Superconductivity Suppression Close to the Metal-Insulator Transition in Strongly Disordered Systems.

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Abstract

On the basis of the recently proposed self-consistent theory of metal- insulator transition in strongly disordered systems, taking into account interaction effects, we study transition temperature T_c suppression in disordered superconductors for the wide disorder interval — from weakly disordered metal up to Anderson insulator, induced by "Coulomb pseudogap" formation in the density of states. It is shown that for a number of systems this theory provides rather satisfactory fit of experimental data.

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The problem of degradation of superconducting transition temperature under strong disordering is relatively old [1]. It is closely connected with the question of superconductivity suppression due to disorder-induced metal-insulator transition [2]. A number of microscopic mechanisms of T_c suppression were proposed, such as the growth of Coulomb pseudopotential [3,4], the influence of Coulomb corrections to the density of states [5] etc. In the majority of papers only small corrections to T_c due to these effects were analyzed.

Recently we proposed [6,7] a theory of metal- insulator transitions which generalize the self-consistent theory of localization [8,9] taking into account the effects of electron-electron interaction. This approach has allowed us to study the behavior of the generalized diffusion coefficient for the wide interval of disorder parameter both for metallic and insulating regions. These results were used in calculations of one-particle density of states with the account of interelectron interactions. These calculations demonstrate the formation and the growth of the "Coulomb pseudogap" in the density of states close to the Fermi level. In metallic region this behavior of the density of states corresponds to the usual square-root Altshuler-Aronov correction [10]. As disorder parameter grows and system moves towards the metal-insulator transition this pseudogap deepens, while the effective region of square-root behavior diminishes, and at the point of the metal-insulator transition the density of states at the Fermi level becomes equal to zero — we obtain a kind of a "Coulomb gap". In the insulating region, for the band of the finite width, we obtain the typical quadratic behavior of the density of states close to the Fermi level, reminiscent of the Coulomb gap due to Efros and Shklovskii [11], widening with the further growth of disorder. Such behavior of the density of states is in qualitative agreement with experiments on the number of disordered systems close to the metal-insulator transition [1], from amorphous alloys [12,13,15,16] to disordered single-crystals of metallic oxides, including high-temperature superconductors [17]. In this paper the results of these calculations of the density of states are used for the numerical study of "Coulomb gap" effects on the T_c suppression for superconductors which are close to the metal-insulator transition.

We shall analyze superconductivity within the framework of the simplest BCS-model. In

the weak coupling approximation the linearized gap-equation takes the following form [2]:

$$\Delta(\xi) = -\int\limits_{-\infty}^{\infty} d\xi' V(\xi,\xi') N(\xi') \frac{1}{2\xi'} th(\frac{\xi'}{2T_c}) \Delta(\xi'), \tag{1}$$

where $N(\xi)$ - is the averaged on disorder density of states which includes the effects of electron-electron interaction, $V(\xi,\xi')$ - is the effective pairing interaction. The only difference with the standard approach is in the account of non-trivial dependence of $N(\xi)$ on the electron energy ξ, close to the Fermi level E_F.

In BCS theory we usually assume the existence of some effective electron- electron attraction, which is determined by the balance of pairing attraction due to electron-phonon interaction and Coulomb repulsion. Thus we consider the effective pairing interaction in the following simple form:

$$V(\xi,\xi') = V_c(\xi,\xi') + V_{ph}(\xi,\xi'), \tag{2}$$

where $V_c(\xi,\xi') = V_c\theta(E_F - |\xi|)\theta(E_F - |\xi'|)$ and $V_{ph}(\xi,\xi') = -V_{ph}\theta(\omega_D - |\xi|)\theta(\omega_D - |\xi'|)$ - are the respectively the electron-electron and electron-phonon interactions, ω_D - is the Debye frequency. The constants $V_c > 0$ and $V_{ph} > 0$ correspond to repulsion and attraction which effectively operate on rather different intervals of energy: $E_F \gg \omega_D$.

After using this expression in Eq.(1) and some transformations using the even-odd properties of the gap function $\Delta(\xi)$ we obtain:

$$\Delta(\xi) = [V_{ph}\theta(\omega_D - \xi) - V_c\theta(E_F - \xi)] \int\limits_0^{\omega_D} d\xi' N(\xi') \frac{1}{\xi'} th(\frac{\xi'}{2T_c}) \Delta(\xi') -$$
$$- V_c\theta(E_F - \xi) \int\limits_{\omega_D}^{E_F} d\xi' N(\xi') \frac{1}{\xi'} th(\frac{\xi'}{2T_c}) \Delta(\xi'). \tag{3}$$

We look for the solution of this equation in the usual two-step form [18]:

$$\Delta(\xi) = \begin{cases} \Delta_{ph}, & |\xi| < \omega_D, \\ \Delta_c, & \omega_D < |\xi| < E_F, \end{cases} \tag{4}$$

where Δ_{ph}, Δ_c - are some constants which are determined from the following system of homogeneous linear equations, which is obtained after the substitution of Eq.(4) into Eq.(3):

$$\{1-(V_{ph}-V_c)N_0(0)K(\frac{\omega_D}{2T_c})\}\Delta_{ph}+V_cN_0(0)[K(\frac{E_F}{2T_c})-K(\frac{\omega_D}{2T_c})]\Delta_c=0,$$
$$V_cN_0(0)K(\frac{\omega_D}{2T_c})\Delta_{ph}+\{1+V_cN_0(0)[K(\frac{E_F}{2T_c})-K(\frac{\omega_D}{2T_c})]\Delta_c=0, \quad (5)$$

where $N_0(0)$ - is the one-electron density of states of noninteracting electrons at the Fermi level and we introduce

$$K(\xi)=\int_0^{\xi}d\xi'\frac{1}{\xi'}th(\xi')\left[\frac{N(2T_c\xi')}{N_0(0)}\right]. \quad (6)$$

Equation for T_c follows from the usual zero-determinant condition for this homogeneous system:

$$(\lambda-\mu^*)K(\frac{\omega_D}{2T_c})=1,$$
$$\mu^*=\mu\{1+\mu[K(\frac{E_F}{2T_c})-K(\frac{\omega_D}{2T_c})]\}^{-1}, \quad (7)$$

where μ^* - is the Coulomb pseudopotential, $\mu=V_cN_0(0)$ - is the Coulomb constant, $\lambda=V_{ph}N_0(0)$ - is the pairing constant due to electron-phonon interaction. In the clean limit, when the density of states at the Fermi level is constant, this reduces to the usual BCS-expression for T_c.

Equation (7) for T_c was solved numerically for disorder parameter changing in wide interval both for metallic and insulating regions. The density of states was calculated taking into account lowest order corrections over electron-electron interaction [6,7]:

$$N(\xi)=-\frac{1}{\pi}Im\int\frac{d^3\mathbf{p}}{(2\pi)^3}G^R(\mathbf{p},\xi), \quad (8)$$

where $G^{R(A)}(\mathbf{p},\xi)=[\xi-\xi_p\pm i\gamma-\Sigma_{ee}^{R(A)}(\mathbf{p},\xi)]^{-1}$ - is the retarded (advanced) electron Green's function, $\Sigma_{ee}^{R(A)}(\mathbf{p},\xi)$ - is "Fock" contribution to electron self- energy [6,10]:

$$\Sigma_{ee}^{R(A)}(\mathbf{p},\xi)\approx 4i\gamma^2\mu N_0^{-1}(0)G_0^{A(R)}(\mathbf{p},\xi)\int_{\xi}^{\infty}\frac{d\omega}{2\pi}\int_{|\mathbf{q}|<k_0}\frac{d^3\mathbf{q}}{(2\pi)^3}\frac{1}{[-i\omega+D(\omega)q^2]^2}. \quad (9)$$

Here $D(\omega)$ - is the generalized diffusion coefficient, which is determined from the following self-consistent nonlinear integral equation [6,7]:

$$\frac{D(\omega)}{D_0} = 1 - \frac{1}{\pi N_0(0)} \frac{D(\omega)}{D_0} \int\limits_{|\mathbf{q}|<k_0} \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{1}{-i\omega + D(\omega)q^2} +$$

$$+ \frac{8i}{3\pi} \frac{\mu D_0}{\pi N_0(0)} \int\limits_{\omega}^{\infty} d\Omega \int\limits_{|\mathbf{q}|<k_0} \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{q^2}{(-i(\Omega+\omega) + D(\Omega+\omega)q^2)(-i\Omega + D(\Omega)q^2)^2}, \qquad (10)$$

where $D_0 = E_F/\ 3m\gamma$ - is the usual Drude diffusion coefficient, $\gamma = 1/2\tau$ - Born scattering rate, τ - mean free time, $k_0 = min\{p_F, l^{-1}\}$ - cut-off in the momentum space, p_F - Fermi momentum, l - mean free path. The data on static conductivity used below were also obtained by numerical solution of Eq.(10) [6,7].

In Fig.1 we show the behavior of the density of states close to the Fermi level which demonstrates the evolution of the "Coulomb pseudogap" as disorder grows. This behavior obviously leads to superconducting T_c suppression as the system moves towards the metal-insulator transition.

Fig.2 demonstrates T_c suppression as disorder parameter $(p_F l)^{-1}$ grows for different values of the Coulomb constant μ and fixed value of pairing constant λ. For large μ and growing disorder $(p_F l)^{-1}$ the value of T_c drops rather fast and becomes zero in metallic region far enough from metal-insulator transition. For smaller values of μ this drop of T_c with growing disorder $(p_F l)^{-1}$ becomes slower and for small μ and large enough λ (dashed curves) we get the possibility of superconductivity persisting even in the insulating region [2]. This possibility is clearly demonstrated at the insert in Fig.2, where we show the T_c dependence on the static conductivity of the system σ for the appropriate values of λ and μ. For large values of μ as conductivity σ drops T_c also drops and superconductivity is completely suppressed rather far from the metal-insulator transition. For small values of μ this drop of T_c with σ becomes slower and for sufficiently large values of λ (dashed curves) T_c remains finite even in the case of $\sigma \to 0$.

Fig.3 demonstrates T_c degradation with the growth of disorder parameter $(p_F l)^{-1}$ for different values of the pairing constant λ for the fixed value of the Coulomb constant μ. For small λ and disorder parameter $(p_F l)^{-1}$ growing the value of T_c drops rather fast and becomes zero in the metallic state far from the metal-insulator transition. As λ grows this

drop of T_c becomes slower and for large enough values of λ superconductivity is completely suppressed only somewhere in the insulating region. At the insert in Fig.3 we show the dependence of the Coulomb pseudopotential μ^* on the disorder parameter $(p_F l)^{-1}$ for the appropriate values of λ and μ demonstrating rather insignificant growth of μ^* with disorder $(p_F l)^{-1}$ close to the point where superconductivity is completely suppressed. Apparently this behavior is natural enough because we neglect all the processes renormalizing the matrix element of Coulomb interaction in Eq.(2) due Anderson localization and electron-electron interactions which can actually lead to rather important growth of Coulomb pseudopotential close to the metal-insulator transition [2].

This kind of behavior of T_c on the static conductivity σ and on disorder parameter was observed experimentally in a number of disordered systems, which remain superconducting close to the disorder induced metal- insulator transition [1], [2], [12] - [17], [19] - [21]. Our results agree rather well with experiments on amorphous alloys of InO_x [14], Nb_xSi_{1-x} [15,16], Au_xSi_{1-x} [19–21].

The authors of Ref. [14] had presented the results of the measurements of disorder parameter $(p_F l)^{-1}$ for the amorphous alloy of InO_x, as well as the data for T_c and static conductivity close to the metal- insulator transition. According to our work [6,7] the static conductivity close to the metal-insulator transition can be expressed as:

$$\sigma = \sigma_0[(p_F l)W_c(\mu) - 1], \tag{11}$$

where σ_0 - is some characteristic scale of conductivity close to the metal-insulator transition, $W_c(\mu)$ - the value of disorder parameter (depending on the Coulomb constant) corresponding to the point of metal- insulator transition. Approximating the experimental data for InO_x by Eq.(11) allows us to estimate the characteristic conductivity scale σ_0 and also, from the value of W_c, the Coulomb constant μ. Satisfactory correlation (Cf. the insert in Fig.3) are obtained for the following values: $\sigma_0 \simeq 324.95\ \Omega^{-1} \cdot cm^{-1}$, and $W_c \simeq 0.606$ giving $\mu \simeq 1.0$.

Fig.3 demonstrates the comparison of our results with experimental data on T_c dependence on static conductivity σ for the amorphous InO_x using the value of $T_{co} = 3.41\ K$,

$\omega_D = 112\ K$ and $E_F = 9.98 \cdot 10^4\ K$, $[\omega_D/E_F] \simeq 1.1 \cdot 10^{-3}$ - for pure In and the given above values of σ_0 and μ, which allows to estimate the pairing constant λ. Satisfactory agreement is obtained for $\lambda \simeq 0.45$. Dashed curves correspond to the values of $\lambda \simeq 0.4$ and 0.5.

Let us discuss now the results for T_c and static conductivity dependence on the Si content for amorphous alloys of Nb_xSi_{1-x} [15,16] and Au_xSi_{1-x} [19–21] close to the metal-insulator transition. Assuming that the disorder parameter in this case is just proportional to Si concentration, so that $(p_F l)^{-1} \sim 1 - x$, we can express Eq.(11) for static conductivity in the following form:

$$\sigma = \sigma_0 \frac{x - x_c}{1 - x}, \tag{12}$$

where x_c - the critical concentration of Nb or Au at the point of metal-insulator transition. Approximating the experimental data for conductivity in Nb_xSi_{1-x} Au_xSi_{1-x} by Eq.(12) allows us to estimate σ_0 and critical concentration x_c. Satisfactory correlation (Cf. inserts in Fig.5 and Fig.6) is obtained for:

Nb_xSi_{1-x}: $\sigma_0 \simeq 1963.9\ \Omega^{-1} \cdot cm^{-1}$, $x_c \simeq 0.115$;

Au_xSi_{1-x}: $\sigma_0 \simeq 2782.13\ \Omega^{-1} \cdot cm^{-1}$, $x_c \simeq 0.14$.

Fig.5 and Fig.6 present the comparison our results with the experimental data for T_c - dependence on conductivity σ for amorphous Nb_xSi_{1-x} and Au_xSi_{1-x}, using for the pure Nb: $T_{co} = 9.26\ K$, $\omega_D = 276\ K$ and $E_F = 6.18 \cdot 10^4\ K$, $[\omega_D/E_F] \simeq 3.0 \cdot 10^{-3}$; while for Au_xSi_{1-x} we assume $T_{co} = T_{cmax} \simeq 0.86\ K$, $\omega_D = 170\ K$ and $E_F = 6.42 \cdot 10^4\ K$, $[\omega_D/E_F] \simeq 0.9 \cdot 10^{-3}$ with the mentioned above values of σ_0, which allows us to estimate the pairing constant λ. Assuming for these systems the Coulomb constant $\mu \simeq 1$, the satisfactory agreement is obtained in case of Nb_xSi_{1-x} for $\lambda \simeq 0.54$ and for Au_xSi_{1-x} with $\lambda \simeq 0.62$.

Surely the results presented above are essentially based upon the simplest BCS-model and are probably oversimplified. More rigorous approach to calculations of T_c must be based upon Eliashberg equations and realistic models of electron-phonon interaction [18]. Especially this is important in case of large enough values of λ, which demonstrate the possibility of superconductivity persisting in the insulating phase. At the same time, in present paper

we were not concerned with the problem of the genesis of the initial value of T_{c0} in pure system, but were studying only the T_c dependence on disorder. In this sense our results may be also qualitatively valid also in the case of strong-coupling superconductivity. We must also note that the more rigorous analysis is also needed taking into account disorder effects in the matrix element of Coulomb repulsion, which lead to the additional mechanism of T_c degradation T_c [2–4]. In the present work we have only taken into account pseudogap effects in the density of states. It is possible that rather satisfactory agreement with experiments can signify the dominating role of pseudogap formation effects in the problem of T_c degradation under disordering, which was claimed (on the level of small corrections) already in Ref. [5].

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Figure Captions:

Fig.1. Density of states in the case of band of finite width $2E_F$ for $\frac{8}{3\pi}\mu = 1.0$ for the different values of disorder parameter $(p_F l)^{-1}$: 1 - 0.1,..., 5 - 0.5 - in metallic region, 7 - 0.7,..., 10 - 1.0 - in insulating region. Dashed curve - 6 corresponds to the point of the metal-insulator transition. Energy ε is in the units of $D_0 k_0^2$.

Fig.2. T_c degradation as a function of disorder parameter $(p_F l)^{-1}$ for the fixed value of the pairing constant λ ($\lambda = 0.5$ - full curves, $\lambda = 1.0$ - dashed curves) for the different values of Coulomb constant $\frac{8}{3\pi}\mu$: 1 - 0.2, ..., 5 - 1.0. At the insert: T_c dependence upon the static conductivity σ for the appropriate values of pairing constant λ and Coulomb constant μ.

Fig.3. T_c degradation as a function of disorder parameter $(p_F l)^{-1}$ for the fixed value of Coulomb constant $\frac{8}{3\pi}\mu = 0.4$ and different values of the pairing constant λ: 1 - 0.3, 2 - 0.4, ..., 8 - 1.0. At the insert: dependence of Coulomb pseudopotential μ^* on disorder parameter $(p_F l)^{-1}$ for the appropriate values of pairing constant λ and Coulomb constant μ. The arrow shows the point of the metal-insulator transition.

Fig.4. T_c dependence on conductivity σ for the amorphous alloys of InO_x. At the insert: approximation of conductivity dependence upon the disorder parameter $(p_F l)^{-1}$.

Fig.5. T_c dependence on conductivity σ for the amorphous alloys of $Nb_x Si_{1-x}$. At the insert: approximation of conductivity dependence upon concentration of Nb.

Fig.6. T_c dependence on conductivity σ for the amorphous alloys of $Au_x Si_{1-x}$. At the insert: approximation of conductivity dependence upon concentration of Au.

REFERENCES

[1] Belitz D., Kirkpatrik T.R. Rev. Mod. Phys., **66**, 261, (1994).

[2] Sadovskii M.V. Superconductivity: Physics, Chemistry, Technology, **8**, 337, (1995); Physics Reports, (1996), (in press).

[3] Anderson P.W., Muttalib K.A., Ramakrishnan T.V . Phys. Rev., **B28**, 117, (1983).

[4] Bulaevskii L.N., Sadovskii M.V. J.Low-Temp.Phys., **59**, 89, (1985).

[5] Belitz D., at all. Phys. Rev., **B40**, 111, (1989).

[6] Kuchinskii E.Z., Sadovskii M.V., Suvorov V.G., Erkabaev M.A. Zh.Eksp.Teor.Fiz.(JETP), **107**, 2027, (1995).

[7] Kuchinskii E.Z., Erkabaev M.A. Fiz.Tverd.Tela (Solid State Physics), (1997), (in press).

[8] Vollhardt D., Wolfle P. In "Electronic Phase Transitions." Ed. by Hanke W., Kopaev Yu., North-Holland, Amsterdam, p.1, (1992).

[9] Sadovskii M.V. Sov. Sci. Rev., Phys., **A7**, 1, (1986).

[10] Altshuler B.L., Aronov A.G. Zh.Eksp.Teor.Fiz.(JETP), **77**, 2028, (1979).

[11] Efros A.L., Shklovskii B.I. - In "Electron-Electron Interaction in Disordered Systems." Ed. by Pollak M., Efros A.L., North-Holland, Amsterdam, p.409, (1984).

[12] McMillan W.L., Mochel J. Phys. Rev. Lett., **46**, 556, (1981).

[13] Imry Y., Ovadyahu Z. Phys. Rev. Lett., **49**, 841, (1982).

[14] Fiory A.T.,Hebard A.F. Phys. Rev. Lett., **52**, 2057, (1984).

[15] Hertel G., Bishop D.J., Spencer E.G., Rowell J.M., Dynes R.C. Phys. Rev. Lett., **50**, 743, (1983).

[16] Bishop D.J., Spencer E.G., Dynes R.C. Sol. Stat. Elect., **28**, 73, (1985).

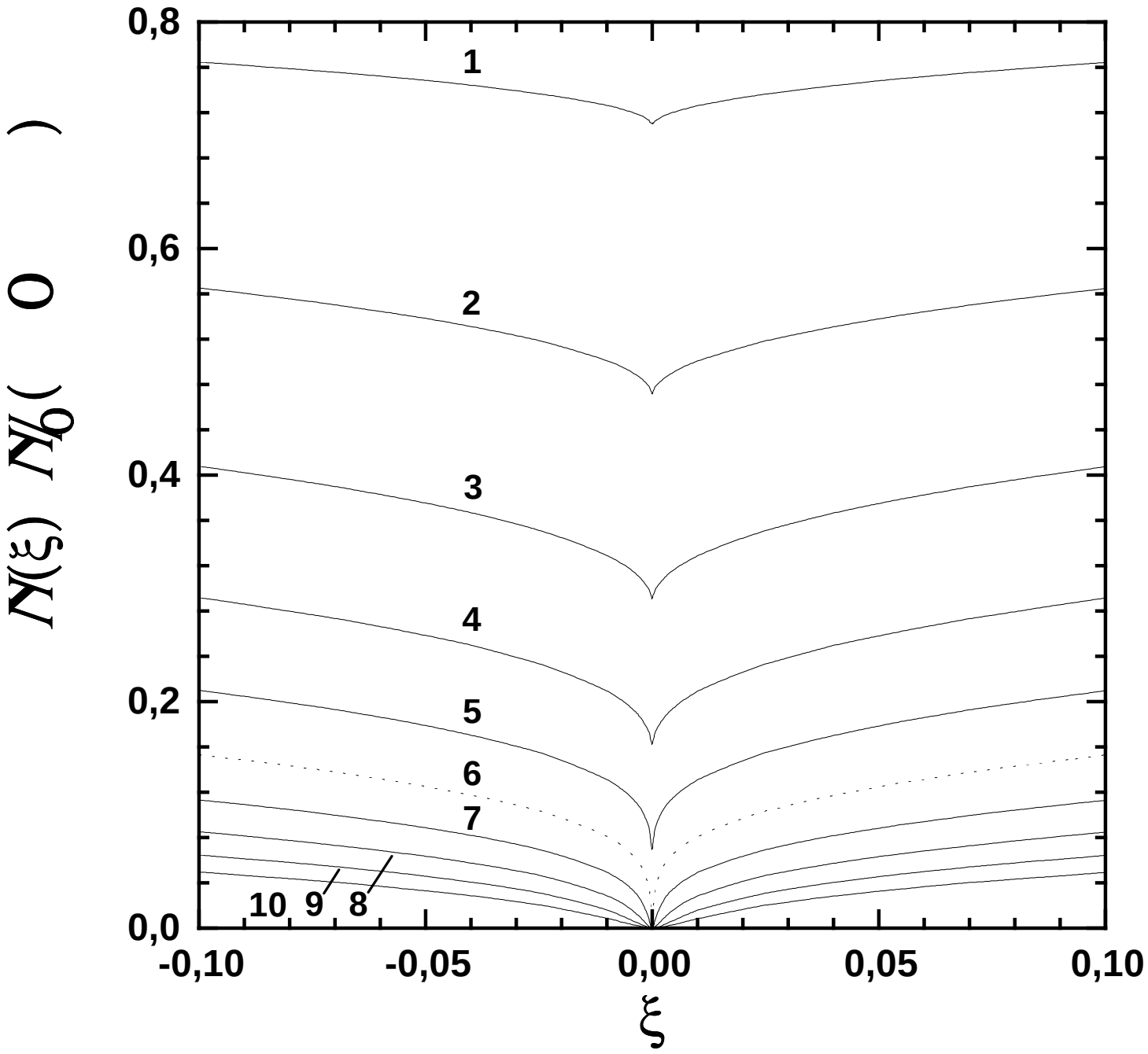
[17] Srikanth H., Rajeev K.P., Shivashankar G.V., Raychaudhuri A.K.
Physica, **C195**, 87, (1992).

[18] Vonsovsky S.V., Izyumov Yu.A., Kurmaev E.Z. Superconductivity of Transition Metals, Their Alloys and Compounds, (Springer Series in Solid State Sciences, Vol.27), Springer Verlag, 1982.

[19] Nishida N., Yamaguchi M., Furubayashi T., Morigaki M., Ishimoto H., Ono K.
Sol. Stat. Comm., **44**, 305, (1982).

[20] Furubayashi T., Nishida N., Yamaguchi M., Morigaki M., Ishimoto H.
Sol. Stat. Comm., **55**, 513, (1985).

[21] Nishida N., Furubayashi T., Yamaguchi M., Morigaki M., Ishimoto H.
Sol. Stat. Elect., **28**, 81, (1985).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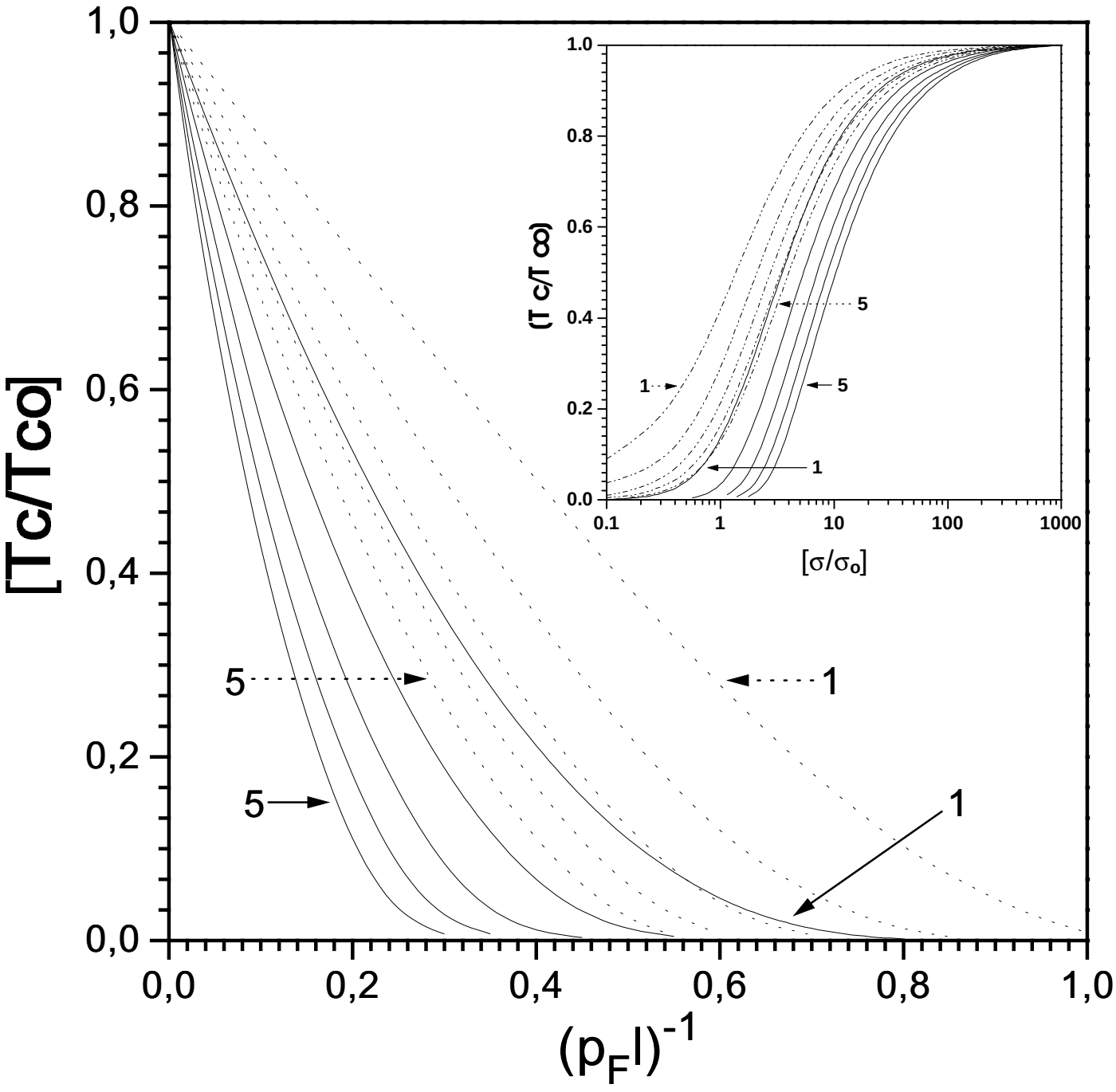
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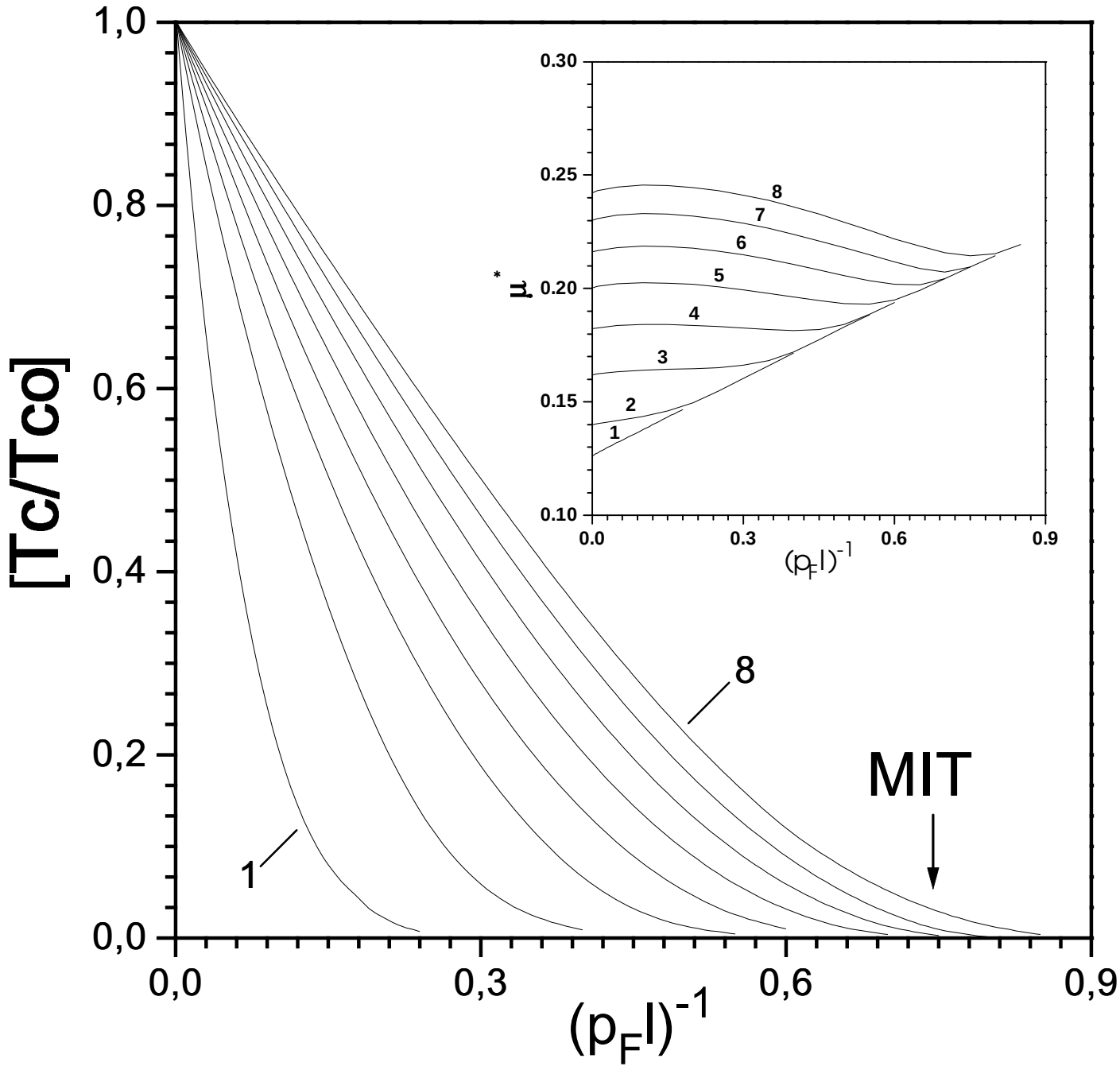
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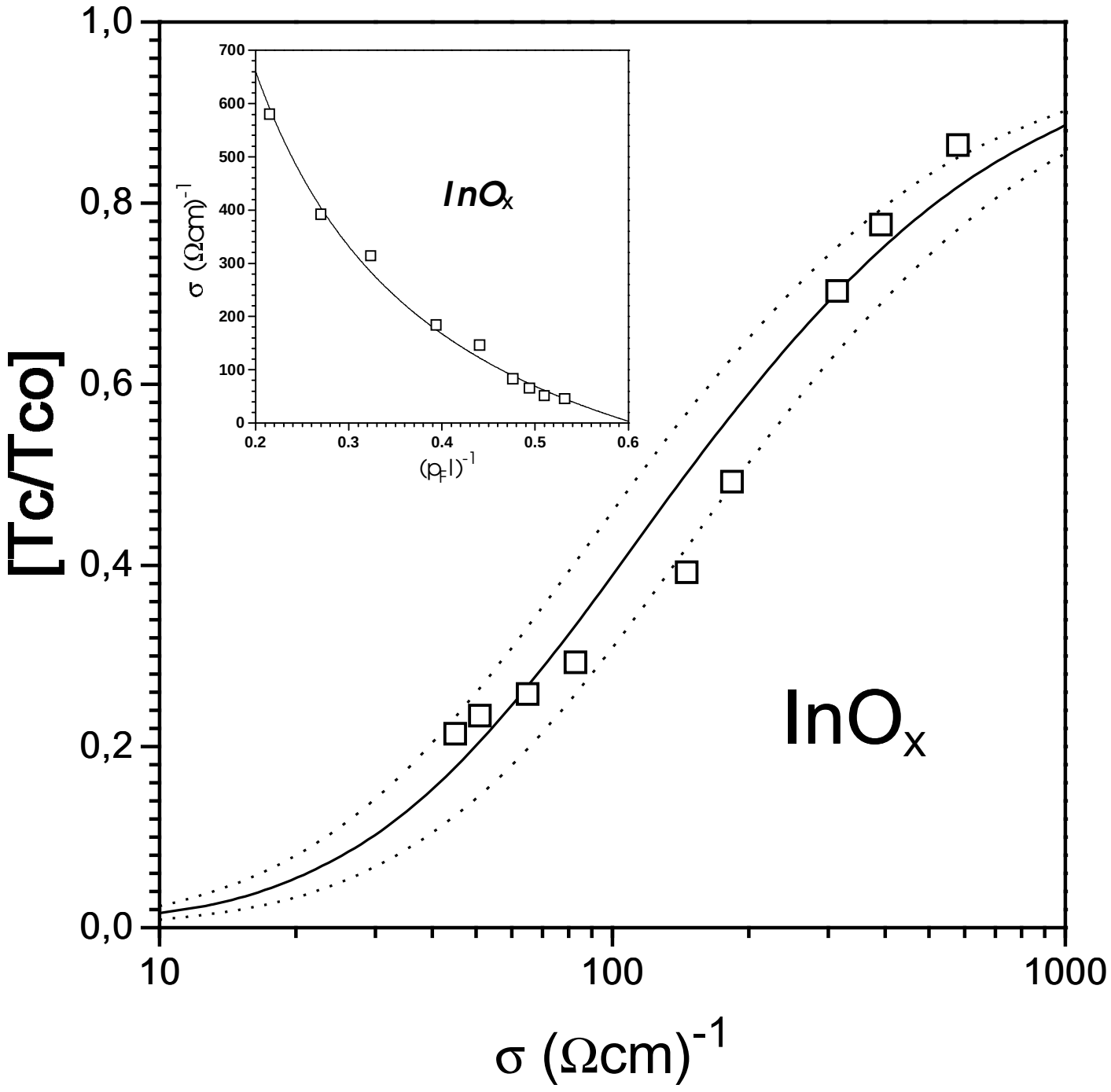
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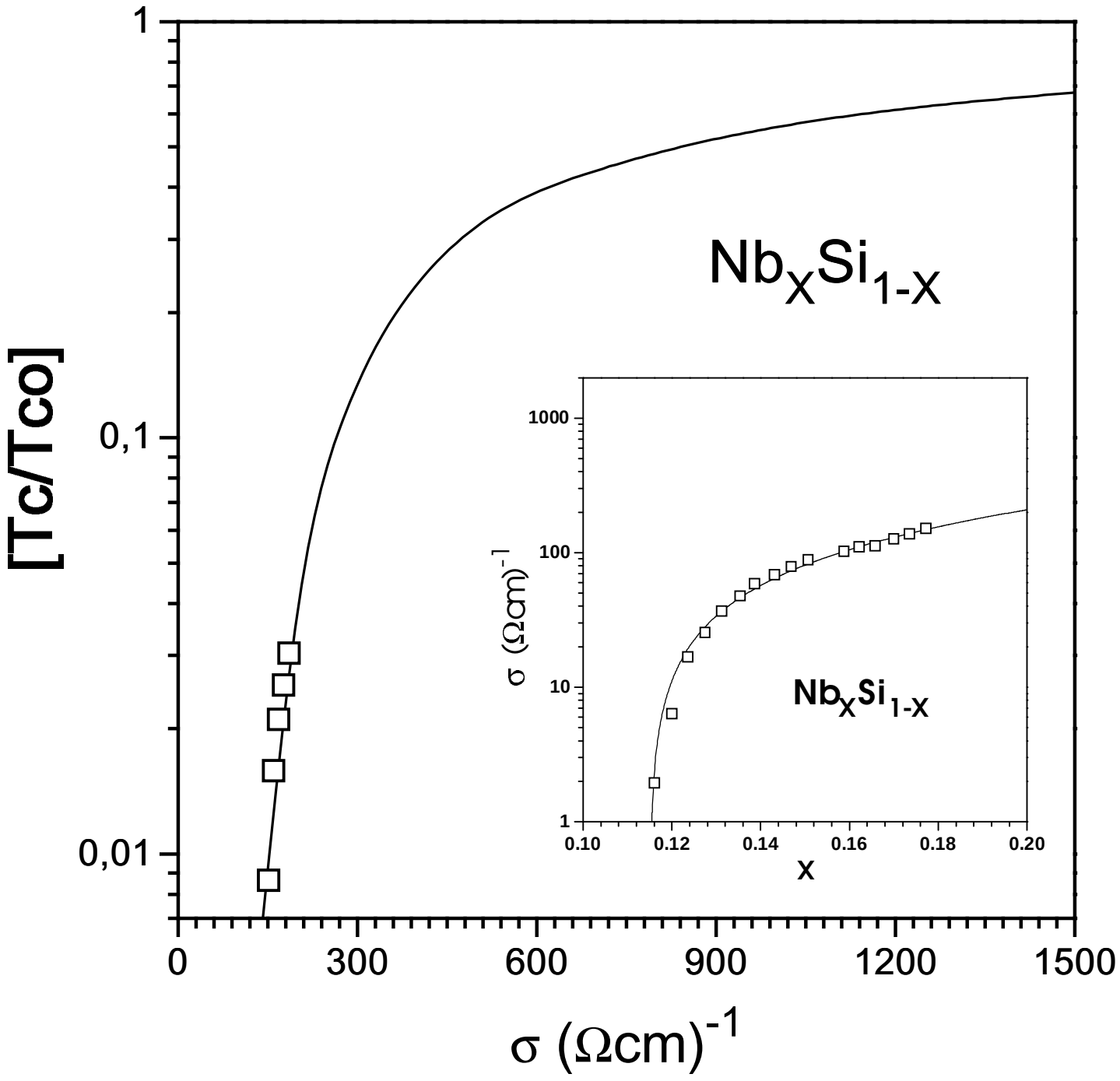
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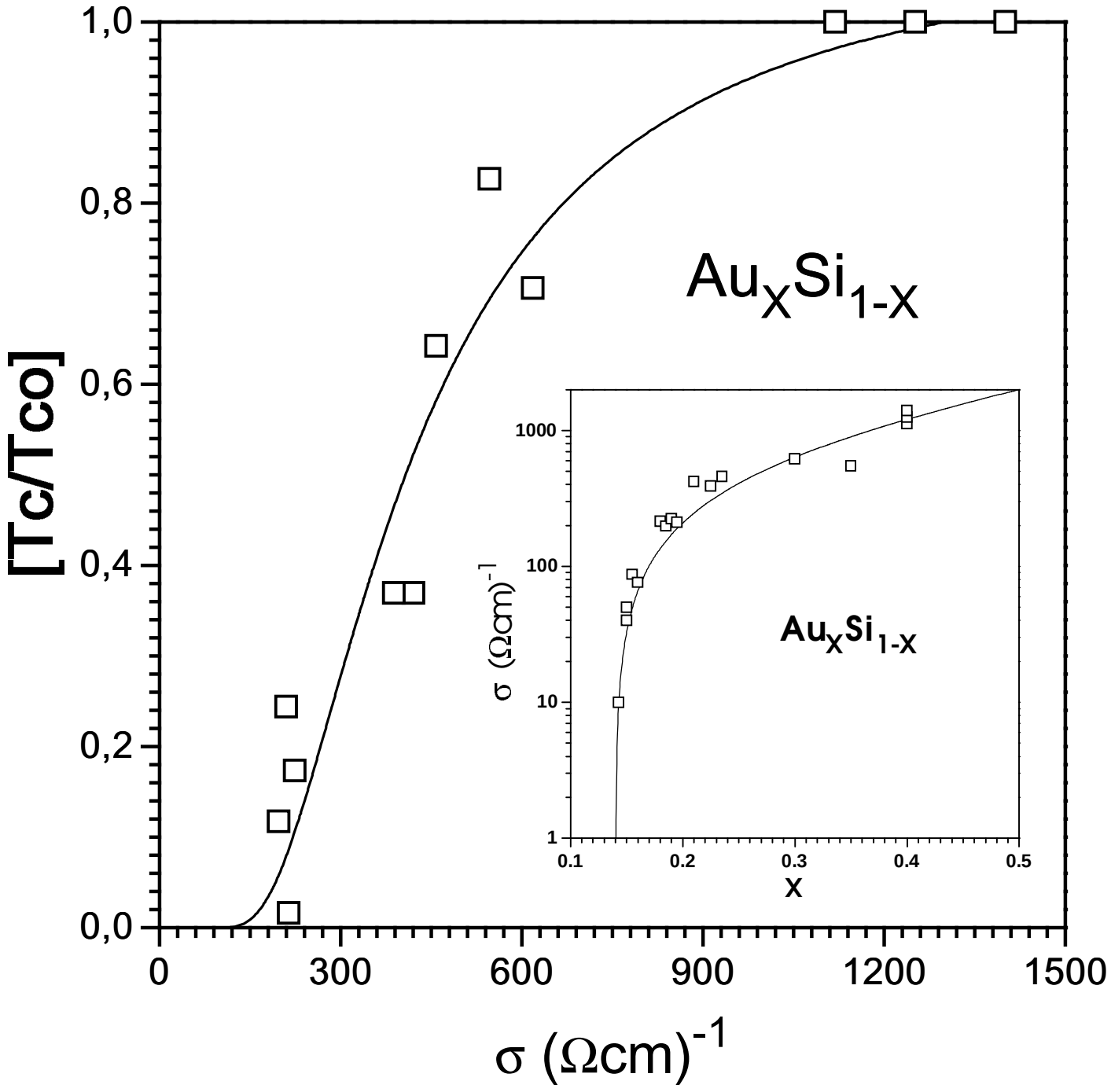
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