OPTICAL CONDUCTIVITY OF HIGH - TEMPERATURE SUPERCONDUCTORS IN "SPIN - BAG" MODEL : EXACT SOLUTION ?

M.V. Sadovskii, A.A. Timofeev

Institute for Electrophysics,
USSR Academy of Sciences,
Ural Branch, Sverdlovsk
620219, USSR

Abstract

Exact summation of all Feynman diagrams for two - particle Green's function is performed for the one - dimensional Gaussian model of static fluctuations of short - range (antiferromagnetic) order with Lorentzian correlator, analogous to "spin - bag" model of Schrieffer et al.

We obtain the general picture of frequency dependence of conductivity for different ranges of correlation length of short - range order. For specific values of parameters we get good qualitative agreement with experimental data on optical absorption in high - T_c superconductors. Anomalous (for 1 - dimensional model) behavior demonstrating the Anderson transition from localized to extended states is also demonstrated.

1. Introduction

It is well known by now that the superconducting state in high - T_c oxides is realized on the phase diagram quite close to region of antiferromagnetic (AFM) insulator. Antiferromagnetic short - range order is being observed also in superconducting phase [1 - 4]. Many theoretical attempts of theoretical description of high - T_c system are based upon this experimental fact. Among different theoretical schemes we address ourselves to "spin - bag" model of Schrieffer et al [5] which is based upon the Hubbard model with intermediate electronic correlations $U \leq W$ (where W is bandwidth). In case of nesting properties of the Fermi surface characteristic for narrow - band two - dimensional models,

---

Talk presented at the International Seminar on High-T_c Superconductivity, Dubna, USSR, July 1990.

antiferromagnetic state is described by the spin - density wave (SDW), while in the metallic phase (for non half-filled band) there appears the attractive interaction between charge carries (holes), leading to Cooper pairing. Most complete analysis for this model was performed for the case of antiferromagnetic phase with long - range order [5]. At the same time from the experimental point of view it is most important to consider the region of short - range antiferromagnetic order fluctuations. For this case only few results were obtained [6]. In particular, within the framework of simplest (RPA - type) approximation it was demonstrated that the pseudo gap appears in electronic spectrum, analogous to that considered earlier for Peierls transition in one - dimension [7]. Some simple estimates of pairing interaction were also presented. In fact the problem of electronic spectrum in case of developed short - range order fluctuations for the model similar to that of Ref. 7 can be solved exactly in one - dimension [8 - 11]. Thus in Refs. 8 - 10 the exact analytic solution were obtained for the case of asymptotically large correlation length both for single - particle and two - particle Green's functions. In Ref. 11 an exact solution in the form of continuous fraction for the single - particle Green's function was presented for the case of arbitrary correlation lengths of short - range order. As a result a full picture of evolution (degradation) of a pseudo gap as correlation length diminishes was obtained.

Here we present the results of an exact solution for the two - particle Green's function, based upon the further development of methods of Ref. 11. Finally we get the picture of evolution of optical properties (frequency depended conductivity) for different ranges of correlation lengths of short - range order fluctuations. It appears that for reasonably short values of this length the frequency dependence of conductivity is in qualitative agreement with experimental data for the superconducting phase of high - T_c oxides[12,13].

2. The Model and its Exact Solution.

Following Refs. 5,6 we start with Hubbard Hamiltonian for two - dimensional (square) lattice:

$$H=-t\sum_{i,j,\sigma}(c^{+}_{i\sigma}c_{j\sigma}+c^{+}_{j\sigma}c_{i\sigma})+U\sum_{i}n_{i\sigma}n_{i\bar{\sigma}}-\mu\sum_{i,\sigma}n_{i\sigma} \qquad (1)$$

were $c^{+}_{i\sigma}$ and $c_{i\sigma}$ - are usual creation and annihilation operators for electron with spin σ at the site i, $n_{i\sigma}=c^{+}_{i\sigma}c_{i\sigma}$, μ - is the chemical potential. As Coulomb repulsion U grows in this model AFM correlation appear between spin on neighboring sites. For half - filled band the free - electron spectrum:

$$\varepsilon_p=-2t(\cos(p_y a)+\cos(p_x a)) \qquad (2)$$

satisfies the nesting condition:

$$\varepsilon_{p+Q}=-\varepsilon_p \qquad (3)$$

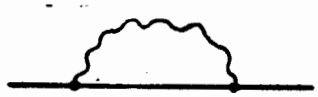
where $Q=(\pi/a,\pi/a)$, which leads to the SDW*) - instability with the wave - vector Q [5]. For non half - filled case AFM order is being destroyed, while short - range order SDW - fluctuations may persist in relatively wide region of phase diagram.

In RPA - approximation the first - order correction for the electron self - energy (Fig. 1) takes the following form [6]:

$$\Sigma(\varepsilon,\xi_p)=-(3/2)iU^2\int(d^2q/4\pi^2)\int(d\omega/2\pi)G(p-q,\varepsilon-\omega)\chi(q,\omega) \qquad (4)$$

where $\chi(q,\omega)$ - is the generalized susceptibility.

*) All the following may be obviously applied for the analogous case near charge - density wave (CDW) instability.



$\varepsilon,p \quad \varepsilon-\omega,p-q \quad \varepsilon,p$

Fig. 1. First order contribution to the electron self - energy. Wavy line - $\chi(q,\omega)$.

In the adiabatic (static) limit, when spin fluctuation are slow in comparison with characteristic electronic frequencies, $\chi(q,\omega)$ is given by [6]:

$$\chi(q,\omega)= \lambda^2\ 2\pi i\delta(\omega)\sum_{Q} \varkappa/((q-Q)^2 + \varkappa^2) \qquad (5)$$

where λ - is some coupling constant, $\varkappa = \xi_{SDW}^{-1}$ is the inverse correlation length of SDW - fluctuations. In case of incommensurate fluctuations $Q=(\mp 2p_F, \mp 2p_F)$ (p_F - is the Fermi momentum). In fact (5) is just the Fourier transform of random potential field correlator $\langle V(r),V(r')\rangle$ with zero average $\langle V(r)\rangle=0$. In the following we assume the Gaussian nature of these fluctuations which is reasonable not very close to SDW - instability. We shall consider only the one - dimensional model of these fluctuations which allows an exact solution. Apparently this model is not very bad for the two - dimensional system in case of perfect nesting.

The problem of electronic spectrum in the random field of this type was first analyzed in Ref. 7, while in Refs. 8 - 11 the exact solution was obtained. This solution is actually based upon remarkable properties of contributions from different Feynman diagrams for the electron self - energy (which are easily demonstrated for the case of Lorentzian correlator (5)) showing the equality of contributions from different classes of diagrams. In particular any contribution from a graph with intersecting interaction lines can be shown to be equal to a contribution from

the certain graph with no intersections of interaction lines. This leads to a universal way to obtain contributions from arbitrary diagrams and relatively simple combinatorics. The details can be found in Ref. 11. Analogous analysis was for the first time used for a different problem [14].

The structure of an exact solution [11] is based on the usual Dyson equation:

$$G^{-1}(\varepsilon,\xi_p) = G_o^{-1}(\varepsilon,\xi_p) - \Sigma_1(\varepsilon,\xi_p) \tag{6}$$

where $\xi_p = \varepsilon_p - \mu \simeq v_F(|p| - p_F)$ and for the self - energy we have the following representation (Fig. 2(a))

$$\Sigma_1(\varepsilon,\xi_p) = \Delta^2\, \Xi_1(\varepsilon,\xi_p)/(\varepsilon + \xi_p + iv_F\varkappa\ \mathrm{sign}\xi_p)^2 =$$

$$= \Delta^2 G_o^2(\varepsilon,-\xi_p - iv_F\varkappa\ \mathrm{sign}\xi_p)\ \Xi_1(\varepsilon,\xi_p) \tag{7}$$

where $\Delta^2 = (3/4)\ \lambda U^2$, and $\Xi_1(\varepsilon,\xi_p)$ is given by the expansion shown in Fig. 2(b), where there are no graphs with intersections in interaction lines, but for the k- th vertex (coming from the left) with "outgoing" interaction line we have to associate the combinatorial factor

$$v(k)=\begin{cases} (k+1)/2, & k=2m+1 \\ k/2 & k=2m \end{cases} \tag{8}$$

for the incommensurate SDW fluctuations and

$$v(k) = k \tag{9}$$

for the commensurate case. The product of these factors gives the total combinatorial factor taking into account the contributions of graphs with intersecting lines. $\Xi_1(\varepsilon,\xi_p)$ can be represented by standard expression:

$$\Xi_1(\varepsilon,\xi_p)=G_o^{-2}(\varepsilon,-\xi_p-iv_F\varkappa\,\mathrm{sign}\xi_p)/\{G_o^{-1}(\varepsilon,-\xi_p-iv_F\varkappa\,\mathrm{sign}\xi_p) - \Sigma_2(\varepsilon,\xi_p)\} \qquad (10)$$

where $\Sigma_2(\varepsilon,\xi_p)$ is expressed as a sum of irreducible diagrams shown in Fig. 2(c)

$$\Sigma_2(\varepsilon,\xi_p)=\Delta^2 G_o^2(\varepsilon,\xi_p+2iv_F\varkappa\ \mathrm{sign}\xi_p)\Xi_2(\varepsilon,\xi_p) \qquad (11)$$

$$\Xi_2(\varepsilon,\xi_p)=G_o^{-2}(\varepsilon,\xi_p+2iv_F\varkappa\,\mathrm{sign}\xi_p)/\{G_o^{-1}(\varepsilon,\xi_p-2iv_F\varkappa\,\mathrm{sign}\xi_p-\Sigma_s(\varepsilon,\xi_p)\} \qquad (12)$$

etc.

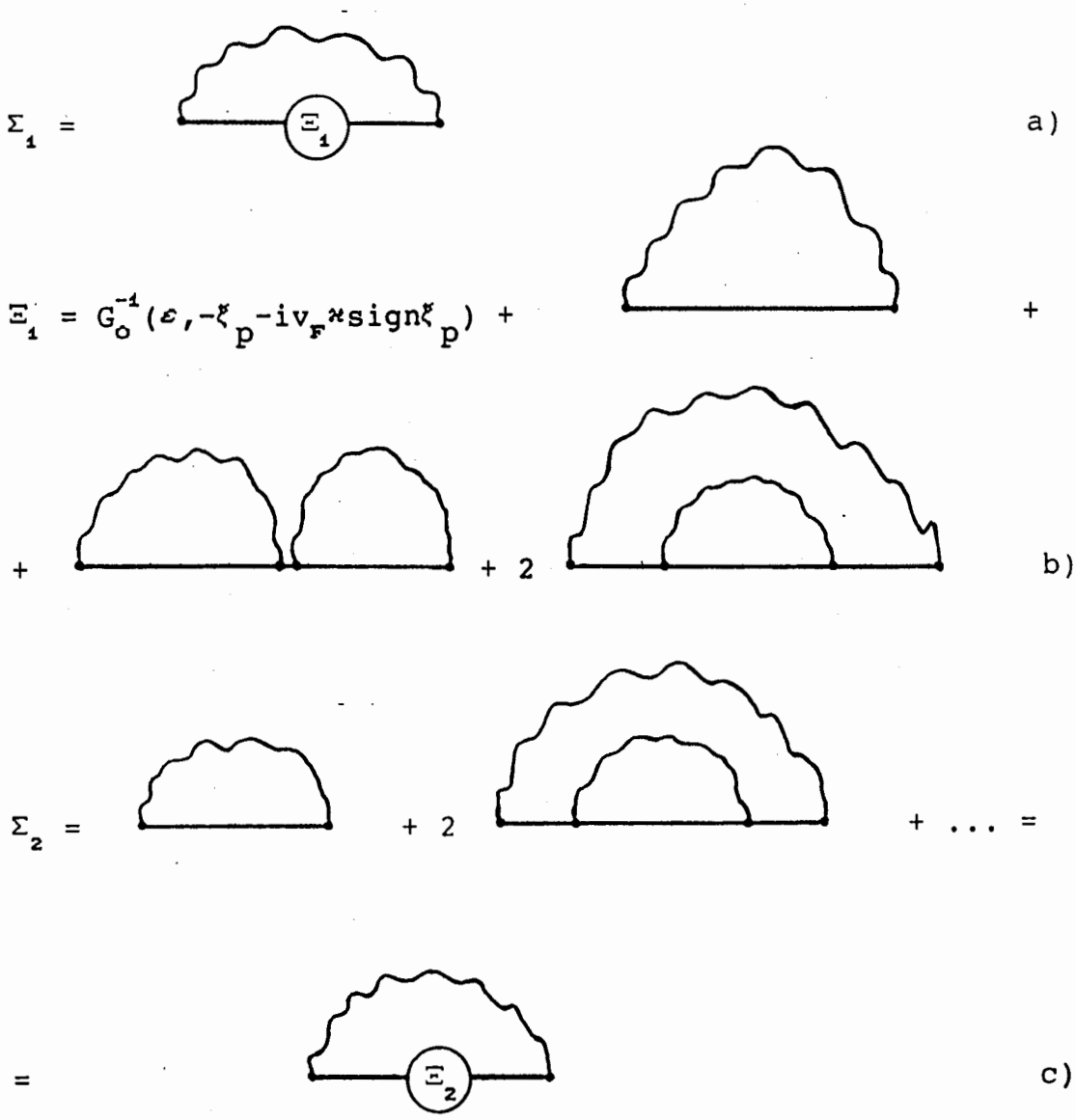


Fig. 2. Structure of Feynman series for self - energy.

Finally:

$$\Sigma_k(\varepsilon,\xi_p)=\Delta^2 G_o^2(\varepsilon,(-1)^k(\xi_p+ikv_F\varkappa\,\mathrm{sign}\xi_p))v(k)\Xi_k(\varepsilon,\xi_p) \tag{13}$$

$$\Xi_k(\varepsilon,\xi_p)=G_o^{-2}(\varepsilon,(-1)^k(\xi_p+ikv_F\varkappa\,\mathrm{sign}\xi_p))*$$

$$* \; 1/\{G_o^{-1}(\varepsilon,(-1)^k(\xi_p+ikv_F\varkappa))-\Sigma_{k+1}(\varepsilon,\xi_p)\} \tag{14}$$

$$\Sigma_k(\varepsilon,\xi_p)=\Delta^2 v(k)/(G_o^{-1}(\varepsilon,(-1)^k(\xi_p+ikv_F\varkappa\,\mathrm{sign}\xi_p))-\Sigma_{k+1}(\varepsilon,\xi_p))\equiv$$

$$\equiv \Delta^2 v(k)G_k(\varepsilon,\xi_p) \tag{15}$$

$$G_k(\varepsilon,\xi_p)=\{\varepsilon-(-1)^k(\xi_p+ikv_F\varkappa\,\mathrm{sign}\xi_p)-\Delta^2 v(k+1)G_{k+1}\}^{-1} \tag{16}$$

$$G_{k=o}(\varepsilon,\xi_p)=G(\varepsilon,\xi_p)$$

These recursion expressions give exact continuous - fraction representation of single - particle Green's function. Results of numerical evolution of the electronic density of states for different values of correlation length $\xi_{SDW}=\varkappa^{-1}$ were given in Ref. 11. In this paper the general discussion of the degradation of pseudo gap for diminishing correlation length was also presented. Here we shall be interested in the possibility to get an exact solution for the two - particle Green's function determining the frequency dependence of conductivity and the general picture of optical absorption in case of pseudo gap in density of states.

Consider the response of our system to the variation of external scalar potential $\delta\varphi_{q\omega}$:

$$\delta G(\varepsilon,\xi_p)=G(\varepsilon,\xi_p)J(\varepsilon,\xi_p;\varepsilon+\omega,\xi_{p+q})G(\varepsilon+\omega,\xi_{p+q})\delta\varphi_{q\omega} \tag{17}$$

where the vertex part

$$J(\omega,\xi_p;\varepsilon+\omega,\xi_{p+q})=-\delta G^{-1}(\varepsilon,\xi_p)/\delta\varphi_{q\omega} \tag{18}$$

for the free - particles is determined by the electronic charge e.

From (6) we have:

$$J(\varepsilon,\xi_p;\varepsilon+\omega,\xi_{p+q}) = e + \delta\Sigma_1(\varepsilon,\xi_p)/\delta\varphi_{q\omega} \equiv$$

$$\equiv e + \Im_1(\varepsilon,\xi_p;\varepsilon+\omega,\xi_{p+q}) \qquad (19)$$

The effective vertex $\Im_1$ is determined by diagram of Fig.3(a). Then using (13) - (16) we can get the following symbolic recursion formulae for vertex part:

$$\Im_1(\varepsilon,\xi_p;\varepsilon+\omega,\xi_{p+q})=\Delta^2 v(1)\delta G_1(\omega,\xi_p)/\delta\varphi_{q\omega}=$$

$$=\Delta^2 v(1)G_1(\varepsilon\xi_p)J_1(\varepsilon,\xi_p;\varepsilon+\omega,\xi_{p+q})G_1(\varepsilon+\omega,\xi_{p+q})$$

$$J_1(\varepsilon,\xi_p;\varepsilon+\omega,\xi_{p+q})=-\delta G_1^{-1}(\varepsilon,\xi_p)/\delta\varphi_{q\omega} = e + \delta\Sigma_2(\varepsilon,\xi_p)/\delta\varphi_{q\omega}\equiv$$

$$\equiv e + \Im_2(\varepsilon,\xi_p;\varepsilon+\omega,\xi_{p+q}) \qquad (20)$$

....

$$\Im_k(\varepsilon,\xi_p;\varepsilon+\omega,\xi_{p+q})=\Delta^2 v(k)\delta G_k(\varepsilon,\xi_p)/\delta\varphi_{q\omega}=$$

$$=\Delta^2 v(k)G_k(\varepsilon,\xi_p)J_k(\varepsilon,\xi_p;\varepsilon+\omega,\xi_{p+q})G_k(\varepsilon+\omega,\xi_{p+q})$$

$$J_k(\varepsilon,\xi_p;\varepsilon+\omega,\xi_{p+q})=-\delta G_k^{-1}(\varepsilon,\xi_p)/\delta\varphi_{q\omega}= e + \delta\Sigma_{k+1}(\varepsilon,\xi_p)/\delta\varphi_{q\omega}\equiv$$

$$\equiv e + \Im_{k+1}(\varepsilon,\xi_p;\varepsilon+\omega,\xi_{p+q}) \ .....$$

$$J_{k=0} \equiv J(\varepsilon,\xi_p;\varepsilon+\omega,\xi_{p+q})$$

To calculate variational derivatives of $\Sigma_k(\varepsilon,\xi_p)$ here it is sufficient to limit ourselves to the diagrams of the type shown in Fig. 3(b), because all graphs with intersecting interaction lines reduce to the corresponding graphs with no intersections [11].

In the following we are mostly interested in the vertex with one Green's function of R - type (retarded) and another of A - type (advanced). From diagram of Fig. 3(b) we can find the

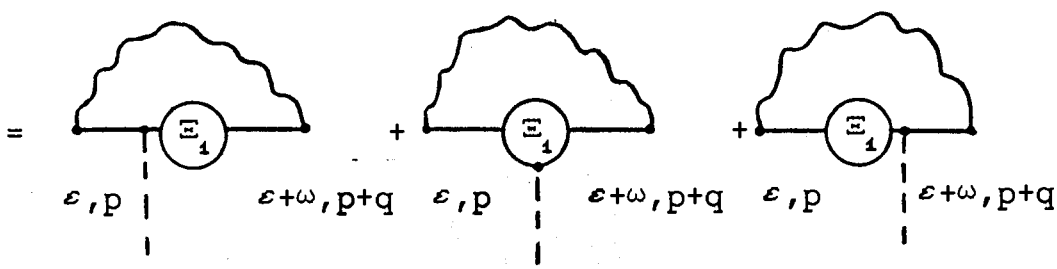
following contribution of RA - type:

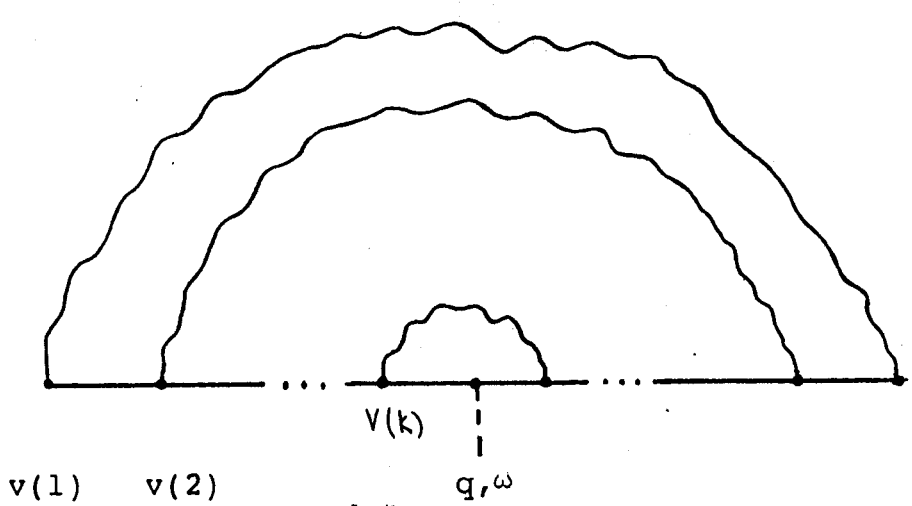
$$(\delta\Sigma_k(\varepsilon,\xi_p)/\delta\varphi_{q\omega})|_{RA}=$$

$$=\{1+2ikv_F\varkappa/(\omega-(-1)^k v_F q+v(k+1)\Delta^2(G^A_{k+1}(\varepsilon,\xi_p)-G^R_{k+1}(\varepsilon+\omega,\xi_{p+q})\}*$$

$$v(k)\Delta^2 G^A_k(\varepsilon,\xi_p)G^R_k(\varepsilon+\omega,\xi_{p+q})J^{RA}_k(\varepsilon,\xi_p;\varepsilon+\omega,\xi_{p+q}) \qquad (21)$$

$$\Im_1 = \delta\Sigma_1/\delta\varphi_{q\omega} = \qquad a)$$





b)

Fig. 3. Diagrams for electronic vertex.

As a result from (18) - (21) we get the fundamental recursion formula for the vertex part:

$$J^{RA}_{k-1}(\varepsilon,\xi_p;\varepsilon+\omega,\xi_{p+q}) = e + \Delta^2 v(k) G^A_k(\varepsilon,\xi_p) G^R_k(\varepsilon+\omega,\xi_{p+q}) * \{1+ \tag{22}$$

$$+2ikv_F \varkappa/(\omega - (-1)^k v_F q + v(k+1)\Delta^2 (G^A_{k+1}(\varepsilon,\xi_p) - G^R_{k+1}(\varepsilon+\omega,\xi_{p+q})))\} J^{RA}_k$$

Remaining analysis can be done numerically: we cut the continuous fractions for G - functions at some far away denominator, where we take $\Sigma_{k+1}=0$, and assume $J_k = e$, then we can sum everything to the limit of k=0.

To calculate the optical conductivity we shall use the general expressions [15,16]. Conductivity is expressed via the retarded density - density response function (polarization operator) $\chi(q,\omega)$:

$$\sigma(\omega) = e^2 \lim_{q \to 0} (-i\omega/q^2)\chi(q,\omega) \tag{23}$$

Accordingly we can determine the dielectric permeability:

$$\begin{aligned} &\mathrm{Re}\ \varepsilon(\omega) - 1 = -(4\pi/\omega)\,\mathrm{Im}\ \sigma(\omega) \\ &\mathrm{Im}\varepsilon(\omega) = (4\pi/\omega)\,\mathrm{Re}\ \sigma(\omega) \end{aligned} \tag{24}$$

The general expression for $\chi(q,\omega)$ is given by:

$$\chi(q,\omega) = \int d\varepsilon \Big\{ [f(\varepsilon+\omega) - f(\varepsilon)]\phi^{RA}(\varepsilon,q,\omega) + f(\varepsilon)\phi^{RR}(\varepsilon,q,\omega) -$$

$$- f(\varepsilon+\omega)\phi^{AA}(\varepsilon,q,\omega)) \Big\} \tag{25}$$

where $f(\varepsilon)$ - is Fermi distribution and we introduced the two - particle Green's functions ϕ^{RA}, ϕ^{RR}, ϕ^{AA}, as determined by graphs like that given in Fig. 4 [16].

$$\phi^{RA}(\varepsilon,q,\omega) = (1/2\pi i)$$

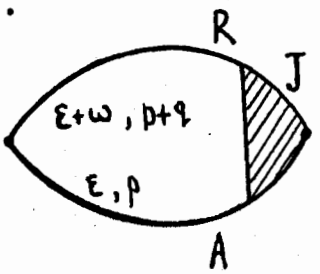


Fig. 4. Diagrammatic definition of $\phi^{RA}(\varepsilon,q,\omega)$.

For T=0 and $\omega \to 0$ it is easy to get:

$$\chi(q,\omega) = \omega\{\phi^{RA}(q\omega) - \phi^{RA}(0,\omega)\} \qquad (26)$$

where only the Fermi surface contributes ($\varepsilon = 0$). Note the important exact relation of the Ward identity type [15,16]:

$$\phi(0,\omega) = -N(\varepsilon_F)/\omega \qquad (27)$$

which expresses the particle conservation law. This relation was used to obtain (26). In (27) $N(\varepsilon_F)$ is the exact (renormalized) value of density of states at the Fermi level.

3. Results and Discussion

Numerical analysis was performed on a standard IBM PC/AT directly using (23), (26) and general recursion relations (6),(15),(16), (22) (Cf. Appendix).

The convergence of iterations for single - particle Green's function (density of states) is very fast (as was already noted in Ref. 11) - typical time to get a value of density of states at a given energy (with rather high accuracy) is less than a minute (starting from a denominator with k = 50 + 100). In case of conductivity situation is more complex and depends on parameters. In the main part of frequency interval $0.5\Delta \leq \omega \leq 3\Delta$, for intermediate values ξ_{SDW} ($0.2\Delta \leq hv_F/\xi_{SDW} \leq 2\Delta$), satisfactory convergence is achieved for a $k < (2 \div 5)10^2$, so to get a value of conductivity at a given frequency we need several minutes. Convergence is much slower outside this region, it is bad in particular for the limit of very small frequencies and also in case of very large correlation lengths (note that in the last case there exists an analytic exact solution [8 - 10]).

The accuracy of both the main procedure is well as a that of numerical calculations can be easily checked using the exact relation (27), comparing the values of $N(\varepsilon_F)$ obtained from the single particle Green's function (6), (15) (Cf. Ref. 11) with the same values obtained via two - particle Green's function $\phi^{RA}(0,\omega)$. For all values of ξ_{SDW} and for $\omega \leq \Delta$ we have practically ideal correspondence, while for $\omega \geq \Delta$ there are some discrepancies, apparently due to our use of relation (26) (valid for $\omega \to 0$) on the new frequency scale of the order of Δ (to use (26) here it is not sufficient to have $\omega \ll \varepsilon_F$). It is more accurate to use here the general expression (25), however the double integration in it makes calculation time unreality for PC/AT. Still our result for $\omega \cong 2\Delta$ are qualitatively quite satisfactory and there are no serious discrepancies from the exact analytic solution for large ξ_{SDW} [8 - 10].

In Fig. 5 we present the results for the case of incommensurate SDW - fluctuations. Conductivity is given in the units of $\omega_p^2/(8\pi\Delta)$ where ω_p - is plasma frequency, correlation length is determined by $\Gamma = hv_F\varkappa \equiv hv_F/\xi_{SDW}$. We can clearly see continuous degradation of intensity of optical transitions through a pseudo gap as ξ_{SDW} diminishes (Γ grows). Especially interesting are curves for $\Gamma \cong 2 \div 4$, which are in good qualitative agreement with experimental picture of optical conductivity of high - T_c oxides in superconducting phase [12,19]. This correlates with small values of $\xi_{SDW} \cong hv_F/\Delta$ (for 2Δ we expect the values of order of 0.1 ÷ 0.2 eV [12,19]), which are reasonable in this region of phase diagram [5, 6]. Apparently the same estimates can follow also from neutron scattering data [1 - 4]. In Fig.5 we compare our curves for $\Gamma = 3.5\ \Delta$ with the experimental data [19]. Dimensionless values of experimental data are given assuming $\omega_p = 3.2$ eV (as determined in Ref. 13) while we take $\Delta = 0.2$ eV. No special fitting of parameters was performed. Thus the model of a soft pseudo gap can qualitatively explain the non - Drudean behavior of optical absorption in high - T_c systems.

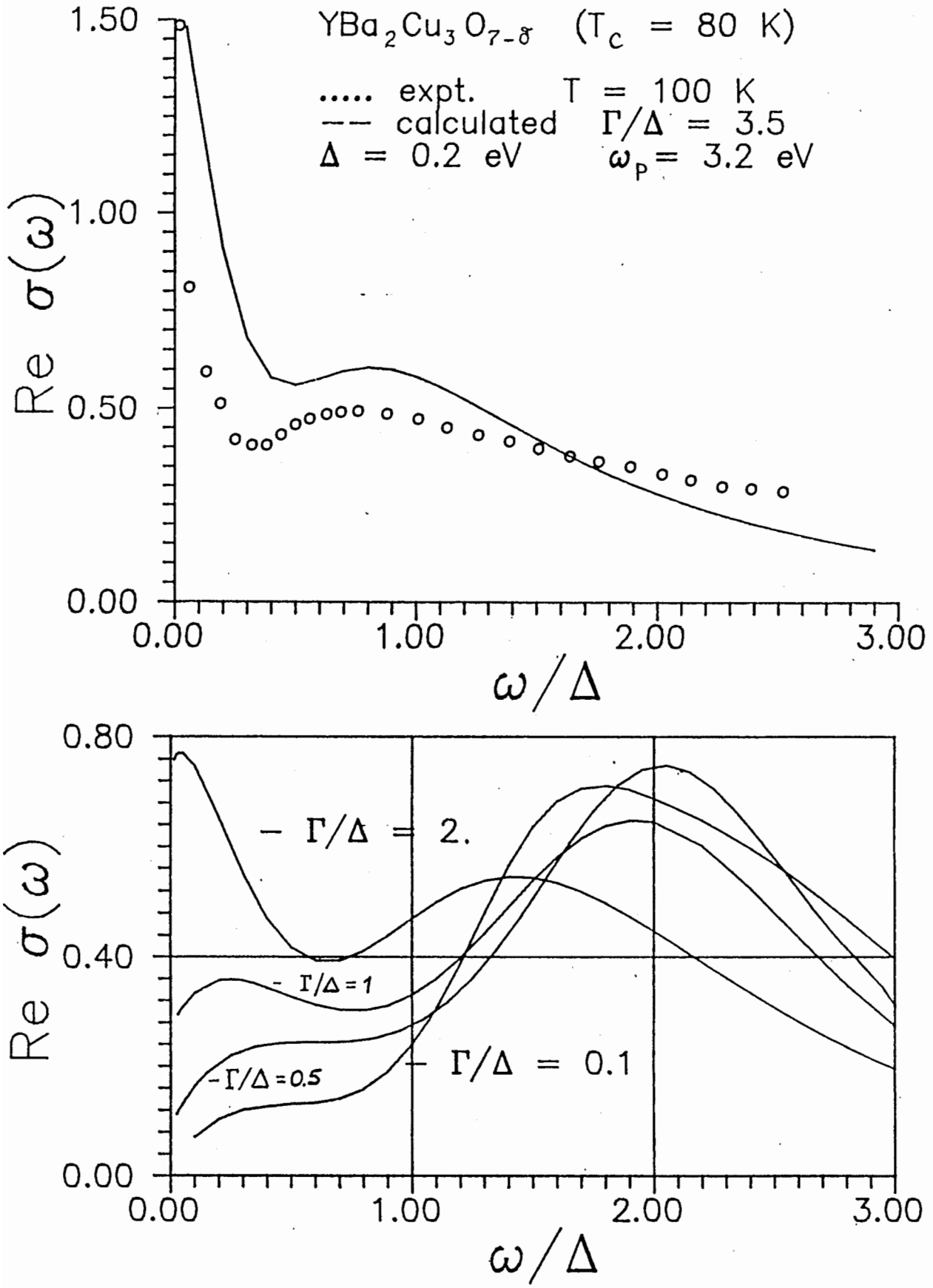


Fig. 5. Frequency dependence of real part of conductivity - incommensurate case.

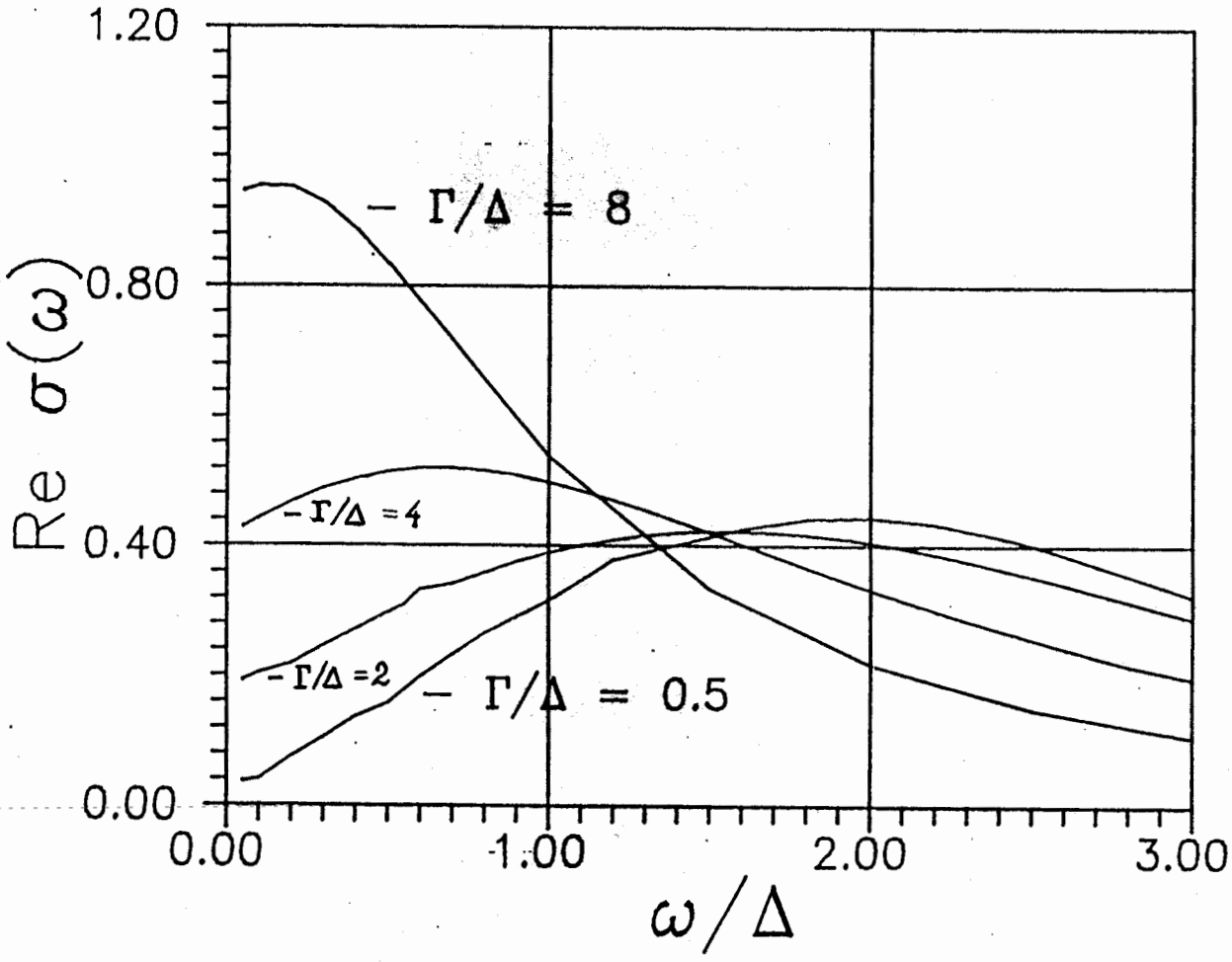


Fig. 6. Frequency dependence of real part of conductivity - commensurate case.

In case of commensurate SDW - fluctuations our results for optical absorption are shown in Fig.6. We can clearly see qualitative difference from incommensurate case. Here we cannot find curves similar to experimental data for high -T_c oxides. If we consider our model seriously this means that SDW - fluctuations in superconductivity phase are incommensurate, though AFM long - range order in the insulating phase is commensurate [1 - 4]. Neutron scattering data for superconducting phase are at present too incomplete to solve this problem definitely. In Fig. 7,8 we gave similar results for Re $\varepsilon(\omega)$.

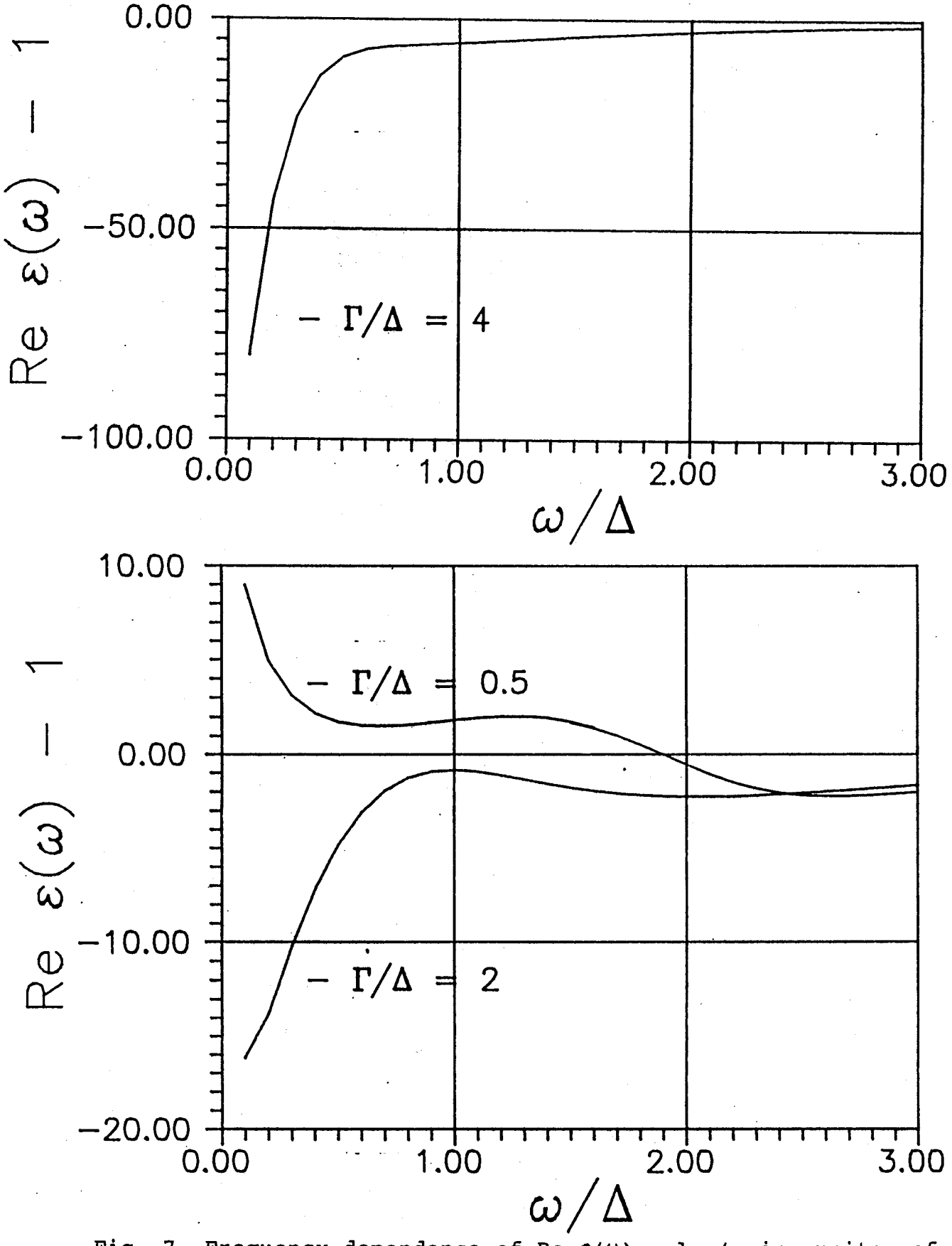


Fig. 7. Frequency dependence of Re $\varepsilon(\omega) - 1$ (in units of $\omega_p^2/8\pi\Delta^2$) - incommensurate case.

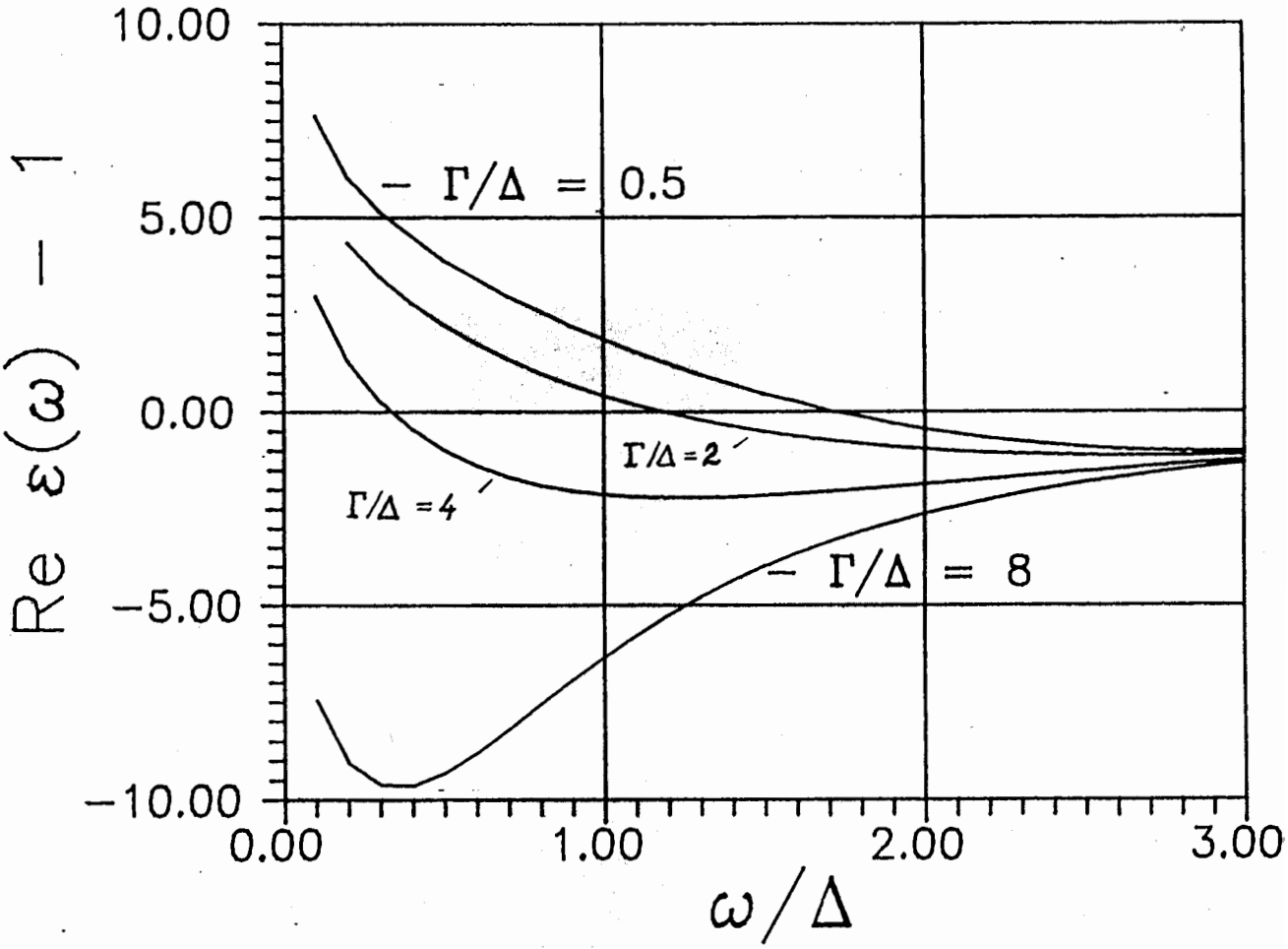


Fig. 8. Frequency dependence of Re $\varepsilon(\omega)$ - 1 (in units of $\omega_p^2/8\pi\Delta^2$) - commensurate case.

From the general theoretical point of view we note that for small frequencies and not so very big values of Γ, Re $\sigma(\omega)$ apparently demonstrated localization behavior (Re $\sigma(\omega \to 0) \to 0$), which can be expected in this (in fact 1 - dimensional) problem [17,18]. Unfortunately we cannot calculate Re $\sigma(\omega \to 0)$ in detail due to bad convergence of our procedure in the limit of small frequencies. However for large enough $\Gamma \geq \Delta$ we definitely see a transition to finite conductivity at $\omega = 0$ at least for incommensurate case (apparently this is also realized for the commensurate fluctuations). For $\omega \gg 2\Delta$ we always have usual Drude - like behavior. This observation of Anderson transition in one - dimension is also unexpected. Apparently it is due to some specific properties of our random - field model, which is quite different from usual "white - noise" limit [17,18]. However at

present we do not have any qualitative explanation of this result.

Now it is probably the moment to explain a question mark in the title of our paper. Any possible inaccuracy of the proposed solution is apparently connected with our use of Eq. (26), which is only valid for $\omega \to 0$, as well as due to rather bad convergence of numerical procedure at small frequencies, where the problem of localization is actually to be solved. Obviously these difficulties are only of technical nature. Physically most problematic is our use of the Gaussian model for SDW - fluctuations. We have already noted that this picture is probably valid rather far from the SDW - instability, near to it we have to take into account diagrammatic contributions of non - Gaussian nature. We also stress that we considered only the one - dimensional model of short - range order fluctuations, while real system of interest are two - dimensional.

4. Conclusion

We presented an exact solution for two - particle Green's functions of SDW (CDW). Numerical analysis was given for frequency dependence of conductivity at different values of correlation length of SDW (CDW) short - range order. Also we gave the similar results for dielectric permeability.

The picture of soft pseudo gap qualitatively explains the non - Drudean behavior of optical absorption in high - T_c systems, which may be considered as some confirmation of the "spin - bag" model of Schrieffer et al. However our results are in fact valid for any model of Gaussian fluctuations of short - range order of SDW (CDW) - type.Our model apparently demonstrates the Anderson transition in one - dimension.

Acknowledgements

Authors are grateful to Prof. J.R. Schrieffer for sending them a preprint of his work on a pseudo gap.

References

1. G. Shirane et al. Phys. Rev. Lett. 1987, v 59, p 1613
2. J. Tranquada et al. Phys. Pev. Lett. 1988, v 60, p 156
3. J. Tranquada et al. Phys. Rev. Lett. 1990, v 64, p 800
4. J. Tranquada, G. Shirane. NATO Advanced Research Workshop "Dynamics of Magnetic Fluctuations in High - T_c Materials", Greece, October 1989, preprint.
5. J.R. Schrieffer, X.G. Wen, S.C. Zhang. Phys. Rev. Lett. 1988, v 60, 944; Phys. Rev. 1989, v B39, p 11663.
6. A. Kampf, J.R. Schrieffer. Los Alamos, preprint, LA-UR-89-3149 (1989)
7. P.A. Lee, T.M. Rice, P.W. Anderson. Phys. Rev. Lett. 1973, v 31, p 462
8. M.V. Sadovskii. JETP, 1974, v 66, p 1720; Fiz. Tverd. Tela, 1974, v 16, p 2504
9. W. Wonneberger, R. Lautenschlager. J. Phys. 1976, v C9, p 2865
10. W. Wonneberger. J Phys. 1977, v C10, p 1074
11. M.V. Sadovskii. JETP. 1979, v 77, p 2070
12. T.Timusk, D.V.Tanner. In "The Physical Properties of High Temperature Superconductors", Ed. by D.M. Ginsberg, World Scientific, Singapore, 1988
13. G.A. Thomas et al. Phys. Rev. Lett. 1988, v 26, p 13130
14. P.V. Elutin. Optica and Spectroscopia, 1977, v 43, p 542
15. S.I. Maleev, B.P. Toperverg. JETP. 1975, v 69, p 1440
16. D. Vollhardt, P. Wolfle, Phys. Rev. 1980, v B22, p 4666
17. V.L. Berezinskii. JETP, 1973, v 65, p 1251
18. A.A. Gogolin. Phys. Reports. 1988, v 166, p 269

Appendix

Introducing dimensionless variables and definitions:

$$h = \omega/\Delta,\ y = v_F q/\Delta,\ x = \xi_p/\Delta,\ W = v_F \varkappa/\Delta,$$

$$S^A[k] \equiv v(k)\,\mathrm{Re}\ G_k^A(\varepsilon-\omega/2,\xi_{p-q/2})*\Delta,$$

$$Q^A[k] \equiv v(k)\,\mathrm{Im}\ G_k^A(\varepsilon-\omega/2,\xi_{p-q/2})*\Delta,$$

$$S^R[k] \equiv v(k)\,\mathrm{Re}\ G_k^R(\varepsilon+\omega/2,\xi_{p+q/2})*\Delta,$$

$$Q^R[k] \equiv v(k)\,\mathrm{Im}\ G_k^R(\varepsilon+\omega/2,\xi_{p+q/2})*\Delta,$$

$$DEN^{A,R}[k+1] \equiv (\mp h/2-(-1)^k(x\mp y/2)-S^{A,R}[k+1])^2+(\pm W*k+Q^{A,R}[k+1])^2,$$

it easy to find

$$S^{A,R}[k] = v(k)(\mp h/2-(-1)^k(x\mp y/2)-S^{A,R}[k+1])/DEN^{A,R}[k+1],$$

$$Q^{A,R}[k] = v(k)(\pm W*k-Q^{A,R}[k+1])/DEN^{A,R}[k+1] \qquad (A.1)$$

and

$$DAR[k+1] \equiv (h-(-1)^k y+S^A[k+1]-S^R[k+1])^2 + (Q^A[k+1]-Q^R[k+1])^2$$

$$D1[k] = ((S^A[k]-S^R[k])(h-(-1)^k y+S^A[k+1]-S^R[k+1]) + (Q^A[k]-Q^R[k])(Q^A[k+1]-Q^R[k+1]))/DAR[k+1],$$

$$D2[k] = ((Q^A[k]-Q^R[k])(h-(-1)^k y+S^A[k+1]-S^R[k+1]) - (S^A[k]-S^R[k])(Q^A[k]-Q^R[k]))/DAR[k+1],$$

write down the following expressions for real and imaginary parts of effective vertex:

$$\mathrm{Re}\ J_{k-1}(\varepsilon-\omega/2,\xi_{p-q/2};\varepsilon+\omega/2,\xi_{p+q/2})=1 + D1[k]*\mathrm{Re}J_k - D2[k]*\mathrm{Im}J_k,$$

$$\mathrm{Im}\ J_{k-1}(\varepsilon-\omega/2,\xi_{p-q/2};\varepsilon+\omega/2,\xi_{p+q/2})=D1[k]*\mathrm{Im}J_k + D2[k]*\mathrm{Re}J_k. \qquad (A.2)$$

Numerical calculations for conductivity were performed by the direct use of general expressions like:

$$\sigma(\omega) = \omega_p^2/(8\pi\Delta)\ \sigma_o(h),$$

$$\sigma_o(h) = \lim_{y\to 0}(1/y^2)\left\{1/2(\sigma_o(h,y) + \sigma_o(h,y))-\sigma(h,0)\right\}, \text{ where}$$

$$\mathrm{Re}\ \sigma_o(h,y) = \int dx \left[(S^A[0]S^R[0]-Q^A[0]Q^R[0])\mathrm{Re}J_o - (S^A[0]Q^R[0]+S^R[0]Q^A[0])\mathrm{Im}J_o\right],$$

$$\mathrm{Im}\ \sigma_o(h,y) = \int dx \left[(S^A[0]S^R[0]-Q^A[0]Q^R[0])\mathrm{Im}J_o + (S^A[0]Q^R[0]+S^R[0]Q^A[0])\mathrm{Re}J_o\right].$$

Typical value of y, at which the procedure converges, is of the order of $10^{-2} + 10^{-3}$ for $h \gg y$.