# Ginzburg-Landau Expansion and the Slope of the Upper Critical Field in Disordered Superconductors with Anisotropic Pairing

A.I.Posazhennikova, M.V.Sadovskii

*Institute for Electrophysics,*

*Russian Academy of Sciences, Ural Branch,*

*Ekaterinburg, 620049, Russia*

*E-mail: sadovski@ief.intec.ru*

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## Abstract

It is demonstrated that the slope of the upper critical field $|dH_{c2}/dT|_{T_c}$ in superconductors with $d$-wave pairing drops rather fast with concentration of normal impurities, while in superconductors with anisotropic $s$-wave pairing $|dH_{c2}/dT|_{T_c}$ grows, and in the limit of strong disorder is described by the known dependences of the theory of "dirty" superconductors. This allows to use the measurements of $H_{c2}$ in disordered superconductors to discriminate between these different types of pairing in high-temperature and heavy-fermion superconductors.

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The main problem of the present day physics of high-temperature superconductors is the determination the nature and type of the Cooper pairing. A number of experiments and theoretical models [1] suggest the realization in these systems of anisotropic pairing with $d_{x^2-y^2}$-symmetry with appropriate zeroes of the gap function at the Fermi surface. At the same time most of these experiments also agree with the so called anisotropic $s$-wave pairing, which follows from some theoretical models [2,3]. In this latest case there again appear the zeroes (with no change of sign) or rather deep minima of the gap function in the same directions in the Brillouin zone as in the case of $d$-wave pairing.

Recently it was shown [4,5] that controlled disordering (introduction of normal impurities) can be an effective method of experimental discrimination between different types of anisotropic pairing. Disordering leads to different behavior of the density of states in superconducting state: $d$-wave pairing superconductor remains gapless, while in an anisotropic $s$-wave superconductor with zeroes of the gap function, small disordering leads to the opening of the finite gap on the Fermi surface.

Gap measurements, especially for the different directions in the Brillouin zone, are difficult enough to perform. The aim of the present paper is to demonstrate that much simpler, in principle, measurements of the upper critical field $H_{c2}$ at different degrees of disorder can also provide an effective method to discern $d$-wave pairing from anisotropic $s$-wave. Surely, the problem under discussion is of interest also for heavy-fermion superconductors.

Following Refs. [4,5], we analyze two-dimensional electronic system with isotropic Fermi surface and separable pairing potential of the form:

$$V(\phi,\phi') = -V\eta(\phi)\eta(\phi') \tag{1}$$

where $\phi$-is a polar angle, determining the electronic momentum direction in the plane, and $\eta(\phi)$ is given by the following model dependence:

$$\eta(\phi) = \begin{cases} cos\,(2\phi) & (\text{d}-\text{ wave }) \\ |cos\,(2\phi)\,| & (\text{ anisotropic s}-\text{ wave }) \end{cases} \tag{2}$$

The pairing constant $V$ is as usual different from zero in some region of the width of $2\omega_c$ around the fermi level ($\omega_c$-is some characteristic frequency of the quanta, responsible for the pairing interaction ). In this case the superconducting gap (order parameter) takes the form: $\Delta(\phi) = \Delta\eta(\phi)$, and positions of its zeroes for $s$ and $d$ cases just coincide.

BCS equations for the impure superconductor are derived in a standard way [6]. Linearized gap equation, determining the transition temperature $T_c$ takes the form3:

$$\Delta(\phi) = -N(0)T_c\sum_{\omega_n}\int_{-\infty}^{\infty}d\xi\int_0^{2\pi}\frac{d\phi'}{2\pi}V(\phi,\phi')\frac{\tilde{\Delta}(\phi')}{\tilde{\omega}_n^2+\xi^2} \tag{3}$$

where

$$\tilde{\Delta}(\phi) = \begin{cases} \Delta\eta(\phi) & \text{(d - wave )} \\ \Delta(\eta(\phi)+2\gamma/\pi|\omega_n|) & \text{( anisotropic s - wave )} \end{cases} \tag{4}$$

$\tilde{\omega}_n = \omega_n + \gamma sign(\omega)$, $\gamma = \pi\rho V_0^2 N(0)$ - is the usual electron damping due to impurity scattering, $V_0$ - impurity potential and $\rho$ - impurity concentration, $N(0)$ - is normal density of states at the Fermi level and $\xi$ - is electronic energy with respect to the Fermi level, $\omega_n = (2n+1)\pi T_c$.

After the traditional analysis $T_c$-equation reduces to [4,5]:

$$ln\left(\frac{T_{c0}}{T_c}\right) = \alpha\left[\Psi\left(\frac{1}{2}+\frac{\gamma}{2\pi T_c}\right)-\Psi\left(\frac{1}{2}\right)\right] \tag{5}$$

where $\alpha = 1$ for the case of $d$-pairing and $\alpha = (1-8/\pi^2)$ for anisotropic $s$-wave pairing, $T_{c0}$-is the transition temperature in the absence of impurities, $\Psi(x)$- is the usual digamma function. The appropriate dependences of $T_c(\gamma/T_{c0})$ are shown in Fig.1. In case of $d$-pairing $T_c$ is completely suppressed for $\gamma = \gamma_c \approx 0.88T_{c0}$. In anisotropic $s$-case the dependence of $T_c$ on $\gamma$ is much weaker, for $\gamma \gg T_{c0}$ we obtain $T_c \sim T_{c0}[1-\alpha ln(\gamma/\pi T_{c0})]$.

Ginzburg-Landau expansion for the free-energy density of a superconducting state up to terms quadratic over $\Delta_q$ can be written as:

$$F_s - F_n = A|\Delta_q|^2 + q^2C|\Delta_q|^2 \tag{6}$$

and is determined by the usual loop-expansion for the free-energy of an electron in the field of random fluctuations of the order-parameter with some small wave vector $q$, shown in

Fig2. Diagramms (c) and (d) are to be subtracted, so that the coefficient $A$ becomes zero for $T = T_c$. All calculations are standard and we only note that for the case of $d$-pairing the contribution of diagramms (b) and (d) actually vanishes up to terms of the order of $q^4$. Finally, the coefficients of Ginzburg-Landau expansion can be written in the following form:

$$A = A_0 K_A; \qquad C = C_0 K_C \tag{7}$$

where $A_0$ and $C_0$ are the usual expressions for the case of isotropic $s$-wave pairing [7]:

$$A_0 = N(0)\frac{T - T_c}{T_c}; \qquad C_0 = N(0)\frac{7\zeta(3)}{48\pi^2}\frac{v_F}{T_c^2} \tag{8}$$

where $v_F$-is electron velocity at the Fermi surface, and all peculiarities of models udder consideration are actually contained in dimensionless coefficients $K_A$ and $K_C$. In the absence of impurities for both models we obtain: $K_A^0 = 1/2$, $K_C^0 = 3/4$. For the impure system we get:

(A) $d$-wave pairing:

$$K_A = \frac{1}{8T_c}\int_{-\omega_c}^{\omega_c}\frac{d\xi}{\xi}\int_{-\infty}^{\infty}\frac{d\omega}{\pi}\frac{\omega+\xi}{ch^2\left(\frac{\omega+\xi}{2T_c}\right)}\frac{\gamma}{\omega^2+\gamma^2} \tag{9}$$

$$K_C = -\frac{3}{56\zeta(3)}\Psi''\left(\frac{1}{2} + \frac{\gamma}{2\pi T_c}\right) \tag{10}$$

(B) anisotropic $s$-wave pairing:

$$K_A = \frac{\gamma}{\pi T_c}\left\{\frac{1}{8}\int_{-\omega_c}^{\omega_c}\frac{d\xi}{\xi}\int_{-\infty}^{\infty}d\omega\frac{\omega+\xi}{ch^2\left(\frac{\omega+\xi}{2T_c}\right)(\omega^2+\gamma^2)} + \frac{\gamma}{\pi}\int_{-\infty}^{\infty}d\omega\frac{1}{ch^2\left(\frac{\omega}{2T_c}\right)(\omega^2+\gamma^2)}\right\} \tag{11}$$

$$K_C = -\frac{3\alpha}{56\zeta(3)}\Psi''\left(\frac{1}{2} + \frac{\gamma}{2\pi T_c}\right) + \frac{24}{7\zeta(3)}\frac{T_c^2}{\alpha\gamma^2}ln\left(\frac{T_c}{T_{c0}}\right) + \frac{6\pi}{7\zeta(3)}\frac{T_c}{\gamma} \tag{12}$$

The appropriate dependences of dimensionless coefficients on disorder parameter $\gamma/T_{c0}$ are shown in Figs.3,4.

Close to $T_c$ the upper critical field $H_{c2}$ is determined from (9):

$$H_{c2} = -\frac{\phi_0}{2\pi}\frac{A}{C} \tag{13}$$

where $\phi_0 = c\pi/e$ — is magnetic flux quantum. Then the slope of the upper critical field close to $T_c$ is:

$$\left|\frac{dH_{c2}}{dT}\right|_{T_c} = \frac{24\pi\phi_0}{7\zeta(3)v_F^2}T_c\frac{K_A}{K_C} \tag{14}$$

Dependence of $|dH_{c2}/dT|_{T_c}$ on $\gamma/T_{c0}$ for both models is shown in Fig.5. We can see that for the case of $d$-wave pairing the slope of $H_{c2}$ drops to zero on the scale of $\gamma \sim T_{c0}$. For the case of anisotropic $s$-wave pairing, on the contrary, the slope grows with disorder and after some transition region of $\gamma \sim T_{c0}$ it crosses over to the usual linear dependence $|dH_{c2}/dT|_{T_c} \sim \gamma$, which is characteristic of the usual theory of "dirty" superconductors with isotropic $s$-wave pairing [8]. In our opinion this sharp difference can be used a simple enough criterion of experimental discrimination of $d$-wave superconductors from anisotropic $s$-wave case. Unfortunately, in case of high-$T_c$ oxides the situation is complicated by the known nonlinearity of temperature dependence of $H_{c2}$, which is observed in rather wide region close to $T_c$. At present the nature of this nonlinearity is unclear and it is probable that it may be due to some inhomogeneity of the samples, though its more fundamental nature can not be excluded.

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Fig.1. Dependence of transition temperature $T_c$ on disorder parameter $\gamma/T_{c0}$. Dashed line - dependence for the case of $d$-wave pairing, full line - the case of anisotropic $s$-wave pairing. At the insert - the same curve for $s$-pairing for wider interval of the parameter $\gamma/T_{c0}$.

Fig.2. Diagrammatic representation of Ginzburg-Landau expansion. Electronic lines are "dressed" by impurity scattering. $\Gamma$-is the impurity vertex calculated in "ladder" approximation. Diagramms (c) and (d) are calculated with $q = 0$ and $T = T_c$.

Fig.3. Dependence of dimensionless coefficients of Ginzburg-Landau expansion on disorder parameter $\gamma/T_{c0}$. The case of $d$-wave pairing.

Fig.4. Dependence of dimensionless coefficients of Ginzburg-Landau expansion on disorder parameter $\gamma/T_{c0}$. The case of anisotropic $s$-wave pairing.

Fig.5. Dependence of normalized slope of the upper critical field $h = \left|\frac{dH_{c2}}{dT}\right|_{T_c} / \left|\frac{dH_{c2}}{dT}\right|_{T_{c0}}$ on disorder parameter $\gamma/T_{c0}$. Dashed line - the case of $d$-wave pairing, full line - the case of anisotropic $s$-wave pairing.

## REFERENCES

[1] Pines D. Physica **C235-240**, 113 (1994).

[2] Chakravarty S., Subdø A., Anderson P.W., Strong S. Science **261**, 337 (1993).

[3] Liechtenstein A.I., Mazin I.I., Andersen O.K. Phys.Rev.Lett. **74**, 2303 (1995).

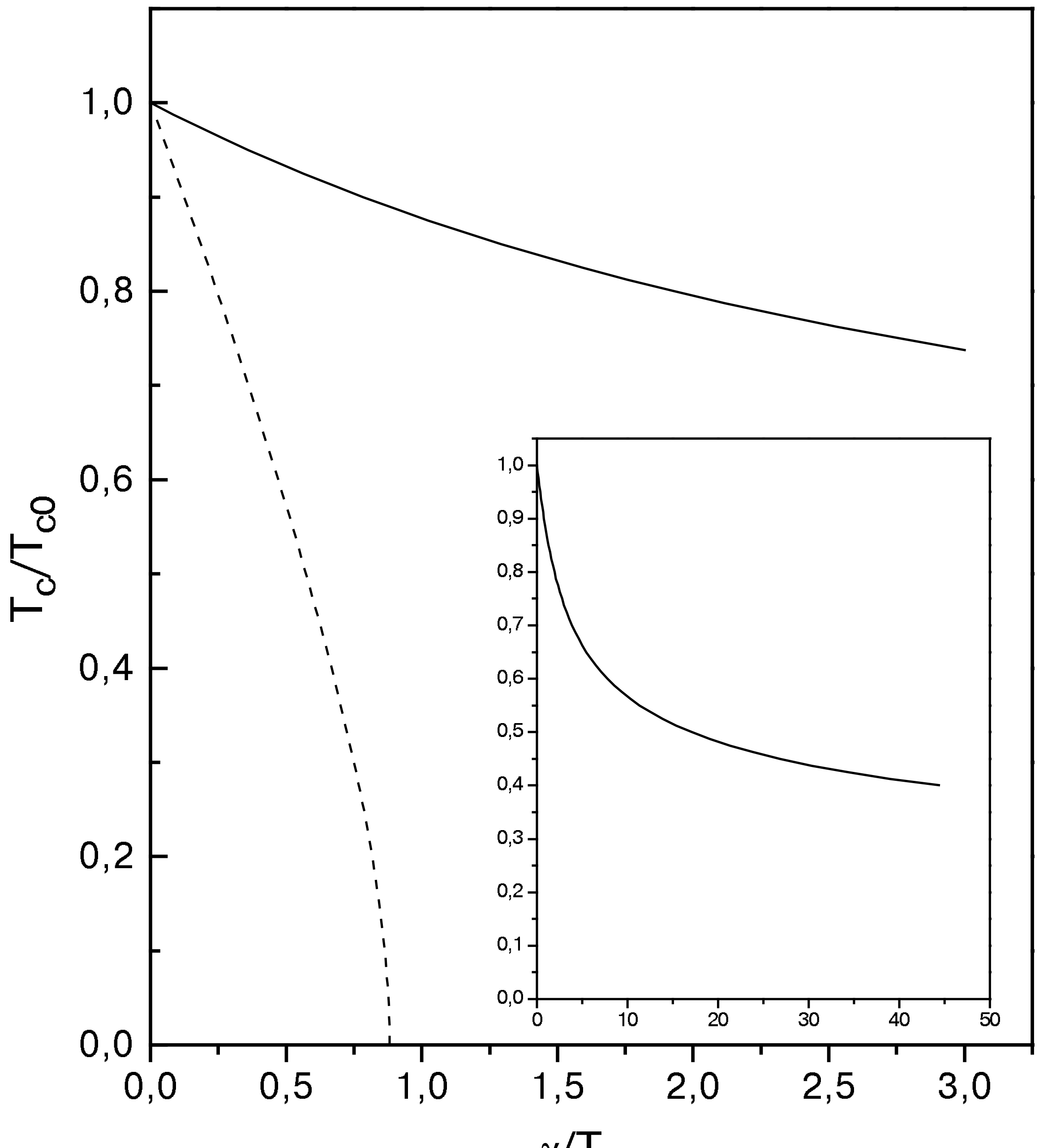
[4] Borkovski L.S., Hirschfeld P.J. Phys.Rev. **B49**, 15404 (1994).

[5] Fehrenbacher R., Norman M.R. Phys.Rev. **B50**, 3495 (1994).

[6] Abrikosov A.A., Gorkov L.P., Dzyaloshinskii I.E. Methods of Quantum Field Theory in Statistical Physics. Pergamon Press, Oxford 1965.

[7] De Gennes P.G. Superconductivity of Metals and Alloys. W.A.Benjamin, N.Y. 1966.

[8] Gorkov L.P. Zh.Eksp.Teor.Phys. (JETP) **37**, 1407 (1959).



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$T_c/T_{c0}$
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$\gamma/T_{c0}$
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Fig.1

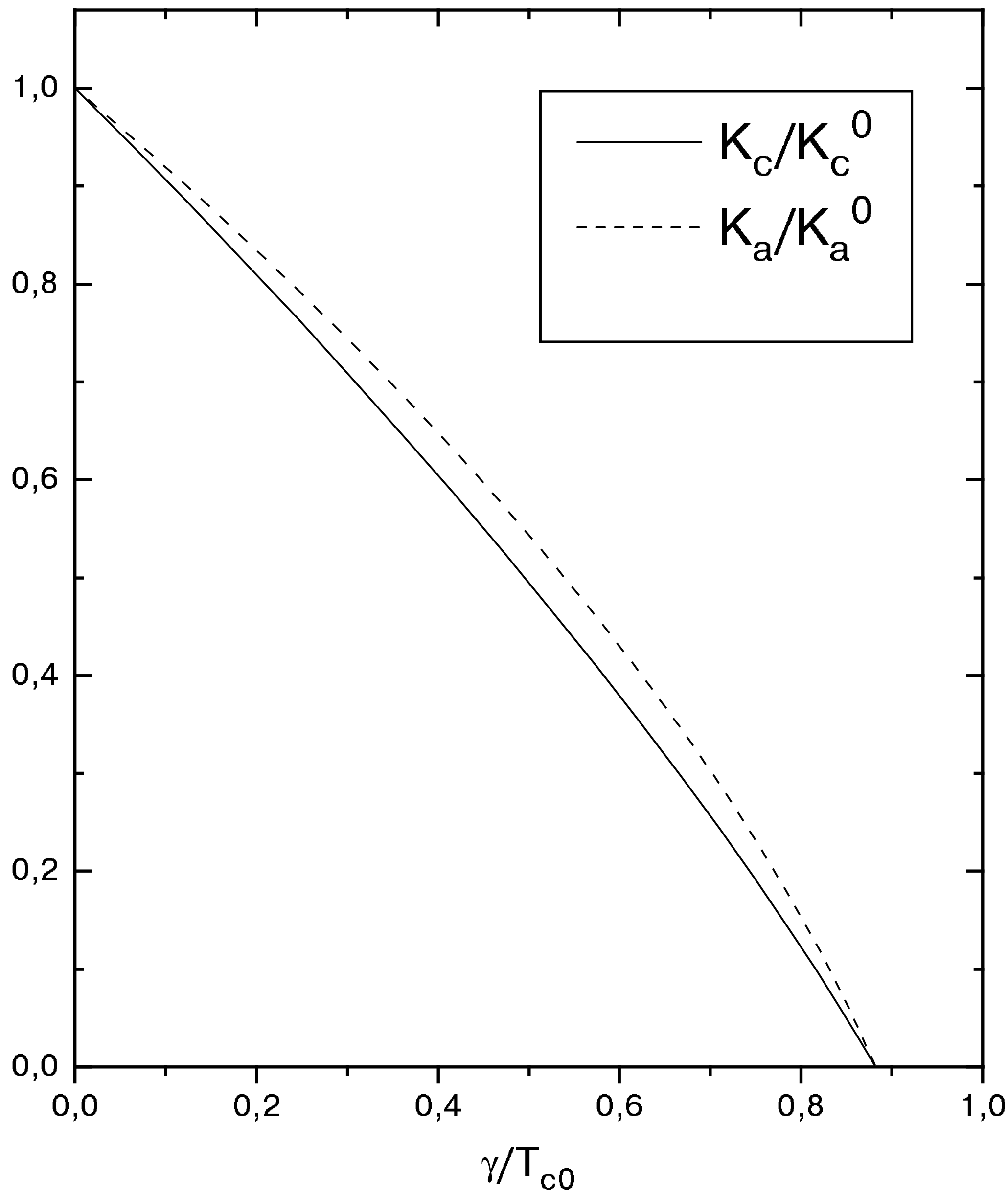


Fig. 3

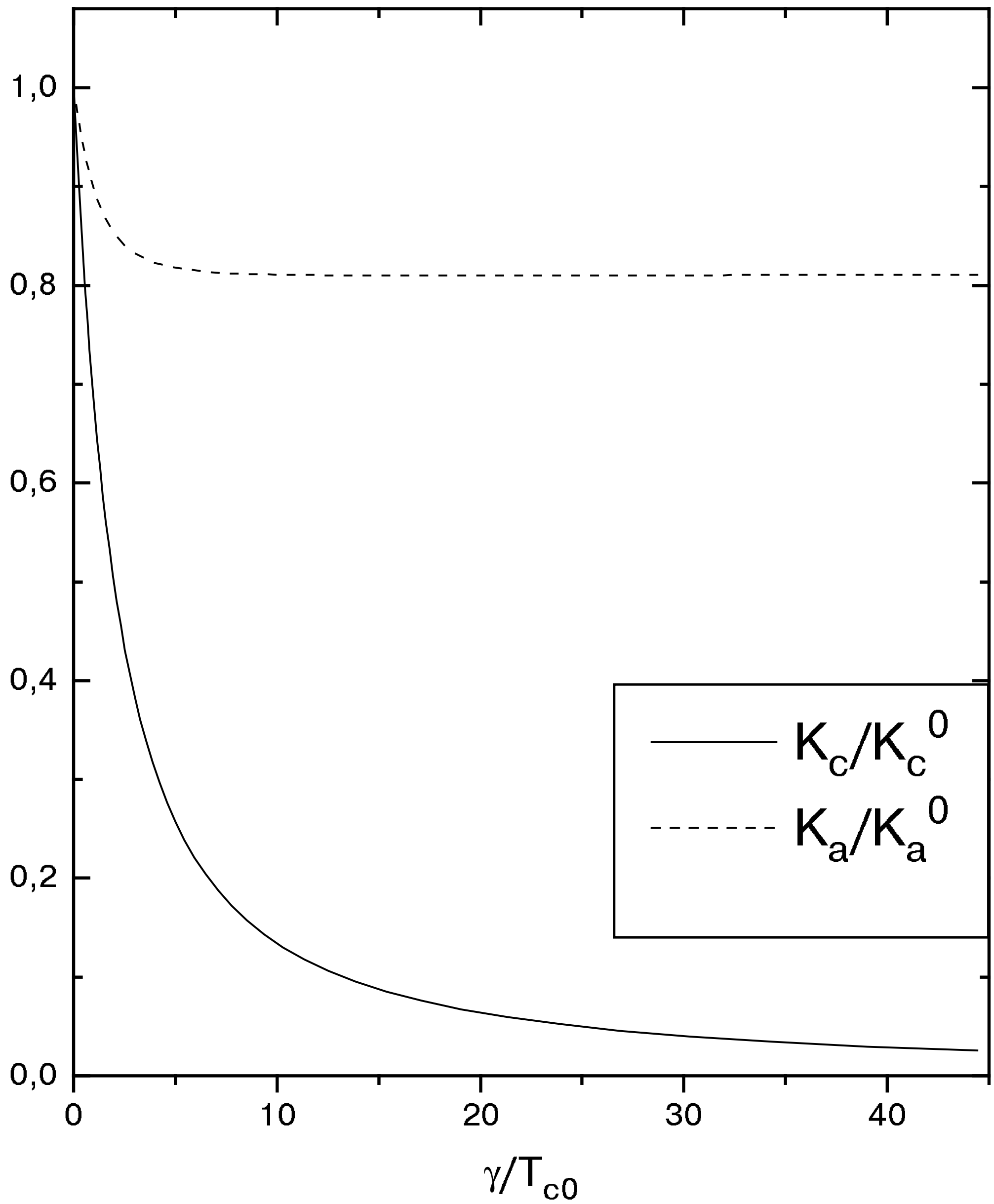


Fig. 4

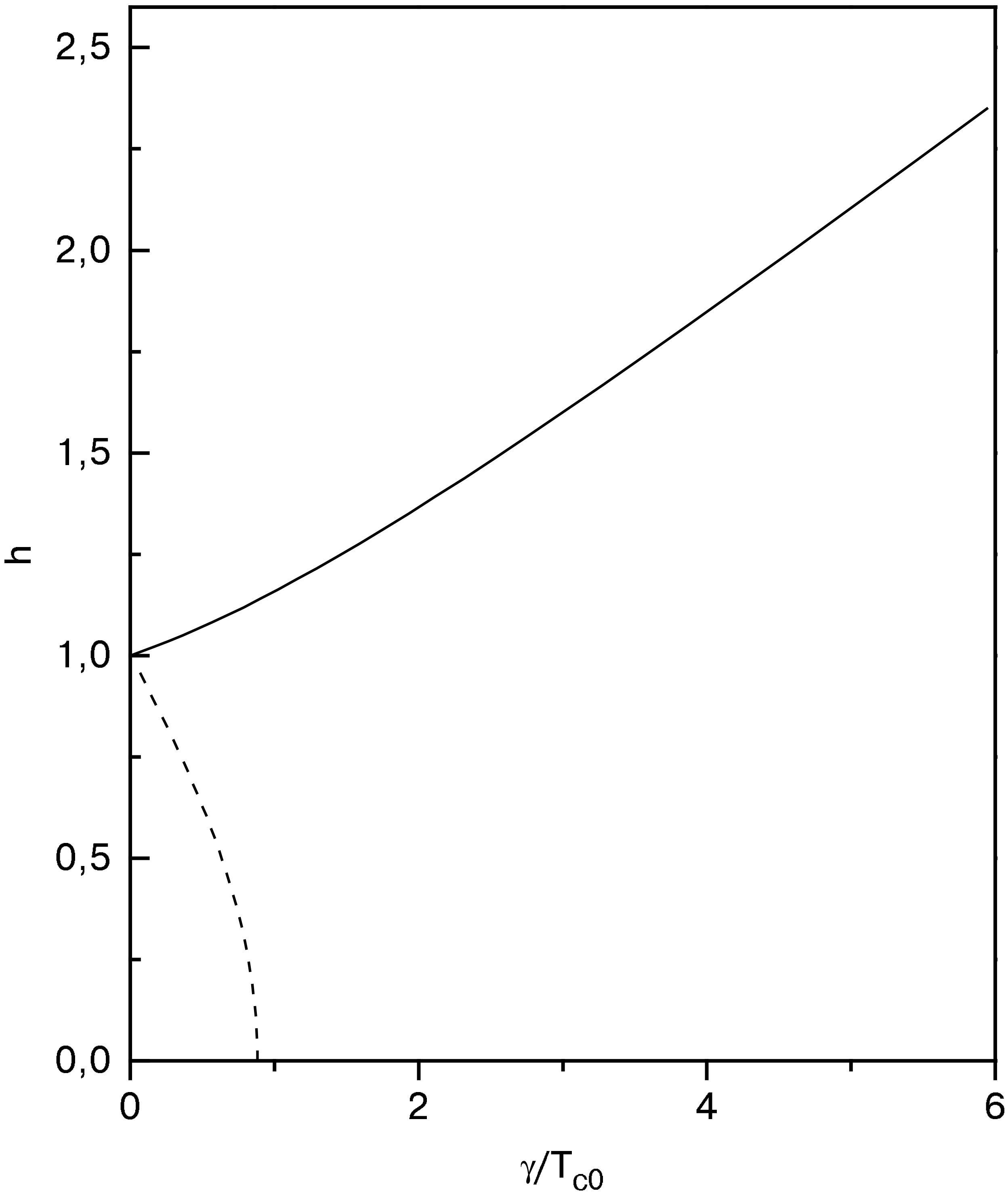


Fig. 5