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### Introduction.

Electronic theory of disordered systems is far at present from the state of relative completeness and clearness typical for the theory of crystalline solids /1/. Most of its qualitative results are based on the concept of electron localization in a random field /2/ and the existence of the so called mobility edges (i.e. critical energies where a transition from localized to extended states takes place). The understanding of the nature of electronic states and transport properties near the mobility edge is the main and still unsolved problem of the theory.

There are two in some sense complementary approaches to this problem. The first is based upon Anderson theory /3-5/ of localization, when the statistical convergence properties of a perturbation series (the most probable one-electron Green function) are investigated. In this approach it is possible to define a localization criterion (the position of mobility edges in a band) and also the spatial behavior of localized wave

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functions /4,6/ near the mobility edge. At the same time it<sup>191</sup> is difficult to define electronic density of states and, more importantly, it seems impossible to calculate conductivity. The second traditional approach, based on a famous work by Edwards /7/, considers the Green functions averaged upon the random configurations of scattering centers or the appropriate random fields /7-11/. This formalism is quite natural for the calculation of different physical quantities such as density of states or conductivity, but it was of very limited success in the studies of the mobility edge region /10,11/.

Recently there were some attempts /6,12,13/ to understand the behavior of electrons near the mobility edge starting from certain analogies of the problem with a well known situation in the critical region of the second-order phase transitions /14/, using a formal equivalence (or rather correspondence) of the problem of an electron in a random field and the problem of phase transition with a zero-component order parameter. This last problem was successfully applied for the solution of the excluded volume problem in the theory of polymer configurations /15,16/. In Ref. /6/ within a framework of Anderson theory /3,4/ it was shown that the spatial behavior of electronic wave functions can be described by characteristic scaling formulae with critical exponents depending only upon the space dimensionality and the number of components ( $n = 0$ ) of the relevant order parameter. In particular, the correlation length of a phase transition /15,16/ defines the behavior of the localization length /6/. The papers by Toulouse /12/ and Thouless /13/ are concerned with the second (Edwards) approach in the theory

of disordered systems. Thouless, in particular, was able to make an analytic continuation of an exact solution of one-dimensional phase transition theory with  $n = 0$  and obtained the known exact results for an electron in one-dimensional random field /13/. It was found that the phase transition itself has nothing to do with the localization problem.

#### I. Anderson model. Scaling at the mobility edge.

Following Anderson we consider an electron propagating in a regular lattice with random energy levels at different lattice sites. The electron become localized for a great enough ratio of the amplitude of random fluctuation of energy levels  $W$  and the amplitude  $V$  of electronic transitions from site to site. Our main interest is to study the electronic states near the critical ratio  $(W/V)_c$ . We follow the approach of Ref. /4,6/.

Consider Anderson Hamiltonian:

$$H = \sum_j E_j a_j^\dagger a_j + \sum_{ij} V_{ij} a_i^\dagger a_j \quad (1.1)$$

Here  $a_i^\dagger$ ,  $a_i$  are electronic creation and destruction operators at the  $i$ -th site,  $E_j$  are random energy levels, distributed according to:

$$P(E_j) = \begin{cases} \frac{1}{W}; & |E_j| < \frac{1}{2} W \\ 0; & |E_j| > \frac{1}{2} W \end{cases} \quad (1.2)$$

The amplitude  $V_{ij}$  is defined to be constant  $V$  for nearest neighbours in the lattice.

The character of electronic states is defined by the Green function:

$$G_{ij}(E) = \left\langle R_i \left| \frac{1}{E - H} \right| R_j \right\rangle \quad (1.3)$$

i.e. the amplitude of an electronic transition (electron with energy equal to  $E$ ) from site  $j$  to site  $i$ . For this Green function a renormalized perturbation series in  $V$  is constructed /3,4/. Then we have:

$$G_{ij}(E) = F_{ij}(E) G_{jj}(E) \quad (1.4)$$

where the series for  $F_{ij}(E)$  is of the form:

$$F_{ij}(E) = \frac{1}{E - E_i - \Delta_i^j(E)} V_{ij} + \sum_{l \neq ij} \frac{1}{E - E_i - \Delta_i^{jl}(E)} V_{il} \frac{1}{E - E_l - \Delta_l^j(E)} V_{lj} + \dots \quad (1.5)$$

and

$$G_{jj}(E) = \frac{1}{E - E_j - \Delta_j(E)} \quad (1.6)$$

For the self-energies  $\Delta_j(E)$ ,  $\Delta_j^{ij}(E)$ ,  $\Delta_j^{il}(E)$  etc. the analogous perturbation expansions can be constructed /3,4/. Note that in (1.5) the repeating indices are suppressed, so that in the  $N$ th order of perturbation theory the number of different terms in a series (1.5) is defined by the number of self-avoiding walks of  $N$  steps going from  $j$  to  $i$ . Anderson has shown /3/ that the localization criterion is just the convergence condition of the series considered (convergence is understood to be convergence in probability /3,5/). In the



localized states region the series converges with a probability equal to unity, and the convergence condition defines the critical ratio  $(W/V)_c$  or the position of mobility edges in a band. The most probable behavior of Green function in the middle of the band ( $E = 0$ ) is described by:

$$G_{ij}(E) \Big|_{E=0} \sim \sum_{N=1}^{\infty} Z_N(R_i - R_j) (2V/W)^N \Psi^N(V/W, K) \quad (1.7)$$

where  $Z_N(R_i - R_j)$  is the number of self-avoiding walks of  $N$  steps linking the sites  $i$  and  $j$ ,  $\Psi$  is a slowly varying (logarithmic) function of  $V/W$  and the connectivity constant  $K$  of the lattice /3/. If we are going to consider localization at some arbitrary energy  $E$  in the band, then we have to replace  $2V/W$  in (1.7) by  $2N(E)V$ , where  $N(E)$  is the electronic density of states /4/.

The critical width of the random level distribution  $W_c$  (or in general case the critical ratio  $(W/V)_c$ ) is defined by the convergence condition /3/:

$$I = (2eV/W_c)K \Psi(V/W_c, K) \quad (1.8)$$

Anderson has used /4/ the  $Z_N(R)$  found in the numerical experiments. We shall use an analytic theory due to de Gennes and des Cloizeaux /14,15,16/ based upon Wilson theory of critical phenomena. The function  $Z_N(R)$  in  $d$ -dimensional space is represented by the inverse Laplace transform:

$$Z_N(R) = \int_{c-i\infty}^{c+i\infty} \frac{d s}{2\pi i} \exp(Ns) G_U(s, R) \quad (1.9)$$

where  $G_U(s, R)$  is an unrenormalized Green function of an Euclidian field theory (Ginzburg-Landau theory of phase transitions) with a Lagrangian:

$$L(x) = 1/2 \sum_{j=1}^n \left\{ (\nabla \phi_j)^2 + m_0^2 \phi_j^2 \right\} + \frac{1}{8} g_0 \left( \sum_{j=1}^n \phi_j^2 \right)^2 \quad (1.10)$$

where  $n$  is the number of components of the  $\phi$ -field, which in the problem under consideration has to be equal to zero. Dimensionless parameter  $s$  is linked with an unrenormalized mass  $s = m_0^2 a^2$ , where  $a$  is a characteristic length of the order of a lattice constant. Phase transition corresponds /14/ to the renormalized mass going to zero with  $s \rightarrow s_c$ :

$$m \sim a^{-1} (s - s_c)^{\nu} \quad (1.11)$$

where  $\nu$  is the correlation length critical exponent.

In (1.9)  $c > s_c$ . The parameter  $s_c$  is linked with the lattice connectivity constant /15-17/:

$$K = \exp(s_c) \quad (1.12)$$

Using (1.9) in (1.7) we obtain: ( $R = R_i - R_j$ )

$$\begin{aligned} G_{ij} &\sim \sum_{N=1}^{\infty} \int_{c-i\infty}^{c+i\infty} \frac{ds}{2\pi i} \exp N(s - s_c) G_U(s, R) (2V/W)^N K^N \Psi^N(V/W, K) \\ &\sim \int_{c-i\infty}^{c+i\infty} \frac{ds}{2\pi i} G_U(s, R) \sum_{N=1}^{\infty} \exp(N(s - s_c) + N \ln W_c/W) = \\ &= G_U(\ln W/W_c + s_c, R) \end{aligned} \quad (1.13)$$

-the principal result showing that the most probable spatial behavior of the one-electron Green function of the Anderson model in the region of localized states ( $W \gtrsim W_c$ ) is the same as that of the correlation function of the phase transition theory (1.10) with  $n = 0$ .  $W = W_c$  is an exact analogue of the critical temperature.

For  $W \gtrsim W_c$  the Green function is exponential /14/:

$$G_{ij} \sim \exp(-|R|/R_{loc}); \quad |R| \gg R_{loc} \quad (1.14)$$

where

$$R_{loc} \sim m^{-1} \sim a \left| \frac{W - W_c}{W_c} \right|^{-\nu} \quad (1.15)$$

is the localization length. Within the framework of Wilson  $\mathcal{E}$ -expansion ( $d = 4 - \mathcal{E}$ ) /14/ for  $n = 0$  we obtain:

$$\nu \approx \frac{1}{2} \left\{ 1 + \frac{\mathcal{E}}{8} + \frac{15}{256} \mathcal{E}^2 + \dots \right\} \approx 0.592 \quad \text{for } \mathcal{E} = 1 \quad (1.16)$$

in excellent agreement with Anderson's  $\nu = 0.6$  /4/ obtained from numerical analysis of self-avoiding walks. For  $W = W_c$ :

$$G_{ij} \sim |R|^{-(d-2+\eta)} \quad (1.17)$$

where:

$$\eta \approx \frac{\mathcal{E}^2}{64} \left\{ 1 + \frac{17}{16} \mathcal{E} \right\} \approx 0.032 \quad \text{for } \mathcal{E} = 1 \quad (1.18)$$

The smallness of  $\eta$  leads to impossibility in the present model of power law localization proposed by Thouless /14/, who used numerically determined values of critical exponents in the preexponential factor of  $Z_N(R)$ , which apparently are very poor.

The above considerations are invalid in one-dimension /19/, because in the present model with  $V$  nonzero only for nearest neighbors, the renormalized perturbation series for the Green function consists of only two terms in one-dimension (one step to the left and one step to the right). The problem of localization is reduced to the study of convergence properties of some continued fraction /19/ and the self-avoiding walks has nothing to do with it. From different considerations the same conclusion was reached by Thouless /13/.

## 2. Electron in the system of random impurities.

Consider an electron in the field of randomly placed scattering centres. The Hamiltonian is of the form:

$$H(r, R_j) = -\frac{1}{2m} \nabla_r^2 + \sum_{j=1}^N V(r - R_j) \quad (2.1)$$

where  $V(r - R_j)$  is the potential of the scatterer placed at  $R_j$ ,  $m$  is the electron mass,  $N$  is the number of scatterers. The spatial distribution function of the scatterers is of the form:

$$P(R_j) = \Omega^{-N} \quad (2.2)$$

where  $\Omega$  is the volume of the system. In the following we always consider an averaged Green function /7,8,10,11/. It can be represented by the following path-integral /20,21/:



$$G(r - r', t) = \int_{r(0)=r'}^{r(t)=r} \mathcal{D}r(\tau) \exp\left\{ \frac{im}{2} \int_0^t d\tau \dot{r}^2(\tau) + \rho \int dR \left[ \exp(-i \int_0^t d\tau V(r(\tau) - R)) - 1 \right] \right\} \quad (2.3)$$

Consider the limit  $\rho \rightarrow \infty$ ,  $V \rightarrow 0$ ,  $\rho V^2 \rightarrow \text{const}$ .

Placing the energy zero at the mean scattering potential we obtain /10,11/:

$$G(r - r', t) = \int_{r(0)=r'}^{r(t)=r} \mathcal{D}r(\tau) \exp\left\{ \frac{im}{2} \int_0^t d\tau \dot{r}^2(\tau) - \frac{\rho}{2} \int_0^t d\tau_1 \int_0^t d\tau_2 W(r(\tau_1) - r(\tau_2)) \right\} \quad (2.4)$$

where

$$W(r(\tau_1) - r(\tau_2)) = \int dR V(r(\tau_1) - R) V(r(\tau_2) - R) \quad (2.5)$$

which is equivalent /11/ to the study of an electron in a Gaussian random field:

$$G(r - r', t) = \langle G(rr', t \{ \emptyset \} ) \rangle_{\phi} \quad (2.6)$$

where

$$G(rr', t \{ \emptyset \} ) = \int_{r(0)=r'}^{r(t)=r} \mathcal{D}r(\tau) \exp\left\{ \frac{im}{2} \int_0^t d\tau \dot{r}^2(\tau) - i \int_0^t d\tau \emptyset[r(\tau)] \right\} \quad (2.7)$$

is the Green function of an electron in the field  $\emptyset(r)$  and averaging in (2.6) is taken upon the distribution of  $\emptyset(r)$  of the form:

$$P(\emptyset(r)) = N \exp\left\{ - \frac{1}{2\mathcal{Q}} \int dr \int dr' \emptyset(r) W^{-1}(r-r') \emptyset(r') \right\} \quad (2.8)$$

where

$$\int dr'' W(r - r'') W^{-1}(r'' - r') = \delta(r - r') \quad (2.8)$$

We use the "white-noise" correlator /8/:

$$W(r - r') = v^2 \delta(r - r') \quad (2.9)$$

corresponding to  $\delta$ -function scattering potential. Green function (2.4) is defined by the well-known Edwards diagramm series /7/, where only diagrams with two interaction lines entering the "cross" are present (see Fig. 1(a-d)), corresponding to Gaussian statistics of the random field (2.8). The diagrams of the type Fig. 1(d) are absent. In the momentum representation two interaction lines with a "cross" correspond to  $\rho V^2$ .

Making in (2.4) an analytic continuation on imaginary time  $t \rightarrow i\beta$ , thus going to a description of thermodynamical properties of the system, we obtain:

$$G(r - r', \beta) = \int_{r(0)=r'}^{r(\beta)=r} \mathcal{D}r(s) \exp\left\{ - \frac{m}{2} \int_0^\beta ds \dot{r}^2(s) + \frac{\rho V^2}{2} \int_0^\beta ds_1 \int_0^\beta ds_2 \delta(r(s_1) - r(s_2)) \right\} \quad (2.10)$$

The path-integral of this type describes the thermodynamics of a polymer chain with an attractive interaction between the monomers /10,11/. Following the method used by de Gennes and des Cloizeaux in the theory of polymers with repulsion (excluded volume problem) /15,16/, within the framework of perturbation theory it is easy to demonstrate that the Green function  $G(r - r', \beta)$  or  $G(p, \beta)$  in the momentum representation is defined by the inverse Laplace transform:

$$G(p, \beta) = \int_{c-i\infty}^{c+i\infty} \frac{d\tau}{2\pi i} \exp(\tau\beta) G(p, \tau) \quad (2.11)$$

where  $G(p)$  is the Green function of the field theory with Lagrangian (compare (1.10)):

$$L(x) = \frac{1}{2} \sum_{j=1}^n \left\{ \frac{1}{2m} (\nabla \phi_j)^2 + \tau \phi_j^2 \right\} - \frac{1}{8} g v^2 \left( \sum_{j=1}^n \phi_j^2 \right)^2 \quad (2.12)$$

Here  $n$  is again the number of components of  $\phi$ , which is supposed to be equal to zero. This condition excludes (at the end of the calculation) "extra" diagrams with closed loops, proportional to  $n$ , which are absent for the electron in a random field. (see Fig.2(c-e)). The diagrams of the type Fig. 2(a,b) are retained.

It can also be easily seen that the Fourier-transformed retarded electron Green function  $G^R(p, E)$  (where  $E = E + i\sigma$ ) can be obtained from  $G(p, \tau)$  of (2.12) via the direct analytic continuation  $\tau \rightarrow -E$ .

Note the "wrong" sign of the coupling constant in (2.12), corresponding to the attractive interaction of the " $\phi$ -particles". This fact /6/ is responsible for the drastic difference of the problem under consideration from that of critical phenomena. In Ref. /28/ this essential point was unfortunately missed and the results claimed are apparently wrong. It is well known that such problem of a field theory is unstable in the sense of absence of the ground state /22/. The physical meaning of this phenomenon is discussed below.

### 3. Weak coupling approximation. "Parquet" solution.

It is well known that in four-dimensional space ( $d = 4$ ) the problem defined by (2.12) may be solved in the "parquet" approximation in case of repulsive interaction /23/. "Parquet" is the dominating sequence of diagrams also for  $d = 4 - \epsilon$ . In the case of attractive interaction it is also valid, because selection of leading diagrams does not depend upon the sign of a coupling constant.

It is easy to check, that the series of "parquet" diagrams for the theory (2.12) is an expansion in powers of a parameters  $u$  where:

$$s = \begin{cases} \ln \frac{1}{\text{Max } a \{ \sqrt{2} m \tau, p_i \}} ; d = 4 \\ \frac{1}{\epsilon} \left\{ \frac{1}{\text{Max } a^\epsilon [ \sqrt{2} m \tau, p_i ]} \right\}^{\epsilon-1} ; d = 4 - \epsilon \end{cases} \quad (3.1)$$

$$U = \frac{m^2 a^\epsilon}{2 \pi^2} g v^2 \quad (3.2)$$

Here  $u$  is the dimensionless coupling constant,  $p_i$  are the external momenta of a given diagram,  $a$  is the constant of elementary length (diverging integrals cutoff), the minimal length scale in the problem. In the following, everywhere there it is possible we are meaning the limit of  $a \rightarrow 0$ .

In our analysis the behavior of four-point vertex part  $\Gamma(s)$  with all external momenta of the same order of magnitude is of prime importance /23/. "Parquet" approximation gives /23/:

$$\Gamma(s) = - \frac{u}{1 - u s} \quad (3.3)$$



Such vertex behavior corresponds to the asymptotic freedom in the theory (2.12). The pole at  $s_c = u^{-1}$  corresponds to the breakdown of the perturbation theory in the region  $s \gg u^{-1}$ . In the renormalization group formalism it corresponds to the growth of the invariant charge of the theory for  $s \rightarrow u^{-1}$ , which is an evidence of the perturbation theory breakdown /24/.

Let us look for the physical consequences of the existence of the pole in (3.3). Introduce:

$$Z_\beta = \int d^d r G(r-r, \beta) = \int_{c-i\infty}^{c+i\infty} \frac{d\tau}{2\pi i} e^{\tau\beta} \int \frac{d^d p}{(2\pi)^d} G(p, \tau) \quad (3.4)$$

which is the partition function for the polymer chain /10/. Going from  $Z_\beta$  to  $Z(t)$  via the analytic continuation  $\beta \rightarrow it$ , we obtain the electronic density of states:

$$N(E) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dt \exp(itE) Z(t) \quad (3.5)$$

Define:

$$c(\tau) = \int \frac{d^d p}{(2\pi)^d} \frac{\partial G(p, \tau)}{\partial \tau} \quad (3.6)$$

which is a specific heat in the theory of critical phenomena. /23/. Graphically it is shown in Fig. 3. Using the Ward identity:

$$\frac{\partial G(p, \tau)}{\partial \tau} = \mathcal{G}(s) G^2(s) \quad (3.7)$$

we find (see also /23/):

$$c(s) = -\frac{m^2}{2\pi^2} \int_0^s dt \mathcal{G}^2(t) \quad (3.8)$$

where

$$\mathcal{G}(s) = \exp\left\{ 2 \int_0^s dt \Gamma(t) \right\} \quad (3.9)$$

In the present problem ( $n = 0$ ) we have:

$$c(s) = -\frac{m^2}{\pi^2 u} (1 - (1 - us)^{1/2}) \quad (3.10)$$

Using the differentiation rule for the Laplace transformation, from (3.10) we obtain:

$$\begin{aligned} Z_\beta &= -\frac{1}{\beta} \int_{c-i\infty}^{c+i\infty} \frac{d\tau}{2\pi i} e^{\tau\beta} \frac{\partial}{\partial \tau} \int \frac{d^d p}{(2\pi)^d} G(p, \tau) \\ &= -\frac{1}{\beta} \int_{c-i\infty}^{c+i\infty} \frac{d\tau}{2\pi i} e^{\tau\beta} c(\tau) \end{aligned} \quad (3.11)$$

Then, using (3.10), (3.1) (for  $p_i = 0$ ):

$$Z_\beta = -\frac{m^2}{\pi^2 u} \frac{1}{\beta} \int_{c-i\infty}^{c+i\infty} \frac{d\tau}{2\pi i} e^{\tau\beta} \left\{ \left[ 1 - \left( \frac{E_{sc}}{\tau} \right)^{\varepsilon/2} \right]^{1/2} - 1 \right\} \quad (3.12)$$

Here  $c > E_{sc}$ ,

$$E_{sc} = \frac{1}{2 m a^2} \left( \frac{u}{\varepsilon} \right)^{2/\varepsilon} \quad (3.13)$$

Expanding the integrand in (3.12) and performing the Laplace transformation, analytic continuation  $\beta \rightarrow it$  and finally the Fourier transformation (3.5) we have:

$$\begin{aligned} N(E) &= \frac{m^2}{\pi^2 u} E \left\{ \frac{1}{2} \frac{1}{\Gamma(\frac{\varepsilon}{2}) \Gamma(2-\frac{\varepsilon}{2})} \left( \frac{E}{E_{sc}} \right)^{-\varepsilon/2} + \right. \\ &\quad \left. + \frac{1}{8} \frac{1}{\Gamma(\varepsilon) \Gamma(2-\varepsilon)} \left( \frac{E}{E_{sc}} \right)^{-\varepsilon} + \frac{1}{16} \frac{1}{\Gamma(\frac{3}{2}\varepsilon) \Gamma(2-\frac{3}{2}\varepsilon)} \left( \frac{E}{E_{sc}} \right)^{-3\varepsilon/2} + \dots \right\} \end{aligned} \quad (3.14)$$

for  $E > E_{sc}$ . For  $\varepsilon \rightarrow 0$ :

$$N(E) = \frac{m^2}{\pi^2 u} E \left\{ \frac{1}{2} \frac{\varepsilon}{2} \left( \frac{E}{E_{sc}} \right)^{-\varepsilon/2} + \frac{1}{8} \varepsilon \left( \frac{E}{E_{sc}} \right)^{-\varepsilon} + \frac{1}{16} \frac{3}{2} \varepsilon \left( \frac{E}{E_{sc}} \right)^{-3\varepsilon/2} + \dots \right\} \quad (3.15)$$



This series is easily summed:

$$N(E) = N_0(E) \frac{1}{\sqrt{1 - \left(\frac{E_{sc}}{E}\right)^{\xi/2}}} \quad (3.16)$$

where

$$N_0(E) = \frac{1}{(4\pi)^2} (2m)^{2-\xi/2} E^{1-\xi/2} \quad (3.17)$$

is the free-electron density of states in  $d = 4 - \xi$ .

The above results can also be derived by the direct analytic continuation  $\zeta \rightarrow -E$  in (3.6) - (3.10).

Thus the "parquet" approximation leads to an unphysical singularity in the density of states at  $E = E_{sc}$ , (see Fig.4, curve a), corresponding to the pole in the vertex (3.3). Broken line in Fig.4 displays the free-electron density of states.  $E_{sc}$  (3.13) is just the width of the Ginzburg critical region of our theory /25/, that is the energy at which all diagrams become relevant. This form of  $E_{sc}$  was recently conjectured by Toulouse /12/. In the region  $E \gg E_{sc}$  the density of states can be expanded in powers of  $(E_{sc}/E)^{\xi/2}$  (3.15), (3.14) which is just the perturbation series for  $u \ll 1$ . "Parquet" approximation (weak coupling) is correct only for  $E \gg E_{sc}$  and breaks for  $E \sim E_{sc}$  due to the growth of the corresponding invariant charge of the theory. In the theory of critical phenomena the invariant charge is small (repulsion) also in the critical region, i.e. perturbation theory is valid. Here the situation is quite analogous to the Kondo problem /26/ where an unphysical singularity also appears in the parquet approximation, at the energies (temperatures) where the perturbation theory breaks down.

#### 4. Strong coupling approximation and "crossover".

In the theory of disordered systems exists the well-known self-consistent field theory of Zittartz-Langer and Edwards /8,10,11/ (or the method of "optimal" fluctuation due to Lifshitz /9/) which enables to find the far asymptotics of the tail of the density of states, due to localized electrons. To compare it with perturbation theory we generalize the relevant results to  $d = 4 - \xi$ .

In the self-consistent field theory /8-10/ it is assumed that an electron in localized state region is bound in an effective potential well of linear dimension  $R$ , and the far asymptotics of the tail of the density of states is determined by the lowest level in this well. Then for the density of states in  $d$ -dimensional space we have /10/:

$$N(E) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dt \exp\left\{-i t \left[ \frac{d\pi^2}{2m R^2} - E \right] - \frac{\rho V^2 t^2}{2R^d}\right\} \\ = \left\{ \frac{R}{2\pi \rho V^2} \right\}^{1/2} \exp\left\{-\frac{1}{2 \rho V^2} \left[ \frac{d\pi^2}{2m R^2} - E \right]^2 R^d\right\} \quad (4.1)$$

The size of the well (localization length)  $R_0$  is determined by the minimization of the exponent:

$$\frac{d}{dR} R^d \left\{ \frac{d\pi^2}{2m R^2} - E \right\}^2 = 0 \quad (4.2)$$

giving:

$$R_0 = \left\{ \frac{2m(-E)}{(4-d)\pi^2} \right\}^{-1/2} \quad (4.3)$$

so that the density of states for  $d = 4 - \xi$  is of the form:

$$N(E) \sim \exp \left( - \frac{\pi^2}{\epsilon} \left( |E| / E_{sc} \right)^{\epsilon/2} \right) \quad (4.4)$$

for  $E < 0$ , (see Fig.4, curve b). This expression is valid for  $|E| \gg E_{sc}$ , where  $E_{sc}$  is again defined by (3.13). In (4.4) we have dropped the preexponential factor which cannot be accurate in this approximation. The presence of an infinite tail is connected with the Gaussian nature of the random field in which very deep fluctuations are present. That is apparently the reason for the absence of the ground state in the field theory (2.12). It is obvious that there is no fundamental problem here (it is not so in relativistic field theory /22/!). (4.4) can be expanded in powers of  $(|E| / E_{sc})^{\epsilon/2}$  which can be interpreted as an expansion in powers of an inverse coupling constant  $u^{-1}$ .

Thus in the region  $E \sim 0$ , i.e. around the mobility edge, there is a region of the width  $2E_{sc}$ , outside which the density of states can be described either by the perturbation theory in powers of  $u$  (weak coupling region  $E \gg E_{sc}$ ), or by the "perturbation" theory in powers of  $u^{-1}$  (strong coupling region  $|E| \gg E_{sc}$ ).

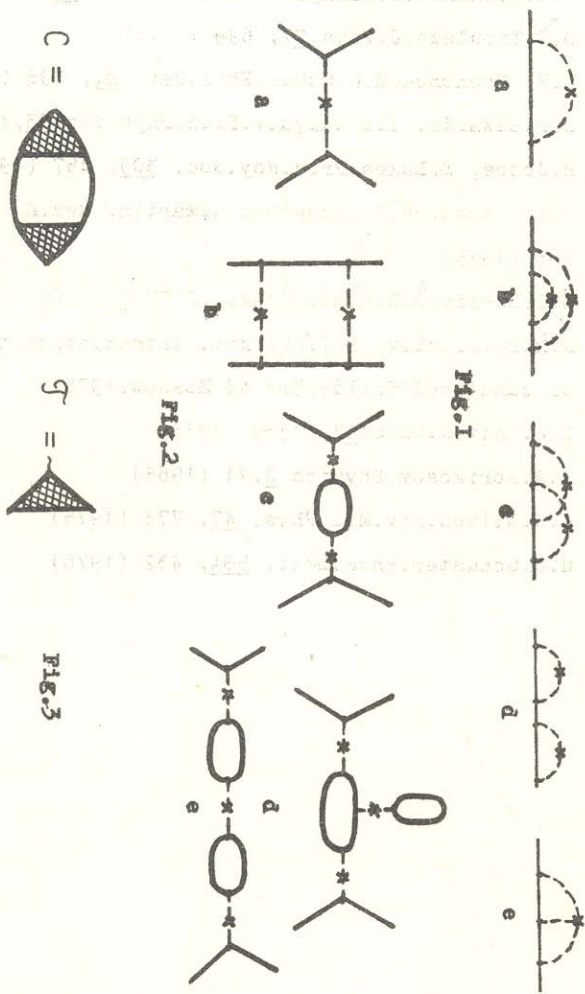
~~violet" and has a "finite" charge in the "infrared".~~

We see that the problem under consideration is similar to the Kondo problem and is characterized by the "crossover" from the weak coupling regime for  $u \ll 1$  towards the strong coupling regime. ~~where the effective coupling constant is of the order  $u/4$ .~~ Analogy with the solution of the Kondo problem /27/ allows one to suppose that such a crossover is continuous (~~as shown by the dotted line in Fig. 5~~). Then the density of states is apparently free of any singularity in the crossover region.

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Introduction

A great deal of effort has been devoted in the last two years to the analysis of the surface features in the electron spectra obtained by photoemission electron spectroscopy. This analysis and other spectroscopic techniques for the study of surfaces have been reported in many cases there are still many problems to be solved. From a theoretical point of view the problem is to calculate the electron density of states  $N(E)$  at the surface. In many cases the local density of states (LDS) at the surface is calculated by the use of the tight-binding method. Such an approach is based on the assumption that the surface atoms are independent of the bulk atoms. This is not always true and in some cases the surface atoms are coupled to the bulk atoms. In this paper I present some calculations recently performed in the photoemission spectroscopy and lead to the same results of more sophisticated theoretical calculations.

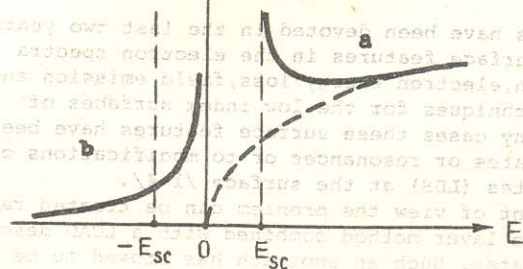


Fig. 4