Temperature dependence of the upper critical field of high-T_c superconductors: localization effects

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The anomalous temperature dependence of the orbital upper critical field H_{c2} observed in epitaxial films of the Bi–Sr–Cu–O system (over a broad temperature range) can be described satisfactorily in terms of localization effects in the 2D (or quasi-2D) case.

Unique data on the temperature dependence of the upper critical field of the $Bi_2Sr_2CuO_y$ system were recently found[1] over the broad temperature range from $T_c \approx 19$ K to $T \approx 0.005T_c$. These results demonstrated a clearly defined anomalous behavior with a negative curvature at all temperatures. Osofsky *et al.*[1] mentioned that this behavior is difficult to understand on the basis of the existing theories. This behavior is sharply different from the standard behavior of the BCS model and also from the behavior predicted by the model of compact charged bosons (bipolarons).[2] In particular, in the latter case the critical field H_{c2} diverges in a power-law fashion as $T \to 0$, while experimentally $H_{c2}(T=0)$ remains finite. Our purpose in the present study was to demonstrate that the observed $H_{c2}(T)$ dependence can be described satisfactorily by localization effects in a 2D (or quasi-2D) model in the limit of a fairly pronounced disorder.[3] The measurements of H_{c2} in Ref. 1 were carried out on epitaxial films of $Bi_2Sr_2CuO_y$, but it is quite probable that the degree of order of these films was not very high. Evidence for this conclusion comes from the fairly large width (~ 7 K) of the superconducting transition. Unfortunately, the corresponding data, in particular, data on the conductivity of the films, were not reported in Ref. 1. This circumstance suggests that it might be worthwhile to interpret the data in the picture of a fairly pronounced disorder, the effects of which would evidently be amplified by the quasi-2D nature of the high-T_c superconducting systems.

We basically restrict the discussion below to a description of a purely 2D case, since the corresponding behavior for quasi-2D systems differs only very slightly from that of purely 2D systems in the parameter region of interest here.[3] The general approach to the study of H_{c2} in highly disordered systems is described in Ref. 4.

In an analysis of highly disordered systems in a magnetic field, knowledge of the two-particle Matsubara Green's functions at small values of q and ω_m in the diffusion and Cooper channels is of governing importance:

$$\begin{Bmatrix} \Phi_E(\mathbf{q}\omega_m=2\epsilon_n) \\ \Psi_E(\mathbf{q}\omega_m=2\epsilon_n) \end{Bmatrix} = -\frac{N(E)}{i|\omega_m| + i\begin{Bmatrix} D_1(\omega_m) \\ D_2(\omega_m) \end{Bmatrix} q^2}. \tag{1}$$

Here $\omega_m = 2\pi m T$ and $\epsilon_n = 2\pi(n + 1/2)T$ are Matsubara frequencies, and $N(E)$ is the density of states at the Fermi level E.

The generalized "diffusion coefficients" D_1 and D_2 are generally not equal to each other in the presence of an external magnetic field, which disrupts the invariance under time reversal. In this case we need to consider a system of coupled equations for the two two-particle functions.

We are interested below in only the case in which the magnetic field is perpendicular to the conducting planes. Repeating the customary analysis of a superconducting transition in an external magnetic field,[5] we find an equation which determines the temperature dependence $H_{c2}(T)$:

$$\ln\frac{T}{T_c} = 2\pi T \sum_{\epsilon_n} \left\{ \frac{1}{2|\epsilon_n| + 2\pi D_2(2|\epsilon_n|)\frac{H}{\Phi_0}} - \frac{1}{2|\epsilon_n|} \right\}. \tag{2}$$

Here $\Phi_0 = \pi c/e$ is the flux quantum, and T_c is the transition temperature from the BCS theory in the absence of a magnetic field.

It can be seen from Eq. (2) that distinctive features in the behavior of the upper critical field stem from particular features of the frequency behavior of the diffusion coefficient. This diffusion becomes nontrivial near the Anderson metal–insulator transition.

In self-consistent localization theory,[6,7] the system of equations for the diffusion coefficients in a magnetic field is, in the 2D case,[8]

$$\begin{aligned} \frac{D_0}{D_2} &= 1 + \frac{1}{\pi N(E)} \sum_{|\mathbf{q}|<q_0} \frac{1}{\omega + D_1 q^2}, \\ \frac{D_0}{D_1} &= 1 + \frac{1}{\pi N(E)} \sum_{|\mathbf{k}|<q_0} \frac{1}{\omega + D_2 k^2}, \end{aligned} \tag{3}$$

where $k^2 = 4m\omega_H(n+1/2)$, $\omega_H = eH/mc$ is the cyclotron frequency, n specifies the Landau level, and q_0 is the cutoff momentum ($q_0 \approx l^{-1}$), determined from the condition $D_0 q_0^2 = 1/2\tau$, where D_0 is the Drude diffusion coefficient, τ^{-1} is the mean free time, and l is the mean free path.

We introduce $d_1 = D_1/D_0$, $d_2 = D_2/D_0$, and the dimensionless disorder parameter $\lambda = 1/2\pi E_\tau$. We can then rewrite system (3) as

$$\begin{aligned} \frac{1}{d_2} &= 1 + \frac{\lambda}{d_1} \ln\left(1 + d_1 \frac{1}{2\omega\tau}\right), \\ \frac{1}{d_1} &= 1 + \frac{\lambda}{d_2} \sum_{n=0}^{N_0} \left(n + \frac{1}{2} + \frac{\omega}{4m\omega_H D_0} \frac{1}{d_2}\right)^{-1}, \end{aligned} \tag{4}$$

where $N_0 = (8m\omega_H D_0 \tau)^{-1}$ is the maximum number of Landau levels, determined by the cutoff.

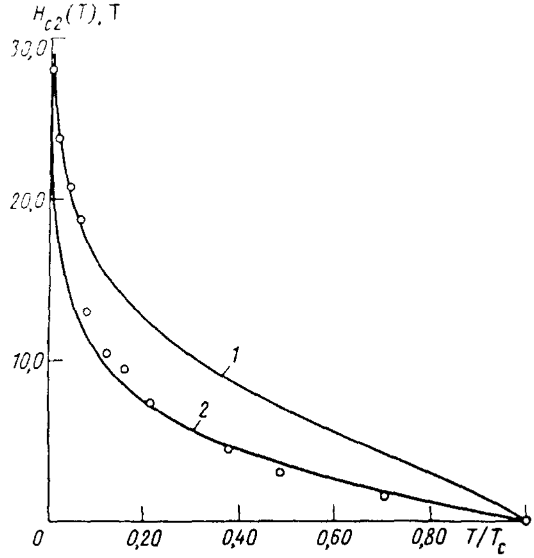

FIG. 1. Temperature dependence of the upper critical field. Theoretical curve 1 is drawn for the case $e^{-1/\lambda}/T_c\tau=2$ with $\lambda=0.18$; curve 2 is drawn for $e^{-1/\lambda}/T_c\tau=20$ with $\lambda=0.032$. The points are experimental data from Ref. 1.

The solution of system (4) for the diffusion coefficient in the Cooper channel can be written for weak magnetic fields, $\omega_H \ll \lambda_e^{-1/\lambda}/\tau$, as follows,[3] at the accuracy required for use in Eq. (2):

$$d_2=\begin{cases} 1, & \omega \gg e^{-1/\lambda}/2\tau, \\ 2\omega\tau e^{1/\lambda}, & \omega \ll e^{-1/\lambda}/2\tau. \end{cases} \tag{5}$$

The effect of the magnetic field on the diffusion can in practice be ignored.

It is easy to see that the particular features of the behavior of the upper critical field associated with the particular features of the frequency behavior of the diffusion coefficient are manifested only at temperatures[3] $T \ll e^{-1/\lambda}/\tau$. At higher temperatures, there is a complete correspondence with the ordinary theory of "dirty" superconductors. A superconductivity may persist in the system with a finite localization length under the inequality[3] $T_c \gg \lambda(e^{-1/\lambda}/\tau)$, which is equivalent to the familiar condition[9] that the length scale of the Cooper pairs be small in comparison with the localization length. The latter is exponentially large in 2D systems with a slight disorder ("slight" in the sense of the inequality $\lambda \ll 1$). Of primary interest for our purposes here is the case of a relatively strong disorder, in which the inequality $T_c \ll e^{-1/\lambda}/\tau$ holds. We are therefore dealing with a region which is fairly narrow along the scale of the parameter λ: $\lambda(e^{-1/\lambda}/\tau) \ll T_c \ll e^{-1/\lambda}/\tau$. In this case the upper critical field is determined by the equation[3] ($\gamma=1.781$)

$$\ln\left(\frac{\gamma}{2\pi}\frac{e^{-1/\lambda}}{\tau T}\right)=\left(1+4\pi\frac{D_0}{\Phi_0}\frac{\tau H_{c2}}{e^{-1/\lambda}}\right)\ln\left[\frac{\gamma}{2\pi}\frac{e^{-1/\lambda}}{\tau T_c}\left(1+4\pi\frac{D_0}{\Phi_0}\frac{\tau H_{c2}}{e^{-1/\lambda}}\right)\right], \tag{6}$$

from which we find the $T(H_{c2})$ dependence explicitly. The corresponding behavior of the upper critical field for two sets of parameters is shown in Fig. 1. The $H_{c2}(T)$ curve is concave, and H_{c2} diverges as $T \to 0$. This weak (logarithmic) divergence stems from our ignoring the inverse effect of the magnetic field on the diffusion.[3] When this effect is taken into account, the divergence of H_{c2} as $T \to 0$ is eliminated, and we find

$$H_{c2}\,(T=0) = \frac{\gamma}{2\pi} \frac{\Phi_0}{D_0} \frac{1}{\tau}. \tag{7}$$

It is here that we are seeing the effect of a disruption of invariance under time reversal. We see that it is important only at extremely low temperatures.[3] We can ignore it below.

In the quasi-2D case, on the insulator side of the Anderson transition, and not too close to this transition, the behavior of the diffusion coefficient differs from the purely 2D behavior only by slight corrections, and the upper critical field can be treated in the purely 2D approach. Near the Anderson transition, for example, in terms of the magnitude of the interplanar transport integral on both the metallic and insulating sides in the parameter region satisfying the inequality $\lambda(e^{-1/\lambda}/\tau) \ll T_c \ll e^{-1/\lambda}/\tau$, the temperature dependence of H_{c2} is actually also very close to the 2D dependence discussed above.[3] Again, distinctions arise only in a fairly narrow region of extremely low temperatures. The details are given in Ref. 3.

The anomalies found in the behavior of H_{c2} ultimately stem from corresponding anomalies in the frequency dependence of the generalized diffusion coefficient near the Anderson transition, and in this sense they reflect a change in the nature of the electronic states near the metal–insulator transition.

Figure 1 also shows experimental values of H_{c2} found in Ref. 1. Theoretical curve 1 is drawn for parameter values which lead to a good agreement with the experimental data at low temperatures. Curve 2 corresponds to parameter values which lead to a good agreement over a broad temperature range except at extremely low temperatures. The mass m is assumed everywhere to be equal to the mass of a free electron. We see that it is possible to achieve a very satisfactory correspondence between theory and experiment. Unfortunately, the values of the ratio $e^{-1/\lambda}/T_c\tau$ which we selected in case 2 correspond to unrealistic values of $T_c\tau$ (too small) at completely plausible values of λ. Such values could hardly be realized in our case with a relatively high T_c. In case 1 the situation is considerably better, although the attenuation is still extremely large at the scale of T_c; this situation corresponds to strong disorder. We note, however, that a detailed discussion of these parameters is hardly possible in the absence of additional data on the films used in Ref. 1. In particular, an independent estimate of the parameter λ would be of major interest. We wish to emphasize that the theoretical parameters used here are rather arbitrary, because of the 2D idealization. A more serious comparison must of course be carried out with formulas of Ref. 3 for the quasi-2D case, which also requires additional information on the system, in particular, reliable data on the anisotropy of the electronic properties.

In our opinion, the good agreement between the experimental data of Ref. 1 and the theoretical predictions found for the 2D (or quasi-2D) case of a disordered system, in which Anderson-localization effects are important, is additional evidence that these effects are important in the physics of high-T_c superconductors.[10]

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