Electron in a random field, theory of phase transitions, and finite-action nonlinear solutions

SSV 128

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(Submitted September 25, 1978)

Fiz. Tverd. Tela (Leningrad) **21**, 743–751 (March 1979)

It is shown that the profile of an electron density-of-states tail in a Gaussian random field is given by the solution of finite-action nonlinear equations for a zero-component scalar field. Ideas of the phase transition theory and the dispersion equation for the coupling constant are used to calculate the preexponential factor in the expression for the density-of-states tail. The applicability of a scaling theory at the mobility edge is discussed.

PACS numbers: 71.25.Mg

1. Ideas of the modern theory of critical phenomena[7] have been used[1–6] to describe the behavior of electron states near the mobility threshold of disordered systems. A formal correspondence between the problem of an electron in a random field and a phase transition with a zero-component order parameter (Euclidean theory of a zero-component scalar field)[8,9] has been used in most papers devoted to this subject (with the exception of Refs. 1 and 3). However, it was pointed out in Refs. 1, 2, 4-6 that the aforementioned correspondence is incomplete since the coupling constant in the corresponding field theory has the "incorrect" sign. Consequently, the standard theory of critical phenomena[7] cannot be applied and the incorrect sign of the coupling parameter indicates that perturbation theory fails in the range of energies of interest.[2,4] The neighborhood of the mobility edge, where the perturbation theory fails, is analogous to the "Ginzburg" critical region in the theory of critical phenomena.[2]

It is our aim to extend Ref. 2 and study in detail the region of localized states (region of negative energies). The present approach is a development of the method proposed by Langer[10] and Zittartz and Langer.[11] It will be shown that the profile of a tail in the electron density of states in a random field is governed by the classical solutions of the field theory studied in Refs. 2, 4, and 5 that are characterized by a finite action.[12,13] We also propose a new method of calculation of the preexponential factor in the density-of-states tail which is based on a dispersion equation for the coupling constant.[14,15] Our approach is analogous to the theory of critical phenomena. Finally, we shall discuss the validity of scaling at the mobility edge.

2. We shall consider an electron in the field of a random distribution of point scatterers and calculate the Fourier transform G(Ep) of the one-electron Green's function averaged over all the configurations of scatterers. In the limit $\rho \to \infty$, $V \to 0$, $\rho V^2 \to$ const, where ρ is the density and V is the scattering potential, the problem under study is equivalent to the motion of an electron in a Gaussian random field with a "white noise" correlation function.[2,11] It was shown in Ref. 2 that such a Green's function can be identified with the Green's function of a scalar field theory with the following Lagrangian (m is the electron mass and E its energy):

$$\mathscr{L} = \frac{1}{2}\sum_{j=1}^{n}\left\{\frac{1}{2m}(\nabla\Phi_j)^2 - E\Phi_j^2\right\} - \frac{1}{8}\rho V^2\left(\sum_{j=1}^{n}\Phi_j^2\right)^2. \tag{1}$$

where n is the number of components of the field Φ (n should be set equal to zero after all the calculations have been carried out), which eliminates the "superfluous" diagrams with loops that do not appear in the problem of an electron in a random field.[2,8,9] We have studied[2] the range of energies $E > 0$, where the standard perturbations theory is applicable (the parquet approximation). The region $E < 0$ (the region of localized states) was discussed in Ref. 2 only qualitatively.[1)] It is our aim to study in detail the region $E < 0$.

The main difficulty of the aforementioned theory is due to negative sign of the coupling constant in Eq. (1), which leads to an instability of the ground state in such a field theory and to a failure of the perturbation theory to describe the electron energies[2,4]

$$E \leqslant E_{sc} = \frac{1}{2ma^2}\left(\frac{u}{4-d}\right)^{\frac{2}{4-d}}, \tag{2}$$

where

$$u = \frac{m^2a^{4-d}}{2\pi^2}\rho V^2 \tag{3}$$

is the dimensionless coupling constant, a is a distance related to the cutoff of divergent integrals (the shortest distance in our problem which is related to the difference between the random field correction function and the correlation function of white noise), and d is the dimensionality of the space considered.

A physically correct approach to such a problem was proposed by Langer,[10] who showed that all the correlation functions should be calculated by an analytic continuation with respect to the coupling constant and exhibit a cut along the negative real axis in the complex plane of the values of the coupling constant. Any correlation function (Green's function) of such a theory can be represented as the following dispersion relation for the coupling constant[14,15] (g is an arbitrary coupling constant):

$$G(g) = \frac{1}{\pi}\int_{-\infty}^{0} dz\,\frac{\Delta(z)}{z-g}, \tag{4}$$

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and

$$\Delta(g)=\frac{1}{2i}[G(g+i\varepsilon)-G(g-i\varepsilon)]=\operatorname{Im}G(g) \tag{5}$$

is a discontinuity over the cut (nonzero for g < 0) which can be obtained from the nonlinear solutions of the classical field theory equation (1) with a finite action.[12-15] We shall always assume that G(g) is the one-particle Green's function.

3. The action of the field theory defined by Eq. (1) is given by

$$S[\Phi]=\int d^dr\,\mathscr{L}(r) \tag{6}$$

and the Green's function is given by the following functional integral:

$$G(r-r'|g)=-Z^{-1}\frac{1}{n}\sum_{j=1}^{n}\int(\delta\Phi(r))\,\Phi_j(r)\,\Phi_j(r')\exp\{-S[\Phi]\}, \tag{7}$$

where

$$Z=\int(\delta\Phi(r))\exp\{-S[\Phi]\}. \tag{8}$$

The minus sign in Eq. (7) is chosen to yield the correct zeroth-order electron Green's function.

The minimization $\delta S[\Phi] = 0$ yields the following classical field equations:

$$\frac{1}{2m}\Delta\Phi_j=-E\Phi_j-\frac{1}{2}\rho V^2\Phi_j\left(\sum_{j=1}^{n}\Phi_j^2\right). \tag{9}$$

We shall seek the solution of the field equations in the form[12,13]

$$\Phi_j(r)=\Phi_0(r)\,u_j, \tag{10}$$

where u is a unit vector ($u^2 = 1$) in the "isospin" space of the theory [0(n) symmetric] considered. Restricting ourselves to the class of spherically symmetric solutions (Refs. 16-18), we obtain from Eq. (9) the following result:

$$\frac{1}{2m}\left\{\frac{d^2\Phi_0}{dr^2}+\frac{d-1}{r}\frac{d\Phi_0}{dr}\right\}=-E\Phi_0-\frac{1}{2}\rho V^2\Phi_0^3. \tag{11}$$

Equation (11) has a trivial solution $\Phi_0 = 0$. We shall consider nontrivial solutions of Eq. (11) with a finite action [i.e., such that the integral in Eq. (6) converges]. For d = 1, it is possible to obtain an exact solution of Eq. (11) (see Ref. 10). Using the results of Refs. 18 and 19, we can show that the required solution appropriate to the problem under study exists only for d < 4. We shall discuss the solutions qualitatively following the method of Ref. 20 (see also Ref. 17).

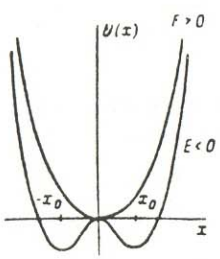

FIG. 1. "Potential energy" corresponding to the equation of motion (13).

We shall introduce new variables

$$\Phi_0(r)=\left(\frac{2|E|}{\rho V^2}\right)^{1/2}x(t),\qquad r=(2m|E|)^{-1/2}t. \tag{12}$$

Equation (11) then assumes the following dimensionless form:

$$\frac{d^2x}{dt^2}+\frac{d-1}{t}\frac{dx}{dt}=\pm x-x^3, \tag{13}$$

where the upper sign corresponds to E < 0 and the lower sign to E > 0.

We can now use an obvious mechanical analogy, i.e., Eq. (13) is an equation of motion for a particle with unit mass in the following potential (Fig. 1):

$$U(x)=\mp\frac{x^2}{2}+\frac{x^4}{4}. \tag{14}$$

The particle in question moves subject to a friction force depending on time as $\sim 1/t$. By considering the "energy"

$$\delta=\frac{1}{2}\left(\frac{dx}{dt}\right)^2+U(x), \tag{15}$$

we can easily demonstrate the dissipative nature of the motion. Using Eq. (13), we obtain

$$\frac{d\delta}{dt}=-\left(\frac{dx}{dt}\right)^2\frac{d-1}{t}<0;\quad d>1. \tag{16}$$

The qualitative behavior of the motion is shown in Fig. 2. The motion in question satisfies the following initial conditions:

$$\left.\begin{aligned}x|_{t=0}&=\text{const},\\ \frac{dx}{dt}\Big|_{t=0}&=0.\end{aligned}\right\} \tag{17}$$

For $t \gg 1$, we can linearize Eq. (13) near the extrema of of U(x), i.e., near the points x = 0, $x = \pm x_0 = \pm 1$ to obtain the asymptotic behavior of the solution defined by Eq. (13). It is quite clear that the solutions of type 2 and 3 shown in Fig. 2 are of no interest since the corresponding action integral defined by Eq. (6) diverges [the field defined by Eq. (12) tends to a constant at infinity]. The asymptotic behavior ($t \gg 1$) of the solution of type 4 in Fig. 2 (E > 0) is given by

$$x(t)\approx\frac{\text{const}}{t^{\frac{d}{2}-1}}J_{\frac{d}{2}-1}(t);\quad t\gg 1, \tag{18}$$

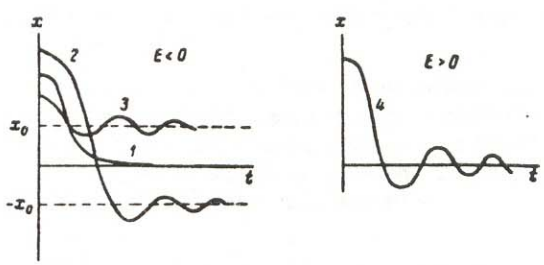

FIG. 2. Qualitative behavior of the solutions of Eq. (13).

[$J_{\frac{d}{2}-1}(t)$ are the Bessel functions] and the integral defined by Eq. (6) also diverges as a function of the upper limit for $d \geq 2$. Therefore, we are left with a unique solution of type 1 shown in Fig. 2 ($E < 0$). The fact that the solution is unique follows from physical considerations. In fact, there is a single point on the curve U(x) which has the property that a particle starting its motion from this point terminates its motion at the point $x = 0$. The asymptotic behavior of the aforementioned unique solution ($t \gg 1$) is given by

$$x(t) \approx \frac{\text{const}}{t^{\frac{d}{2}-1}} K_{\pm\left(\frac{d}{2}-1\right)}(t) \approx \frac{\text{const}}{t^{\frac{d-1}{2}}} \exp(-t); \quad t \gg 1, \tag{19}$$

[$K_{\pm\left(\frac{d}{2}-1\right)}(t)$ is the modified Bessel function] and the corresponding action integral defined by Eq. (6) converges.

Using Eq. (12), we obtain

$$S[\Phi_0] = \int d^d r \mathscr{L}(\mathbf{r} \mid \Phi_0(\mathbf{r})) = A_d \frac{m^{-d/2}}{\rho V^2} |E|^{2-d/2}. \tag{20}$$

The constant A_d, which depends on the dimensionality, is determined by dimensionless integrals of x(t). The calculation of A_d requires numerical integration of the equation of motion (13) supplemented by the initial conditions (17).

4. The functional integral in Eq. (7) can be evaluated by the steepest descent method near the classical solutions with a finite action defined by Eq. (12) (see Refs. 10, 12-15). For $E > 0$, only the trivial solution $\Phi_0 = 0$ exists and the steepest descent method yields the standard perturbation theory,[10,15] which was used in Ref. 2. For $E < 0$, there is a nontrivial solution with a finite action defined by Eqs. (12), (17), and (19). The field $\Phi(\mathbf{r})$ can be expanded near $\Phi_0(\mathbf{r})$ as follows:

$$\Phi(\mathbf{r}) = \Phi_0(\mathbf{r} - \mathbf{R}_0) + \varphi(\mathbf{r}). \tag{21}$$

It is then possible to perform all the calculations in the lowest order of a perturbation theory with respect to $\varphi(\mathbf{r})$. All the correlation functions will contain a factor $\exp(-S[\Phi_0])$ which is nonanalytic in the coupling constant and also a preexponential factor which is obtained in the evaluation of the Gaussian integral in the variable $\varphi(\mathbf{r})$. The problems related to the negative sign of the coupling constant, to the arbitrary choice of the location of the solution $\Phi_0(\mathbf{r}-\mathbf{R}_0)$ in space (arbitrary choice of $\mathbf{R}_0$), and to the arbitrary orientation of the vector $\mathbf{u}$ in the isotopic space introduced in Eq. (10) [O(n) symmetry] require special discussion. All the required calculations are analogous to the calculations of Refs. 10, 12-15, 17. The imaginary part of the one-electron Green's function is given by

$$\operatorname{Im} G(Ep \mid -\rho V^2) = C(|E|, p) \exp\left\{-\frac{A(E)}{\rho V^2}\right\} \frac{\theta(-\rho V^2)}{(\rho V^2)^{\frac{d+1}{2}}}, \tag{22}$$

where $C(|E|, p)$ is a function of E and p which is independent of the coupling constant ρV^2,

$$A(E) = A_d m^{-d/2} |E|^{2-\frac{d}{2}}, \tag{23}$$

the theta function in Eq. (22) indicates that the imaginary part of the Green's function is nonzero only for negative values of the coupling constant in the field theory with the Lagrangian defined by Eq. (1). The power of the coupling constant in the preexponential factor in Eq. (22) can be easily understood. The translation invariance (arbitrary choice of $\mathbf{R}_0$) yields a factor[15,17] $(\rho V^2)^{-d/2}$ (there are d translation "zero" modes); an additional factor $(\rho V^2)^{-\frac{n-1}{2}}$ is related to arbitrary orientation of the vector $\mathbf{u}$ (n − 1 rotational "zero" modes); and the factor $(\rho V^2)^{-\nu/2}$ is related to the product of ν fields which appears in the definition of the $\nu/2$-th Green's function[15] (in the case considered, $\nu = 2$). These results are independent of the actual form of the classical solutions $\Phi_0(\mathbf{r} - \mathbf{R}_0)$ (see Refs. 15, 17); the type of solution determines $C(|E|, p)$.

The Green's function can be calculated from $\operatorname{Im} G(Ep|-\rho V^2)$ (i.e., from the discontinuity across the cut in the complex coupling constant plane) via the dispersion integral (4).

$$G(Ep \mid g) = \frac{1}{\pi} C(|E|, p) \int_{-\infty}^{0} dz \frac{\exp\left\{\frac{A(E)}{z}\right\}}{(z-g)(-z)^{\frac{d+1}{2}}}, \tag{24}$$

where g is an arbitrary coupling constant for an electron in a random field $g = -\rho V^2$. The integral in Eq. (24) can be easily evaluated:

$$G(Ep \mid g) = -\frac{1}{\pi} C(|E|, p) g^{-\frac{d+1}{2}} \exp\left\{\frac{A(E)}{g}\right\} \Gamma\left(\frac{d+1}{2}\right) \Gamma\left(\frac{1-d}{2}, \frac{A(E)}{g}\right), \tag{25}$$

where $(\alpha, x) = \int_x^\infty dt e^{-t} t^{\alpha-1}$ is the incomplete gamma function.

The Green's function of an electron in a random field represents the analytic continuation of Eq. (25) from the region $g > 0$ to negative values $g = -\rho V^2$ (see Ref. 21).

It follows from Eq. (22) that our preexponential factor in the tail of the density of states is correct.[11,22,23] The preexponential factor is completely determined by the classical solutions of the field theory defined by Eq. (1) with a finite action. The main advantage of our method is that it yields automatically correct results and does not introduce the additional assumptions employed in Refs. 11, 22, and 23 such as the assumption that the first level of the fluctuation well is dominant. Different treatments of the energy ranges $E > 0$ and $E < 0$ are also introduced automatically since the classical solutions with a finite action exist only for $E < 0$. Nonanalytic dependences on the coupling constant also arise quite naturally in the present method (breakdown of the standard perturbation theory).

The condition of validity of our results can be formulated as $S[\Phi_0] \gg 1$, i.e., our results hold when the method of steepest descent used in the evaluation of the functional integral in Eq. (7) is justified. In other words, the following condition should be satisfied:

$$\frac{A(E)}{\rho V^2} = \frac{A_d}{2\pi^2} \frac{1}{4-d} \left(\frac{|E|}{E_{ec}}\right)^{2-\frac{d}{2}} \gg 1,$$

i.e.,

$$|E| \gg E_{sc}. \tag{25a}$$

which reduces to the condition obtained in Ref. 2. The condition of validity of the "perturbation theory" near the classical solution with a finite action defined by Eq. (21) is the same as the criterion of validity of the standard perturbation theory in the energy range $E > 0$. It was noted in Ref. 2 that an interval of width $2E_{SC}$ about $E = 0$ is an analog of the "Ginzburg" critical region in the theory of critical phenomena. However, in contrast to the theory of critical phenomena, perturbation theory, when applied to the case considered, fails even for space of dimensionality $d = 4 - \varepsilon$.

5. The preexponential factor $C(|E|, p)$ in Eq. (22) can be evaluated provided the classical solutions with a finite action are known explicitly. For $d > 1$, such solutions can be obtained only numerically. We shall now develop a method of calculation of the preexponential factor based on the analogy with the theory of phase transitions, which makes it possible to avoid numerical calculations.

For $g > 0$, the Green's function defined by Eq. (25) corresponds to the correlation function of a stable field theory (the theory of second-order phase transitions). Far from the critical region, the aforementioned correlation function is well known,[7] i.e., the correlation function is given by the standard Ornstein–Zernike expression. For $|E| \gg E_{SC}$, we obtain

$$G(E_p | g>0) \approx -\frac{1}{|E| + \frac{p^2}{2m}}. \tag{26}$$

On the other hand, using the asymptotic expression[23,24] for the incomplete gamma function, we find that Eq. (25) yields ($|E| \gg E_{SC}$; $E < 0$)

$$G(E_p | g>0) \approx -\frac{1}{\pi}\Gamma\left(\frac{d+1}{2}\right)[A(E)]^{-\frac{d+1}{2}} C(|E|, p). \tag{27}$$

Comparing Eqs. (26) and (27), we obtain

$$C(|E|, p) \approx \frac{\pi A_d^{\frac{d+1}{2}}}{\Gamma\left(\frac{d+1}{2}\right)} m^{-\frac{d(d+1)}{2}} \frac{|E|^{(d+1)\left(1-\frac{d}{4}\right)}}{|E| + \frac{p^2}{2m}}, \quad |E| \gg E_{sc}. \tag{28}$$

The imaginary part of the electron Green's function is then given by ($|E| \gg E_{SC}$)

$$\operatorname{Im} G(E_p | -\rho V^2) \approx \pm \frac{\pi A_d^{\frac{d+1}{2}}}{\Gamma\left(\frac{d+1}{2}\right)} \frac{|E|^{(d+1)\left(1-\frac{d}{4}\right)}}{|E| + \frac{p^2}{2m}} \frac{1}{\left(m^{\frac{d}{2}}\rho V^2\right)^{\frac{d+1}{2}}} \exp\left\{-\frac{A(E)}{\rho V^2}\right\}. \tag{29}$$

We can now calculate the density of electron states in the tail region including the preexponential factor. We find that ($|E| \gg E_{SC}$; $E < 0$)

$$N(E) = -\frac{1}{\pi}\int \frac{d^d p}{(2\pi)^d} \operatorname{Im} G^R(E_p | -\rho V^2)$$

$$\approx K_d \frac{A_d^{\frac{d+1}{2}}}{\Gamma\left(\frac{d+1}{2}\right)} \frac{|E|^{(d+1)\left(1-\frac{d}{4}\right)}}{\left(m^{\frac{d}{2}}\rho V^2\right)^{\frac{d+1}{2}}} \exp\left\{-\frac{A(E)}{\rho V^2}\right\} \int_0^{1/a} dp\, p^{d-1} \frac{1}{|E| + \frac{p^2}{2m}}, \tag{30}$$

where $K_d = 2^{-(d-1)}\pi^{-\frac{d}{2}} \frac{1}{\Gamma\left(\frac{d}{2}\right)}$. For $d = 1$, we can take the limit $a \to 0$ and Eq. (30) yields

$$N(E) = K_1 \frac{\pi A_1}{\sqrt{2}} \frac{|E|}{\rho V^2} \exp\left\{-A_1 \frac{|E|^{3/2}}{m^{1/2}\rho V^2}\right\}. \tag{31}$$

It follows from Ref. 11 that $A_1 = 4\sqrt{2}/3$ [Eq. (11) for $d = 1$ can be solved exactly] and Eq. (31) reduces to the exact result of Refs. 25 and 11 with an accuracy up to the factor $3/\pi$. For $d \geq 2$, the divergent integral in Eq. (30) is cut off at a momentum $\sim 1/a$, the cut-off momentum being related to the reciprocal of the range of the correlation function of random fields. Our calculations are valid for energies $|E| \ll E_0 = 1/2ma^2$. For $|E| \gg E_0$, the tail of the density of states is governed by the quasiclassical approximation.[26–28] For $d = 2$, Eq. (30) yields ($E_{SC} \ll |E| \ll E_0$)

$$N(E) \approx \text{const} \frac{|E|^{3/2}}{m^{1/2}(\rho V^2)^{3/2}} \ln \frac{E_0}{|E|} \exp\left\{-A_2 \frac{|E|}{m\rho V^2}\right\}, \tag{32}$$

for $d = 3$, we obtain

$$N(E) \approx \text{const} \frac{|E| E_0^{1/2}}{m^{3/2}(\rho V^2)^2} \exp\left\{-A_3 \frac{|E|^{3/2}}{m^{3/2}\rho V^2}\right\}. \tag{33}$$

For $2 < d < 4$, the tail in the density of states is given by

$$N(E) \approx K_d \left(\frac{A_d}{2\pi^2(4-d)}\right)^{\frac{d+1}{2}} \frac{2m}{\Gamma\left(\frac{d+1}{2}\right)} \frac{(2mE_0)^{\frac{d-2}{2}}}{(d-2)} \times \left(\frac{|E|}{E_{sc}}\right)^{(d+1)\left(1-\frac{d}{4}\right)} \exp\left\{-A_d \frac{m^{-\frac{d}{2}}}{\rho V^2} |E|^{2-\frac{d}{2}}\right\}. \tag{34}$$

We believe that the aforementioned expressions yield (with an accuracy up to a constant factor) exact expressions for the preexponential factor of the density of states in the energy range considered.

6. The energy range $|E| \ll E_{SC}$ lies outside the region of validity of our theory. It follows from the theory of critical phenomena that, for $g > 0$ and for energies $|E| \ll E_{SC}$, the correlation function obeys the standard scaling

$$G(E_p | g>0) \approx C|E|^{-\gamma} D(p^2\xi^2); \quad \xi \sim |E|^{-\nu}. \tag{35}$$

Here, γ and ν are the critical indices of the susceptibility and correlation length ξ, $D(x)$ is a universal (independent of the details of the interaction) function, and C is a nonuniversal factor. Since it is well known that the discontinuity across the cut in the dispersion relation (4) [$\Delta(z)$ in Eq. (4) is unique for arbitrary g] is universal, this seems to indicate that the Green's function should exhibit an analogous universal behavior for $|E| \ll E_{SC}$ irrespective of the sign of g. Applying formally Eq. (25) to the region $|E| \ll E_{SC}$ and using $\Gamma(\alpha, x) \to \Gamma(\alpha)$ for $x \to 0$ ($\alpha \neq 0, -1, -2, \ldots$), and also $A(E) \to 0$ for $E \to 0$ and $\Gamma(1/2 +$

$d/2)\Gamma(1/2 - d/2) = \pi/\cos(\pi d/2)$, we obtain

$$G(Ep \mid g>0) \approx -\frac{1}{\cos\frac{\pi d}{2}} g^{-\frac{d+1}{2}} C(|E|, p); \quad |E| \ll E_{sc}. \tag{36}$$

where $d \neq 1, 3$ but the values $d = 2$ and $d = 4 - \varepsilon$ are admissible. Comparing Eqs. (35) and (36), we obtain

$$C(|E|, p) \sim |E|^{-\gamma} D(p^2\xi^2). \tag{37}$$

Equation (22) then yields

$$\operatorname{Im} G(Ep \mid -pV^2) \approx B|E|^{-\gamma} D(p^2\xi^2), \tag{38}$$

where B is a (nonuniversal) constant (independent of E and p). It must be understood that Eqs. (36)-(38) represent an extrapolation of Eq. (25) beyond its range of validity. However, mere assumption that the discontinuity across the cut in the dispersion equation (4) can be factorized leads to a result similar to that defined by Eq. (38), i.e.,

$$\Delta(Ep \mid z) = \operatorname{Im} G(Ep \mid z) \approx C(|E|, p) f(z) \tag{39}$$

for $|E| \ll E_{SC}$. Such a factorization holds when Eq. (25) is applied formally to the region $|E| \ll E_{SC}$ and implies the scaling defined by Eq. (37) irrespective of the sign of g. unfortunately, we are unable to prove Eq. (39). However, if we assume the validity of Eq. (39), we find that Eq. (35) yields Eqs. (37) and (38) and the density of states for $|E| \ll E_{SC}$ is given by

$$N(E) \approx -\frac{B}{\pi}|E|^{-\gamma}\int \frac{d^d p}{(2\pi)^d} D(p^2\xi^2) = D(\text{const} + |E|^{1-\alpha}), \tag{40}$$

where α is the specific-heat critical index. For $d = 4 - \varepsilon$ and $n = 0$, we obtain

$$\alpha \approx \frac{\varepsilon}{4} + O(\varepsilon^2), \tag{41}$$

where D is a constant (independent of E). As a result, we obtain

$$\frac{dN(E)}{dE} \sim |E|^{-\alpha}; \quad |E| \to 0. \tag{42}$$

The density of states in the limit $|E| \to 0$ exhibits a kink and its derivative diverges as the specific heat in the theory of critical phenomena.

Consequently, assuming the factorization defined by Eq. (39), we obtain a scaling at the mobility edge which holds for the average electron Green's function in a random field. If this is the case, the well-known discrepancy between the Anderson result that considers the "most probable" electron Green's function[1] and the standard approach due to Edwards based on the average Green's function disappears. An alternative approach is to treat the neighborhood of the mobility edge as an analog of the transition region in the Kondo problem, where the Anderson and Edwards treatments are complementary.[2]

The author is grateful to L. V. Keldysh for his discussions and interest in the present work and to D. V. Shirkov for making available a preprint of Ref. 15.

[1)]Equation (26) of Ref. 2 contains an error and the resulting equation (27), which states that the effective interaction reaches a constant value for large negative energies, is incorrect.

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Translated by D. Mathon