

# ON THE THEORY OF “ODD-PAIRING” SUPERCONDUCTORS

M. V. Sadovskii, E. Z. Kuchinskii, and M. A. Erkaev<sup>a \*</sup>

<sup>a</sup>Institute for Electrophysics, Russian Academy of Sciences, Ural Branch, Ekaterinburg 620219, Russia

We consider the model of superconducting pairing with the energy gap function which is odd over  $k - k_F$ . In this case superconductivity is possible even in the presence of an arbitrarily large point-like repulsion between electrons. We discuss mainly a model pairing interaction for which the BCS equations can be solved exactly, allowing the complete analysis of the interplay of the usual (“even”) and “odd” pairing. “Odd” pairing dominates for strong enough repulsion and pairing interactions. We show that the normal impurities (disorder) lead to very strong degradation of the “odd” pairing, which is even more rapid than in case of magnetic impurities in traditional superconductors.

In a recent paper Mila and Abrahams proposed an interesting model, which allows the existence of superconducting pairing even in the case of infinitely strong point-like repulsion between electrons[1]. Naturally this model is of great interest as a basis for a possible mechanism of high-temperature superconductivity in metallic oxides. The model is based upon the demonstration of the existence of nontrivial solution of BCS-like gap equation:

$$\Delta(\xi) = -N(0) \int_{-\infty}^{\infty} d\xi' V(\xi, \xi') \times \frac{\Delta(\xi')}{2\sqrt{\xi'^2 + \Delta^2(\xi')}} th \frac{\sqrt{\xi'^2 + \Delta^2(\xi')}}{2T} \quad (1)$$

with the gap function  $\Delta(\xi) = -\Delta(-\xi)$  (i.e. odd in  $k - k_F$ ,  $\xi = v_F(k - k_F)$ ) in case of the presence in  $V(\xi, \xi')$  of an attractive interaction  $-V_2(\xi, \xi') < 0$  (which is non-zero for  $|\xi|, |\xi'| < \omega_c$  and  $|\xi - \xi'| < \omega_c$ ) despite the existence of a strong (infinite) point-like repulsion  $V_1(\xi, \xi') = U > 0$  (for  $|\xi|, |\xi'| < E_F$ ). In case of the odd gap function  $\Delta(\xi)$  the repulsive interaction in Eq. (1) drops out, while the attractive part  $V_2(\xi, \xi')$  may produce pairing with unusual properties (gap function is zero at the Fermi surface, which leads to the gapless superconductivity).

\*The research described in this publication was made possible in part by the Grant  $N^o$  RGL000 from the International Science Foundation as well as by the Grant  $N^o$  93-02-2066 from the Russian Foundation for Fundamental Research.

If the normal (nonmagnetic) impurities are present the equations for normal and anomalous Green’s functions take the usual form[2], with renormalized frequency and the gap function given by:

$$\tilde{\omega} = \omega - \frac{\gamma}{\pi} \int_{-\infty}^{\infty} d\xi \frac{\tilde{\omega}}{\tilde{\omega}^2 + \xi^2 + |\tilde{\Delta}(\xi)|^2} \quad (2)$$

$$\tilde{\Delta}(\xi) = \Delta(\xi) + \frac{\gamma}{\pi} \int_{-\infty}^{\infty} d\xi \frac{\tilde{\Delta}(\xi)^*}{\tilde{\omega}^2 + \xi^2 + |\tilde{\Delta}(\xi)|^2} \quad (3)$$

Here  $\omega = (2n + 1)\pi T$ ,  $\gamma = \pi c V_0^2 N(0)$ —is the scattering rate due to point-like impurities with potential  $V_0$ , chaotically distributed in space with concentration  $c$ . The integral term in Eq. (3) vanishes due to the odd nature of  $\Delta(\xi)$  and the gap renormalization is absent. This fact explains the strong impurity suppression of the “odd” pairing.

Close to the transition temperature  $T_c$  Eqs. (2) and (3) may be linearized over  $\Delta(\xi)$ , and after the standard calculations we obtain the following linear gap equation, which determines  $T_c$ :

$$\Delta(\xi) = N(0) \int_{-\infty}^{\infty} d\xi' V_2(\xi, \xi') \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{1}{\xi'} \times th \left( \frac{\omega + \xi'}{2T} \right) \frac{\gamma}{\omega^2 + \gamma^2} \Delta(\xi') \quad (4)$$

In the following we shall use the model interaction:

$$V_2(\xi, \xi') = \begin{cases} V[\cos \frac{\pi}{2} \frac{\xi - \xi'}{\omega_c} + 1]; & |\xi - \xi'| < \omega_c \\ 0 & \text{for } |\xi|, |\xi'| > \omega_c; |\xi - \xi'| > \omega_c \end{cases} \quad (5)$$

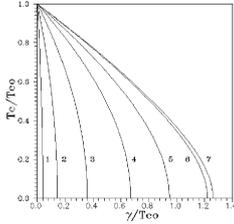


Figure 1.  $T_c$  dependence on the scattering rate  $\gamma$  for the different values of pairing constant  $g$ : 1— $g = 1.22$ ; 2—1.24; 3—1.30; 4—1.5; 5—2.0; 6—5.0, 7—10.0.

The main attractive property of this choice is that it allows the reduction of the integral gap equation to a simple transcendental equation which can be easily solved. The main qualitative results obtained below do not depend on the choice of the model potential.

The  $T_c$ -equation reduces now to:

$$1 = N(0)V \int_0^{\omega_c} \frac{d\xi'}{\xi'} \sin^2 \left( \frac{\pi \xi'}{2\omega_c} \right) \int_{-\infty}^{\infty} \frac{d\omega}{\pi} \times \text{th} \left( \frac{\omega + \xi'}{2T_c} \right) \frac{\gamma}{\omega^2 + \gamma^2} \quad (6)$$

In the “pure” limit of ( $\gamma \rightarrow 0$ ) pairing exists for  $g > g_c = 1.213$ . In Fig. 1 we show the dependence of  $T_c$  on  $\gamma$  for a number of characteristic values of the pairing constant  $g$ . It is clearly seen that normal impurities strongly suppress the “odd” pairing. Superconductivity vanishes for  $\gamma \sim T_{c0}$  and this suppression is even stronger than in case of magnetic impurities in traditional superconductors[3]. This is reflected in particular by the disappearance of superconductivity region on the “phase diagram” in Fig. 1 for  $g \rightarrow g_c$  and the absence of the universal behavior which is characteristic for the case of magnetic impurities.

The critical scattering rate  $\gamma_c$ , corresponding to

$T_c(\gamma \rightarrow \gamma_c) \rightarrow 0$ , is determined by the following equation:

$$\Delta(\xi) = N(0) \int_{-\infty}^{\infty} d\xi' V_2(\xi, \xi') \times \frac{1}{\pi \xi'} \text{arctg} \left( \frac{\xi'}{\gamma_c} \right) \Delta(\xi') \quad (7)$$

which for the model interaction of Eq. (6) reduces to:

$$1 = \frac{2}{\pi} N(0)V \int_0^{\omega_c} \frac{d\xi'}{\xi'} \sin^2 \left( \frac{\pi \xi'}{2\omega_c} \right) \text{arctg} \left( \frac{\xi'}{\gamma_c} \right) \quad (8)$$

It is easily shown that for  $g \gg g_c$  we have the universal result:  $\gamma_c/T_{c0} = 4/\pi \approx 1.273$ . It is also not difficult to see that this result as well as the dependence of  $T_c$  on  $\gamma$  for  $g \gg g_c$  do not depend at all on the choice of the model potential  $V_2(\xi, \xi')$ . This behavior is clearly seen in Fig. 1.

For the model potential:

$$V_2(\xi, \xi') = \begin{cases} V(|\xi - \xi'|/\omega_c)^{-2/3}; & |\xi - \xi'| < \omega_c \\ 0 & \text{for } |\xi|, |\xi'| > \omega_c; |\xi - \xi'| > \omega_c \end{cases} \quad (9)$$

which was extensively used in[1], the critical coupling constant  $g_c$  is apparently zero and our numerical data for  $T_c(\gamma)$  show no qualitative change in this dependence in comparison with the data presented above.

It is well known that high- $T_c$  state in cuprates is very sensitive to the structural disordering[4], and is destroyed close to the metal-insulator transition induced by disordering i.e. for  $\gamma \sim E_F$ , but not for  $\gamma \sim T_{c0} \ll E_F$ . This fact makes the model of an “odd” pairing rather improbable candidate for the explanation of high- $T_c$  superconductivity in cuprates.

## REFERENCES

- [1] Mila F., Abrahams E. Phys. Rev. Lett. **67**, 2379 (1991)
- [2] Abrikosov A.A., Gorkov L.P. Zh. Eksp. Teor. Fiz. **35**, 1158 (1958); **36**, 319 (1959)
- [3] Abrikosov A.A., Gorkov L.P. Zh. Eksp. Teor. Fiz. **39**, 1781 (1960)
- [4] Aleksashin B.A. et al. Zh. Eksp. Teor. Fiz. **95**, 678 (1989)