Disorder Effects in Superconductors with Anisotropic Pairing: From Cooper Pairs to Compact Bosons

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In weak coupling BCS-theory normal impurities do not influence superconducting T_c in case of s-wave pairing. In case of d-wave pairing these are strongly pair-breaking. This fact is in rather strong contradiction with many experiments on disordered high-T_c superconductors. With the growth of electron attraction within the Cooper pair the system smoothly crosses over from BCS-pairs to compact Boson picture of superconductivity. Significant deviations from universal Abrikosov-Gorkov dependence of T_c on disorder are expected in this crossover region with superconducting state becoming more stable than in the weak coupling case. As high-T_c superconductors are actually in this intermediate region, we can understand the relative stability of their d-wave pairing state under disordering.

In the weak-coupling BCS-approximation normal impurities do not influence superconducting T_c in case of isotropic s-wave pairing (Anderson theorem) [1]. However in case of d-wave pairing normal impurities are strongly pair-breaking [2] and the universal dependence of T_c on disorder is expressed by Abrikosov-Gorkov equation:

$$ln\left(\frac{T_{c0}}{T_c}\right) = \left[\Psi\left(\frac{1}{2}+\frac{\gamma}{2\pi T_c}\right) - \Psi\left(\frac{1}{2}\right)\right] \qquad (1)$$

where $\Psi(x)$ is digamma function, γ is the scattering rate of electrons, due to impurities with some potential V, chaotically distributed in space with density n_{imp}. From Eq.(1) it follows that T_c is completely suppressed at the critical scattering rate $\gamma = 0.88T_{c0}$, which determines the critical impurity concentration or residual resistivity of the normal state:

$$\rho_{AG} = \frac{2m\gamma_c}{ne^2} = \frac{8\pi\gamma_c}{\omega_p^2} \qquad (2)$$

where n and m are electron concentration and mass, ω_p is plasma frequency of electrons [2].

At present there is an emerging consensus on the d-wave nature of pairing state in high-temperature superconducting copper oxides [3]. However the scale of the critical scattering rate of $\gamma_c \sim T_{c0}$ is in rather strong contradiction with the large amount of data on disorder suppression of T_c in these systems [4], which apparently demonstrate superconducting state being conserved up to disorder induced metal-insulator transition, i.e $\gamma \sim E_F \gg T_{c0}$.

Consider the limit of extremely strong pairing interaction, leading to compact Boson formation [5]. In this case T_c is determined by temperature of Bose condensation of free Bosons. In case of impure system condensation point can be determined by the following equation [6]:

$$\mu_p - \Sigma(0) = 0 \qquad (3)$$

where μ_p is the chemical potential of pairs and $\Sigma(0)$ is the zero-frequency limit of Boson self-energy due to impurity scattering, which in the weak scattering approximation reduces to:

$$\Sigma(\varepsilon_n) = n_{imp}V^2\int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{i\varepsilon_n - \frac{\mathbf{p}^2}{2m^\star}+\mu_p} \qquad (4)$$

where $m^\star = 2m$ is the mass of the pair, and we assume temperatures $T > T_c$. Direct calculations give:

$$\Sigma(0) = Re\tilde{\Sigma}(0) + E_{0c} \qquad (5)$$

where $E_{0c} = -\frac{m^\star}{\pi^2}n_{imp}V^2p_0$ is the band-edge shift due to impurity scattering [7] (p_0 - is some microscopic cut-off in momentum space) and

$$Re\tilde{\Sigma}(0) = \frac{1}{\sqrt{2}\pi}n_{imp}V^2m^{\star 3/2}\sqrt{|\mu_p|} \qquad (6)$$

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Actually, E_{0c} leads just to renormalization of the chemical potential: $\tilde{\mu} = \mu_p - E_{0c}$, so that in renormalized form Eq.(3) reduces to:

$$\tilde{\mu}\left(1 - \frac{1}{\sqrt{2|\tilde{\mu}|}\pi} n_{imp} V^2 m^{\star 3/2} sign\tilde{\mu}\right) = 0 \qquad (7)$$

with the only relevant solution of $\tilde{\mu} = 0$, i.e. $\mu_p - E_{0c} = 0$, determining the Bose condensation temperature of the impure system by the standard equation:

$$\frac{n}{2} = g \int_{-\infty}^{\infty} d\varepsilon N(\varepsilon) \frac{1}{\exp(\frac{\varepsilon}{T_c}) - 1} \qquad (8)$$

where $g = 2s+1$ (for Bosons of spin s), $N(\varepsilon)$ is the impurity averaged density of states. Obviously we obtain the standard expression for T_c:

$$T_c = \frac{3.31}{g^{2/3}} \frac{(n/2)^{2/3}}{m^\star} \qquad (9)$$

which is *independent of disorder*. The only possible disorder effect may be connected with exponentially small “Lifshits tail” in the density of states in Eq.(8) due to localization, which is neglected in our simplest approximation of Eq.(4).

It was shown rather long ago by Nozieres and Schmitt-Rink [5] for non impure superconductor that as the strength of the pairing interaction grows, there is a smooth crossover of T_c from the weak-coupling BCS-picture to that of compact Bosons. It is clear that in the impure case we shall also obtain smooth crossover in T_c dependence on disorder, interpolating between the BCS and compact Boson limits discussed above. In case of d-wave pairing the universal dependence of T_c on disorder defined by Eq.(1) ceases to be valid starting from the crossover region from large Cooper pairs to compact Bosons. The physical reason for this is quite clear — depairing mechanism of T_c suppression by disorder ceases to operate with the growth of attractive interaction within pairs. Qualitative behavior of T_c dependence on disorder is shown in Fig.1. It is clear that for d-wave system belonging to transitional region we can easily obtain superconducting state persisting for rather large disorder with $\rho > \rho_{AG}$. Superconducting copper oxides belong precisely to this transition region [8]. This can explain their relative stability to disordering [4], despite the possible d-wave symmetry of the pairing state.

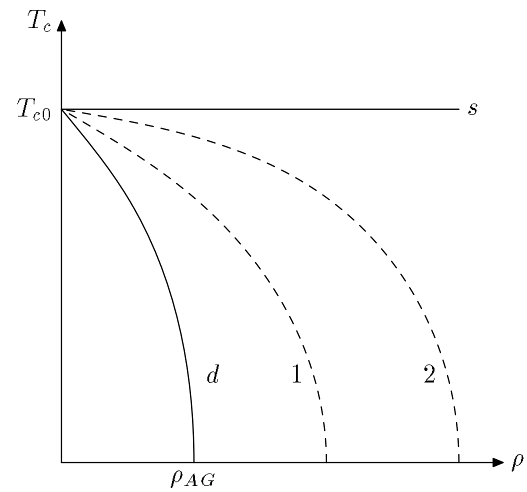

Figure 1. Qualitative dependence of transition temperature T_c on normal state resistivity ρ. Curve d — universal Abrikosov-Gorkov dependence. Curve s — isotropic s-wave pairing. Dashed curves — d-wave pairing in crossover region. Pairing strength grows from curve 1 to 2.

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