Ginzburg – Landau expansion and the upper critical field in disordered attractive Hubbard model.

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We present a short review of our studies of disorder influence upon Ginzburg – Landau expansion coefficients in Anderson – Hubbard model with attraction in the framework of the generalized DMFT+ Σ approximation. A wide range of attractive potentials U is considered – from weak coupling limit, where superconductivity is described by BCS model, to the limit of very strong coupling, where superconducting transition is related to Bose – Einstein condensation (BEC) of compact Cooper pairs, which are formed at temperatures significantly higher than the temperature of superconducting transition, as well as the wide range of disorders – from weak to strong, when the system is in the vicinity of Anderson transition. For the same range of parameters we study in detail the temperature behavior of orbital and paramagnetic upper critical field $H_{c2}(T)$, which demonstrates the anomalies both due to the growth of attractive potential and the effects of strong disordering.

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1. INTRODUCTION

The studies of disorder influence upon superconductivity have rather long history. In classic papers by Abrikosov and Gor'kov [1, 2, 3, 4] the weak disorder limit $(p_F l \gg 1, \text{ where } p_F \text{ is Fermi}$ momentum and l is the mean free path) was considered for the case of weak coupling superconductivity, which is well described by BCS theory. The notorious Anderson theorem" on the critical temperature T_c of superconductors with "normal" (nonmagnetic) disorder [5, 6] is also related to this limit. The generalization of the theory of "dirty" superconductors to the case of strong enough disorder $(p_F l \sim 1)$ (and further, up to the vicinity of Anderson transition) was made in Refs. [7, 8, 9, 10], where superconductivity was also considered in the weak coupling limit.

The problem of BCS theory generalization to the region of very strong coupling is also analyzed for a long time. Significant progress in this direction was achieved in a paper by Nozieres and Schmitt-Rink [11], who proposed an effective method to study the crossover from BCS behavior in the weak coupling limit to Bose – Einstein condensation (BEC) in the region of strong coupling. At the same time, the problem of superconductivity in disordered systems in the limit of strong coupling and in the region of BCS – BEC crossover is pretty poorly studied.

One of the simplest models to study the BCS – <u>BEC crossover is the</u> Hubbard model with attraction. The most successful approach to study Hubbard model, both to describe the strongly correlated systems for the case of repulsive interactions and to study BCS BEC crossover, is the dynamical mean field theory (DMFT) [12, 13, 14]. In recent years we have developed the generalized DMFT+ Σ approach to Hubbard model [15, 16, 17, 18, 19, 20, 21], which is quite convenient for the studies the role of different external (with respect to those taken into account by DMFT) interactions. In Ref. [22] we used this approach to analyze the single – particle properties and optical conductivity of the Hubbard model with attraction. Further on, the DMFT+ Σ method was used by us in Ref. [23, 24] to study disorder influence on the temperature of superconducting transition, which was calculated within Nozieres - Schmitt-Rink approach.

Starting with the classic paper by Gor'kov [3] it is well known that the Ginzburg – Landau expansion is of fundamental importance in the theory of "dirty" superconductors, allowing the effective studies of the behavior of different physical properties dependencies close to critical temperature on disorder [6]. The generalization of this theory to the region of strong disorder (up to Anderson metal – insulator transition) was also based on microscopic derivation of the coefficients of this expansion [7, 8, 9, 10]. However, this analysis, as noted above, was always done in the weak coupling limit of BCS theory.

In this paper we shall present a short review of the results obtained in our papers [25, 26, 27], devoted to microscopic derivation of the coefficients of Ginzburg – Landau expansion, taking into account the role of

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disorder in the wide region of BCS – BEC crossover and including the region of strong disorder in the vicinity of Anderson transition. We shall also review the closely related results of Refs. [28, 29] on the temperature dependence of orbital and paramagnetic upper critical magnetic fields in the region of this crossover and for different levels of disordering.

2. TEMPERATURE OF SUPERCONDUCTING TRANSITION

Consider disordered nonmagnetic Hubbard model with attraction and the Hamiltonian:

$$H = -t \sum_{\langle ij \rangle \sigma} a_{i\sigma}^{\dagger} a_{j\sigma} + \sum_{i\sigma} \epsilon_i n_{i\sigma} - U \sum_i n_{i\uparrow} n_{i\downarrow}, \quad (1)$$

where t > 0 is the transfer amplitude between the nearest neighbors, U is onsite attraction potential, $n_{i\sigma} = a^{\dagger}_{i\sigma}a_{i\sigma}$ is onsite number of electrons operator, $a_{i\sigma}(a^{\dagger}_{i\sigma})$ is annihilation (creation) operator of an electron with spin σ . Local energies ϵ_i are assumed to be independent random variables on different lattice sites. We assume the Gaussian distribution of energy levels ϵ_i :

$$\mathcal{P}(\epsilon_i) = \frac{1}{\sqrt{2\pi}W} \exp\left(-\frac{\epsilon_i^2}{2W^2}\right).$$
(2)

Parameter W here serves as the measure of disorder strength and the Gaussian random field of energy levels creates "impurity" scattering, which is considered within the standard approach, based upon calculations of the averaged Green's functions [30, 31].

The generalized DMFT+ Σ approach [15, 16, 17, 20] extends the standard dynamical mean field theory (DMFT) [12, 13, 14] by the addition of "external" self – energy part (SEP) $\Sigma_{\mathbf{p}}(\varepsilon)$ (in general momentum dependent), which is due to any interaction outside DMFT and provides an effective calculation method both for single – particle and two – particle properties [18, 19].

For an "external" SEP entering DMFT+ Σ loop, for the case of scattering by disorder analyzed here, we use the simplest self – consistent Born approximation, neglecting the "crossing" diagrams for impurity scattering:

$$\Sigma_{imp}(\varepsilon) = W^2 \sum_{\mathbf{p}} G(\varepsilon, \mathbf{p}), \qquad (3)$$

where $G(\varepsilon, \mathbf{p})$ is the full single – electron Green's function in DMFT+ Σ approximation.

To solve the effective single Anderson impurity problem of DMFT we used the numerical renormalization group (NRG) [32].

In the following we consider the "bare" band with semielliptic density of states (per unit cell and per single spin projection):

$$N_0(\varepsilon) = \frac{2}{\pi D^2} \sqrt{D^2 - \varepsilon^2} \tag{4}$$

where D defines the halfwidth of conduction band, which is a good approximation for for three – dimensional case. In Ref. [24] we have shown that in DMFT+ Σ approach for the model with semielliptic density of states all the influence of disorder upon *single* – *particle* properties is reduced to the widening of band by disorder, i.e. to the substitution $D \rightarrow D_{eff}$, where D_{eff} is the effective halfwidth of the "bare" band in the absence of electronic correlations (U = 0), widened by disorder:

$$D_{eff} = D\sqrt{1 + 4\frac{W^2}{D^2}}.$$
 (5)

The "bare" (in the absence of U) density of states, "dressed" by disorder:

$$\tilde{N}_0(\xi) = \frac{2}{\pi D_{eff}^2} \sqrt{D_{eff}^2 - \varepsilon^2} \tag{6}$$

remains semielliptic in the presence of disorder.

All calculations below were done for the case of quarter – filled band (number of electrons per lattice site n=0.5).

To consider superconductivity in a wide interval of pairing interaction U, following Refs. [22, 24] we use Nozieres – Schmitt-Rink approximation [11], which allows qualitatively correct (though approximately) describe the BCS – BEC crossover region. In this approach the critical temperature T_c is determined [24] by the usual BCS – like equation:

$$1 = \frac{U}{2} \int_{-\infty}^{\infty} d\varepsilon \tilde{N}_0(\varepsilon) \frac{th \frac{\varepsilon - \mu}{2T_c}}{\varepsilon - \mu},\tag{7}$$

where the chemical potential μ for different values of U and W is obtained from the standard equation for the number of electrons (band filling), determined from the full Green's function, calculated in B DMFT+ Σ approximation. This allows to find T_c for a wide interval of the values of parameters of the theory, including the BCS – BEC crossover region and the limit of strong coupling, as well as for different levels of disorder. It is the essence of interpolation scheme of Nozieres and Schmitt-Rink — in the weak coupling region transition temperature is controlled by the equation for Cooper



Fig. 1. Universal dependence of the temperature of superconducting transition on the strength of Hubbard attraction for different levels of disorder.

instability (7), while in the strong coupling limit it is determined as the temperature of BEC, which is controlled by the chemical potential. In Ref. [24] we have shown that disorder influence on the critical temperature T_c in the model with semielliptic bare density of states is universal and is reduced just to the change of the effective bandwidth. In Fig. 1, as an illustration of this, we show the universal dependence of critical temperature T_c on Hubbard attraction for different levels of disorder, which demonstrates the validity of the generalized Anderson theorem [23, 24]. In the weak coupling region the temperature of superconducting transition is well described by BCS model (to compare in Fig.1 we show the dashed line corresponding to BCS model, when T_c is determined by Eq. (7) with chemical potential independent of U and determined by quarter – filling of the "bare" band), while in the strong coupling region the critical temperature is mainly determined by BEC condition for Cooper pairs and drops with the growth of U as t^2/U , passing the maximum at $U/2D_{eff} \sim 1$. The review of these and some other results obtained for disordered Hubbard model in DMFT+ Σ approximation can be found in Ref. |21|.

3. GIBZBURG - LANDAU EXPANSION

Ginzburg – Landau expansion for the difference of free energies in superconducting and normal states can be written in a standard form [31]:

$$F_s - F_n = A|\Delta_{\mathbf{q}}|^2 + q^2 C|\Delta_{\mathbf{q}}|^2 + \frac{B}{2}|\Delta_{\mathbf{q}}|^4,$$
 (8)

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Fig. 2. Diagrammatic form of Ginzburg – Landau expansion.

where $\Delta_{\mathbf{q}}$ is the amplitude of the Fourier component of order parameter. Expansion (8) is determined by diagrams of the loop expansion for free energy in the field of fluctuations of order parameter (denoted by dashed lines) with small wave vector \mathbf{q} [31], shown in Fig. 2 [31].

Within Nozieres – Schmitt-Rink approach [11] we use weak coupling approximation to analyze Ginzburg – Landau coefficients, so that the loops with two and four Cooper vertexes shown in Fig. 2 do not contain contributions from Hubbard attraction and are "dressed" only by impurity scattering. However, as in the case of calculation of T_c , the chemical potential, which is essentially dependent on coupling strength and in the strong coupling limit determines the condition of Bose condensation of Cooper pairs, should be calculated within full DMFT+ Σ procedure. In Ref. [25] we have shown that in this approach the coefficients A and B are given by the following expressions:

$$A(T) = \frac{1}{U} - \int_{-\infty}^{\infty} d\varepsilon \tilde{N}_0(\varepsilon) \frac{th\frac{\varepsilon-\mu}{2T}}{2(\varepsilon-\mu)},\tag{9}$$

$$B = \int_{-\infty}^{\infty} \frac{d\varepsilon}{2(\varepsilon - \mu)^3} \left(th \frac{\varepsilon - \mu}{2T} - \frac{(\varepsilon - \mu)/2T}{ch^2 \frac{\varepsilon - \mu}{2T}} \right) \tilde{N}_0(\varepsilon),$$
(10)

For $T \to T_c$ coefficient A(T) takes the following form:

$$A(T) \equiv \alpha (T - T_c). \tag{11}$$

In BCS limit for coefficients α and B we obtain the standard result [31]:

$$\alpha_{BCS} = \frac{\tilde{N}_0(\mu)}{T_c} \qquad B_{BCS} = \frac{7\zeta(3)}{8\pi^2 T_c^2} \tilde{N}_0(\mu).$$
(12)

Thus the coefficients A and B are determined only by the density of states $\tilde{N}_0(\varepsilon)$ widened by disorder and by the chemical potential. For semielliptic bare density of states the dependence of these coefficients on disorder is due only to substitution $D \to D_{eff}$, so that in the presence of disorder we get the universal dependencies of α and B (made dimensionless by the effective bandwidth) on $U/2D_{eff}$ [25]. Actually the coefficients α and B drop fast with the growth of



Fig. 3. The universal dependence of specific heat discontinuity on $U/2D_{eff}$ for different levels of disorder.

coupling strength $U/2D_{eff}$. It should be noted that Eqs. (9) and (10) for coefficients A and B were obtained in Ref. [25] using exact Ward identities and remain valid also in case of strong disorder (Anderson localization).

The universal dependence on disorder related to widening of the band $D \rightarrow D_{eff}$ appears also for specific heat discontinuity at transition temperature [25], which is determined by coefficients α and B:

$$\Delta C \equiv C_s(T_c) - C_n(T_c) = T_c \frac{\alpha^2}{B}.$$
 (13)

This universal dependence of specific heat discontinuity on $U/2D_{eff}$ is shown in Fig. 3. In BCS limit specific heat discontinuity grows with coupling strength, while in BEC limit it drops, passing through a maximum at $U/2D_{eff} \approx 0.55$. This behavior of specific heat discontinuity is determined mainly by the behavior of T_c (cf. Fig.1), while the ratio $\frac{\alpha^2}{B}$ in Eq. (13) smoothly depends on the coupling strength.

Now we shall follow Refs. [26, 27] to analyze the coefficient C. From diagrammatic representation of Ginzburg – Landau expansion, shown in Fig.2, it is clear that C is determined as a coefficient before q^2 in Cooper – like two – particle loop (first term in Fig. 2). Thus we obtain the following expression:

$$C = -T \lim_{q \to 0} \sum_{n, \mathbf{p}, \mathbf{p}'} \frac{\Psi_{\mathbf{p}\mathbf{p}'}(\varepsilon_n, \mathbf{q}) - \Psi_{\mathbf{p}\mathbf{p}'}(\varepsilon_n, 0)}{q^2}, \quad (14)$$

where $\Psi_{\mathbf{p},\mathbf{p}'}(\varepsilon_n,\mathbf{q})$ is two – particle Green's function in Cooper channel, "dressed" (in Nozieres – Schmitt-Rink approximation) only by impurity scattering.

In BCS limit and in the absence of disorder the coefficient C takes the following form [31]:

$$C_{BCS} = \frac{7\zeta(3)}{16\pi^2 T_c^2} N_0(\mu) \frac{v_F^2}{d},$$
 (15)

where v_F is velocity at the Fermi surface, d is space dimensionality. Disorder influence on coefficient C is not reduced only to the substitution $N_0 \to \tilde{N}_0$, so that in the presence of disorder, in contrast to coefficients α and B (cf. (12)), even in BCS limit we can not obtain any compact expression for C similar to Eq. (15),

After rather cumbersome analysis [26, 27] we get the following general expression for the coefficient C:

$$C = -\frac{1}{8\pi} \int_{-\infty}^{\infty} d\varepsilon \frac{th\frac{\varepsilon}{2T}}{\varepsilon} Im\left(\frac{iD(2\varepsilon)\sum_{\mathbf{p}}\Delta G_{\mathbf{p}}(\varepsilon)}{\varepsilon + i\delta}\right) = \\ = -\frac{1}{8\pi} \int_{-\infty}^{\infty} d\varepsilon \frac{th\frac{\varepsilon}{2T}}{\varepsilon^2} Re(D(2\varepsilon)\sum_{\mathbf{p}}\Delta G_{\mathbf{p}}(\varepsilon)) - \\ -\frac{1}{16T} Im(D(0)\sum_{\mathbf{p}}\Delta G_{\mathbf{p}}(0)), (16)$$

where $\Delta G_{\mathbf{p}}(\varepsilon) = G^{R}(\varepsilon, \mathbf{p}) - G^{A}(-\varepsilon, \mathbf{p})$ and $D(\omega)$ is the frequency dependent generalized diffusion coefficient [31, 33, 34, 35, 36, 38, 37, 39], which is determined within generalization of the self – consistent theory of localization by the following self – consistency equation [19]:

$$D(\omega) = i \frac{\langle v \rangle^2}{d} \left(\omega - \Delta \Sigma_{imp}^{RA}(\omega) + W^4 \sum_{\mathbf{p}} (\Delta G_{\mathbf{p}}(\varepsilon))^2 \sum_{\mathbf{q}} \frac{1}{\omega + iD(\omega)q^2} \right)^{-1}, \quad (17)$$

where $\omega = 2\varepsilon$, $\Delta \Sigma_{imp}^{RA}(\omega) = \Sigma_{imp}^{R}(\varepsilon) - \Sigma_{imp}^{A}(-\varepsilon)$, d – space dimensionality, and velocity $\langle v \rangle$ is determined by the following expression:

$$\langle v \rangle = \frac{\sum_{\mathbf{p}} |\mathbf{v}_{\mathbf{p}}| \Delta G_{\mathbf{p}}(\varepsilon)}{\sum_{\mathbf{p}} \Delta G_{\mathbf{p}}(\varepsilon)}; \mathbf{v}_{\mathbf{p}} = \frac{\partial \varepsilon(\mathbf{p})}{\partial \mathbf{p}}.$$
 (18)

Taking into account applicability limits of diffusion approximation, summation over q in Eq. (17) should be limited by [31, 38]:

$$q < k_0 = Min\{l^{-1}, p_F\},\tag{19}$$

where l is the mean – free path due to elastic scattering by disorder, p_F is Fermi momentum.

Thus we obtain an interpolation scheme to determine the coefficient C, which in the weak disorder limit reproduces the results of "ladder" approximation,

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while in the strong disorder limit it takes into account the effects of Anderson localization (in the framework of self – consistent theory of localization).

It was shown [19, 20] that in DMFT+ Σ approximation for Anderson – Hubbard model the critical disorder for Anderson metal – insulator transition W/2D = 0.37 (for the choice of cutoff as in Eq. (19)), so that in this approximation it does not depend on the value of Hubbard interaction U. The approach developed above allows determination of coefficient C including the region of Anderson insulator with disorder W/2D > 0.37.

4. PHYSICAL PROPERTIES CLOSE TO THE TEMPERATURE OF SUPERCONDUCTING TRANSITION

The coherence length at a given temperature $\xi(T)$ determines the characteristic scale of inhomogeneities of superconducting order parameter:

$$\xi^2(T) = -\frac{C}{A}.\tag{20}$$

From Eq. (11) we have: $A = \alpha (T - T_c)$, то

$$\xi(T) = \frac{\xi}{\sqrt{1 - T/T_c}},\tag{21}$$

where we have introduce the coherence length of a superconductor as:

$$\xi = \sqrt{\frac{C}{\alpha T_c}},\tag{22}$$

which in the weak coupling limit and in the absence of disorder has the standard form [31]:

$$\xi_{BCS} = \sqrt{\frac{C_{BCS}}{\alpha_{BCS}T_c}} = \sqrt{\frac{7\zeta(3)}{16\pi^2 d}} \frac{v_F}{T_c}.$$
 (23)

The penetration depth of magnetic field into superconductor is defined as:

$$\lambda^2(T) = -\frac{c^2}{32\pi e^2} \frac{B}{AC}.$$
 (24)

Thus:

$$\lambda(T) = \frac{\lambda}{\sqrt{1 - T/T_c}},\tag{25}$$

where we have introduced:

$$\lambda^2 = \frac{c^2}{32\pi e^2} \frac{B}{\alpha CT_c},\tag{26}$$

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which in the absence of disorder and in the weak coupling limit is:

$$\lambda_{BCS}^2 = \frac{c^2}{32\pi e^2} \frac{B_{BCS}}{\alpha_{BCS} C_{BCS} T_c} = \frac{c^2}{16\pi e^2} \frac{d}{N_0(\mu) v_F^2}.$$
 (27)

Note that λ_{BCS} does not depend on T_c , i.e. on the coupling strength, and it can be conveniently used to normalize penetration depth λ (26) at arbitrary U and W.

Close to T_c the upper critical magnetic field H_{c2} is defined via Ginzburg – Landau coefficients as:

$$H_{c2} = \frac{\Phi_0}{2\pi\xi^2(T)} = -\frac{\Phi_0}{2\pi}\frac{A}{C},$$
 (28)

where $\Phi_0 = c\pi/e$ is the magnetic flux quantum. Then the slope of the upper critical field close to T_c is given by:

$$\frac{dH_{c2}}{dT} = \frac{\Phi_0}{2\pi} \frac{\alpha}{C}.$$
(29)

Coefficient C is essentially a two – particle entity, thus it is not universally dependent on disorder in contrast to coefficients A and B and disorder influence upon it does not reduce only to effective band widening by disorder. Let us now discuss the main results of our calculations for this coefficient (for more details cf. Refs. [26, 27]). Coefficient C rapidly decreases with the growth of coupling strength. Especially strong drop is observed in the weak coupling region. Localization corrections become important in the limit of strong enough disorder (W/2D > 0.25). For such disorder level localization corrections significantly suppress the coefficient C in the weak coupling region, while in the strong coupling region for U/2D > 1 localization corrections in fact do not change the value of the coefficient, even in the limit of strong disorder with W/2D > 0.37, when the system becomes Anderson insulator. This is apparently due to the fact, that in the region of strong coupling the (pseudo)gap is opened in the density of states at the Fermi level [22], so that there are no states to localize in the vicinity of the Fermi level at all. In Fig. 4 we show the dependencies of coefficient C on disorder strength for different values of coupling U/2D. In this figure (and all that follow in this Section) the filled symbols and continuous lines correspond to calculations taking into account localization corrections, while the empty symbols and dashed lines correspond to "ladder" approximation. In the weak coupling limit (U/2D = 0.1) we observe fast enough drop of the coefficient C with disorder growth in the region of weak impurity scattering. At the same time in the region of strong enough disorder in "ladder" approximation we can observe the increase



Fig. 4. Dependence of the coefficient C, normalized by its value in the absence of disorder, on disorder for different values of Hubbard attraction U. Dashed line – "ladder" approximation, full lines – results obtained with account of localization corrections.

of the coefficient C with the growth of disorder, which is mainly due the noticeable widening of the band by this disorder and corresponding drop of the effective coupling strength $U/2D_{eff}$. However, localization corrections which become important for strong disorder W/2D > 0.25, lead to suppression of C while disorder grows, also in the limit of strong impurity scattering. In the region of intermediate coupling (U/2D = 0.4 -0.6) coefficient C in "ladder" approximation is rather insignificantly increasing with disorder growth. In BEC limit (U/2D > 1) coefficient C in fact is independent of impurity scattering both in the "ladder" approximation and with the account of localization corrections. Localization corrections in BEC limit in fact do not change the value of coefficient C as compared to "ladder" approximation. As Ginzburg – Landau coefficients α and B are universally dependent on disorder, Anderson localization has no influence upon them at all, and coefficient C, which is strongly dependent on localization correction in the weak coupling limit, in BEC limit is in fact independent of these corrections. Correspondingly, the physical properties depending on coefficient C, are also significantly dependent on localization corrections in the weak coupling limit, but in fact do not feel Anderson localization in BEC limit.

In Fig. 5 we show the dependence of coherence length ξ on the level of disorder for different values of coupling strength. In the weak coupling limit coherence length ξ drops fastly with the growth of U at any disorder level, reaching the values of the order of lattice parameter a in intermediate coupling region of



Fig. 5. Dependence of coherence length on disorder for different values of Hubbard attraction. Coherence length is normalized by lattice parameter a. At the insert: dependence of coherence length on disorder in the weak coupling limit.

 $U/2D \sim 0.4 - 0.6$. Further growth of coupling strength only slightly changes the coherence length. In BCS limit, i.e. for the weak coupling and weak enough impurity scattering we observe (cf. insert at Fig. 5) the standard dependence for "dirty" superconductors $\xi \sim l^{1/2}$, i.e. the coherence length rapidly drops with the growth of disorder. However, at strong enough disorder in ladder approximation (dashed lines) coherence length grows with disorder, which is mainly due to noticeable widening of the bare band and corresponding suppression of $U/2D_{eff}$. Localization corrections are important only for large disorder (W/2D > 0.25)and lead to significant drop of coherence length in BCS limit of weak coupling and practically does not change coherence length in BEC limit. Taking into account localization corrections leads to noticeable drop of coherence length as compared to "ladder" approximation in the limit of strong disorder restoring the suppression of ξ with the growth of disorder in this limit. In standard BCS model with the bare band of infinite width in the limit weak disorder the coherence length drops with disorder $\xi \sim l^{1/2}$, and close to And erson transition ξ drops even faster as $\xi \sim l^{2/3}$ [7, 8, 9], in contrast to our model, where close to Anderson transition the coherence length rather weakly depends on disorder, which is related to a significant widening of the band by disorder. With the growth of the coupling strength $U/2D \ge 0.4$ -0.6 the coherence length ξ becomes of the order of lattice parameter and becomes almost disorder independent. In particular, in BEC limit of very strong coupling U/2D = 1.4,



Fig. 6. Dependence of penetration depth normalized by its BCS value in the weak coupling limit on the strength of Hubbard attraction U for different levels of disorder.

1.6, the growth of disorder up to very strong values (W/2D = 0.5) leads to a factor two drop of coherence length, so that in the limit of strong coupling the account of localization corrections becomes irrelevant.

In Fig. 6 we show the dependence of penetration depth, normalized by its BCS value in the absence of disorder (27) on Hubbard attraction strength U for different levels of disorder. In the absence of impurity scattering penetration depth grows with coupling strength. In the weak coupling limit, in accordance with the usual theory of "dirty" superconductors, disorder leads to a fast growth of penetration depth ($\lambda \sim l^{-1/2}$, where l is the mean free path). With increase of the coupling strength the growth of penetration depth with disorder slows down and in the limit of very strong coupling fro U/2D = 1.4, 1.6 penetration depth even slightly decreases with the growth of disorder. Thus, in presence of disorder we observe the drop of penetration depth with the growth of Hubbard attraction in the region of relatively weak coupling and the growth of λ with U in BEC strong coupling limit. The account of localization corrections is relevant only in the limit of strong disorder (W/2D > 0.25)and leads significant growth of penetration depth as compared with results of the "ladder" approximation in weak coupling limit. However, qualitatively the dependence of penetration depth on disorder does not change. In BEC limit localization influence on penetration depth is insignificant. Similar dependence on disorder is observed also for dimensionless Ginzburg – Landau parameter $\kappa = \lambda/\xi$. In weak coupling limit Ginzburg - Landau parameter rapidly grows with



Fig. 7. Dependence of the slope of the upper critical field, normalized by it value in the absence of disorder for different values of Hubbard attraction. At the insert: the growth of the slope with disorder in the weak coupling limit.

disorder in accordance with the theory of "dirty" superconductors, where $\kappa \sim l^{-1}$. With the increase of the coupling strength the growth of Ginzburg – Landau parameter with disorder slows down and in the strong coupling limit of U/2D > 1 parameter κ is practically disorder independent. The account of localization corrections leads quantitatively to a noticeable increase of Ginzburg – Landau parameter in Anderson insulator phase ($W/2D \geq 0.37$) for the weak coupling. In the limit of strong coupling the account of localization is again irrelevant.

HIn Fig. 7 we show the dependence of the slope of the upper critical magnetic field on disorder. In the weak coupling limit we again observe the behavior typical for "dirty" superconductors - the slope of the upper critical field grows with disorder (cf. insert at Fig. 7). Taking into account localization corrections in the weak coupling limit greatly increases the slope of the upper critical field as compares with the "ladder" approximation in Anderson insulator (W/2D > 0.37). As a result in Anderson insulator the slope of the upper critical field grows with impurity scattering much faster, than in "ladder" approximation. At intermediate couplings (U/2D = 0.4 - 0.8) the slope of the upper critical field is practically independent of impurity scattering at weak disorder. In the "ladder" approximation this behavior is conserved also in the region of strong disorder. However, the account of localization corrections leads to significant growth of the slope with disorder in Anderson insulator phase. In the limit of very strong coupling the slope of the upper critical field can even slightly decreases with disorder, but for strong disorder the slope grows with the growth of impurity scattering. In BEC limit the account of localization corrections becomes irrelevant and only slightly changes the slope of the upper critical field as compared with the "ladder" approximation.

5. TEMPERATURE DEPENDENCE OF THE ORBITAL UPPER CRITICAL FIELD

Most vividly the influence of disordering is manifested in the behavior of the upper critical field in the theory of "dirty" superconductors. As disorder grows both the slope of the temperature dependence of the upper critical field at T_c [6] and $H_{c2}(T)$ at all temperatures increase [40, 41]. Effects of Anderson localization in the limit of strong disorder also are most explicit in the temperature dependence of the upper critical field. Precisely at the point of Anderson metal - insulator transition localization effects lead to lead to sharp increase of H_{c2} at low temperatures and the temperature dependence of $H_{c2}(T)$ is qualitatively different from Werthamer, Helfand, Hohenberg (WHH) dependence [40, 41], which is characteristic for the theory of "dirty" superconductors $-H_{c2}(T)$ dependence becomes concave [7, 8, 9].

Let us consider disorder influence on the temperature dependence of the upper critical field $H_{c2}(T)$ in a wide region of attraction strength U, including the BCS – BEC crossover region, as well as for the wide interval of disorders, up to the vicinity of Anderson transition [28]. In Nozieres – Schmitt-Rink approach used here the critical temperature of superconducting transition is determined by a joint solution of equation for Cooper instability in Cooper particle – particle channel in weak coupling approximation and equation for the chemical potential of the system, which is defined for the whole interval of the values of Hubbard interaction from the condition of quarter – filling of the band within DMFT+ Σ approximation. The usual condition for Cooper instability has the form:

$$1 = -U\chi(\mathbf{q}),\tag{30}$$

where $\chi(\mathbf{q})$ is Cooper susceptibility, determined by the loop in Cooper channel. In the presence of an external magnetic field the total momentum \mathbf{q} in Cooper channel acquires the additional contribution from vector potential \mathbf{A} [6, 40]

$$\mathbf{q} \to \mathbf{q} - \frac{2e}{c} \mathbf{A}.$$
 (31)

As our model assumes an isotropic spectrum, Cooper instability $\chi(\mathbf{q})$ depends on \mathbf{q} only through q^2 . The minimal eigenvalue of an operator $(\mathbf{q} - \frac{2e}{c}\mathbf{A})^2$, defining the upper critical magnetic field $H = H_{c2}$ is [42]

$$q_0{}^2 = 2\pi \frac{H}{\Phi_0},\tag{32}$$

where $\Phi_0 = \frac{ch}{2e} = \frac{\pi\hbar}{e}$ is magnetic flux quantum. Then the equation for $T_c(H)$ or $H_{c2}(T)$ remains as usual:

$$1 = -U\chi(q^2 = q_0^2). \tag{33}$$

In further analysis we shall neglect the relatively weak influence of magnetic field on diffusion (noninvariance with respect to time reversal), which is reflected in nonequality of the loops in Cooper and diffusion channels. This influence of magnetic field was analyzed in Refs. pa6orax [9, 10, 43, 44], where it was demonstrated, that the account of this, even close to Anderson metal – insulator transition, only slightly decreases the value of $H_{c2}(T)$ in low temperature region. Under the condition of invariance to time reversal and equivalence of the loops in Cooper and diffusion channels, Cooper instability is determined by the loop in diffusion channel. As a result Eq. (33) for the orbital critical field $H_{c2}(T)$ takes the form [28]:

$$1 = -\frac{U}{2\pi} \int_{-\infty}^{\infty} d\varepsilon Im \left(\frac{\sum_{\mathbf{p}} \Delta G_{\mathbf{p}}(\varepsilon)}{2\varepsilon + iD(2\varepsilon)2\pi \frac{H_{c2}}{\Phi_0}} \right) th \frac{\varepsilon}{2T}.$$
(34)

The generalized diffusion coefficient is again determined in the framework of self – consistent theory of localization as described above.

In Fig. 8 we show temperature dependencies of the upper critical field for different degrees of disorder in three regions of coupling strength of interest to us: in BCS weak coupling limit (U/2D = 0.2), in BCS – BEC crossover region (intermediate coupling U/2D = 1.0) and in BEC limit of strong coupling (U/2D = 1.6).

In strong coupling region (Fig.8(a)) the growth of disorder leads to increase of the upper critical field at all temperatures in weak disorder limit (W/2D < 0.19), in this case the temperature dependencies have negative curvature and are close in form to the standard WHH dependence [40, 41]. With further growth of disorder and without account of localization corrections the upper critical field at all temperatures starts to decrease. However, the account of localization corrections in weak coupling limit at strong disorder $(W/2D \ge$ 0.37) significantly increases the upper critical field and qualitatively changes its temperature behavior, so that the dependencies. of $H_{c2}(T)$ acquire the positive



Fig. 8. Temperature dependence of the upper critical field for different values of disorder: (a) – BCS weak coupling limit (U/2D = 0.2); (b) – BCS – BEC crossover region, intermediate coupling (U/2D = 1.0); (c) – BEC limit of strong coupling (U/2D = 1.6). Filled symbols and lines correspond to calculations with account of localization corrections. Empty symbols and dashed lines correspond to "ladder" approximation for impurity scattering.

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curvature. The upper critical field fastly increases with disorder at all temperatures.

For intermediate coupling (Fig. 8(b)) in the limit of weak disorder the temperature dependence of the upper critical field becomes practically linear. The upper critical field at all temperatures increases with the growth of disorder. In the limit of strong disorder $(W/2D \geq 0.37)$ localization corrections. as in the weak coupling limit, increase the upper critical field at all temperatures. The dependencies of $H_{c2}(T)$ acquire positive curvature. However, in the region of intermediate coupling the influence of localization effects is significantly weaker, than in the limit of weak coupling being relevant only in the low temperature region.

In BEC limit of strong coupling (Fig. 8(c)) in weak disorder region $H_{c2}(T)$ dependencies are in fact linear. The upper critical filed grows with increasing disorder at all temperatures. In the limit of strong disorder at the point of Anderson transition itself (W/2D = 0.37)the dependence $H_{c2}(T)$ remains linear and taking into account localization corrections in fact does not change the temperature dependence of the upper critical field. Further increase of disorder leads to the growth of $H_{c2}(T)$. Deep in Anderson insulator phase (W/2D)0.5) $H_{c2}(T)$ dependence acquires positive curvature and the account of Anderson localization increases $H_{c2}(T)$ in low temperature region, while close to T_c localization corrections become irrelevant even at such a strong disorder. Thus, the strong coupling significantly decreases the influence of localization effects on the temperature dependence of the upper critical field.

Thus, the increase of coupling strength U leads to a rapid growth of $H_{c2}(T)$, especially in low temperature region. In BEC limit and in BEC - BCS crossover region $H_{c2}(T)$ dependence becomes practically linear. Disordering at any coupling strength also leads to the growth of $H_{c2}(T)$. In BCS limit of weak coupling increasing disorder leads to the growth of both the slope of the upper critical field close to $T = T_c$ and $H_{c2}(T)$ in low temperature region. In the limit of strong disorder, in the vicinity of Anderson transition localization corrections lead to the additional sharp increase of the upper critical field in low temperature region, so that the $H_{c2}(T)$ dependence becomes concave, acquiring the positive curvature. In BCS - BEC crossover region and in BEC limit weak disorder influence on the slope of the upper critical field at T_c is negligible, though strong disorder in the vicinity of Anderson transition leads to noticeable increase of the slope of the upper critical field with disorder. In low temperature region $H_{c2}(T)$ significantly grows with increasing disorder, especially in the vicinity of Anderson transition, where localization corrections noticeably increase $H_{c2}(T = 0)$ and $H_{c2}(T)$ dependence becomes concave, instead of linear, characteristic for the strong coupling at weak disorder.

In the model under discussion the values of the upper critical field at low temperatures can reach extreme values, up to (or even formally exceeding) $\frac{\Phi_0}{2\pi a^2}$. This requires further analysis of the model, both taking into account inevitable quantization of electronic spectrum in magnetic field and paramagnetic effect.

6. TEMPERATURE DEPENDENCE OF PARAMAGNETIC CRITICAL FIELD

In weal coupling region and for weak disorder the upper critical magnetic field of a superconductor is determined by orbital effects and is usually much lower than paramagnetic limit. However, the growth of the coupling strength and disorder, as was shown above, lead to a rapid increase of the orbital H_{c2} possibly exceeding the paramagnetic limit. In this Section we shall consider the behavior of paramagnetic critical field for a wide region of coupling strength U, including the region of BCS – BEC crossover and the limit of very strong coupling, with the account of disorder (including rather strong one).

It is well known that in BCS weak coupling limit paramagnetic effects (spin splitting effects) lead to the existence at low temperatures a region on the phase diagram of a superconductor in magnetic field, where paramagnetic critical magnetic field H_{cp} decreases with lowering temperature. This behavior is an evidence of instability, leading to a first order phase transition to Fulde – Ferrell – Larkin – Ovchinnikov (FFLO) phase [45, 46, 47] with Cooper pairs with finite momentum q and periodic in space order parameter. Further on, our analysis will be limited only to a second order phase transition and superconducting order parameter will be assumed spatially homogeneous, allowing us to determine the border of instability towards first order transition in the regions of BCS – BEC crossover and strong coupling, also for different levels of disorder. The problem of stability of FFLO state under these conditions will not be considered.

Within Nozieres – Schmitt-Rink approach the critical temperature in the presence of spin splitting in external magnetic field (neglecting orbital effects) or paramagnetic critical magnetic field H_{cp} at temperature $T < T_c$ is determined by the following BCS – like equation [29]:

$$1 = \frac{U}{4} \int_{-\infty}^{\infty} d\varepsilon \frac{\tilde{N}_0(\varepsilon)}{\varepsilon - \mu} \left(th \frac{\varepsilon - \mu - \mu_B H_{cp}}{2T} + th \frac{\varepsilon - \mu + \mu_B H_{cp}}{2T} \right), \quad (35)$$

where the chemical potential μ for different values of U and W is determined from DMFT+ Σ calculations, i.e. from the standard equation for the number of electrons in the band. It should be noted that Eq. (35) is obtained from an exact Ward identity [29] and remains valid in the presence of strong disorder, including the vicinity of Anderson transition. Eq. (35) demonstrates, that all of disorder influence on H_{cp} reduces to renormalization of the bare semielliptic density of states by disorder, that is for bare band with semielliptic density of states the influence of disorder on H_{cp} is universal and reduces only to band widening by disorder, i.e. to the substitution $D \to D_{eff}$. It is clear that paramagnetic critical field will be in general rising with the growth of coupling strength U as it becomes more and more difficult fro magnetic field to break pairs of strongly coupled electrons [29].

In Fig. 9 we show the results on disorder influence of temperature dependence of paramagnetic critical magnetic field. In BCS weak coupling limit (Fig. 9(a)) disorder growth leads both to the decrease of critical temperature in the absence of magnetic field T_{c0} (cf. [23, 24]) and to the decrease of the critical magnetic field at all temperatures. Instability region corresponding to first order transition is conserved also in the presence of disorder. In fact, as was noted before, disorder influence upon $H_{cp}(T)$ is universal and related only to the substitution $D \rightarrow D_{eff}$. As a result, the growth of disorder leads to the decrease of effective coupling strength, which is determined by dimensionless parameter $U/2D_{eff}$. This leads to a substantial widening of a relative temperature $T/T_c(H)$ region of the first order transition.

For intermediate coupling (U/2D = 0.8) in the region of BCS – BEC crossover (Fig. 9(b)) the growth of disorder only weakly changes the critical temperature T_{c0} (cf. [23, 24]), leading to some increase of $H_{cp}(T)$. As all influence of disorder is related only to the substitution $D \rightarrow D_{eff}$, the increase of disorder here again leads to the decrease of effective coupling strength $U/2D_{eff}$ and restoration of instability region of the first order transition.

In BEC limit of strong coupling the growth of disorder leads to significant growth the critical temperature T_{c0} (cf. [23, 24]). At the same time the critical magnetic field in low temperature region is rather weakly increasing with disorder. In BEC limit



Fig. 9. Temperature dependence of paramagnetic critical magnetic field for different levels of disorder: (a) - BCS weak coupling limit (U/2D = 0.2); (b) - BCS - BEC crossover region (intermediate coupling: U/2D = 0.8); (c) - BEC limit of strong coupling (U/2D = 1.6).



Fig. 10. Universal temperature dependence of paramagnetic critical magnetic field on disorder. (a) – weak coupling $U/2D_{eff} = 0.2$, W = 0 and W = 0.11 (b) – strong coupling $U/2D_{eff} = 1.6$, W = 0 and W = 0.11

the instability region of first order transition does not appear even in the presence of very strong disorder (W/2D = 0.5). In fact in BEC limit the influence of disorder is also universal and related only to the substitution $D \rightarrow D_{eff}$. As a result, if we normalize spin splitting and temperature by effective bandwidth $2D_{eff}$ and fix the effective coupling strength $U/2D_{eff}$, we shall obtain the universal temperature dependence of paramagnetic critical magnetic field. In Fig. 10 we show examples of such universal behavior for typical cases of weak and strong coupling both in presence and in the absence of disorder.

In the absence of disorder in BEC limit of strong coupling U/2D = 1.6 for $T \to 0$ we have $2\mu_B H_{cp}/2D \approx$ 0.125, which for a characteristic bandwith $2D \sim 1$ eV gives $H_{cp} \sim 10^7$ Gauss. For orbital critical magnetic field (cf. [28]) in the same model, for the same coupling strength and $T \to 0$, for a characteristic value of lattice parameter $a = 3.3 * 10^{-8}$ cm, we obtain $H_{c2} \approx 1.6 * 10^{8}$ Gauss. Thus, the orbital critical magnetic field at low temperatures increases with coupling strength much faster than paramagnetic field and in BEC limit the main contribution to the upper critical magnetic field at low temperatures will be due to paramagnetic effect. The growth of disorder leads to a large increase of orbital critical magnetic field [28], while $H_{cp}(T \to 0)$ in BCS – BEC crossover region and in BEC – limit is relatively weakly dependent on disorder. Then, also in the presence of disorder in BEC limit the main contribution to the upper critical magnetic field at low temperatures will come essentially from paramagnetic effect.

Thus, the growth of the coupling strength U leads to rapid increase of $H_{cp}(T)$ and disappearance of the instability region of first order transition at low temperatures in BCS - BEC crossover region and in BEC limit, which appears at low temperatures in BCS limit of weak coupling. Physically this is related to the fact, that it is more difficult for magnetic field to break strongly coupled pairs. The growth of disorder on BCS limit of weak coupling leads both to the decrease of critical temperature and to the decrease of $H_{cp}(T)$. Instability region of first order transition at low temperatures in the presence of disorder is conserved. In the region of intermediate coupling (U/2D = 0.8)disorder influence on both critical temperature and $H_{cp}(T)$ is rather weak. However, the growth of disorder leads to restoration of low temperature region of instability of the first order transition, which is not observed in the absence of disorder. This rather unexpected conclusion is due to specifics of attractive Hubbard model, where the effective dimensionless parameter $U/2D_{eff}$ controls the coupling strength in disordered case.

In BEC limit at low temperatures, for reasonable parameters of the model, paramagnetic critical magnetic field is noticeably lower than the orbital one, so that the upper critical field in this region is determined essentially by paramagnetic critical field. In the presence of disorder this conclusion is also even more valid, as the orbital critical field rapidly grows with disorder, while paramagnetic critical field in this limit only weakly dependent on disorder.

7. CONCLUSION

In this paper, within Nozieres – Schmitt-Rink approximation and $DMFT+\Sigma$ generalization of

dynamic mean field we have studied the influence of disordering, including the strong one (Anderson localization region), on Ginzburg – Landau expansion and the behavior of related physical properties close to T_c , and also the upper critical magnetic field (both orbital and paramagnetic) in disordered Anderson – Hubbard model with attraction, for a wide range of the values of attraction potential U, from the region of weak coupling, where instability of the normal phase and superconductivity are well described by BCS model, up to the limit of strong coupling, where superconducting transition is related to Bose condensation of compact Cooper pairs, which are formed at temperatures much higher than superconducting transition temperature.

Due to size limitations of this review above we have presented only a part of our results. Further details, as well as more detailed derivations of the main equations can be found in original papers [25, 26, 27, 28, 29].

Note that all results obtained in this work implicitly used an assumption of self – averaging superconducting order parameter entering Ginzburg – Landau expansion. It is well known [9] that this assumption becomes, in general case, invalid close to Anderson metal – insulator transition, which is due to development in this region of strong fluctuations of the local density of states, leading to strong spatial fluctuations of the order parameter [48] and inhomogeneous picture of superconducting transition [49]. This problem is of great interest in the context of superconductivity in BCS – BEC crossover and in the region of strong coupling, and deserves further studies.

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Список литературы

- A.A. Abrikosov, L.P. Gor'kov. Zh. Eksp. Teor. Fiz. 36, 319 (1958) [Sov. Phys. JETP 9, 220 (1959)].
- A.A. Abrikosov, L.P. Gor'kov. Zh. Eksp. Teor. Fiz. 35, 1158 (1958) [Sov. Phys. JETP 9, 1090 (1959)].
- L.P. Gor'kov. Zh. Eksp. Teor. Fiz. 36, 1918 (1959) [Sov. Phys. JETP 36, 1364 (1959)].
- A.A. Abrikosov, L.P. Gor'kov. Zh. Eksp. Teor. Fiz. **39**, 1781 (1960) [Sov. Phys. JETP **12**, 1243 (1961)].
- 5. P.W. Anderson. J. Phys. Chem. Solids 11, 26 (1959).
- P.G. De Gennes. Superconductivity of Metals and Alloys. W.A. Benjamin, NY 1966.
- L.N. Bulaevskii, M.V. Sadovskii. Pis'ma Zh. Eksp. Teor. Fiz. **39**, 524 (1984) [JETP Letters **39**, 640 (1984)].
- L.N. Bulaevskii, M.V. Sadovskii. J.Low.Temp.Phys. 59, 89 (1985).
- 9. M.V. Sadovskii. Physics Reports 282, 226 (1997).

- M.V. Sadovskii. Superconductivity and Localization. World Scientific, Singapore 2000
- P. Nozieres and S. Schmitt-Rink, J. Low Temp. Phys. 59, 195 (1985).
- Th. Pruschke, M. Jarrell, J. K. Freericks. Adv. Phys. 44, 187 (1995).
- A. Georges, G. Kotliar, W. Krauth, M. J. Rozenberg. Rev. Mod. Phys. 68, 13 (1996).
- D. Vollhardt in "Lectures on the Physics of Strongly Correlated Systems XIV", eds. A. Avella and F. Mancini, AIP Conference Proceedings vol. 1297 (AIP, Melville, New York, 2010), p. 339; ArXiV: 1004.5069.
- E.Z.Kuchinskii, I.A.Nekrasov, M.V.Sadovskii. Pis'ma Zh. Eksp. Teor. Fiz. 82, 217 (2005) [JETP Letters 82, 198 (2005)].
- M.V. Sadovskii, I.A. Nekrasov, E.Z. Kuchinskii, Th. Prushke, V.I. Anisimov. Phys. Rev. B 72, No 15, 155105 (2005)
- E.Z. Kuchinskii, I.A. Nekrasov, M.V. Sadovskii. Fizika Nizkikh Temperatur **32**, 528 (2006) [Low Temp. Phys. **32**, 398 (2006)].
- E.Z. Kuchinskii, I.A. Nekrasov, M.V. Sadovskii. Phys. Rev. B 75, 115102-115112 (2007).
- E.Z. Kuchinskii, I.A. Nekrasov, M.V. Sadovskii, Zh. Eksp. Teor. Fiz. **133**, 670 (2008) [JETP **106**, 581 (2008)].
- E.Z. Kuchinskii, I.A. Nekrasov, M.V. Sadovskii. Usp. Fiz. Nauk **182**, 345 (2012) [Physics Uspekhi **53**, 325 (2012)].
- E.Z. Kuchinskii, M.V. Sadovskii. Zh. Eksp. Teor. Fiz. 149, 589 (2016) [JETP 122, 509 (2016)].
- N.A. Kuleeva, E.Z. Kuchinskii, M.V. Sadovskii. Zh. Eksp. Teor. Fiz. **146**, 304 (2014) [JETP **119**, 264 (2014)].
- E.Z. Kuchinskii, N.A. Kuleeva, M.V. Sadovskii. Pis'ma Zh. Eksp. Teor. Fiz. **100**, 213 (2014) [JETP Letters **100**, 192 (2014)].
- E.Z. Kuchinskii, N.A. Kuleeva, M.V. Sadovskii. Zh. Eksp. Teor. Fiz. **147**, 1220 (2015) [JETP **120**, 1055 (2015)].
- E.Z. Kuchinskii, N.A. Kuleeva, M.V. Sadovskii. Zh. Eksp. Teor. Fiz. **149**, 430 (2016) [JETP **122** 375 (2016)].
- E.Z. Kuchinskii, N.A. Kuleeva, M.V. Sadovskii. Fizika Nizkikh Temperatur 43, 22 (2017) [Low Temp. Phys. 42 No. 1 (2017)].
- 27. E.Z. Kuchinskii, N.A. Kuleeva, M.V. Sadovskii. Zh. Ekap. Teor. Fiz. **152**, 133 (2017)[JETP **125**, 111 (2017)].
- E.Z. Kuchinskii, N.A. Kuleeva, M.V. Sadovskii. Zh. Eksp. Teor. Fiz. **152**, 1321 (2017) [JETP **125**, No.6, 1127 (2017)].
- E.Z. Kuchinskii, N.A. Kuleeva, M.V. Sadovskii. Zh. Eksp. Teor. Fiz. **154**, 881 (2018) [JETP **127**, 753 (2018)].

 [A.A. Abrikosov, L.P. Gor'kov, I.E. Dzyaloshinskii. Quantum Field Theoretical Methods in Statistical Physics. Pergamon Press, Oxford, 1965.

13

- M.V. Sadovskii. Diagrammatics. World Scientific, Singapore 2019.
- R. Bulla, T.A. Costi, T. Pruschke, Rev. Mod. Phys. 60, 395 (2008).
- D. Vollhardt and P. Wölfle, Phys. Rev. B 22, 4666 (1980)
- D. Vollhardt and P. Wölfle, Phys. Rev. Lett. 48, 699 (1982).
- P. Wölfle and D. Vollhardt, in Anderson Localization, eds. Y. Nagaoka and H. Fukuyama, Springer Series in Solid State Sciences, vol. 39, p.26. Springer Verlag, Berlin 1982.
- A.V. Myasnikov, M.V. Sadovskii, Fiz. Tverd. Tela 24, 3569 (1982) [Sov. Phys.-Solid State 24, 2033 (1982)].
- 37. E.A. Kotov, M.V. Sadovskii. Zs. Phys. B 51, 17 (1983).
- M.V. Sadovskii, in Soviet Scientific Reviews Physics Reviews, ed. I.M. Khalatnikov, vol. 7, p.1. Harwood Academic Publ., NY 1986.
- D. Vollhardt, P. Wölfle, in *Electronic Phase Transitions*, eds. W. Hanke and Yu.V. Kopaev, vol. 32, p. 1. North– Holland, Amsterdam 1992.
- N.R. Werthamer, E. Helfand. Phys.Rev. 147, 288 (1966).
- N.R. Werthamer, E. Helfand, P.C. Hohenberg. Phys.Rev.147, 295 (1966).
- E.M. Lifshits, L.P. Pitaevskii. Statistical Physics. Part
 Ch. 5, Pergamon Press, Oxford, 1980.
- E.Z. Kuchinskii, M.V. Sadovskii. Sverkhprovodimast', Fizika, Khimija, Tekhnika 4, 2278 (1991) [Superconductivity: Physics, Chemistry, Technology 4, 2278 (1991)].
- E.Z. Kuchinskii, M.V. Sadovskii. Physica C185-189, 1477 (1991).
- 45. P. Fulde, R.A. Ferrell. Phys. Rev. A135, 550 (1964).
- A.I. Larkin, Yu. N. Ovchinnikov. Zh. Eksp. Teor. Fiz. 47, 1136 (1964).
- D. Saint-James, G. Sarma, E.J. Thomas. Type II Superconductivity. Pergamon Press, Oxford, 1969].
- L.N. Bulaevskii, M.V. Sadovskii. Pis'ma Zh. Eksp. Teor. Fiz. 43, 76 (1986) [JETP Letters 43, 99 (1986)].
- L.N. Bulaevskii, S.V. Panyukov, M.V. Sadovskii. Zh. Eksp. Teor. Fiz. **92**, 672 (1987) [JETP **65**, 380 (1987)].