Disorder Effects in Superconductors with Anisotropic Pairing: From Cooper Pairs to Compact Bosons

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Abstract

In the weak coupling BCS-approximation normal impurities do not influence superconducting T_c in significant manner in case of isotropic s-wave pairing. However, in case of d-wave pairing these are strongly pair-breaking. This fact is in rather strong contradiction with many experiments on disordered high-T_c superconductors assuming the d-wave nature of pairing in these systems. With the growth of electron attraction within the Cooper pair the system smoothly crosses over from BCS-pairs to compact Boson picture of superconductivity. As pairing strength grows and pairs become compact significant deviations from universal Abrikosov-Gorkov dependence of T_c on disorder appear in case of d-wave pairing with superconducting state becoming more stable than in the weak coupling case. As high-T_c superconductors are actually in the intermediate region with Cooper pairs size of the order of few interatomic lengths, these results can explain the relative stability of d-wave pairing under rather strong disordering.

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It is well known that in the usual weak-coupling BCS-approximation normal impurities do not influence superconducting T_c in case of isotropic s-wave pairing (Anderson theorem) [1]. In case of the so called anisotropic s-wave pairing T_c reduction due to disorder is also relatively weak [2,3]. However in case of d-wave pairing normal impurities are strongly pair-breaking [2–4] and the universal dependence of T_c on disorder is expressed by the famous Abrikosov-Gorkov equation:

$$ln\left(\frac{T_{c0}}{T_c}\right) = \left[\Psi\left(\frac{1}{2} + \frac{\gamma}{2\pi T_c}\right) - \Psi\left(\frac{1}{2}\right)\right] \tag{1}$$

where $\Psi(x)$ is digamma function, $\gamma = \pi n_{imp} v^2 N(E_F)$ is the usual scattering rate of electrons, due to impurities with point-like potential v, which are chaotically distributed in space with some density n_{imp}, $N(E_F)$ - density of states at the Fermi level E_F. From Eq.(1) it follows directly that T_c is completely suppressed at some critical scattering rate $\gamma = 0.88T_{c0}$, which determines the appropriate critical impurity concentration or residual resistivity of the normal state

$$\rho_{AG} = \frac{2m\gamma_c}{ne^2} = \frac{8\pi\gamma_c}{\omega_p^2} \tag{2}$$

where n and m are electron concentration and mass, ω_p is plasma frequency of electrons [4].

At present there is an emerging consensus on the d-wave nature of the pairing state in high-temperature superconducting copper oxides [5]. However the scale of the critical scattering rate of $\gamma_c \sim T_{c0}$ is in rather strong contradiction with the large amount of data on disorder suppression of T_c in these systems [6], which apparently demonstrate superconducting state being conserved up to disorder induced metal-insulator transition, i.e $\gamma \sim E_F \gg T_{c0}$. The aim of the present report is to propose some possible explanation of this discrepancy.

Consider the (opposite to the usual BSC-picture) limit of extremely strong pairing interaction, leading to compact Boson formation [7]. In this case T_c is determined by the temperature of Bose condensation of free Bosons. In case of impure system condensation point can be determined by the following equation [8]:

$$\mu_p - \Sigma(0) = 0 \tag{3}$$

where μ_p is the chemical potential of pairs and $\Sigma(0)$ is the zero-frequency limit of Boson self-energy due to impurity scattering, which in the weak scattering approximation reduces to the one-loop expression, corresponding to diagram shown in Fig.1:

$$\Sigma(\varepsilon_n) = n_{imp}v^2 \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{i\varepsilon_n - \frac{\mathbf{p}^2}{2m^\star} + \mu_p} \tag{4}$$

where $\varepsilon_n = 2\pi nT$ is the even Matsubara frequency, $m^\star = 2m$ is the mass of the pair, and we assume temperatures $T > T_c$. In the following we consider only three-dimensional systems. Direct calculations give:

$$\Sigma(0) = Re\tilde{\Sigma}(0) + E_{0c} \tag{5}$$

where $E_{0c} = -\frac{m^\star}{\pi^2} n_{imp}v^2 p_0$ is the band-edge shift due to impurity scattering [9] (p_0 - is some cut-off in momentum space of the order of inverse lattice spacing a^{-1}) and

$$Re\tilde{\Sigma}(0) = \frac{1}{\sqrt{2}\pi} n_{imp}v^2 m^{\star 3/2}\sqrt{|\mu_p|} \tag{6}$$

Actually, E_{0c} leads just to renormalization of the chemical potential: $\tilde{\mu} = \mu_p - E_{0c}$, so that in renormalized form Eq.(3) reduces to:

$$\tilde{\mu}\left(1 - \frac{1}{\sqrt{2|\tilde{\mu}|}\pi} n_{imp}v^2 m^{\star 3/2} sign\tilde{\mu}\right) = 0 \tag{7}$$

with the only relevant ($\tilde{\mu} < 0$ for Bosons at $T > T_c$) solution of $\tilde{\mu} = 0$, i.e. $\mu_p - E_{0c} = 0$, determining the Bose condensation temperature of the impure system by the standard equation:

$$\frac{n}{2} = g \int_{-\infty}^{\infty} d\varepsilon N(\varepsilon) \frac{1}{e^{\frac{\varepsilon}{T_c}} - 1} \tag{8}$$

where $g = 2s+1$ (for Bosons of spin s), $N(\varepsilon)$ is the impurity averaged density of states, which in case of the simplest approximation of Eq.(4) just reduces to $N(E - E_{0c})$ - the usual free particle expression with energy ε calculated with respect to the shifted band-edge. Obviously we obtain the standard expression for T_c [10]:

$$T_c = \frac{3.31}{g^{2/3}} \frac{(n/2)^{2/3}}{m^\star} \tag{9}$$

which is *independent of disorder*. The only possible disorder effect may be connected with exponentially small "Lifshits tail" in the density of states in Eq.(8) due to localization [11], which is neglected in our simplest approximation of Eq.(4). Thus, our conclusion is that in case of extremely strong pairing interaction (compact Boson picture of superconductivity) T_c is practically disorder independent for *any* value of the spin of Cooper pair, e.g. s-wave, d-wave etc.

It was shown rather long ago by Nozieres and Schmitt-Rink [7] for non impure superconductor that as the strength of the pairing interaction grows, there is a smooth crossover of T_c from the weak-coupling BCS-picture to that of compact Bosons. In the impure case similar analysis for T_c can be performed solving the following coupled system of equations generalizing similar equations of Ref. [7] — the usual equation for BCS instability:

$$1 - \chi(0,0) = 0 \tag{10}$$

and the equation for Fermion density (chemical potential of electrons μ):

$$\frac{1}{2}(n - n_f) = \int \frac{d^3\mathbf{q}}{(2\pi)^3} \int \frac{d\omega}{\pi} \frac{1}{exp(\frac{\omega}{T_c}) - 1} \frac{\partial}{\partial\mu}\delta(\mathbf{q}\omega) \tag{11}$$

where $n_f(\mu, T_c)$ is the free Fermion part of density,

$$\delta(\mathbf{q}\omega) = arctg\frac{Im\chi(\mathbf{q}\omega)}{1 - Re\chi(\mathbf{q}\omega)}, \tag{12}$$

and Cooper susceptibility $\chi(\mathbf{q}\omega)$ is determined by diagrams shown in Fig.2. In this figure the vertices contain the symmetry factors for different types of pairing, e.g. in case of cubic lattice [12]:

$$\begin{aligned} \psi_s(\mathbf{p}) = 1 \qquad & \text{(isotropic } s\text{-wave)} \\ \psi_{s'}(\mathbf{p}) = \cos p_x a + \cos p_y a + \cos p_z a \qquad & \text{(anisotropic } s\text{-wave)} \\ \psi_{d_{x^2-y^2}}(\mathbf{p}) = \cos p_x a - \cos p_y a \qquad & (d\text{-wave}) \\ \psi_{d_{3z^2-r^2}}(\mathbf{p}) = 2\cos p_z a - \cos p_x a - \cos p_y a \qquad & \text{etc.} \end{aligned} \tag{13}$$

Pairing interaction is assumed to have the following form:

$$V_i(\mathbf{p},\mathbf{p}') = V_{\mathbf{p}\mathbf{p}'}\psi_i(\mathbf{p})\psi_i(\mathbf{p}') \tag{14}$$

with $\psi_i(\mathbf{p})$ defined as above and pairing potential

$$V_{\mathbf{p}\mathbf{p}'} = -\frac{V_0}{\sqrt{\left(1+\frac{p^2}{p_0^2}\right)\left(1+\frac{p'^2}{p_0^2}\right)}} \tag{15}$$

similar to that used in Ref. [7] with $p_0 \sim a^{-1}$.

Numerical work required to solve Eqs.(10),(11) is very heavy even for non impure case [7]. However, it is clear that these equations will produce also the smooth crossover in T_c dependence on disorder, interpolating between the BCS and compact Boson limits discussed above. In isotropic s-wave case T_c will remain practically independent from disorder, i.e. the Anderson theorem remains valid also for compact Boson limit. In case of d-wave pairing the universal dependence of T_c on disorder defined by Eq.(1) ceases to be valid in the crossover region from large Cooper pairs to compact Bosons. The physical reason for this is quite clear — depairing mechanism of T_c suppression by disorder ceases to operate with the growth of attractive interaction within pairs, and in the strong coupling region T_c is determined by Bose condensation of pairs in impure system. Qualitative behavior of T_c dependence on disorder is shown in Fig.3. It illustrates the smooth crossover in T_c dependence on normal state resistivity from universal Abrikosov-Gorkov dependence (curve d) to T_c independent on disorder (curve s). Dashed lines correspond to transition region and the values of coupling constant V_0 growing from curve 1 to curve 2. It is clear that for d-wave system belonging to this transitional region we can easily obtain superconducting state persisting for rather large disorder with $\rho > \rho_{AG}$.

Crossover region is qualitatively defined by the simple inequality introduced in Ref. [13]: $\pi^{-1} < p_F\xi < 2\pi$, where p_F is Fermi momentum and ξ is superconducting coherence length. It appears that high-temperature superconductors lie on the the so-called Uemura plot [14] near the "instability" line $p_F\xi = 2\pi$ [13]. This can explain deviations of T_c dependence on disorder in these systems from universal Abrikosov-Gorkov curve and their relative stability to disordering [6], despite the possible d-wave symmetry of the pairing state.

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Figure Captions:

Fig.1. One-loop Boson self-energy due to random impurity scattering.

Fig.2. (a) Diagram representation of Cooper susceptibility $\chi(\mathbf{q}\omega)$. V — pairing potential. Γ — impurity scattering vertex-part in Cooper channel, defined by the "ladder" approximation (b).

Fig.3. Qualitative dependence of transition temperature T_c on disorder (normal state residual resistivity ρ). Curve d — universal Abrikosov-Gorkov dependence of Eq.(1). Curve s — the case of isotropic s-wave pairing. Dashed curves — d-wave pairing in crossover region from BCS pairs to compact Bosons.

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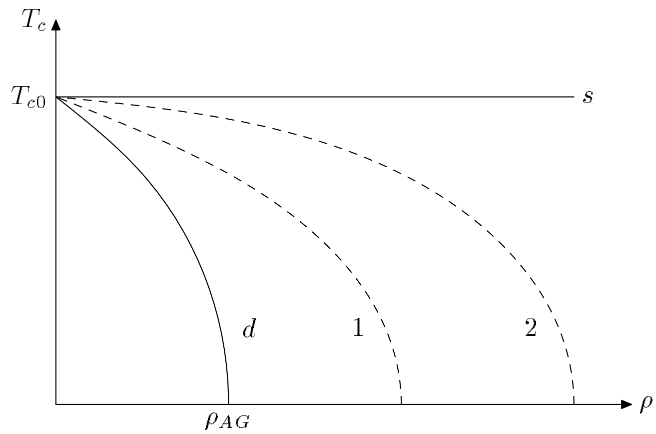
T_c
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1
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