow -\cos(2\phi)$. As a result we obtain for the coefficient C_0 an isotropic expression:

$$C_0 = N_0(0) \frac{7\zeta(3)}{32\pi^2 T_c^2} \langle |\mathbf{v}(\mathbf{p})|^2 e^2(\mathbf{p}) \rangle, \quad (\text{A9})$$

which for the case of isotropic s -wave pairing and spherical Fermi surface becomes the standard one:

$$C_{BCS} = N_0(0) \frac{7\zeta(3)v_F^2}{32\pi^2 T_c^2} \quad (\text{A10})$$

Coefficient B defined by the diagram shown in Fig. 6(b), in the absence of pseudogap fluctuations ($W = 0$) and for $\mathbf{q} = 0$ takes the following form:

$$\begin{aligned}
B_0 &= T_c \sum_{\varepsilon_n} \sum_{\mathbf{p}} \frac{1}{(\varepsilon_n^2 + \xi_{\mathbf{p}}^2)^2} e^4(\mathbf{p}) = T_c \sum_{\varepsilon_n} \int_{-\infty}^{\infty} d\xi \frac{1}{(\varepsilon_n^2 + \xi^2)^2} \sum_{\mathbf{p}} \delta(\xi - \xi_{\mathbf{p}}) e^4(\mathbf{p}) \approx \\
&\approx N_0(0) T_c \sum_{\varepsilon_n} \int_{-\infty}^{\infty} d\xi \frac{1}{(\varepsilon_n^2 + \xi^2)^2} \frac{\sum_{\mathbf{p}} \delta(\xi_{\mathbf{p}}) e^4(\mathbf{p})}{N_0(0)} = B_{BCS} \langle e^4(\mathbf{p}) \rangle \quad (\text{A11})
\end{aligned}$$

where

$$B_{BCS} = N_0(0) \frac{7\zeta(3)}{8\pi^2 T_c^2} \quad (\text{A12})$$

is the standard expression for B in case of isotropic s -wave pairing.

FIGURES

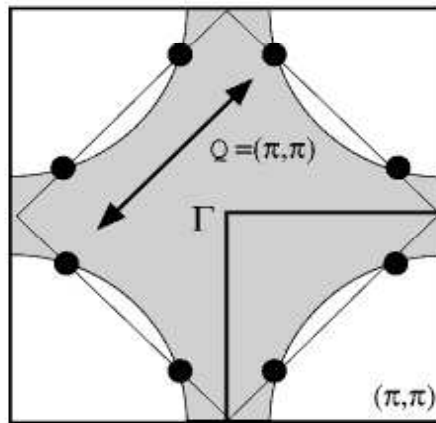


FIG. 1. Fermi surface with “hot spots” connected by scattering vector of the order of $\mathbf{Q} = (\frac{\pi}{a}, \frac{\pi}{a})$.

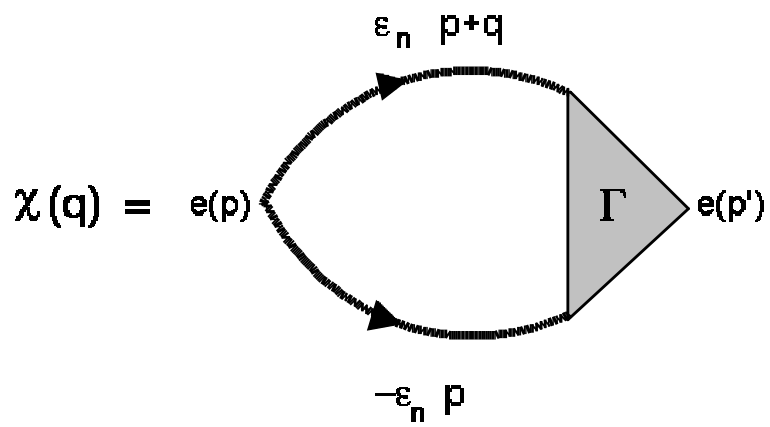


FIG. 2. Diagram for the generalized susceptibility $\chi(\mathbf{q})$ in Cooper channel.

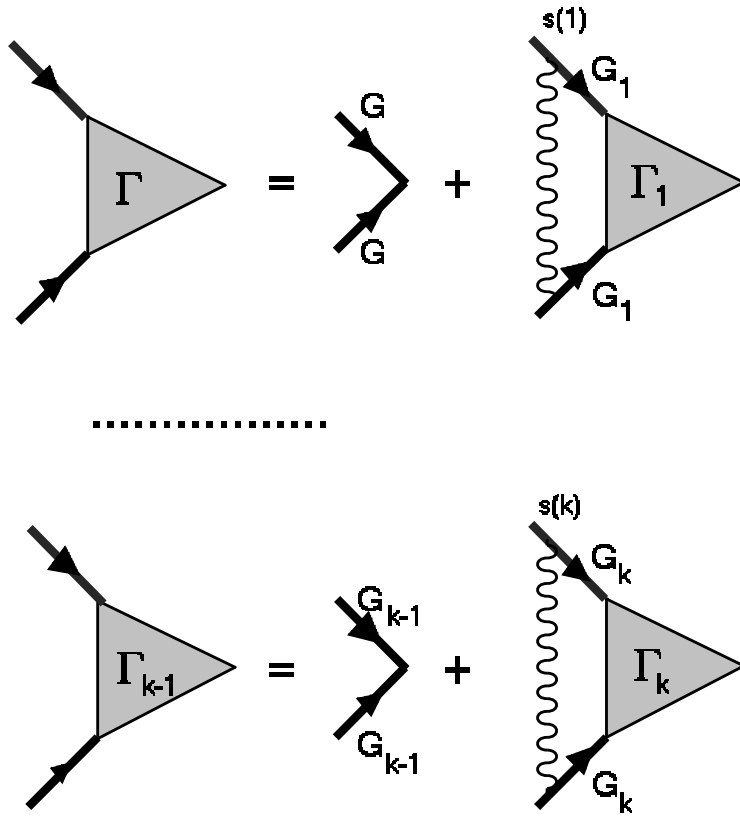


FIG. 3. Recurrence equations for the vertex part.

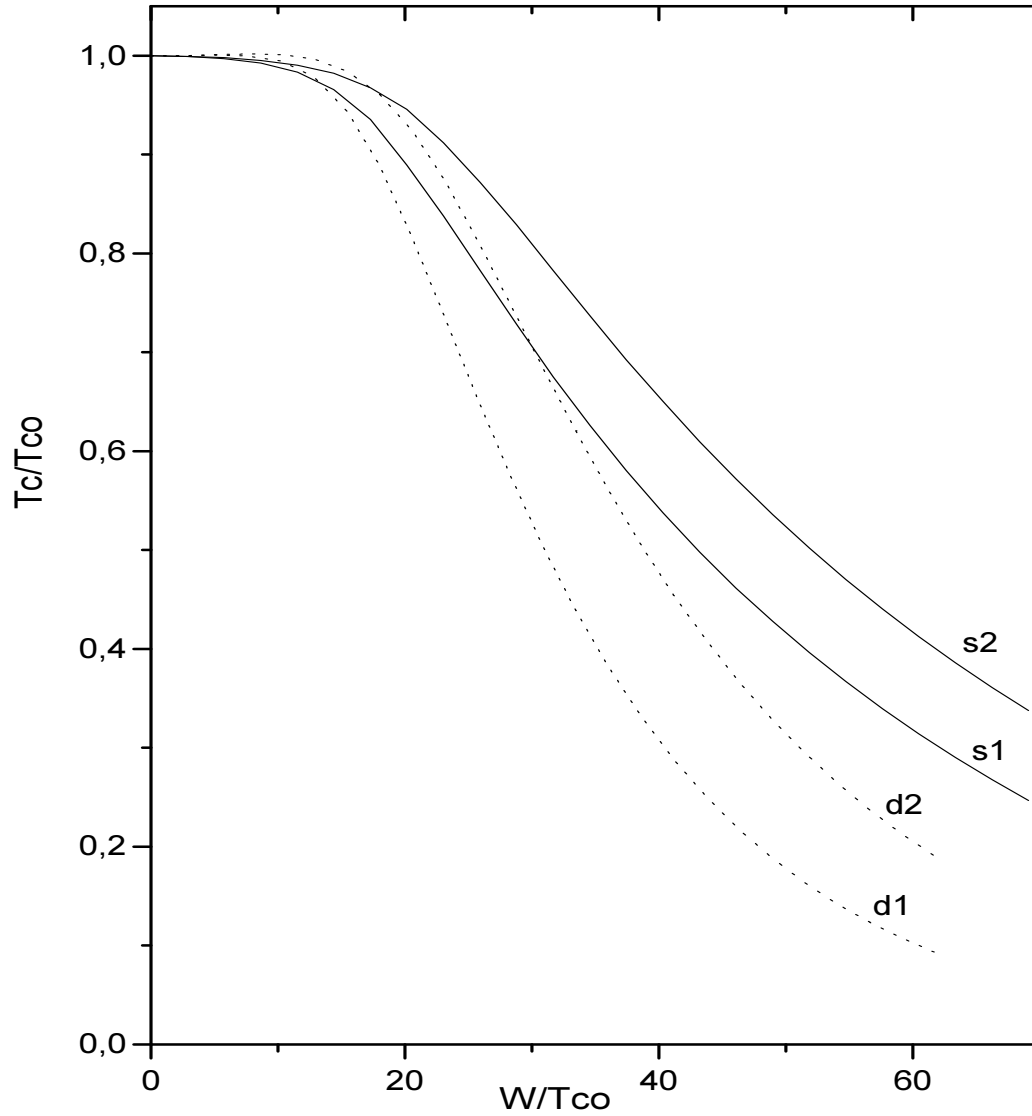


FIG. 4. Dependence of superconducting transition temperature T_c/T_{c0} on the effective width of the pseudogap W/T_{c0} for s -wave pairing and scattering by charge (CDW) fluctuations (curves s1 and s2) and for $d_{x^2-y^2}$ -wave pairing and scattering by spin (AFM(SDW)) fluctuations (curves d1 and d2). Data are given for the values of inverse correlation length $\kappa a = 0.2$ (s1 and d1) and $\kappa a = 0.5$ (s2 and d2).

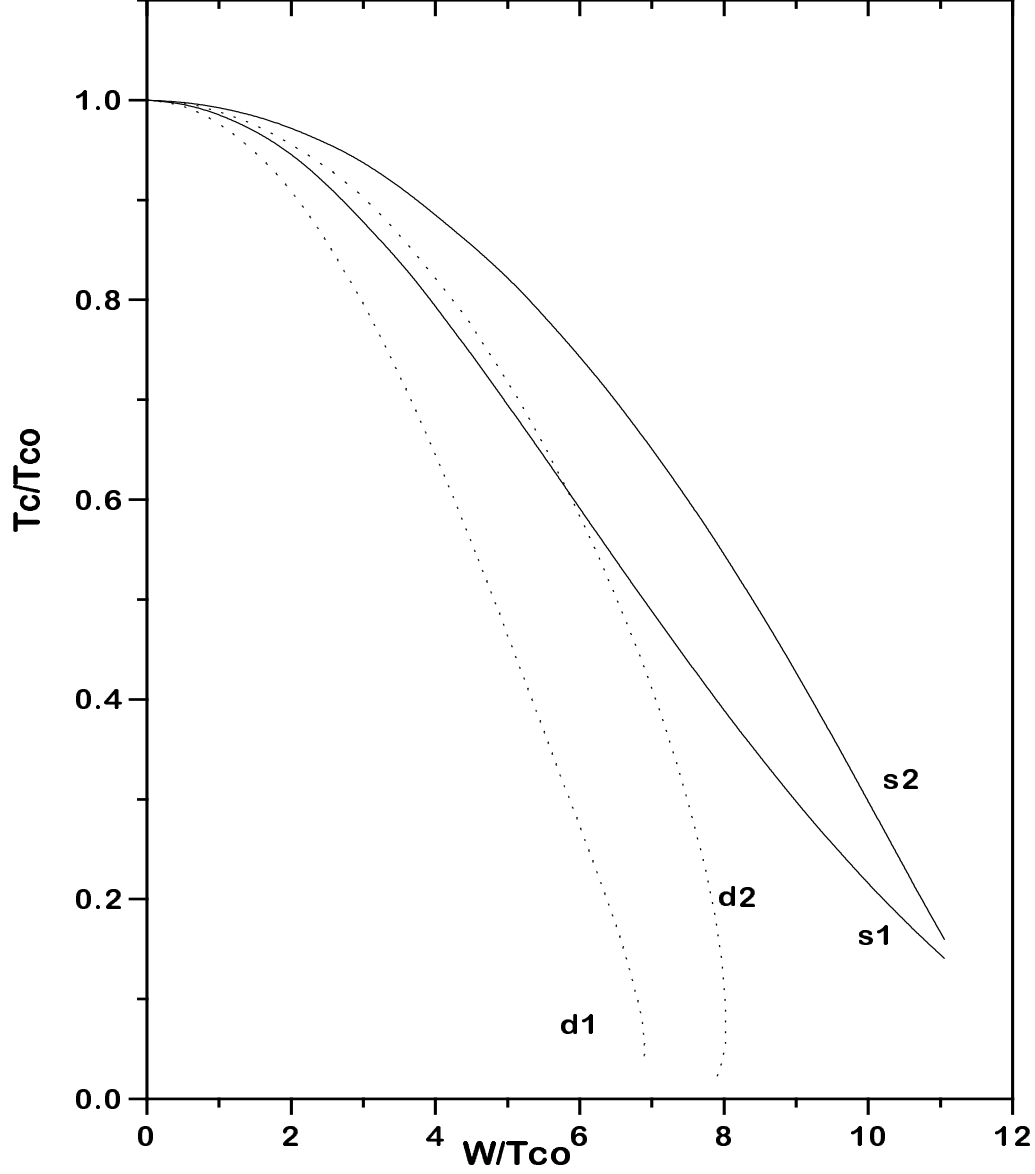


FIG. 5. Dependence of superconducting transition temperature T_c/T_{c0} on the effective width of the pseudogap W/T_{c0} for s -wave pairing and scattering by spin (AFM(SDW)) fluctuations (curves s1 and s2) and for $d_{x^2-y^2}$ -wave pairing and scattering by charge (CDW) fluctuations (curves d1 and d2). Data are given for the values of inverse correlation length $\kappa a = 0.2$ (s1 and d1) and $\kappa a = 1.0$ (s2 and d2).

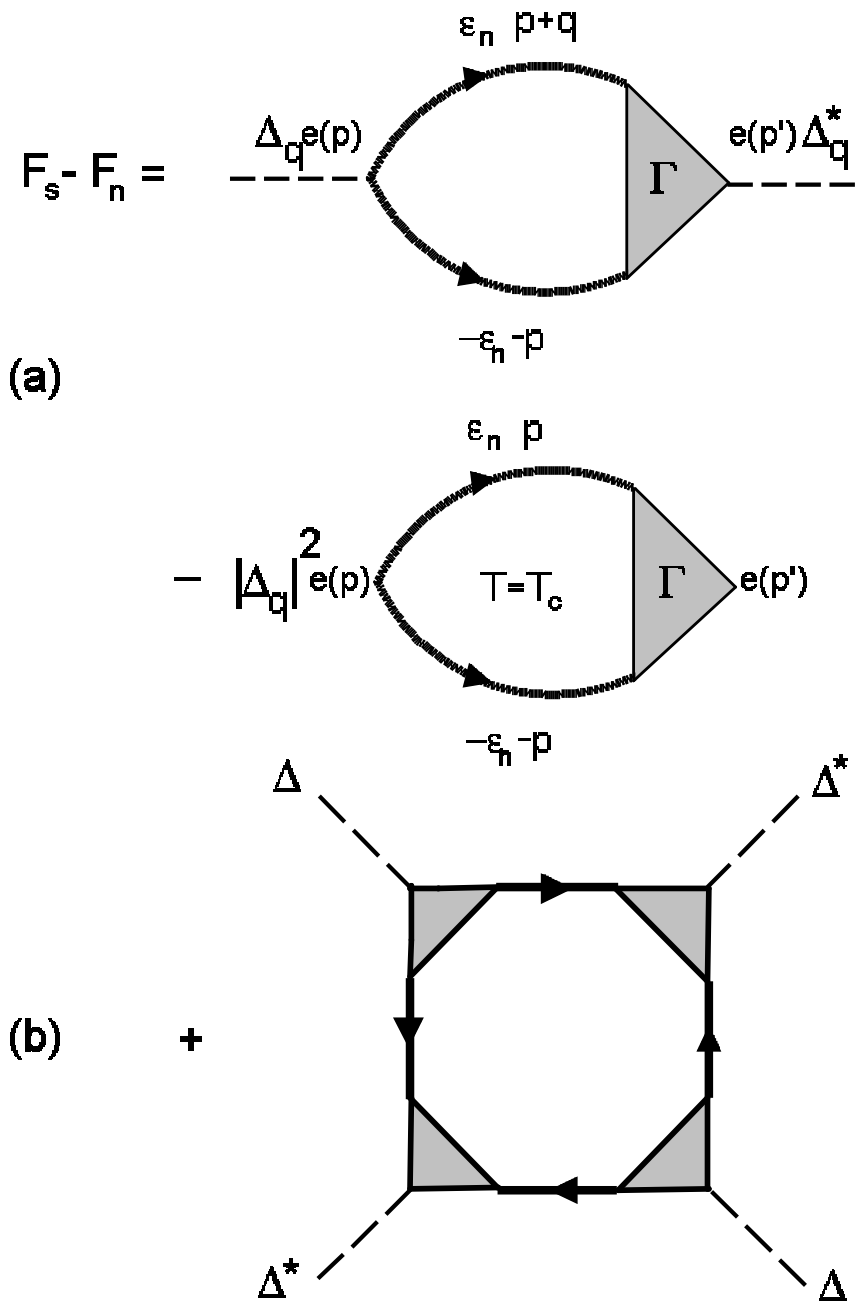


FIG. 6. Diagrammatic representation of Ginzburg – Landau expansion.

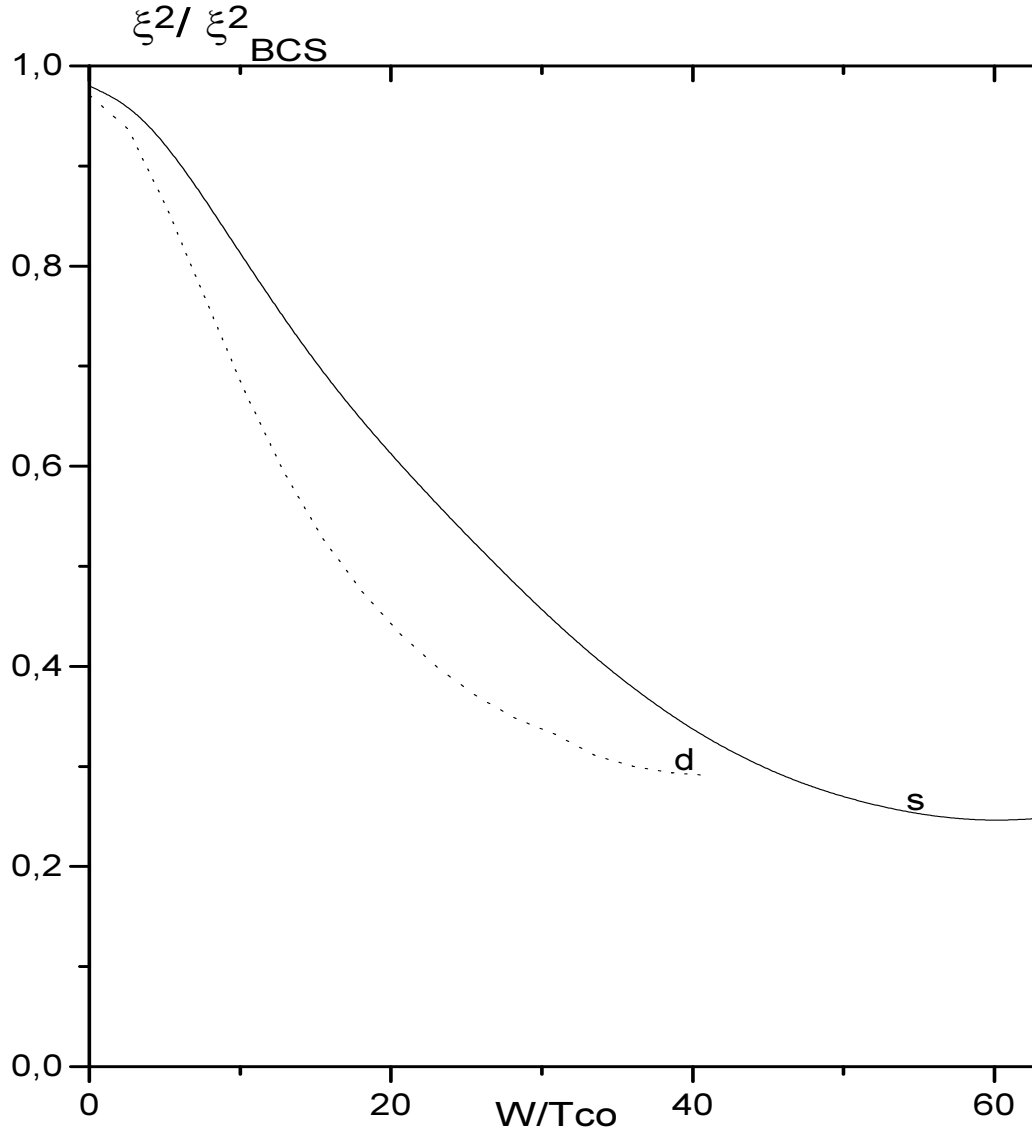


FIG. 7. Dependence of the square of the coherence length ξ^2/ξ_{BCS}^2 on the effective width of the pseudogap W/T_{c0} for s -wave pairing and scattering by charge (CDW) fluctuations (full curve) and for $d_{x^2-y^2}$ -wave pairing and scattering by spin (AFM(SDW)) fluctuations (dotted curve). Data are given for the value of inverse correlation length $\kappa a = 0.2$.

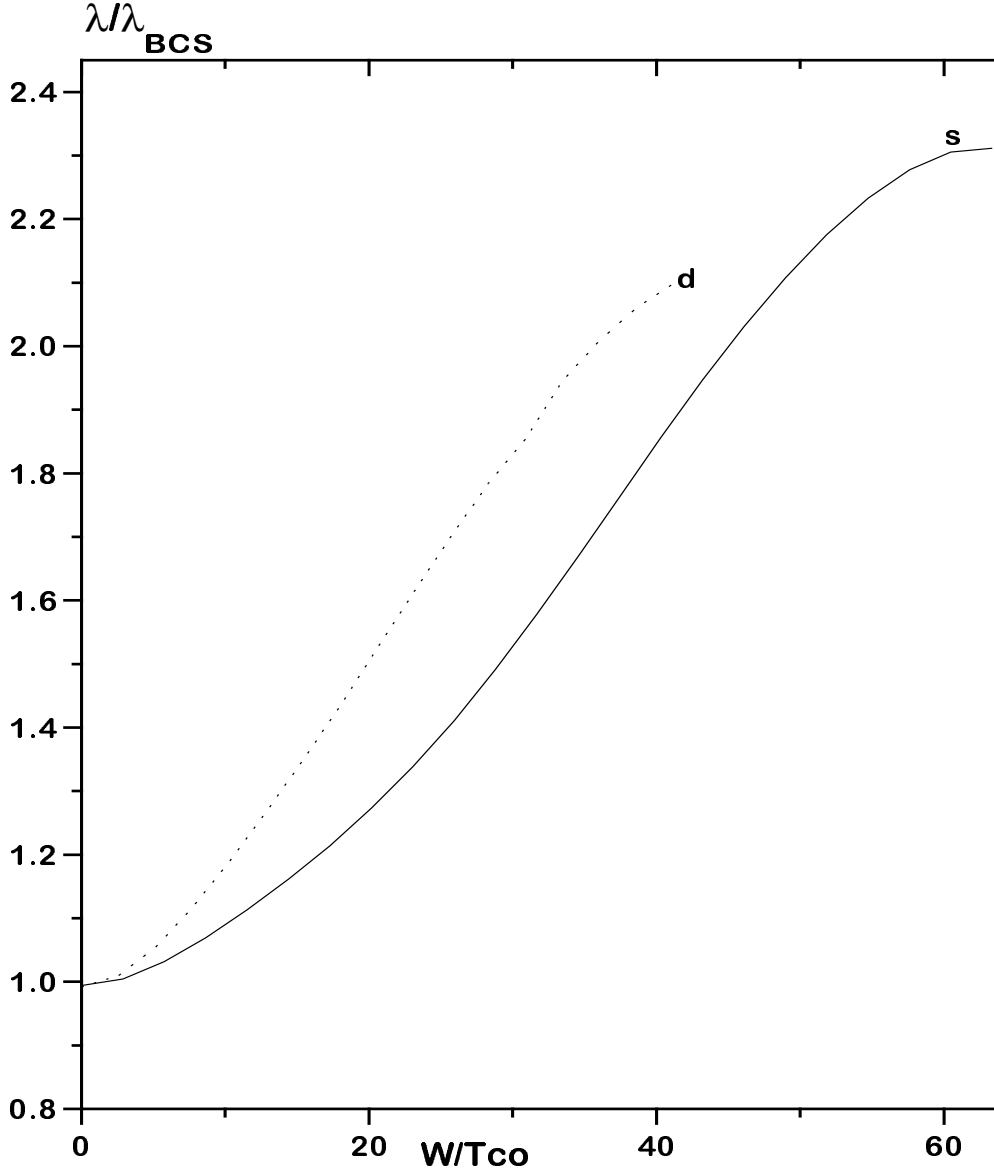


FIG. 8. Dependence of penetration depth λ/λ_{BCS} on the effective width of the pseudogap W/T_{c0} for s -wave pairing and scattering by charge (CDW) fluctuations (full curve) and for $d_{x^2-y^2}$ -wave pairing and scattering by spin (AFM(SDW)) fluctuations (dashed curve). Data are given for the value of inverse correlation length $\kappa a = 0.2$.

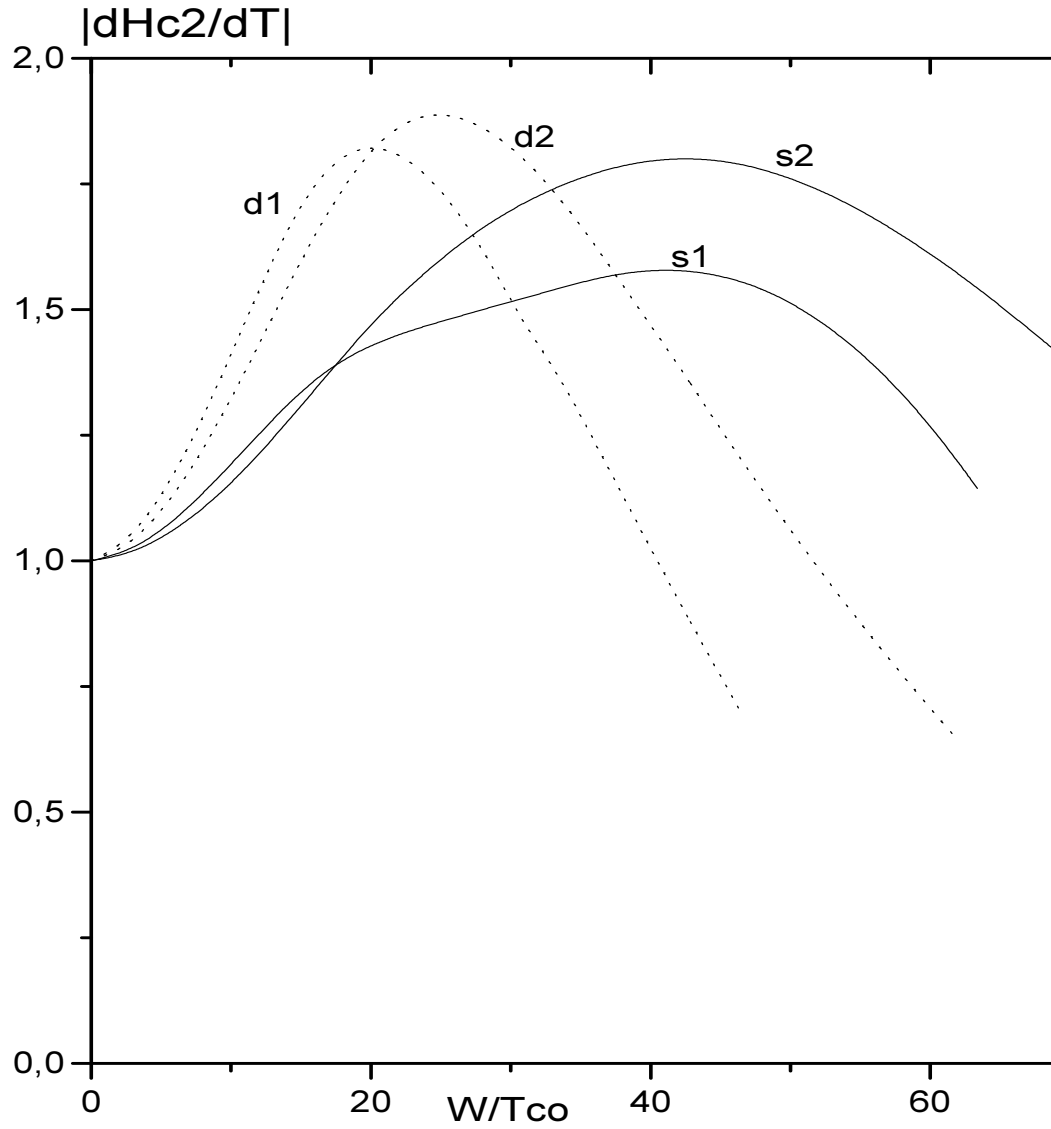


FIG. 9. Dependence of the derivative (slope) of the upper critical field (normalized by its value for $W = 0$) on the effective width of the pseudogap for s -wave pairing and scattering by charge (CDW) fluctuations (curves s1 and s2) and for $d_{x^2-y^2}$ -wave pairing and scattering on spin (AFM(SDW)) fluctuations (curves d1 and d2). Data are given for the values of inverse correlation length $\kappa a = 0.2$ (s1 and d1) and $\kappa a = 0.5$ (s2 and d2).

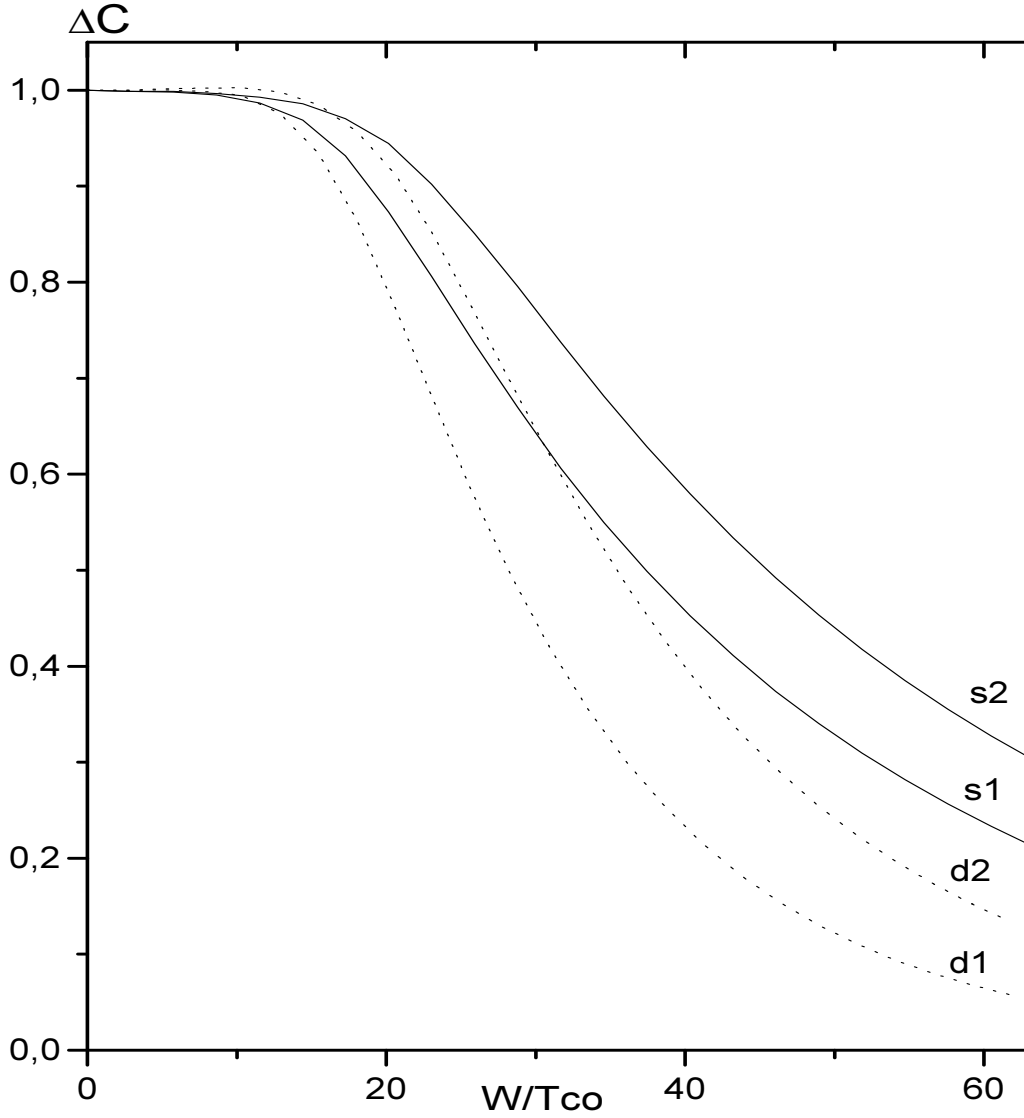


FIG. 10. Dependence of specific heat discontinuity at superconducting transition on the effective width of the pseudogap W/T_{c0} for s -wave pairing and scattering by charge (CDW) fluctuations (curves s1 and s2) and for $d_{x^2-y^2}$ -wave pairing and scattering by spin (AFM(SDW)) fluctuations (curves d1 and d2). Data are given for the values of inverse correlation length $\kappa a = 0.2$ (s1 and d1) and $\kappa a = 0.5$ (s2 and d2).

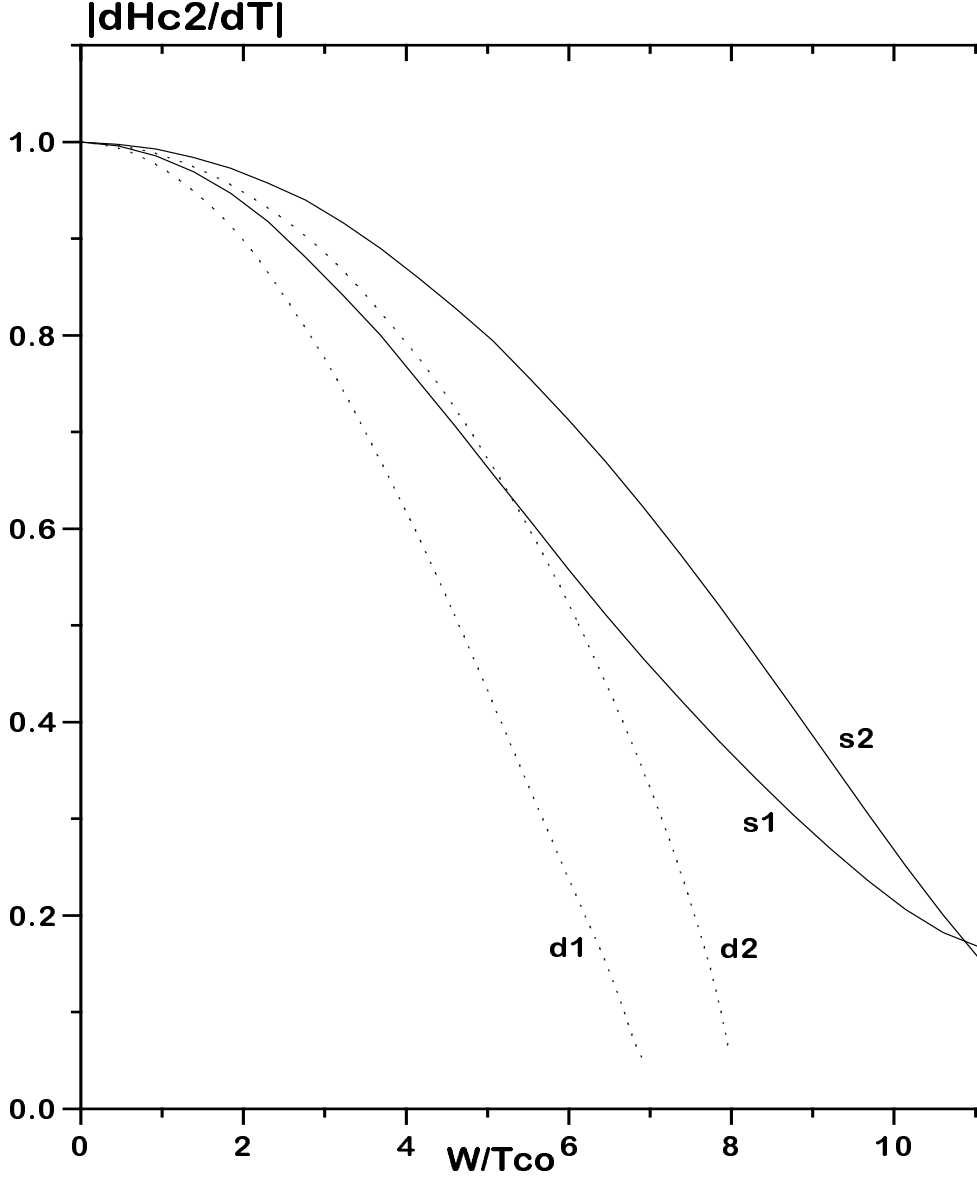


FIG. 11. Dependence of the derivative (slope) of the upper critical field (normalized by its value for $W = 0$) on the effective width of the pseudogap W/T_{c0} for s -wave pairing and scattering by spin (AFM(CDW)) fluctuations (curves s1 and s2) and for $d_{x^2-y^2}$ -wave pairing and scattering by charge (CDW) fluctuations (curves d1 and d2). Data are given for the values of inverse correlation length $\kappa a = 0.2$ (s1 and d1) and $\kappa a = 1.0$ (s2 and d2).

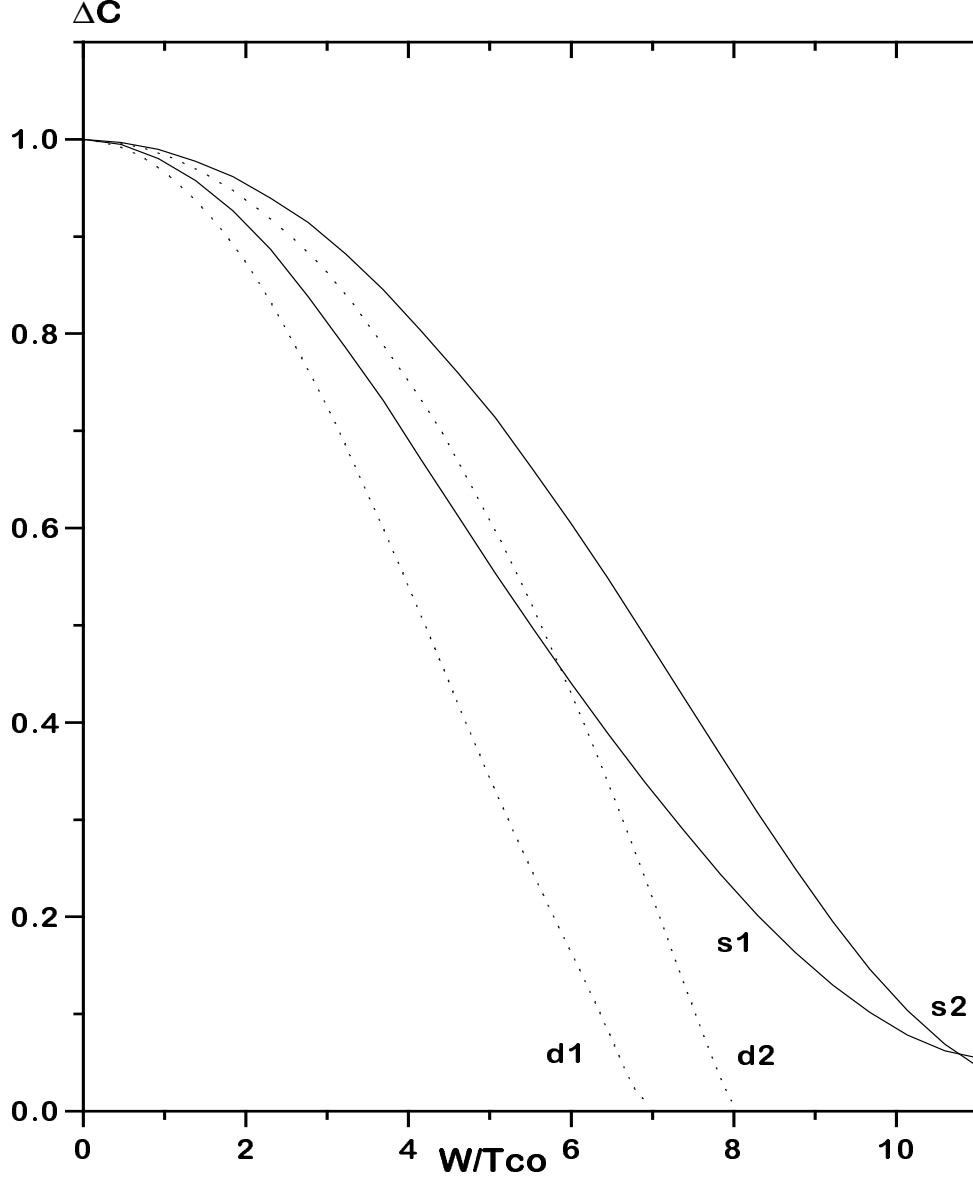


FIG. 12. Dependence of specific heat discontinuity at superconducting transition on the effective width of the pseudogap W/T_{c0} for s -wave pairing and scattering by spin (AFM(CDW)) fluctuations (curves s1 and s2) and for $d_{x^2-y^2}$ -wave pairing and scattering on charge (CDW) fluctuations (curves d1 and d2). Data are given for the values of inverse correlation length $\kappa a = 0.2$ (s1 and d1) and $\kappa a = 1.0$ (s2 and d2).

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