

Superconductivity in a Toy Model of the Pseudogap State

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Abstract

We analyze superconducting state (both s and d – wave) in a simple exactly solvable model of pseudogap state, induced by short – range order fluctuations (e.g. antiferromagnetic), which is based upon model Fermi – surface with “hot patches”. It is shown that superconducting energy gap averaged over these fluctuations is non – zero even for the temperatures larger than mean – field T_c of superconducting transition in a sample as a whole. For temperatures $T > T_c$ superconductivity apparently exists within separate regions (“drops”). We study the spectral density and the density of states and demonstrate that superconductivity signals itself in these already for $T > T_c$, while at T_c itself nothing special happens from this point of view. These anomalies are in qualitative agreement with a number experiments on underdoped cuprates.

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I. INTRODUCTION

Among different anomalies of electronic properties of high – temperature superconductors especially interesting are those of the pseudogap state observed, mainly, in the underdoped region of the phase diagram [1,2]. Pseudogap anomalies are seen in a number of experiments, such as optical conductivity, NMR, inelastic neutron scattering, angle – resolved photoemission (ARPES) etc. [1]. Especially striking evidence of the existence of this state is observed in ARPES – experiments [1,3], which demonstrate essentially anisotropic changes of the spectral density of current carriers in a wide temperature region both in normal and superconducting state of these systems. Maximum of these anomalies is observed around $(\pi, 0)$ point of the Brillouin zone, while in the direction of its diagonal (close to (π, π) – point) these are practically absent. Qualitatively speaking these anomalies signify the complete “destruction” of the Fermi surface in the vicinity of $(\pi, 0)$ – point, while Fermi liquid behavior is conserved in the direction of diagonal. In this sense it is usually stated that the pseudogap possesses “ d – wave” symmetry, similar to that of the superconducting gap in these compounds [1–3]. At the same time, the fact that pseudogap anomalies are observed up to temperatures $T \sim T^*$ which are significantly larger than superconducting T_c , can be an evidence for quite different nature of these anomalies, not connected to superconducting pairing. This conclusion may be further supported by the fact that pseudogap state is observed mainly in underdoped cuprates, i.e. for compositions which are close to antiferromagnetic phase.

There are two major approaches to theoretical understanding of the pseudogap state of high – temperature superconductors. One is based upon rather popular idea of Cooper pair formation above the temperature of superconducting transition [2,4–7]. The other assumes that the pseudogap state is somehow induced by fluctuations of antiferromagnetic short – range order (see e.g. [8–12]).

The majority of theoretical works are devoted to the study of pseudogap state in the normal phase of cuprates at $T > T_c$. In a recent paper [13] a very simple exactly solvable model of the pseudogap state was proposed, based upon the picture of “hot” (nesting) patches on the Fermi surface. Within this model a derivation of Ginzburg – Landau expansion was given for different types of Cooper pairing with qualitative analysis of pseudogap effects (induced by AFM short – range order fluctuations) on the main superconducting properties close to T_c . The present work extends this model to the study of anomalies of superconducting state within the pseudogap for all temperatures $T < T_c$.

II. MODEL OF THE PSEUDOGAP STATE.

We shall consider a greatly simplified model of the pseudogap state [13], which is based on the idea of well – developed fluctuations of antiferromagnetic (AFM, SDW) short – range order ¹, which is qualitatively similar to the “hot spots” model of Ref. [11]. We assume that

¹Note that our analysis can also be applied to the case of CDW short – range order and other similar models.

the Fermi surface of two – dimensional system of electrons has the form shown in Fig. 1. Similar Fermi surface was in fact observed in a number of ARPES experiments on cuprate superconductors (e.g. see quite recent papers [14,15]). Fluctuations of short – range order are assumed to be static and Gaussian with the following correlation function: (cf. [8]):

$$S(\mathbf{q}) = \frac{1}{\pi^2} \frac{\xi^{-1}}{(q_x - Q_x)^2 + \xi^{-2}} \frac{\xi^{-1}}{(q_y - Q_y)^2 + \xi^{-2}} \quad (1)$$

where ξ - correlation length of these fluctuations, and the scattering vector is taken to be either $Q_x = \pm 2k_F$, $Q_y = 0$ or $Q_y = \pm 2k_F$, $Q_x = 0$. We also assume that these fluctuations interact only with electrons from the flat (“hot”) patches on the Fermi surface shown in Fig. 1, and this scattering is in fact of one – dimensional nature. The effective interaction with fluctuations will be described by the value of $(2\pi)^2 W^2 S(\mathbf{q})$, where parameter W of dimensions of energy is defining the energy scale (width) of the pseudogap ². The choice of the scattering vector $\mathbf{Q} = (\pm 2k_F, 0)$ or $\mathbf{Q} = (0, \pm 2k_F)$ implicitly assumes that fluctuations are incommensurate (generalization to the commensurate case is also possible [13], but we shall not discuss it here).

In the limit of $\xi \rightarrow \infty$ this model allows an exact solution by methods proposed (for one – dimensional case) in Ref. [16]. For the case of finite ξ a “nearly” exact solution may be constructed [11,12], directly generalizing a one – dimensional Ansatz of Refs. [17,18]. In this work we consider only maximally simplified variant of this model with $\xi \rightarrow \infty$, when effective interaction with fluctuations (1) takes the simplest possible form ³:

$$(2\pi)^2 W^2 \{ \delta(q_x \pm 2p_F) \delta(q_y) + \delta(q_y \pm 2p_F) \delta(q_x) \} \quad (2)$$

In this case we can easily sum all diagrams of the perturbation series for an electron scattered by these fluctuations [16] and obtain one – particle Green’s function in the following form [13]:

$$G(\epsilon_n, p) = \int_0^\infty dD \mathcal{P}(D) \frac{i\epsilon_n + \xi_p}{(i\epsilon_n)^2 - \xi_p^2 - D(\phi)^2}, \quad (3)$$

where $\xi_p = v_F(|\mathbf{p}| - p_F)$ (v_F - Fermi velocity), $\epsilon_n = (2n + 1)\pi T$, and fluctuating dielectric gap $D(\phi)$ is different from zero only on the “hot” patches:

$$D(\phi) = \begin{cases} D & , 0 \leq \phi \leq \alpha, \frac{\pi}{2} - \alpha \leq \phi \leq \frac{\pi}{2} \\ 0 & , \alpha \leq \phi \leq \frac{\pi}{2} - \alpha \end{cases} \quad (4)$$

where $\alpha = \arctg(\frac{p_y^0}{p_x^0})$, ϕ - is polar angle, defining the direction of vector \mathbf{p} in (p_x, p_y) – plane. For other values of ϕ the value of $D(\phi)$ is defined similarly to (4) by obvious symmetry considerations.

²We can say that we introduce an effective electron – fluctuations coupling constant of the form: $W_{\mathbf{p}} = W[\theta(p_x^0 - p_x)\theta(p_x^0 + p_x) + \theta(p_y^0 - p_y)\theta(p_y^0 + p_y)]$.

³Let us stress that due to the Gaussian nature of fluctuations the limit of $\xi \rightarrow \infty$ does not assume any kind of long – range order in the system.

The amplitude of dielectric gap D is random and distributed according to Rayleigh [17] (its phase is also random and distributed homogeneously on the interval $(0, 2\pi)$):

$$\mathcal{P}(D) = \frac{2D}{W^2} \exp\left(-\frac{D^2}{W^2}\right) \quad (5)$$

Thus on the “hot” patches this Green’s function has the form of “normal” Gor’kov’s function, averaged over fluctuations of dielectric gap D , distributed according to (5). “Anomalous” Gor’kov’s functions on these “dielectrized” patches are equal to zero (due to random phases of dielectric gap D) in accordance with the absence of any long – range order. However, the average values of the pairs of these functions are non zero and contribute to the two – particle Green’s function [16,13]. Varying α in (4) within the interval $0 \leq \alpha \leq \frac{\pi}{4}$, we actually change the size of “hot” patches on the Fermi surface where the nesting condition $\xi_{p-Q} = -\xi_p$ is fulfilled. In particular, the value of $\alpha = \pi/4$ corresponds to the square – like Fermi surface. Outside “hot” patches (second inequality in (4)) the Green’s function (3) just coincides with that of free electrons.

Our results for electronic density of states and spectral density following from (3) were given in Ref. [13] and demonstrated the the pseudogap of characteristic width of $\sim 2W$ as well as non-Fermi liquid behavior on the “hot” patches. In the case of finite correlation lengths ξ the Green’s function on these patches is expressed via certain continuous fraction [19] (cf. similar results in Refs. [17,18,11,12]) and spectral density demonstrates more “smeared” (in comparison with the case of $\xi \rightarrow \infty$) behavior with diminishing ξ , which is described in detail in Refs. [18,11,12]. In Ref. [19] this model was applied to calculations of optical conductivity of two – dimensional system in the pseudogap state.

III. SUPERCONDUCTIVITY IN THE PSEUDOGAP STATE.

Consider now the case for superconductivity in this model. Let us assume that superconducting pairing is induced by an attractive interaction of the following simplest form [13]:

$$V(\mathbf{p}, \mathbf{p}') = V(\phi, \phi') = -V e(\phi) e(\phi'), \quad (6)$$

where ϕ – is as above an angle defining the direction of electronic momentum \mathbf{p} in the plane, while for $e(\phi)$ we take the simplest model dependence:

$$e(\phi) = \begin{cases} 1 & (s\text{-wave pairing}) \\ \sqrt{2} \cos(2\phi) & (d\text{-wave pairing}) \end{cases}. \quad (7)$$

Coupling constant V , as usual, is taken to be non zero in some energy interval of the order of $2\omega_c$ around the Fermi level (where ω_c – is characteristic frequency of the quanta responsible for attraction). In this case superconducting energy gap has the following form:

$$\Delta(\mathbf{p}) \equiv \Delta(\phi) = \Delta e(\phi). \quad (8)$$

Let us first consider superconductivity in a system with *fixed* dielectric gap D on “hot” patches of the Fermi surface. The question of superconductivity in a system with partial

dielectrization of electronic spectrum was studied in a number of works (see e.g. [20,21]). The case very similar to that considered here it was analyzed in a paper by Bilbro and McMillan [22], from which we can immediately use some of the results for rather simple generalization for the case under study.

In particular, for the case of s -wave pairing, superconducting gap Δ equation is:

$$1 = \lambda \int_0^{\omega_c} d\xi \left\{ \tilde{\alpha} \frac{th \frac{\sqrt{\xi^2 + D^2 + \Delta^2(D)}}{2T}}{\sqrt{\xi^2 + D^2 + \Delta^2(D)}} + (1 - \tilde{\alpha}) \frac{th \frac{\sqrt{\xi^2 + \Delta^2(D)}}{2T}}{\sqrt{\xi^2 + \Delta^2(D)}} \right\} \quad (9)$$

where $\lambda = VN_0(0)$ – is the dimensionless pairing coupling constant ($N_0(0)$ – is the density of states of free electrons at the Fermi level), while parameter $\tilde{\alpha} = 4\alpha/\pi$ defines a fraction of “hot” patches on the Fermi surface.

First term in Eq. (9) corresponds to the contribution of “hot” (dielectrized) patches, where the electronic spectrum is [22]: $E_p = \sqrt{\xi_p^2 + D^2 + \Delta^2}$, while the second term gives the contribution of “cold” (metallic) patches, where the spectrum is the usual BCS – like: $E_p = \sqrt{\xi_p^2 + \Delta^2}$. Equation (9) defines superconducting gap $\Delta(D)$ for the fixed value of dielectric gap D , which is non zero at “hot” patches.

In case of d -wave pairing, analogous equation is:

$$1 = \lambda \frac{4}{\pi} \int_0^{\omega_c} d\xi \left\{ \int_0^\alpha d\phi e^2(\phi) \frac{th \frac{\sqrt{\xi^2 + D^2 + \Delta^2(D)e^2(\phi)}}{2T}}{\sqrt{\xi^2 + D^2 + \Delta^2(D)e^2(\phi)}} + \int_\alpha^{\pi/4} d\phi e^2(\phi) \frac{th \frac{\sqrt{\xi^2 + \Delta^2(D)e^2(\phi)}}{2T}}{\sqrt{\xi^2 + \Delta^2(D)e^2(\phi)}} \right\} \quad (10)$$

From these equations it can be seen that $\Delta(D)$ diminishes with the growth of D , while $\Delta(0)$ coincides with Δ_0 in the absence of dielectrization on flat patches which appears at temperature $T = T_{c0}$, defined by the equation:

$$1 = \lambda \int_0^{\omega_c} d\xi \frac{th \frac{\xi}{2T_{c0}}}{\xi} \quad (11)$$

both for s -wave and d -wave pairing.

For $D \rightarrow \infty$ first terms in Eqs. (9), (10) tend to zero, so that equations for $\Delta_\infty = \Delta(D \rightarrow \infty)$ are:

$$1 = \lambda \int_0^{\omega_c} d\xi (1 - \tilde{\alpha}) \frac{th \frac{\sqrt{\xi^2 + \Delta_\infty^2}}{2T}}{\sqrt{\xi^2 + \Delta_\infty^2}} \quad (s\text{-wave pairing}) \quad (12)$$

$$1 = \lambda \frac{4}{\pi} \int_0^{\omega_c} d\xi \int_\alpha^{\pi/4} d\phi e^2(\phi) \frac{th \frac{\sqrt{\xi^2 + \Delta_\infty^2 e^2(\phi)}}{2T}}{\sqrt{\xi^2 + \Delta_\infty^2 e^2(\phi)}} \quad (d\text{-wave pairing}) \quad (13)$$

Eq. (12) coincides with gap equation for $D = 0$ with “renormalized” coupling constant $\tilde{\lambda} = \lambda(1 - \tilde{\alpha})$, so that for the case of s -wave pairing:

$$\Delta_{\infty} = \Delta_0(\tilde{\lambda} = \lambda(1 - \tilde{\alpha})) \quad (14)$$

and non zero superconducting gap for $D \rightarrow \infty$ appears at $T < T_{c\infty}$:

$$T_{c\infty} = T_{c0}(\tilde{\lambda} = \lambda(1 - \tilde{\alpha})). \quad (15)$$

In case of d -wave pairing from Eq.(13) we get:

$$T_{c\infty} = T_{c0}(\tilde{\lambda} = \lambda(1 - \alpha_d)) \quad (16)$$

where

$$\alpha_d = \tilde{\alpha} + \frac{\sin \pi \tilde{\alpha}}{\pi} \quad (17)$$

defines an “effective” fraction of flat patches in case of d -wave pairing. Thus, for $T < T_{c\infty}$ superconducting gap is non zero for arbitrary values of D and diminishes from Δ_0 to Δ_{∞} with the growth of D . For $T_{c\infty} < T < T_{c0}$ the gap is different from zero only for $D < D_{max}$. Appropriate dependences of Δ on D can be easily found by numerical solution of Eqs.(9) and (10).

In our model of the pseudogap state dielectric gap D is not fixed but random and is distributed according to (5). Accordingly the above equations should be averaged over these fluctuations. We can, for example, directly calculate the superconducting gap $\langle \Delta \rangle$ averaged over the fluctuations of D :

$$\langle \Delta \rangle = \int_0^{\infty} dD \mathcal{P}(D) \Delta(D) = \frac{2}{W^2} \int_0^{\infty} dD D e^{-\frac{D^2}{W^2}} \Delta(D) \quad (18)$$

Here, dependences of $\Delta(D)$ described above immediately lead to the conclusion that the averaged gap (18) is in fact non zero up to temperature $T = T_{c0}$, i.e. superconducting transition temperature in the absence of any pseudogap anomalies. However, it is obvious that transition temperature T_c for a superconductor with pseudogap is lower than T_{c0} [13]. Thus, an apparently paradoxical behavior of $\langle \Delta \rangle$ signifies, probably, the appearance in the system of local regions with $\Delta \neq 0$ (superconducting “drops”) induced by fluctuations of D for all temperatures $T_c < T < T_{c0}$, while coherent superconducting state appears in the sample only for $T < T_c$. Of course the complete justification of these qualitative picture can be obtained only in more realistic model with finite correlation length ξ of AFM – fluctuations⁴. At the same time the simplicity of the current model with $\xi \rightarrow \infty$ allows immediately to obtain an exact result for $\langle \Delta \rangle$.

To determine superconducting transition temperature T_c in a sample as a whole, we shall use the standard mean – field approach (compare e.g. the analogous approach for a superconductor with impurities [24]), assuming the self – averaging of the superconducting

⁴Qualitatively this situation resembles the appearance of inhomogeneous superconducting state induced by strong fluctuations of the local density of states close to the Anderson metal – insulator transition [23,24].

gap over fluctuations of D (i.e. in fact independence of Δ on D – fluctuations). Then the equations for mean – field gap Δ_{mf} take the following form:

$$1 = \lambda \int_0^{\omega_c} d\xi \left\{ \tilde{\alpha} \frac{2}{W^2} \int_0^\infty dD D e^{-\frac{D^2}{W^2}} \frac{th \frac{\sqrt{\xi^2 + D^2 + \Delta_{mf}^2}}{2T}}{\sqrt{\xi^2 + D^2 + \Delta_{mf}^2}} + (1 - \tilde{\alpha}) \frac{th \frac{\sqrt{\xi^2 + \Delta_{mf}^2}}{2T}}{\sqrt{\xi^2 + \Delta_{mf}^2}} \right\} \quad (19)$$

for the case of s -wave pairing, and

$$1 = \lambda \frac{4}{\pi} \int_0^{\omega_c} d\xi \left\{ \int_0^\infty dD D e^{-\frac{D^2}{W^2}} \int_0^\alpha d\phi e^{2(\phi)} \frac{th \frac{\sqrt{\xi^2 + D^2 + \Delta_{mf}^2 e^{2(\phi)}}}{2T}}{\sqrt{\xi^2 + D^2 + \Delta_{mf}^2 e^{2(\phi)}}} + \int_\alpha^{\pi/4} d\phi e^{2(\phi)} \frac{th \frac{\sqrt{\xi^2 + \Delta_{mf}^2 e^{2(\phi)}}}{2T}}{\sqrt{\xi^2 + \Delta_{mf}^2 e^{2(\phi)}}} \right\} \quad (20)$$

for the case of d -wave pairing.

From Eqs. (19), (20) it is easy to obtain also appropriate equations for T_c . For the case of s -wave pairing we get:

$$1 = \lambda \left\{ \tilde{\alpha} \frac{2}{W^2} \int_0^\infty dD D e^{-\frac{D^2}{W^2}} \int_0^{\omega_c} d\xi \frac{th \frac{\sqrt{\xi^2 + D^2}}{2T_c}}{\sqrt{\xi^2 + D^2}} + (1 - \tilde{\alpha}) \int_0^{\omega_c} d\xi \frac{th \frac{\xi}{2T_c}}{\xi} \right\} \quad (21)$$

For the case of d -wave pairing in Eq. (21) we have only to replace $\tilde{\alpha}$ by “effective” α_d from (17). These equations for T_c coincide with those obtained in microscopic derivation of Ginzburg – Landau expansion for this model in Ref. [13], where these equations were studied in detail. In general we always have $T_{c\infty} < T_c < T_{c0}$.

Temperature dependences of average gap $\langle \Delta \rangle$ and mean – field gap Δ_{mf} , obtained numerically from equations of our model for the case of s -wave pairing, are shown in Fig. 2⁵. Mean – field gap Δ_{mf} goes to zero at $T = T_c < T_{c0}$, while $\langle \Delta \rangle$ is non zero up to $T = T_{c0}$, the “tails” in temperature dependences of $\langle \Delta \rangle$ in the region of $T_c < T < T_{c0}$ apparently signifying the existence of local superconducting “drops” in the sample, while superconductivity in a sample as a whole is absent. Note that temperature dependences of $\langle \Delta(T) \rangle$ shown in Fig. 2 qualitatively resemble those observed in underdoped cuprates in ARPES [3,25] and specific – heat experiments [26], if we assume that real T_c observed in these samples corresponds to our mean – field T_c , while “drops” with $\langle \Delta \rangle \neq 0$ exist for all temperatures $T > T_c$ up to T_{c0} , which is significantly larger than T_c . This interpretation implicitly assumes that in the “absence” of the pseudogap underdoped cuprates would have possessed much larger temperature of superconducting transition, than those observed in the experiment.

Despite the fact that superconductivity in a sample as a whole for $T_c < T < T_{c0}$ (according to our interpretation) is absent, the presence of non zero average gap $\langle \Delta \rangle$ leads, as

⁵In case of d -wave pairing temperature dependences of $\langle \Delta \rangle$ and Δ_{mf} are qualitatively similar to those obtained for s -wave pairing.

will be shown below, to the appearance of a number anomalies in experimentally measurable characteristics, such as tunnelling density of states and spectral density measured in ARPES – experiments.

IV. SPECTRAL DENSITY AND DENSITY OF STATES.

Retarded Green's function of an electron in the vicinity of “hot” patch on the Fermi surface in superconducting state is given by:

$$G^R(E, \xi_p) = \int_0^\infty dD \mathcal{P}(D) \frac{E + \xi_p}{(E + i0)^2 - \xi_p^2 - D^2 - \Delta^2(D)e^2(\phi)} \quad (22)$$

From this we obtain spectral density as:

$$A(E, \xi_p) = -\frac{1}{\pi} \text{Im} G^R(E, \xi_p) = \frac{2}{W^2} \int_0^\infty dD D e^{-\frac{D^2}{W^2}} (E + \xi_p) \delta(\xi_p^2 + D^2 + \Delta^2(D)e^2(\phi) - E^2) \quad (23)$$

Using mean – field approach, assuming $\Delta = \Delta_{mf}$ which is independent of D , we obtain:

$$A_{mf}(E, \xi_p) = \frac{|E| + \xi_p \text{Sign} E}{W^2} \exp\left(\frac{\xi_p^2 + \Delta_{mf}^2 e^2(\phi) - E^2}{W^2}\right) \theta(E^2 - \xi_p^2 - \Delta_{mf}^2 e^2(\phi)) \quad (24)$$

In this approximation spectral density acquires a gap for $|E| < \Delta_{mf}$ which disappears for $T \rightarrow T_c$ ($\Delta_{mf} \rightarrow 0$). In fact we have already seen, that Δ possesses a significant dependence on the value of dielectric gap D (cf.(9),(10)), so that from (23) we get:

$$A(E, \xi_p) = \sum_i \frac{|E| + \xi_p \text{Sign} E}{W^2} e^{-\frac{D_i^2}{W^2}} \frac{1}{\left|1 + \frac{d\Delta^2(D)}{dD^2}\Big|_{D=D_i} e^2(\phi)\right|} \quad (25)$$

where D_i – are the positive roots of the equation $D^2 + \xi_p^2 + \Delta^2(D)e^2(\phi) - E^2 = 0$. Energy dependences of spectral density for $\xi_p = 0$, i.e. for an electron momentum on the Fermi surface (we restrict ourselves only to this case) are given in Fig. 3 and Fig. 4 for the case of s -wave and d -wave pairing respectively.

For $T_{c\infty} < T < T_{c0}$ spectral density acquires a discontinuity at $E = D_{max}$ due to discontinuity of the derivative $d\Delta^2(D)/dD^2$ at $D = D_{max}$ (i.e. at maximal D for which $\Delta(D)$ is non zero). Effects of finite correlation length of fluctuations ξ will obviously lead to the smearing of this discontinuity, however there will remain a characteristic dip after the main peak of the spectral density. Similar dip is observed in ARPES – experiments [1,3] and there is no accepted interpretation of it up to now.

In the case d -wave pairing the value of $D^2 + \Delta^2(D)$ grows with the growth of D , thus the equation $D^2 + \Delta^2(D) - E^2 = 0$ acquires roots only for $|E| > \Delta_0$. Thus, the gap in the spectral density appears for $|E| < \Delta_0$, so that the width of this gap is determined by Δ_0 , and not by Δ_{mf} . Besides, the gap in the spectral density appears for $T = T_{c0}$, and there is no qualitative changes in the spectral density at $T = T_c$.

In case of d -wave pairing, for small enough width of the pseudogap W and small fraction of flat patches α_d , the value of $D^2 + \Delta^2(D)e^2(\phi)$ grows with the growth of D and the width

of the gap in spectral density becomes equal to $\Delta_0 e(\phi)$, analogously to the case of s -wave pairing. However, with the growth of the width of the pseudogap W and the fraction of flat patches, the value of $D^2 + \Delta^2(D) e^2(\phi)$ drops with the growth of D for small enough D , leading to the width of the gap in spectral density smaller than Δ_0 , while for $E = \Delta_0$ there is a discontinuity in spectral density (discontinuity at $E = D_{max}$ remains).

Consider now the tunnelling density of states $N(E)$. In case of s -wave pairing we obtain:

$$\frac{N(E)}{N_0(0)} = \frac{2}{W^2} \int_0^\infty dD D e^{-\frac{D^2}{W^2}} \left\{ \tilde{\alpha} \frac{|E|}{\sqrt{E^2 - D^2 - \Delta^2(D)}} \theta(E^2 - D^2 - \Delta^2(D)) + (1 - \tilde{\alpha}) \frac{|E|}{\sqrt{E^2 - \Delta^2(D)}} \theta(E^2 - \Delta^2(D)) \right\} \quad (26)$$

Assuming self-averaging nature of the gap we have $\Delta = \Delta_{mf}$ which does not depend on fluctuations of D , then:

$$\frac{N_{mf}(E)}{N_0(0)} = \left\{ \tilde{\alpha} \frac{2}{W^2} \int_0^{\sqrt{E^2 - \Delta_{mf}^2}} dD D e^{-\frac{D^2}{W^2}} \frac{|E|}{\sqrt{E^2 - D^2 - \Delta_{mf}^2}} + (1 - \tilde{\alpha}) \frac{|E|}{\sqrt{E^2 - \Delta_{mf}^2}} \right\} \theta(E^2 - \Delta_{mf}^2). \quad (27)$$

In this approximation for $|E| < \Delta_{mf}$ there is a gap in the density of states which vanishes for $T \rightarrow T_c (\Delta_{mf} \rightarrow 0)$, however there remains the pseudogap due to AFM - fluctuations:

$$\frac{N(E)}{N_0(0)} = \tilde{\alpha} \frac{2}{W^2} \int_0^E dD D e^{-\frac{D^2}{W^2}} \frac{|E|}{\sqrt{E^2 - D^2}} + (1 - \tilde{\alpha}) \quad (28)$$

discussed previously in Ref. [13]. In fact $\Delta(D)$ in (26) depends on D according to (9). From Eq. (26) and appropriate dependence $\Delta(D)$ it can be seen that for $T < T_{c\infty}$ there is a gap in the density of states for $E < \Delta_\infty$, while for $T > T_{c\infty}$ there is no gap in the density of states, but certain contribution to the pseudogap due to superconducting pairing remains. For $T_c < T < T_{c0}$ the gap $\Delta(D)$ is different from zero for $D < D_{max}$, thus the difference from pseudogap behavior due only to AFM - fluctuations is observed already for $T_c < T < T_{c0}$, and only for $T > T_{c0}$ we observe purely AFM - pseudogap (28).

In Fig. 5 we show energy dependence of the density of states in case of s -wave pairing for different temperatures. There is a cusp in the density of states at $|E| = \Delta_0$, besides for $T > T_{c\infty}$ there is another cusp at $|E| = D_{max} > \Delta_0$, though this last cusp is only visible for high enough temperatures $T \sim T_{c0}$. Density of states changes qualitatively only at $T = T_{c0}$ while nothing special happens at mean - field T_c .

For d -wave pairing density of states becomes:

$$\frac{N(E)}{N_0(0)} = \frac{4}{\pi} \frac{2}{W^2} \int_0^\infty dD D e^{-\frac{D^2}{W^2}} \left\{ \int_0^\alpha d\phi \frac{|E|}{\sqrt{E^2 - D^2 - \Delta^2(D) e^2(\phi)}} \theta(E^2 - \Delta^2(D) e^2(\phi) - D^2) + \int_\alpha^{\pi/4} d\phi \frac{|E|}{\sqrt{E^2 - \Delta^2(D) e^2(\phi)}} \theta(E^2 - \Delta^2(D) e^2(\phi)) \right\} \quad (29)$$

Assuming self – averaging $\Delta = \Delta_{mf}$ and is independent of D . Then the width of superconducting pseudogap in the density states is of the order of Δ_{mf} and the appropriate contribution vanishes for $T \rightarrow T_c$ while the pseudogap (28) due to AFM – fluctuations remains. However, in reality in (29) we have $\Delta = \Delta(D)$ defined by Eq. (10).

The behavior of the density of states in case of d -wave pairing is shown in Fig. 6. As for s -wave pairing we observe significant difference between the exact density of states and that obtained in the mean – field approach, which is due to fluctuations of superconducting gap (superconducting “drops”) induced by AFM short – range order. Exact density of states does not “feel” superconducting transition in a sample as a whole which takes place at $T = T_c$. Characteristic width of the pseudogap in the density states is given by Δ_0 , not by Δ_{mf} , as in mean – field approximation. Appropriate contributions become observable already at $T = T_{c0} > T_c$.

V. CONCLUSION.

In this work we have continued our studies of anomalies of superconducting state in quite simplified exactly solvable model of the pseudogap in two – dimensional system of electrons [13]. The main simplifying assumption of our model (in addition to the static nature of fluctuations) is the use of the limit of $\xi \rightarrow \infty$ for correlation length of fluctuations of AFM short – range order, which allows us to obtain main results in analytic form. In particular, in this limit we can obtain an exact average superconducting gap (18). Though it is clear, in principle, that this model can be generalized to the case of finite correlation lengths [11,12,19], it is less clear how we will be able to analyze superconducting state in this generalized model outside the limits of mean – field approach, similar to what was done above for the case of $\xi \rightarrow \infty$. Qualitatively it is clear that the effects of finite ξ will lead to some smearing of cusps and discontinuities which appear in the model with $\xi \rightarrow \infty$, as well as relatively smooth dependence of T_c and other characteristics of superconducting state on the value of ξ .

The results obtained above show that the pseudogap state induced by AFM short – range order fluctuations (or similar CDW fluctuations) leads (in addition to the anomalies of the normal state [11,12,19]) also to rather unusual properties of superconducting state, related to partial dielectrization (non Fermi – liquid behavior) of electronic spectrum on the “hot” patches of the Fermi surface. These properties correlate well with a number of anomalies observed in the underdoped state of HTSC – cuprates. It is obvious that more serious comparison with experiments can only be performed in more realistic approach, taking into account, first of all, the effects of the finite correlation lengths ξ , which is relatively small in real systems. At low temperatures it is also important to take into account the dynamic nature of AFM fluctuations.

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Figure Captions:

Fig.1. Model Fermi surface of two – dimensional system. “Hot” patches are shown by thick lines of the width of $\sim \xi^{-1}$.

Fig.2. Temperature dependences of superconducting gaps Δ_{mf} (points), $\langle \Delta \rangle$ (full lines) and Δ_0 (dashed line) in case of s -wave pairing.

1.— $\lambda = 0, 4$; $\tilde{\alpha} = 2/3$; $\omega_c/W = 3$ ($T_c/T_{c0} = 0, 42$).

2.— $\lambda = 0, 4$; $\tilde{\alpha} = 0, 2$; $\omega_c/W = 1$ ($T_c/T_{c0} = 0, 71$).

Fig.3. Spectral density on the Fermi surface in case of s -wave pairing for different values of T/T_{c0} : 1.-0,8; 2.-0,4; 3.-0,1.

(a) — $\lambda = 0, 4$; $\tilde{\alpha} = 0, 2$; $\omega_c/W = 1$ ($T_c/T_{c0} = 0, 71$, $T_{c\infty}/T_{c0} = 0, 54$).

Points: mean – field approximation for the spectral density $A_{mf}(E)$ for $T/T_{c0} = 0, 4$.

(b) — $\lambda = 0, 4$; $\tilde{\alpha} = 2/3$; $\omega_c/W = 3$ ($T_c/T_{c0} = 0, 42$, $T_{c\infty}/T_{c0} = 7 \cdot 10^{-3}$).

Points: mean – field approximation for $A_{mf}(E)$ at $T/T_{c0} = 0, 1$

Fig.4. Spectral density on the Fermi surface for $\phi = 0$ in case of d -wave pairing.

(a) — $\lambda = 0, 4$; $\tilde{\alpha} = 0, 2$; $\omega_c/W = 1$ ($T_c/T_{c0} = 0, 42$, $T_{c\infty}/T_{c0} = 0, 2$),

$T/T_{c0} =$: 1.-0,8; 2.-0,6; 3.-0,1.

(b) — $\lambda = 0, 4$; $\tilde{\alpha} = 2/3$; $\omega_c/W = 5$ ($T_c/T_{c0} = 0, 48$, $T_{c\infty}/T_{c0} \sim 10^{-18}$),

$T/T_{c0} =$: 1.-0,8; 2.-0,3; 3.-0,1. Points: mean – field spectral density $A_{mf}(E)$ for $T/T_{c0} = 0, 1$

Fig.5. Density of states in case of s -wave pairing.

(a) — $\lambda = 0, 4$; $\tilde{\alpha} = 0, 2$; $\omega_c/W = 1$ ($T_c/T_{c0} = 0, 71$, $T_{c\infty}/T_{c0} = 0, 54$),

$T/T_{c0} =$: 1.-0,8; 2.-0,71; 3.-0,54; 4.-0,1.

Points: mean – field density of states $N_{mf}(E)$ for $T/T_{c0} = 0, 4$

At the insert: density of states for $T/T_{c0} = 0, 4$

(b) — $\lambda = 0, 4$; $\tilde{\alpha} = 2/3$; $\omega_c/W = 3$ ($T_c/T_{c0} = 0, 42$, $T_{c\infty}/T_{c0} = 7 \cdot 10^{-3}$),

$T/T_{c0} =$: 1.-0,8; 2.-0,42; 3.-0,2; 4.-0,05.

Points: mean field density of states $N_{mf}(E)$ for $T/T_{c0} = 0, 1$. Dashed line: pseudogap in the density of states for $T > T_{c0}$.

Fig.6. Density of states in case of d -wave pairing.

(a) — $\lambda = 0, 4$; $\tilde{\alpha} = 0, 2$; $\omega_c/W = 1$ ($T_c/T_{c0} = 0, 42$, $T_{c\infty}/T_{c0} = 0, 2$),

$T/T_{c0} =$: 1.-0,8; 2.-0,42; 3.-0,2.

Points: mean field density of states $N_{mf}(E)$ for $T/T_{c0} = 0, 2$

At the insert: density of states for $T/T_{c0} = 0, 2$.

(b) — $\lambda = 0, 4$; $\tilde{\alpha} = 2/3$; $\omega_c/W = 5$ ($T_c/T_{c0} = 0, 48$, $T_{c\infty}/T_{c0} \sim 10^{-18}$),

$T/T_{c0} =$: 1.-0,8; 2.-0,48; 3.-0,1.

Points: mean field density of states $N_{mf}(E)$ for $T/T_{c0} = 0, 1$. Dashed line: pseudogap in the density of states for $T > T_{c0}$.

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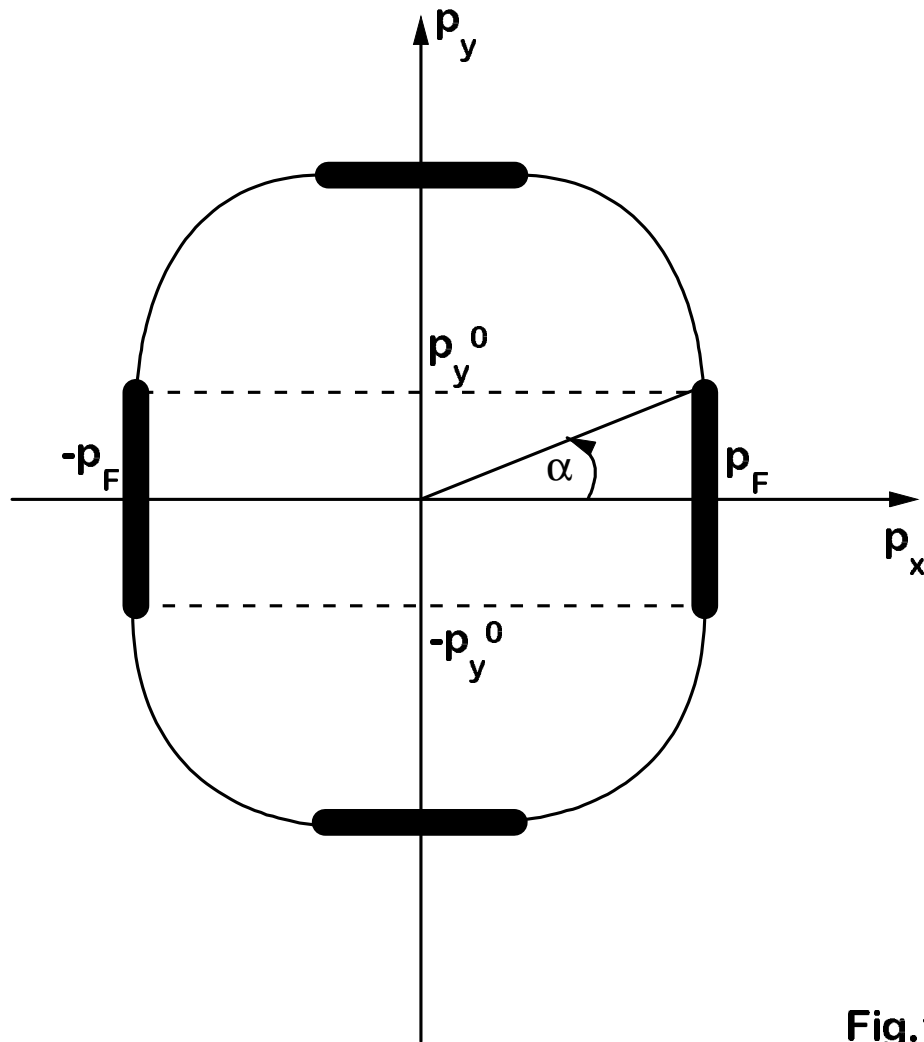


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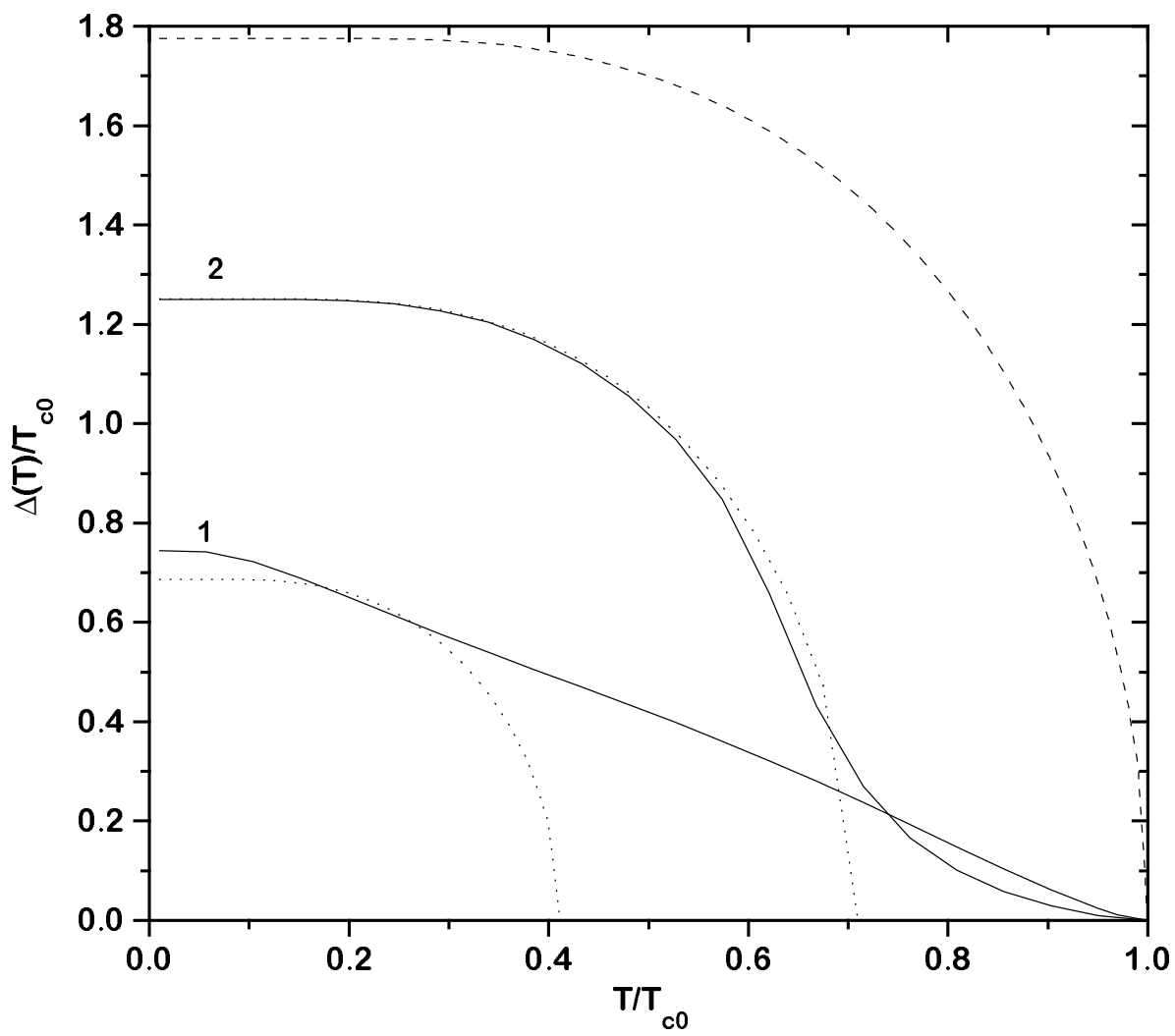


Fig.2

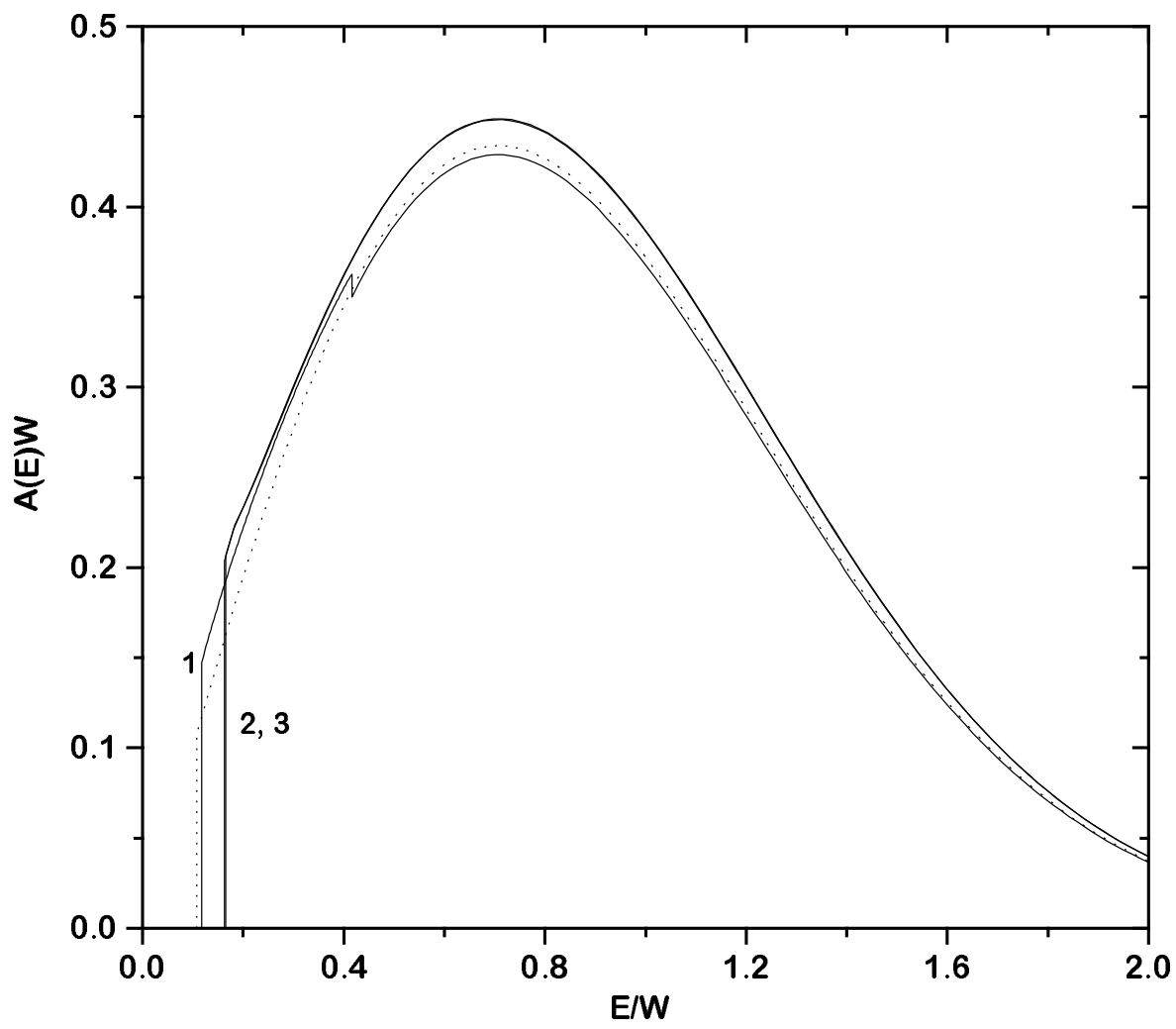


Fig.3(a)

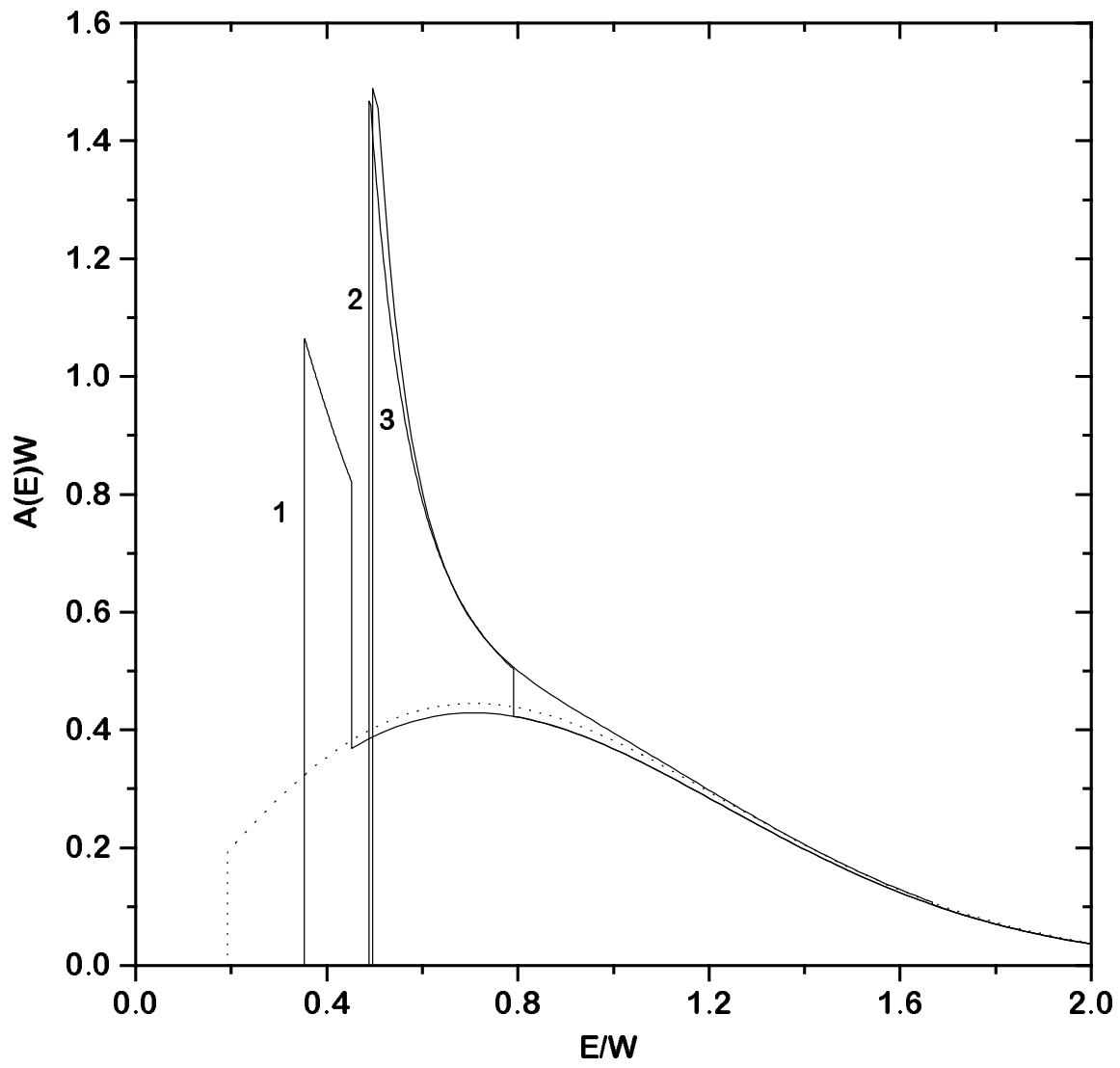


Fig.3(b)

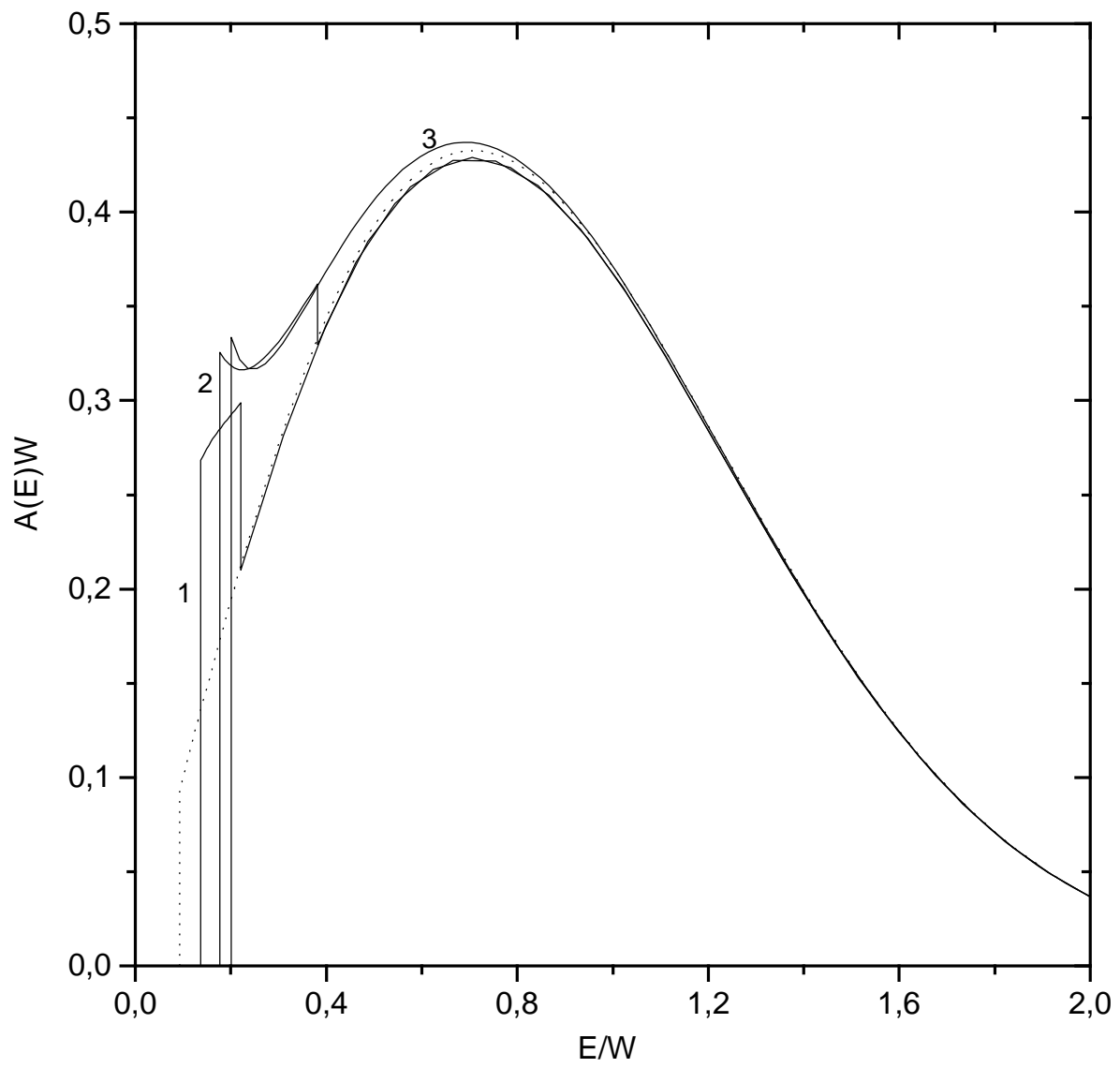


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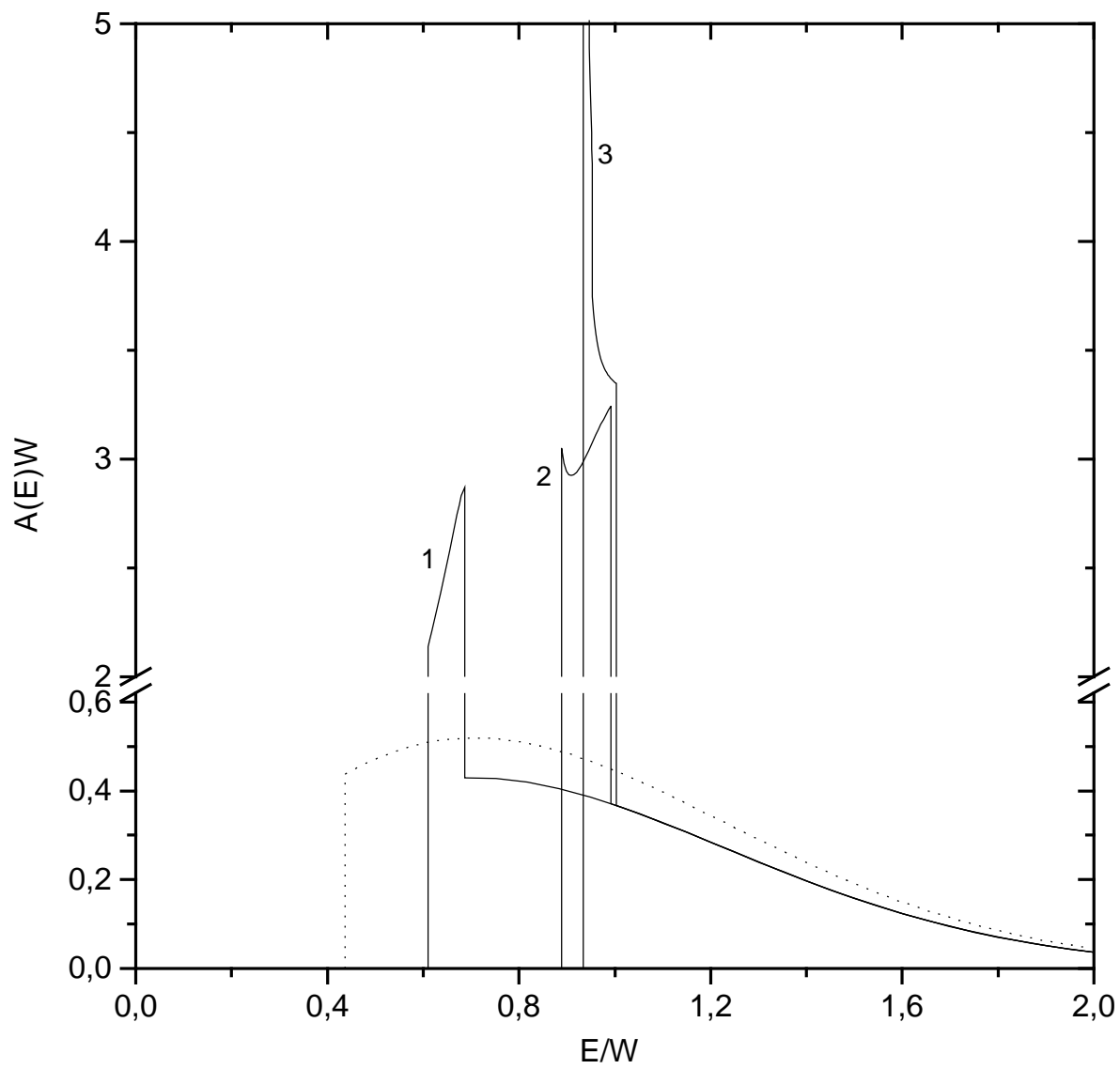


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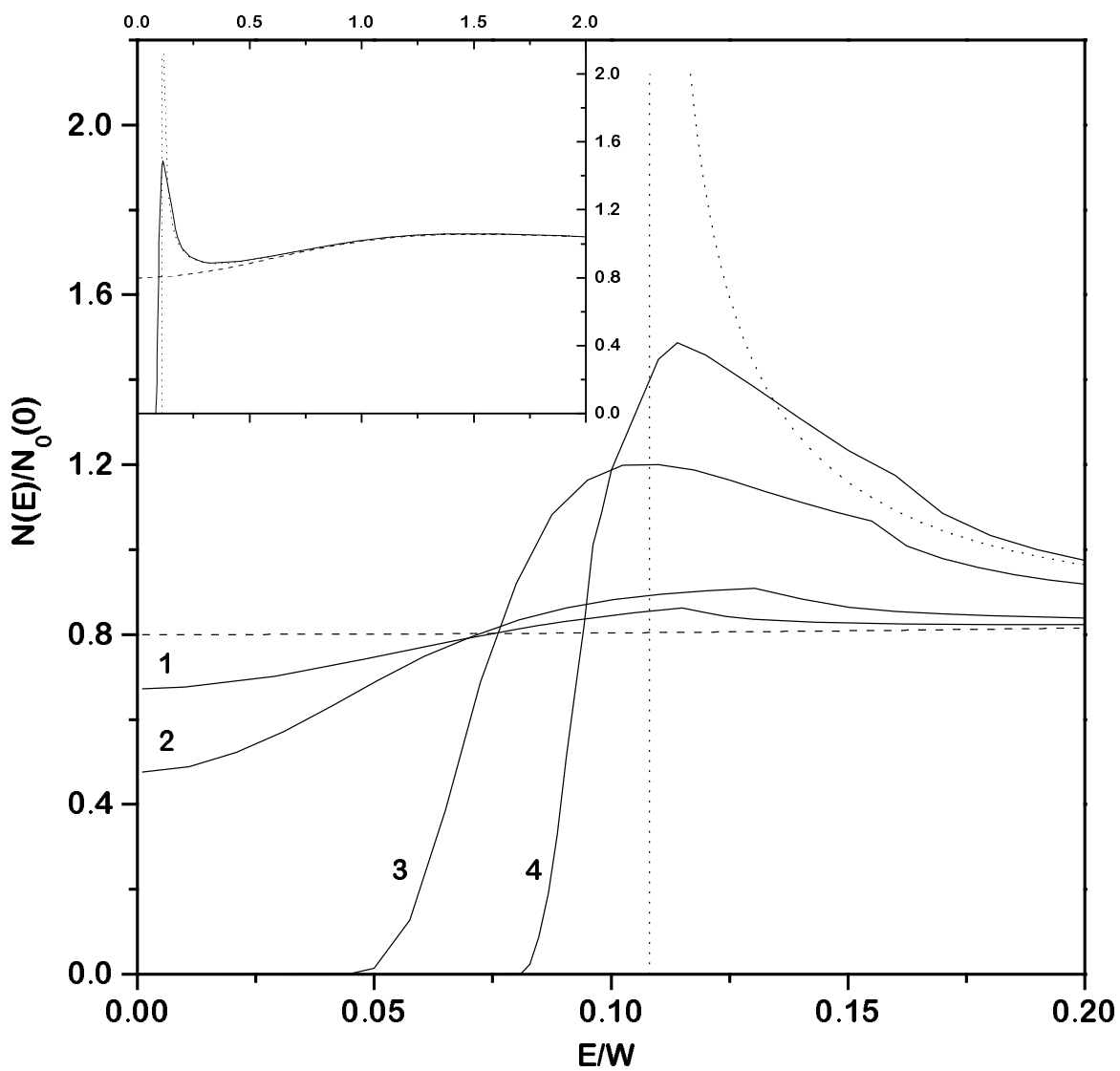


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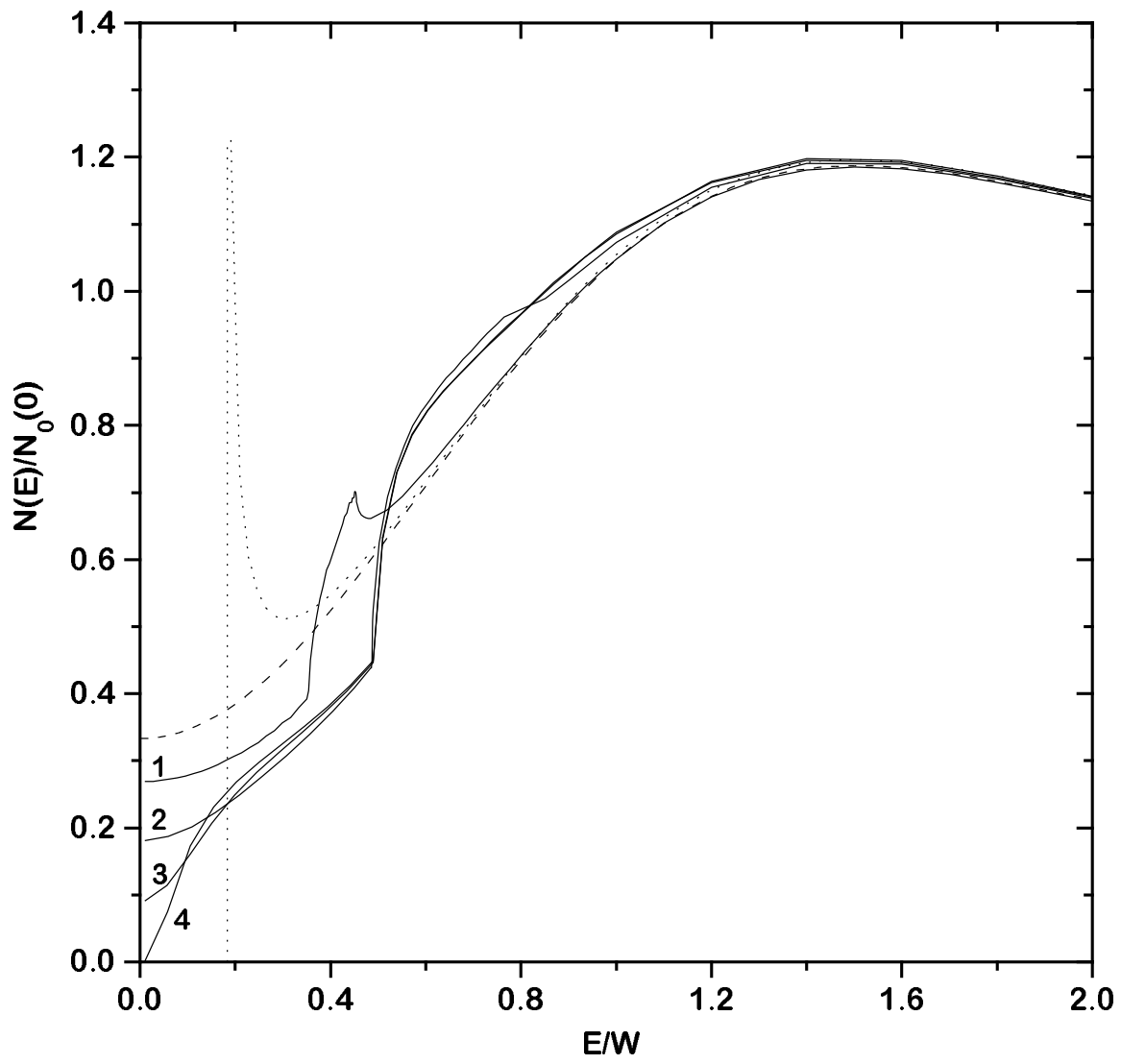


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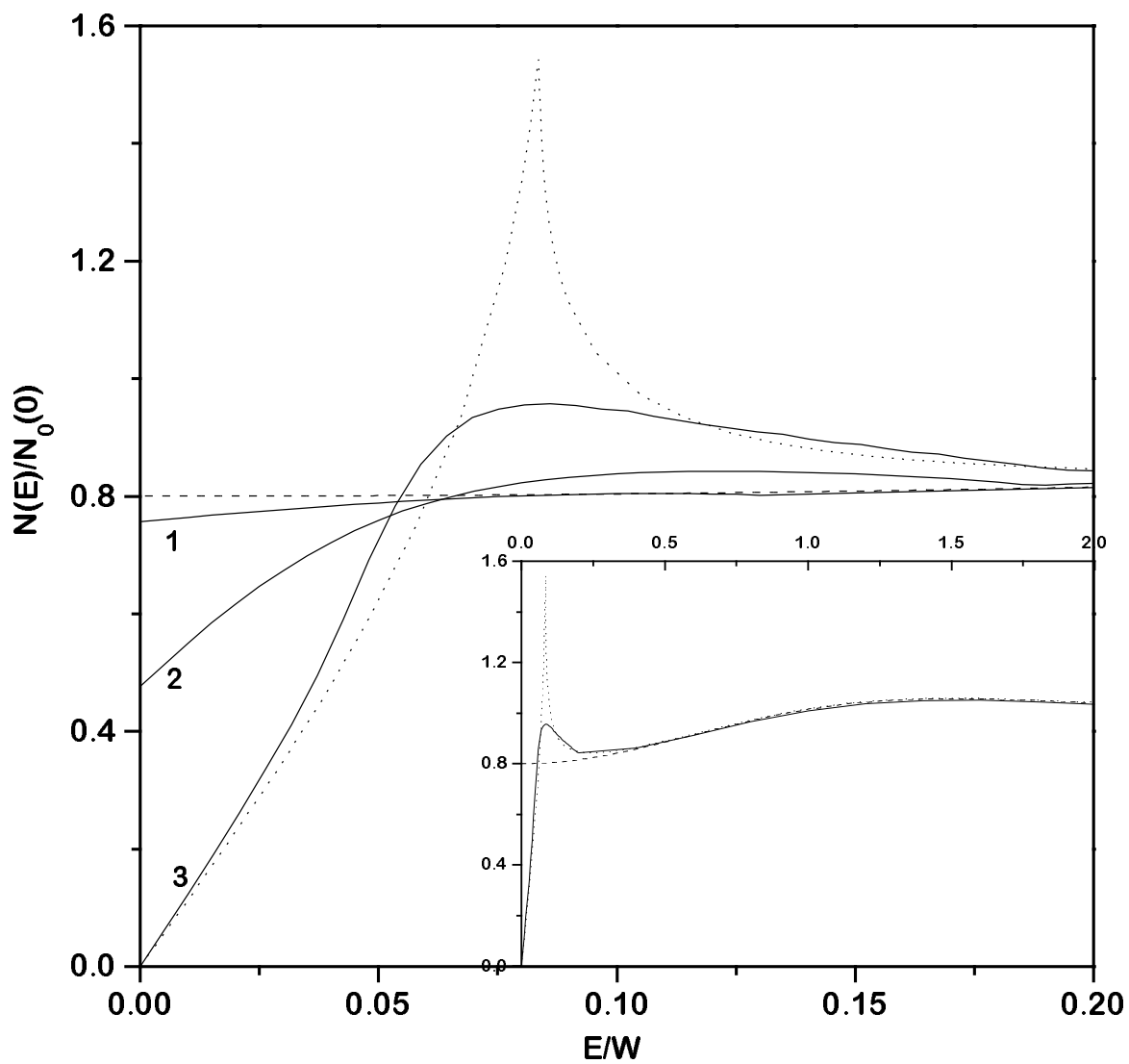


Fig.6(a)

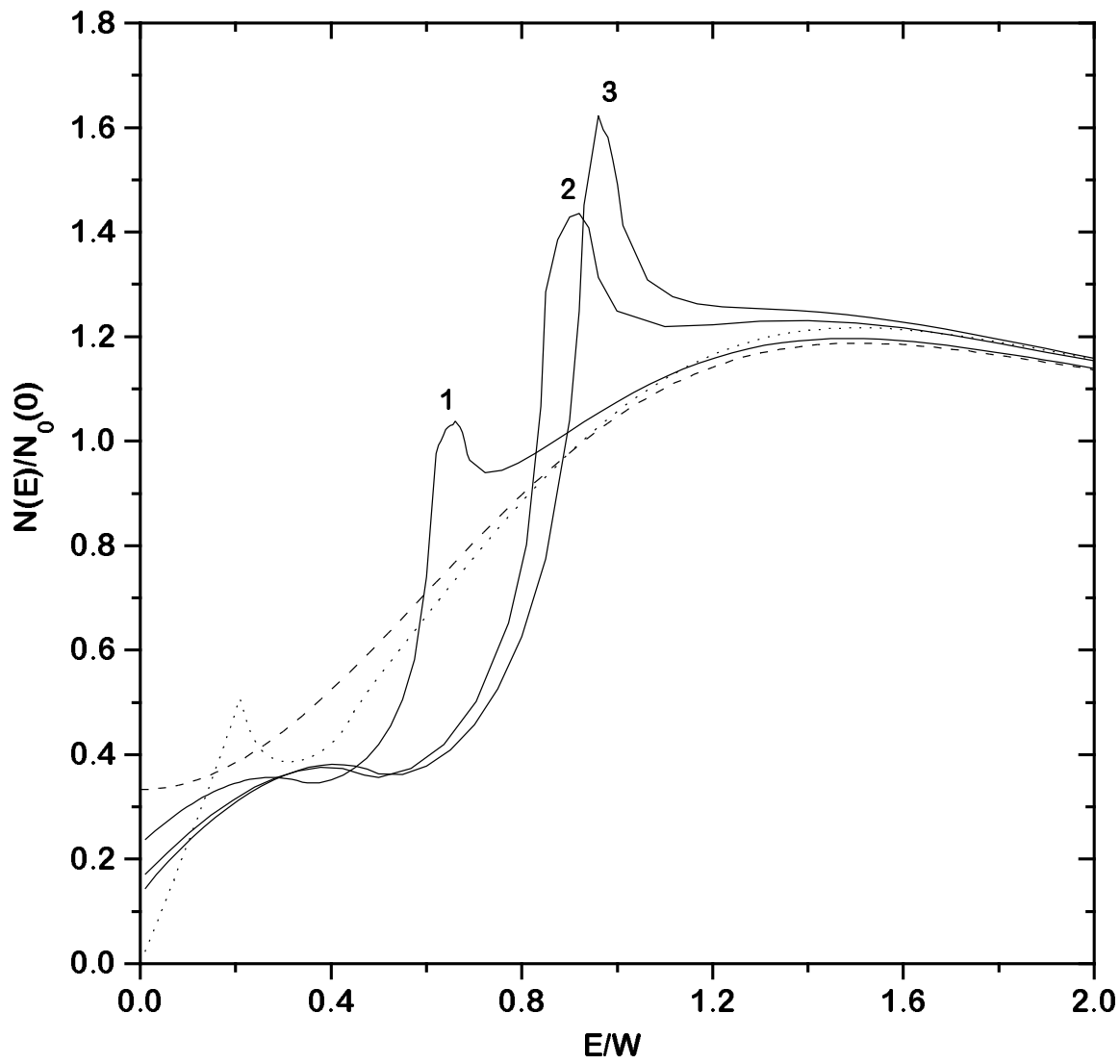


Fig.6(b)