

Temperature Dependence of Paramagnetic Critical Magnetic Field in Disordered Attractive Hubbard Model

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Within the generalized DMFT+ Σ approach we study disorder effects in the temperature dependence of paramagnetic critical magnetic field $H_{cp}(T)$ for Hubbard model with attractive interaction. We consider the wide range of attraction potentials U – from the weak coupling limit, when superconductivity is described by BCS model, up to the limit of very strong coupling, when superconducting transition is related to Bose – Einstein condensation (BEC) of compact Cooper pairs. The growth of the coupling strength leads to the rapid growth of $H_{cp}(T)$ at all temperatures. However, at low temperatures paramagnetic critical magnetic field H_{cp} grows with U much slower, than the orbital critical field, and in BCS limit the main contribution to the upper critical magnetic field is of paramagnetic origin. The growth of the coupling strength also leads to the disappearance of the low temperature region of instability towards type I phase transition and Fulde – Ferrell – Larkin – Ovchinnikov (FFLO) phase, characteristic for BCS weak coupling limit. Disorder leads to the rapid drop of $H_{cp}(T)$ in BCS weak coupling limit, while in BCS – BEC crossover region and BEC limit $H_{cp}(T \rightarrow 0)$ dependence on disorder is rather weak. Within DMFT+ Σ approach disorder influence on $H_{cp}(T)$ is of universal nature at any coupling strength and related only to disorder widening of the conduction band. In particular, this leads to the drop of the effective coupling strength with disorder, so that disordering restores the region of type I transition in the intermediate coupling region.

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I. INTRODUCTION

In the weak coupling region and for the weak disorder the upper critical magnetic field of a superconductor is determined by orbital effects and usually is much lower than the paramagnetic limit. However, the growth of the coupling strength and disordering lead to the rapid growth of the orbital H_{c2} possibly overcoming the paramagnetic limit.

In this paper we study the behavior of paramagnetic critical field in the region of very strong coupling of electrons of the Cooper pair and in the crossover region from BCS – like behavior for the weak coupling to Bose – Einstein condensation (BEC) in the strong coupling region [1], taking disorder into account (including the strong enough).

The simplest model to study the BCS – BEC crossover is Hubbard model with attractive interaction. Most successful approach to the studies of Hubbard model, both to describe the strongly correlated systems in the case of repulsive interactions and to study the BCS – BEC crossover for the case of attraction, is the dynamical mean – field theory (DMFT) [2–4].

In recent years we have developed the generalized DMFT+ Σ approach to Hubbard model [5–11], which is quite effective for the studies of the influence of different external (outside those taken into account by DMFT) interactions. This DMFT+ Σ method was used by us in Refs. [12–14] to study the disorder influence on the temperature of superconducting transition. In particular, for the case of semi – elliptic initial density of states, adequate to describe three – dimensional systems, it was

demonstrated that disorder influence on the critical temperature (in the whole region of interaction strengths) is related only to the general widening of the initial conduction band (density of states) by disorder (the generalized Anderson theorem). In Ref. [15], using the combination of the Nozieres – Schmitt-Rink approximation and DMFT+ Σ in attractive Hubbard model we have analyzed the influence of disordering on the temperature dependence of the orbital upper critical field $H_{c2}(T)$ both for the wide region of coupling strengths U , including the BCS – BEC crossover region, and in the wide region of disorder up to the vicinity of Anderson transition. Both the growth of the coupling strength and disorder lead to the rapid growth of H_{c2} , leading in the BEC – limit to unrealistically high values of $H_{c2}(T \rightarrow 0)$, significantly overcoming the paramagnetic limit.

In this work we perform the detailed analysis of disorder influence on the temperature dependence of paramagnetic critical magnetic field of a superconductor for the wide range of coupling strengths U , including the BCS – BEC crossover region and the limit of very strong coupling.

It is well known, that in BCS weak coupling limit paramagnetic effects (spin splitting) lead to the existence of a low temperature region at the phase diagram of a superconductor in magnetic field, where paramagnetic critical field H_{cp} decreases with further lowering of the temperature. This behavior signifies the instability leading to the region of type I phase transition, where also the so called Fulde – Ferrell – Larkin – Ovchinnikov (FFLO) phase may appear [16–18] with Cooper pairs with finite momentum \mathbf{q} and spatially periodic superconducting or-

der parameter. In the following we limit ourselves to the analysis of type II transition and homogeneous superconducting order parameter, allowing us to determine the border of instability towards type I transition in BCS – BEC crossover and strong coupling regions at different disorder levels. The problem of stability of FFLO phase under these conditions is not analyzed here.

II. HUBBARD MODEL WITHIN DMFT+ Σ APPROACH IN NOZIERES – SCHMITT-RINK APPROXIMATION

We are considering the disordered Hubbard model with attractive interaction, taking into account spin – splitting by external magnetic field H , and described by the Hamiltonian:

$$H = -t \sum_{\langle ij \rangle \sigma} a_{i\sigma}^\dagger a_{j\sigma} + \sum_{i\sigma} \epsilon_i n_{i\sigma} - U \sum_i n_{i\uparrow} n_{i\downarrow} - \mu_B H \sum_{i\sigma} \sigma n_{i\sigma}, \quad (1)$$

where $t > 0$ – is transfer amplitude between nearest neighbors, U – is the onsite Hubbard attraction, $n_{i\sigma} = a_{i\sigma}^\dagger a_{i\sigma}$ – is electron number operator on a given site, $a_{i\sigma}$ ($a_{i\sigma}^\dagger$) – electron annihilation (creation) operator, $\sigma = \pm 1$, $\mu_B = \frac{e\hbar}{2mc}$ – Bohr magneton, and local energies ϵ_i are assumed to be independent and random on different lattice sites. We assume Gaussian distribution for energy levels ϵ_i at a given site:

$$\mathcal{P}(\epsilon_i) = \frac{1}{\sqrt{2\pi}\Delta} \exp\left(-\frac{\epsilon_i^2}{2\Delta^2}\right) \quad (2)$$

Distribution width Δ represents the measure of disorder, and Gaussian random field of energy levels (independent on different lattice sites) produces “impurity” scattering, which is analyzed within the standard approach, based on calculations of the averaged Green’s functions.

The generalized DMFT+ Σ approach [5–9] extends the standard dynamical mean field theory (DMFT [2–4] by addition of an “external” self – energy $\Sigma_{\mathbf{p}}(\varepsilon)$ (in general case momentum dependent) due to any kind of interaction outside the DMFT, which gives an effective calculation method both for single – particle and two – particle properties [8, 10]. This approach conserves the standard system of self – consistent DMFT equations [2–4], with “external” self – energy $\Sigma_{\mathbf{p}}(\varepsilon)$ being recalculated at each iteration step using some approximate scheme, corresponding to the type of additional interaction, while the local Green’s function of DMFT is “dressed” by $\Sigma_{\mathbf{p}}(\varepsilon)$ at each step of the standard DMFT procedure.

In the problem of disorder scattering under discussion here [10, 11] for “external” self – energy we are using the simplest (self – consistent Born) approximation, neglecting diagrams with “crossing” interaction lines due to impurity scattering. Such an “external” self – energy remains momentum independent (local).

To solve the single – impurity Anderson problem of DMFT in this paper, as in our previous works, we use

quite efficient method of numerical renormalization group (NRG) [19].

In the following we assume the “bare” conduction band with semi – elliptic density of states (per unit cell with lattice parameter a and single spin projection), which gives a good approximation for three – dimensional case:

$$N_0(\varepsilon) = \frac{2}{\pi D^2} \sqrt{D^2 - \varepsilon^2}, \quad (3)$$

where D defines the half – width of the conduction band..

In Ref. [14] we have shown, that in DMFT+ Σ approach in the model with semi – elliptic density of states all the effects of disorder on single – particle properties reduce only to widening of conduction band by disorder, i.e. to the replacement $D \rightarrow D_{eff}$, where D_{eff} – is the effective “bare” band half – width in the absence of correlations ($U = 0$), widened by disorder:

$$D_{eff} = D \sqrt{1 + 4 \frac{\Delta^2}{D^2}}. \quad (4)$$

The “bare” (in the absence of U) density of states, “dressed” by disorder

$$\tilde{N}_0(\xi) = \frac{2}{\pi D_{eff}^2} \sqrt{D_{eff}^2 - \xi^2}, \quad (5)$$

remains semi – elliptic also in the presence of disorder.

It is necessary to note, that in other models of the “bare” band disorder not only widens the band, but also changes the form of the density of states. In general, there is no complete universality of disorder influence on single – particle properties, which reduces to the replacement $D \rightarrow D_{eff}$. However, in the limit of strong enough disorder the “bare” band becomes almost semi – elliptic and this universality is restored [14].

All calculations in the present paper, as in our previous works, were performed for rather typical case of quarter – filled band (electron number per lattice site $n=0.5$).

To analyze superconductivity for the wide range of pairing interactions U , following Ref. [14], we use Nozieres – Schmitt-Rink approximation [1], which allows qualitatively correct (though approximate) description of BCS – BEC crossover. In this approach, to determine the critical temperature T_c (in the absence of H) we use [14] the conventional BCS weak coupling equation, but the chemical potential of the system μ for different values of U and Δ is determined from DMFT+ Σ calculations, i.e. from the standard equation for the number of electrons in conduction band, which allows us to find T_c for the wide range of model parameters, including the BCS – BEC crossover region, as well as for different levels of disorder. This reflects the physical meaning of Nozieres – Schmitt-Rink approximation: in the weak coupling region transition temperature is controlled by the equation for Cooper instability, while in the strong coupling limit it is determined as BEC temperature, which is controlled by chemical potential. It was demonstrated, that such an approach guarantees the correct interpolation between

the limits of weak and strong couplings, including also the effects of disorder [1, 12, 14]. In particular, in Refs. [12, 14] it was shown, that disorder influence on critical temperature T_c and single – particle characteristics (e.g. density of states) in the model with semi – elliptic “bare” density of states is universal and is reduced only to the changes of the effective bandwidth.

III. MAIN RESULTS

In the framework of Nozieres – Schmitt-Rink approach the critical temperature in the presence of spin – splitting of electron level in external magnetic field (and neglecting the orbital effects) or paramagnetic critical magnetic field H_{cp} at temperatures $T < T_c$ is determined by the following BCS – like equation:

$$1 = \frac{U}{4} \int_{-\infty}^{\infty} d\varepsilon \frac{\tilde{N}_0(\varepsilon)}{\varepsilon - \mu} \left(th \frac{\varepsilon - \mu - \mu_B H_{cp}}{2T} + th \frac{\varepsilon - \mu + \mu_B H_{cp}}{2T} \right), \quad (6)$$

where the chemical potential μ for different values of U and Δ is determined from DMFT+ Σ – calculations, i.e. from the standard equation for the number of electrons in conduction band. The general derivation of Eq. (6) in the presence of disorder is given in the Appendix. Note that Eq. (6) is derived from the exact Ward identity and remains valid even in the case of strong disorder, including the vicinity of Anderson transition. Eq. (6) explicitly demonstrates, that all disorder effects on H_{cp} are reduced to the renormalization of the initial density of states by disorder, so that for the case of initial band with semi – elliptic density of states disorder influence on H_{cp} is universal and is only due to the band widening by disorder, i.e. to the replacement $D \rightarrow D_{eff}$.

In Fig.1 we show the temperature dependence of paramagnetic critical magnetic field for different values of coupling strength. Chemical potential entering Eq. (6) is, in general, dependent not only on the coupling strength, but also on the values of magnetic field and temperature. In Figs.1 (a-e), for the sake of comparison, dashed lines show the results of calculations with chemical potential taken at $H = 0$ and $T = T_c$ for the given value of $U/2D$, while continuous curves with symbols represent the results of full calculations with $\mu = \mu(H, T)$.

In the weak coupling limit ($U/2D = 0.2$) we obtain the standard behavior of temperature dependence of paramagnetic critical field of BCS theory [18]. At low temperatures we observe the region of decreasing H_{cp} as temperature diminishes, with maximum H_{cp} at finite temperature. It is well known, that in this region the system is unstable with respect to type I phase transition [18], where is also a possibility of transition to FFLO phase [16, 17] with Cooper pairs with finite momentum ($\mathbf{q} \neq 0$) and inhomogeneous superconducting order parameter. Critical field in BCS limit is relatively weakly dependent on the value of chemical potential, so that we can neglect weak field and temperature dependence of

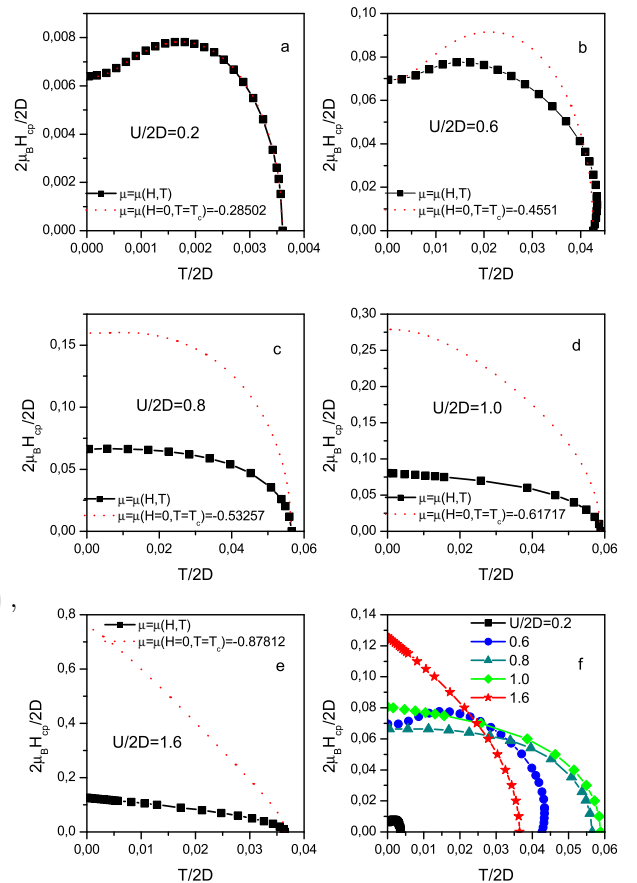


FIG. 1: Dependence of paramagnetic critical magnetic field on temperature for different values of coupling strength. Dashed lines were obtained neglecting the dependence of chemical potential on temperature and magnetic field at given U .

$\mu(H, T)$ (dashed line in Fig. 1 (a) in fact coincides with the result of an exact calculation). With the growth of the coupling strength the region of instability towards type I transition shrinks (cf. Fig.1 (b),(c)) and it completely disappears with further increase of coupling (Fig. 1 (d),(e)). With the increase of coupling strength the critical magnetic field becomes more and more dependent on the value of the chemical potential, so that the account of its temperature and magnetic field dependence $\mu(H, T)$ becomes very important (cf. Fig. 1 (c-e)).

At intermediate coupling ($U/2D = 0.6$) the account of temperature and magnetic field dependence of μ leads to small changes of the critical field, however we observe significant qualitative changes for $T \sim T_c$. The small growth of chemical potential with increase of H at weak fields leads to noticeable growth of T_c , which overcomes the decrease of T_c with the growth of magnetic field due to explicit H – dependence in Eq. (6), leading to some increase of $T_c(H)$ at small H .

In Fig. 1(f) we show temperature dependencies of the critical magnetic field for different values of U . It is known that the critical temperature T_{c0} grows with coupling strength in BCS limit and decreases in BEC

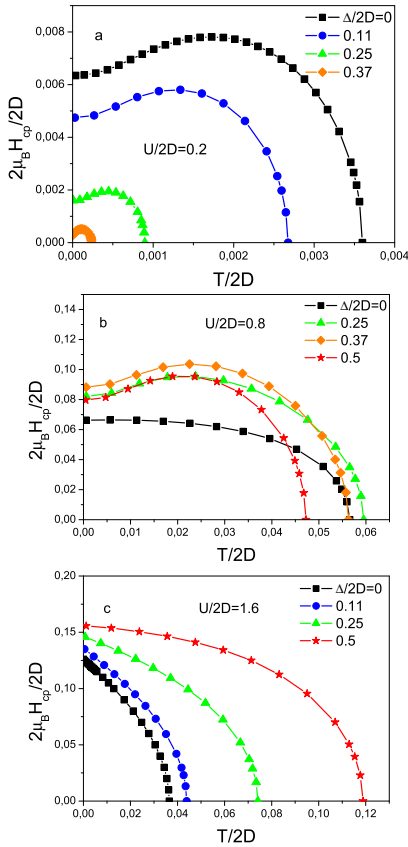


FIG. 2: Temperature dependence of paramagnetic critical magnetic field for different levels of disorder: (a) — BCS weak coupling limit ($U/2D = 0.2$); (b) — BCS – BEC crossover region (intermediate coupling: $U/2D = 0.8$); (c) — BEC strong coupling region ($U/2D = 1.6$).

strong coupling limit, passing through a maximum at $U/2D = 1$ [12–14]. The critical magnetic field at low temperatures grows with coupling strength both in BCS and BEC limits, though in BCS – BEC crossover region ($U/2D = (0.6 - 1)$) we observe rather weak dependence of the critical magnetic field on coupling strength.

The physical reason of the growth of paramagnetic critical field with coupling strength is pretty obvious — it is more difficult for magnetic field to break the pairs of strongly coupled electrons.

In Fig. 2 we present our results on disorder influence on temperature dependence of paramagnetic critical magnetic field. In BCS weak coupling limit (Fig. 2(a)) the increase of disorder leads both to decrease of the critical temperature in the absence of magnetic field T_{c0} (cf. [13, 14]) and to decrease of the critical magnetic field at all temperatures. The region of instability to type I transition is conserved also in the presence of disorder. In fact, as was noted above, disorder influence on $H_{cp}(T)$ is actually universal and related only to the replacement $D \rightarrow D_{eff}$. As a result, disorder growth leads to decrease of the effective coupling, which is defined by dimensionless parameter $U/2D_{eff}$. This leads to the increase of

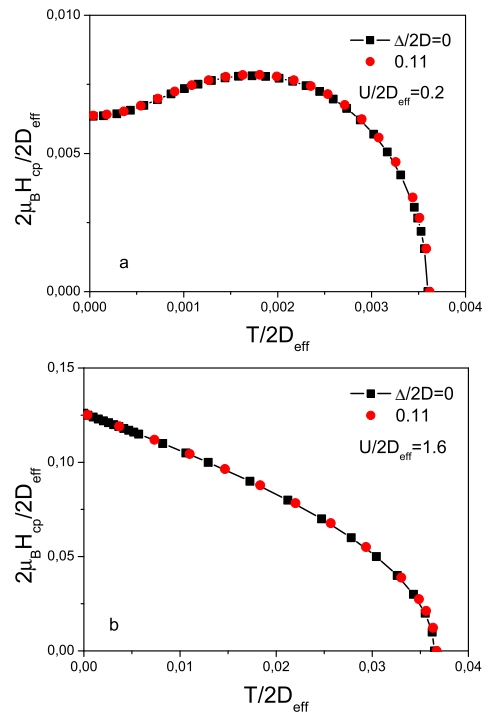


FIG. 3: Universality of temperature dependence of paramagnetic critical magnetic field on disorder. (a) — weak coupling $U/2D_{eff} = 0.2$, $\Delta = 0$ $\Delta = 0.11$ (b) — strong coupling $U/2D_{eff} = 1.6$, $\Delta = 0$ $\Delta = 0.11$

the relative width $T/T_c(H)$ of the temperature region of type I transition.

At intermediate coupling ($U/2D = 0.8$) in BCS – BEC transition region (Fig. 2(b)) disorder growth relatively weakly changes the critical temperature T_{c0} (cf. [13, 14]), leading to some increase of $H_{cp}(T)$. As all the effects of disordering are due to the replacement $D \rightarrow D_{eff}$, the increase of disorder again leads to the decrease of the effective coupling strength $U/2D_{eff}$ and restoration of the region of instability towards type I transition.

In BEC – limit of strong coupling the growth of disorder leads to significant increase of the critical temperature T_{c0} (cf. [13, 14]). At the same time, the critical magnetic field at low temperatures only weakly increases with increasing disorder. In BEC – limit instability to type I transition does not appear even in the presence of very strong disorder ($\Delta/2D = 0.5$). In fact, in BEC – limit disorder influence is again universal and related only to the replacement $D \rightarrow D_{eff}$. As a result, if we make the spin splitting and temperature dimensionless dividing both by the effective bandwidth $2D_{eff}$ and keep the effective coupling strength $U/2D_{eff}$ fixed, we obtain the universal temperature dependence of paramagnetic critical magnetic field. In Fig. 3 we show examples of such universal behavior for typical cases of weak and strong coupling in the absence and in the presence of disorder.

In the absence of disorder in BEC strong coupling limit with $U/2D = 1.6$ for $T \rightarrow 0$ we have (cf. Fig.1)

$2\mu_B H_{cp}/2D \approx 0.125$, so that for characteristic value of the bandwidth $2D \sim 1$ eV we get $H_{cp} \sim 10^7$ Gauss. For orbital critical magnetic field (cf. [15]) in the same model and for the same coupling strength, for $T \rightarrow 0$ and typical value of lattice parameter $a = 3.3 * 10^{-8}$ cm, we obtain $H_{cp} \approx 1.6 * 10^8$ Gauss. Thus, the orbital critical magnetic field at low temperatures grows with increase of the coupling strength much faster, than paramagnetic critical field, and in BEC strong coupling limit the main contribution to the upper critical field at low temperatures is actually due to the paramagnetic effect. The growth of disorder leads to significant growth of the orbital critical magnetic field [15], while $H_{cp}(T \rightarrow 0)$ in the region of BCS – BEC crossover and in BEC limit is relatively weakly dependent on disorder. Thus, also in the presence of disorder in BEC limit the main contribution to the upper critical field at low temperatures comes from paramagnetic effect.

IV. CONCLUSION

In this paper, within the combination of Nozieres – Schmitt-Rink and DMFT+ Σ approximations, we have studied disorder influence on temperature behavior of paramagnetic critical magnetic field. Calculations were done for a wide range of the values of attractive potential U , from the weak coupling region $U/2D \ll 1$, where superconductivity is well described by BCS model, up to the limit of strong coupling $U/2D \gg 1$, where superconducting transition is due to Bose condensation of compact Cooper pairs, which are formed at temperatures much exceeding the temperature of superconducting transition.

The growth of coupling strength U leads to a fast increase of $H_{cp}(T)$ and disappearance, both in the region of BCS – BEC crossover and in BEC limit, of the region of instability, leading to type I transition, which appears at low temperatures in BCS weak coupling region. Physically this is due to the fact, that it becomes more and more difficult for magnetic field to break pairs of strongly coupled electrons.

The growth of disorder in BCS weak coupling limit leads both to decrease of critical temperature and decrease of $H_{cp}(T)$. The region of instability to type I transition at low temperatures remains also in the presence of disorder. In the intermediate coupling region ($U/2D = 0.8$) disorder only weakly affects both the critical temperature and $H_{cp}(T)$. However, the growth of disorder leads to restoration of the low temperature region of instability to type I transition, which is not observed in the absence of disorder. This, rather unexpected, conclusion is related to specifics of the attractive Hubbard model, which in disordered case is controlled by dimensionless coupling parameter $U/2D_{eff}$. As was shown in our previous works, in BEC strong coupling limit the growth of disorder leads to noticeable growth of the critical temperature in the absence of magnetic field. However, the value of $H_{cp}(T \rightarrow 0)$ in this model

is relatively weakly dependent on disorder. In BEC limit at low temperatures and for reasonable values of model parameters paramagnetic critical magnetic field is much smaller, than the orbital critical field, so that the upper critical field in this region is mainly determined by paramagnetic critical field. In the presence of disorder this conclusion is even more valid, as the orbital critical field rapidly grows with increasing disorder, while paramagnetic critical field is weakly disorder dependent in this limit.

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Appendix A: Appendix: Equation for paramagnetic critical magnetic field

In general case the Nozieres – Schmitt-Rink approach [1] assumes, that corrections from strong pairing interaction significantly change the chemical potential of the system, but possible vertex corrections from this interaction in Cooper channel are irrelevant, so that to analyze Cooper instability we can use the weak coupling approximation (ladder approximation). In this approximation the condition of Cooper instability in disordered attractive Hubbard model is written as:

$$1 = U\chi_0(q = 0, \omega_m = 0) \quad (A1)$$

where

$$\chi_0(q = 0, \omega_m = 0) = T \sum_n \sum_{\mathbf{p}\mathbf{p}'} \Phi_{\mathbf{p}\mathbf{p}'}(\varepsilon_n) \quad (A2)$$

is two – particle loop in Cooper channel “dressed” only by impurity scattering, while $\Phi_{\mathbf{p}\mathbf{p}'}(\varepsilon_n)$ is the averaged over impurities two – particle Green’s function in Cooper channel at Matsubara frequencies $\varepsilon_n = \pi T(2n + 1)$.

To obtain $\sum_{\mathbf{p}\mathbf{p}'} \Phi_{\mathbf{p}\mathbf{p}'}(\varepsilon_n)$ we use an exact Ward identity, derived by us in Ref. [8]:

$$G_{\uparrow}(\varepsilon_n, \mathbf{p}) - G_{\downarrow}(-\varepsilon_n, -\mathbf{p}) = - \sum_{\mathbf{p}'} \Phi_{\mathbf{p}\mathbf{p}'}(\varepsilon_n) (G_{0\uparrow}^{-1}(\varepsilon_n, \mathbf{p}') - G_{0\downarrow}^{-1}(-\varepsilon_n, -\mathbf{p})), \quad (A3)$$

Here $G_{0\uparrow,\downarrow}(\varepsilon_n, \mathbf{p}) = (i\varepsilon_n + \mu - \varepsilon(\mathbf{p}) \pm \mu_B H)^{-1}$ is the “bare” Green’s function and $G_{\uparrow,\downarrow}(\varepsilon_n, \mathbf{p})$ is averaged over impurities (but not “dressed” by Hubbard interaction!) single – particle Green’s function. Using the symmetry $\varepsilon(\mathbf{p}) = \varepsilon(-\mathbf{p})$ we obtain from Ward identity (A3):

$$\sum_{\mathbf{p}\mathbf{p}'} \Phi_{\mathbf{p}\mathbf{p}'}(\varepsilon_n) = - \frac{\sum_{\mathbf{p}} G_{\uparrow}(\varepsilon_n, \mathbf{p}) - \sum_{\mathbf{p}} G_{\downarrow}(-\varepsilon_n, \mathbf{p})}{2i\varepsilon_n + 2\mu_B H}, \quad (A4)$$

so that for Cooper susceptibility (A2) we get:

$$\begin{aligned} \chi_0(q=0, \omega_m=0) &= \\ &= -\frac{T}{2} \sum_n \frac{\sum_{\mathbf{p}} G_{\uparrow}(\varepsilon_n, \mathbf{p}) - \sum_{\mathbf{p}} G_{\downarrow}(-\varepsilon_n, \mathbf{p})}{i\varepsilon_n + \mu_B H} = \\ &= -\frac{T}{2} \sum_n \left(\frac{\sum_{\mathbf{p}} G_{\uparrow}(\varepsilon_n, \mathbf{p})}{i\varepsilon_n + \mu_B H} + \frac{\sum_{\mathbf{p}} G_{\downarrow}(\varepsilon_n, \mathbf{p})}{i\varepsilon_n - \mu_B H} \right). \end{aligned} \quad (\text{A5})$$

Performing the standard summation over Fermion Matsubara frequencies, we obtain:

$$\begin{aligned} \chi_0 &= -\frac{1}{8\pi i} \int_{-\infty}^{\infty} d\varepsilon \left(\frac{\sum_{\mathbf{p}} G_{\uparrow}^R(\varepsilon, \mathbf{p}) - \sum_{\mathbf{p}} G_{\uparrow}^A(\varepsilon, \mathbf{p})}{\varepsilon + \mu_B H} + \right. \\ &\quad \left. + \frac{\sum_{\mathbf{p}} G_{\downarrow}^R(\varepsilon, \mathbf{p}) - \sum_{\mathbf{p}} G_{\downarrow}^A(\varepsilon, \mathbf{p})}{\varepsilon - \mu_B H} \right) th \frac{\varepsilon}{2T} = \\ &= \frac{1}{4} \int_{-\infty}^{\infty} d\varepsilon \left(\frac{\tilde{N}_{0\uparrow}(\varepsilon)}{\varepsilon + \mu_B H} + \frac{\tilde{N}_{0\downarrow}(\varepsilon)}{\varepsilon - \mu_B H} \right) th \frac{\varepsilon}{2T}, \end{aligned} \quad (\text{A6})$$

where $\tilde{N}_{0\uparrow, \downarrow}(\varepsilon)$ is the “bare” ($U = 0$) density of states for different spin projections, “dressed” by impurity scattering. Spin splitting can be considered as an addition to chemical potential, so that introducing the “bare” density of states “dressed” by disorder in the absence of external magnetic field $\tilde{N}_0(\varepsilon)$, we obtain the final result for

Cooper susceptibility:

$$\chi_0 = \frac{1}{4} \int_{-\infty}^{\infty} d\varepsilon \frac{\tilde{N}_0(\varepsilon)}{\varepsilon} \left(th \frac{\varepsilon + \mu_B H}{2T} + th \frac{\varepsilon - \mu_B H}{2T} \right) \quad (\text{A7})$$

In Eq. (A7) energy ε is counted from the chemical potential level. If we count it from the middle of the conduction band we have to replace $\varepsilon \rightarrow \varepsilon - \mu$ and the condition of Cooper instability (A1) leads to the equation defining critical temperature depending on the external magnetic field, which gives the equation for paramagnetic critical magnetic field (6). The chemical potential for different values of U and Δ should be determined from DMFT+ Σ calculations, i.e. from the standard equation for electron number (band filling), which allows us to find H_{cp} for the wide range of model parameters, including the region of BCS – BEC crossover and the limit of strong coupling at different levels of disorder. This reflects the physical meaning of Nozieres – Scmitt-Rink approximation — in the weak coupling region the temperature of superconducting transition is controlled by the equation for Cooper instability (6), while in the strong coupling limit it is defined as the temperature of BEC, which is controlled by chemical potential. The joint solution of Eq. (6) and the equation for the chemical potential guarantees the correct interpolation for H_{cp} in the region of BCS – BEC crossover.

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