Localization of one-particle spin excitations in a ferromagnet with a random easy-axis anisotropy

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Spin wave excitations in a ferromagnet with a random easy-axis anisotropy are studied. It is shown that anomalous damping of magnons near the edge of the spin wave band takes place and this damping is attributed to the localization of magnons. It is shown that the problem of localization of magnons is equivalent to the localization of electrons in the Anderson model with a diagonal disorder. The position of the mobility edge is calculated.

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1. The effect of fluctuations of the uniaxial anisotropy parameter on the spectrum of spin waves of an amorphous ferromagnet was studied in Ref. 1 on the basis of the phenomenological Landau–Lifshitz equation. It was suggested in Ref. 1 that the spin wave spectroscopy methods could be used to detect the resulting modification in the dispersion law of spin waves and thus estimate the fluctuations in the anisotropy parameter at different sites and determine their spatial correlation. It is of interest to investigate this problem within a lattice model of an amorphous ferromagnet and compare the spin wave spectrum calculated by a perturbation theory method with the results on the position of the mobility edge of spin wave excitations (in the spirit of the Anderson theory of localization of electrons[2]).

We shall, therefore, consider a model of a uniaxial Heisenberg ferromagnet in which only the anisotropy parameter $K(n) \geq 0$ characterizing an easy-axis anisotropy is a random quantity and all the other parameters are regular

$$H = -\frac{1}{2} J \sum_{n=1}^{N} \sum_{\Delta=1}^{Z} \mathbf{S}_n \mathbf{S}_{n+\Delta} - \sum_{n=1}^{N} K(n) (S_n^z)^2. \tag{1}$$

Here, $J > 0$; Z is the number of nearest neighbors, and we assume that the uniaxial properties of a crystal manifest themselves only by a uniaxial single-ion anisotropy but do not influence the lattice parameters or the exchange interaction. It follows that the magnetic lattice can be well approximated by a cubic lattice. The condition $K(n) \geq 0$ indicates that the ordering of all the spins in the ground state $|\Psi_0\rangle$ is ferromagnetic and the energy of the ground state is given by

$$E_0 = -\frac{1}{2} JNZS^2 - S^2 \sum_n K(n).$$

We shall now write the Schrödinger equation for a state $|\Psi_1\rangle$ corresponding to a single spin deviation (the total z component of the spin moment of the crystal is given by $S^z_{\mathrm{sum}} = NS - 1$)

$$H|\Psi_1\rangle = E_1|\Psi_1\rangle. \tag{2}$$

The wave function $|\Psi_1\rangle$ can be expanded in terms of the basis of one-particle spin deviations localized at the lattice sites

$$|\Psi_1\rangle = \sum_{n=1}^{N} c_n |n\rangle; \quad |n\rangle = (2S)^{-1/2} S_n^- |\Psi_0\rangle. \tag{3}$$

As a result, we obtain either homogeneous equations for the coefficients

$$\{E - JSZ - (2S-1) K(n)\} c_n + JS \sum_{\Delta=1}^{Z} c_{n+\Delta} = 0 \tag{4}$$

or inhomogeneous equations for the corresponding Green's function

$$\{E - JSZ - (2S-1) K(n)\} G_{np} + JS \sum_{\Delta=1}^{Z} G_{n+\Delta,\,p} = \delta_{np} \tag{5}$$

(the energy $E = E_1 - E_0$ is measured from the ground state energy). Here, $G_{np}(E + i0^+)$ is the Fourier transform of the retarded Green's function

$$G_{np}(t) = -i\theta(t)(2S)^{-1} \langle \Psi_0 | S_n^+(t) S_p^-(0) | \Psi_0 \rangle.$$

We can calculate the self-energy corrections to the spectrum of spin waves $\varepsilon_{\mathbf{q}}^0$ in the mean-field approximation

$$\varepsilon_{\mathbf{q}}^0 = (2S-1) \bar{K}(n) + JS\left(Z - \sum_{\Delta} e^{i\mathbf{q}\Delta}\right) \tag{6}$$

using the Edwards–Jones method,[3] which yields the following dispersion law in the Born approximation:

$$\varepsilon_{\mathbf{q}} \simeq (2S-1) \bar{K}\left[1 - \left(\frac{2S-1}{2S}\right) \frac{D\{K\}}{\bar{K}J} \zeta\right] + JSa^2q^2\left[1 - \left(\frac{2S-1}{2S}\right)^2 \frac{D\{K\}}{J^2} \eta\right]. \tag{7}$$

The damping of spin waves is given by

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$$\Gamma_q \approx (2S-1)^2 D\{K\} \pi g_0(\varepsilon_q^0) \approx (2S-1)^2 \frac{D\{K\}}{\pi JS} aq \qquad (8)$$

in the long-wavelength limit $aq \ll 1$ (a is the lattice parameter and g_0 is the spin wave density of states in a crystal in the mean-field approximation).

We have used in the derivation of Eqs. (7) and (8) the assumption that the fluctuations of the anisotropy parameter at different lattice sites are statistically independent, i.e., that the averaging over the disorder is performed as follows:

$$\overline{K(n)K(m)} = [\overline{K^2}(n) - (\overline{K})^2]\delta_{np} + (\overline{K})^2 \equiv D\{K\}\delta_{np} + (\overline{K})^2, \qquad (9)$$

where D(K) is the dispersion of the anisotropy parameter. The approximation which neglects all spatial correlations of the fluctuations of the anisotropy is not necessary but it is convenient for our comparison with the theory of localization. The numerical coefficients ζ and η in Eq. (7) are given by

$$\zeta = \frac{1}{N}\sum_q \frac{2}{Z - \sum_\Delta e^{iq\Delta}} \approx 0.51 \text{ for simple lattice,}$$

$$\eta = \lim_{a\to 0^+} \frac{1}{N}\sum_q \frac{4}{\left(Z - 2a - \sum_\Delta e^{iq\Delta}\right)^2} \approx 0.05 \text{ for cubic lattice.}^4$$

Since the ratio $(2S-1)/2S$ is of the order of unity, the requirement that corrections to the mean-field theory should be small assumes the form $D\{K\}/\overline{K}J \ll 1$ and $D\{K\}\eta/J^2 \ll 1$. It can be seen that the fluctuations of the anisotropy parameter reduce the gap in the spin wave spectrum and the spin wave stiffness. It should be noted that the gap in the spin wave spectrum is defined as $\varepsilon_{q=0}$ rather than the actual gap in the density of states of single-particle spin excitations corresponding to the lowest Lifshitz boundary of the one-particle spectrum for $\varepsilon_{min} = (2S-1)\min\{K(n)\}$. The density of states of one-particle excitations increases rapidly at energies $\sim\varepsilon_{q=0}$.

The most interesting physical result which can be deduced from Eqs. (7) and (8) can be formulated as follows. The dispersion curve near $\varepsilon_{q=0}$ is not well defined due to damping $\Gamma_q \sim aq$ and the change in the excitation energy near the gap is given by $\varepsilon_q - \varepsilon_{q=0} \sim a^2q^2$. It follows that the dispersion curve of spin waves in the long-wavelength limit is well defined only if the condition

$$\frac{\Gamma_q}{\varepsilon_q - \varepsilon_{q=0}} \approx \frac{1}{\pi}\left(\frac{2S-1}{2S}\right)^2 \frac{D\{K\}}{J^2}\frac{1}{aq} \ll 1 \qquad (10)$$

is satisfied, i.e., for $D\{K\}/\overline{J}^2 \ll aq \ll 1$. It appears that this important result was first noted by Korenblit and Shender[5] for an asperomagnet with a random distribution of the easy-magnetization axes.

It should be noted that the Goldstone gapless mode does not appear in our model. The existence of such a mode is due to continuous degeneracy of the ground state which does not occur in systems with an easy-axis anisotropy. However, the situation is quite different for systems with an easy-plane anisotropy [K(n) < 0 in Eq. (1)] where the ground state is invariant with respect to rotations in the plane of the easy magnetization.

2. Since there is a region in which the perturbation theory breaks down near the energies corresponding to the bottom of the spin wave band calculated in the mean field approximation, spin wave excitations can become localized near the bottom of the band.

Introducing the notation

$$\varepsilon_n \equiv (2S-1)K(n) + JSZ, \quad V \equiv JS, \qquad (11)$$

we can see that Eqs. (4) and (5) demonstrate that the present modél is equivalent to the Anderson model with diagonal disorder.[2,6,7] The quantity ε_n plays the role of a random electron energy at the n-th site and V is the amplitude of the electron hopping between sites. Consequently, we can apply to our model the criteria developed for the mobility edge of electrons. We shall not require a great numerical accuracy in the calculation of the mobility edge and restrict ourselves to a qualitative analysis. Consequently, we can use the Ziman criterion of localization of excitations[8]

$$Z\exp\left\{\overline{\ln\left|\frac{JS}{E - JSZ - (2S-1)K(n)}\right|}\right\} \leqslant 1. \qquad (12)$$

We shall consider a uniform distribution of the anisotropy parameter in the interval

$$\overline{K} - \frac{W}{2} < K(n) < \overline{K} + \frac{W}{2}, \quad W \leqslant 2\overline{K}. \qquad (13)$$

Performing the averaging in Eq. (12), we find that the one-particle spin excitations become localized provided the condition

$$Ze\frac{1}{|x^2 - y^2|^{1/2}}\left|\frac{x-y}{x+y}\right|^{x/2y} \leqslant 1 \qquad (14)$$

is satisfied, where $x = [E - JSZ - (2S-1)\overline{K}]/JS$ is the dimensionless energy and $y = (2S-1)W/2JS$ is the dimensionless scatter of the random values of the anisotropy parameter.

The equality in Eq. (14) yields an equation for the calculation of the mobility edge of spin excitations. Since Eq. (14) is invariant under the substitution $x \to -x$, it follows that the mobility edges are symmetrically localized with respect to the point $x = 0$ [or with respect to $E = (2S-1)\overline{K} + JSZ$, which represents the center of the spin wave band in the mean field approximation]. Setting $x = 0$, we find the following condition for the scatter in the anisotropy which is required for the localization in the whole band:

$$\frac{2(2S-1)\overline{K}}{2JS} > \frac{(2S-1)W}{2JS} > Ze. \qquad (15)$$

However, the case when the dispersion $D\{K\} = W^2/12$ is small compared with $\overline{K}J$ and J^2 is more interesting. Equation (14) yields $x = \pm Z$ for $y \to 0$. For $y/x \approx y/Z \ll 1$, we then obtain

$$|x| \approx Z\left[1 + \frac{1}{6}\left(\frac{y}{x}\right)^2\right] \qquad (16)$$

which has the following solution:

$$x \approx \pm Z\left[1 + \frac{1}{6}\left(\frac{y}{Z}\right)^2\right]. \qquad (17)$$

The lowest value of the mobility edge ε_{loc} is then given by

$$\frac{\varepsilon_{\text{loc}}}{2JS}=\frac{(2S-1)K}{2JS}\left[1-\frac{1}{Z}\left(\frac{2S-1}{2S}\right)\frac{D\{K\}}{KJ}\right]$$
$$=\frac{\varepsilon_{q=0}}{2JS}+\left(\frac{2S-1}{2S}\right)^2\left(\zeta-\frac{1}{Z}\right)\frac{D\{K\}}{J^2}, \quad (18)$$

where we have used, for comparison, the energy of the gap $\varepsilon_{\mathbf{q}=0}$ given by Eq. (7) and $\zeta - 1/Z \approx 0.34$ for a simple cubic lattice.

The method of Ref. 9 yields analogous results with a different numerical factor

$$\frac{\varepsilon_{\text{loc}}}{2JS}=\frac{(2S-1)K}{2JS}\left[1-\frac{2}{K_c}\left(\frac{2S-1}{2S}\right)\frac{D\{K\}}{KJ}\right]$$
$$=\frac{\varepsilon_{q=0}}{2JS}+\left(\frac{2S-1}{2S}\right)^2\left(\zeta-\frac{2}{K_c}\right)\frac{D\{K\}}{J^2}, \quad (19)$$

where K_c is the lattice connectivity constant[10] and $\zeta - 2/K_c \simeq 0.08$ for a simple cubic lattice.

We find that, when the dispersion of the anisotropy $D\{K\}/J^2 \ll 1$ is small compared with the exchange interaction, the region in which the spin waves are not well defined extends approximately up to energies $\sim D^2\{K\}/J$ from the bottom of the spin wave band of the unperturbed "average" crystal, and the lowest value of the mobility edge lies at a distance $\sim D\{K\}/J$ below the gap in the spin wave spectrum of the unperturbed crystal (however, it lies above the gap $\varepsilon_{\mathbf{q}=0}$ calculated in the perturbation theory). An estimate based on the Ziman theory[8] yields a somewhat higher position of the mobility edge than the estimate due to Abou-Chacra and Thouless.[9]

It follows that fluctuations of the anisotropy parameter of a crystal with an easy-axis anisotropy lead to a rapid increase in the damping near the bottom of the spin wave band and to a localization of magnons in the region of anomalous damping. Unfortunately, the region in question cannot be studied by the resonance method because of a sharp increase in the damping (for example, by the spin wave resonance). However, we may assume that the existence of localized spin excitations should manifest itself in the transport effects (for example, it should influence the magnitude of the magnon contribution to the thermal conductivity, etc.).

The appearance of a region of anomalous damping $\Gamma_{\mathbf{q}} \sim a q$ near the bottom of the spin wave band is typical of disordered magnetic materials with an easy-axis anisotropy and, in particular, it manifests itself in asperomagnets with randomly oriented axes of the easy magnetization[5] and in ferromagnets with a regular easy-axis anisotropy but with a random distribution of exchange integrals of different signs.[11] Consequently, the relationship between the anomalous behavior of the damping of magnons and their localization which was demonstrated in the present problem indicates that the localization of magnons near the bottom of the one-particle spin excitations should occur in all such cases. Such localization should be most important for the thermodynamic and transport properties of magnetic materials. For disordered magnetic materials with an isotropic exchange interaction or with an easy-plane anisotropy, spin excitations should become localized in the upper part of the energy band. This was recently demonstrated for an isotropic Heisenberg spin glass (Ref. 12). It appears that the problem of localization of magnons near the bottom of the energy band requires separate discussion for each model system because of the possible appearance of a low-lying impurity band of local spin excitations with their polarization opposite to the polarization of the spin excitations of the matrix of the unperturbed magnetic crystal.

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