Electron–electron interaction in a self-consistent localization theory

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(Submitted June 23, 1983)

Fiz. Tverd. Tela (Leningrad) **25**, 3372–3382 (November 1983)

First-order corrections to the density of states due to electron-electron interactions are evaluated within a self-consistent theory. A generalization of the well-known results of Al'tshuler and Aronov to the insulating phase is obtained. A kink in the density of states at the Fermi level is flattened, but the resultant correction to the density of states exhibits a logarithmic singularity at the Fermi level throughout the whole region of localized states. Screening in the insulating phase is discussed.

PACS numbers: 71.50. + t, 71.20. + c, 71.30. + h

Localization of electrons in disordered systems is usually studied neglecting the electron–electron interactions.[1] However, the importance of this interaction both for "dirty" metals[2–4] and for strongly localized electrons[5,6] has been emphasized. There have been several studies[7–12] attempting to treat consistently the interaction effects near the metal–insulator transition in disordered systems. Only the metallic (or "quasimetallic" for two-dimensional systems) phase has been considered but the approach to the Anderson (or Mott) transition, i.e., the insulating phase, has not been studied. Although there are serious contradictions between various treatments, all the results indicate that correlations play an important if not the dominant role in the description of the metal–insulator transition in disordered systems. Several problems remain unresolved, for example, the most fundamental problem of the existence of localization in systems with electron–electron interactions. The situation is complicated by the well-known difficulties[1] which arise in a theoretical description of the Anderson transition even in the one-electron approximation. It is our opinion that great progress in qualitative understanding of the physics of localization was made by Vollhardt and Wolfle (Refs. 13–17), who developed a new version of a self-consistent localization theory. The main advantage of their theory is that it is possible to calculate all the relevant physical quantities in the whole range of variation of the parameters of the problem, i.e., ranging from a "good" metal to an Anderson insulator. It appears that such a theory provides a qualitatively correct interpolation scheme which describes the Anderson transition, although it does not contain a well-defined small parameter in the three-dimensional case.[16] Another advantage of a self-consistent theory is that it could be generalized to various types of interaction and it could also include applied fields.

We shall study the effect of first-order perturbation corrections to the electron–electron interaction within a self-consistent localization theory. We shall mainly consider corrections to the density of states. It will be shown that the approach of Aronov and Al'tshuler[2–4] can be naturally included in a self-consistent theory and a simple generalization of their results can be thus obtained for the insulating side of the Anderson transition. We shall assume that the concept of localization remains valid even in systems with interactions. In fact, such an assumption is necessary for a treatment based on the first-order corrections in the interaction to be valid within the context of a general theory of the transition described by a self-consistent localization theory.

1. PRINCIPAL RESULTS OF A SELF-CONSISTENT LOCALIZATION THEORY

The self-consistent localization theory is based on the Bethe–Salpeter equation in d-dimensional space

$$\Phi^{RA}_{\mathbf{pp}'}(\mathbf{q}\omega) = G^R(E_F+\omega p_+)\,G^A(E_F p_-)\left\{-\frac{1}{2\pi i}\delta(\mathbf{p}-\mathbf{p}') + \int\frac{d^d\mathbf{p}''}{(2\pi)^d}\,U_{\mathbf{pp}''}(\mathbf{q}\omega)\,\Phi^{RA}_{\mathbf{p}''\mathbf{p}'}(\mathbf{q}\omega)\right\}, \tag{1}$$

$$\Phi^{RA}_{\mathbf{pp}'}(\mathbf{q}\omega) = -\frac{1}{2\pi i}\left\langle G^R(\mathbf{p}_+\mathbf{p}'_+E_F+\omega)\,G^A(\mathbf{p}'_-\mathbf{p}_-E_F)\right\rangle;\quad \mathbf{p}_\pm = \mathbf{p}\pm\frac{1}{2}\mathbf{q} \tag{2}$$

 0038-5654/83/11 1942-06 $03.90

is the two-particle electron Green function and averaging over a random distribution of scatterers is indicated by angular brackets. Moreover, $U_{pp'}(q\omega)$ is the irreducible (in the RA channel) vertex part and E_F is the Fermi energy. Graphical representation of Eq. (1) using an equation for the total vortex part $\Gamma_{pp'}(q\omega)$ is shown in Fig. 1. Using Eq. (1), we can obtain (Refs. 13-15) the following approximate expression for the Green function:

$$\Phi^{RA}(\mathbf{q}\omega)=\int\frac{d^d\mathbf{p}}{(2\pi)^d}\int\frac{d^d\mathbf{p}'}{(2\pi)^d}\Phi^{RA}_{\mathbf{pp}'}(\mathbf{q}\omega)$$
$$=-N_0\frac{\omega+M_{E_F}(\mathbf{q}\omega)}{\omega^2+\omega M_{E_F}(\mathbf{q}\omega)-\frac{2E_F}{dm}\mathbf{q}^2}=-N_0\frac{1}{\omega+iD_{E_F}(\mathbf{q}\omega)\,\mathbf{q}^2} \tag{3}$$

(N_0 is the one-electron density of states at the Fermi level). Here, we introduced a generalized diffusion coefficient

$$D_{E_F}(\mathbf{q}\omega)=i\frac{2E_F}{dm}\frac{1}{M_{E_F}(\mathbf{q}\omega)}, \tag{4}$$

where m is the electron mass and $M_{E_F}(\mathbf{q}\omega)$ is a relaxation kernel which satisfies the following self-consistency equation[13-15] in the limit $\mathbf{q}\to 0$:

$$M_{E_F}(\omega)=2i\gamma\left\{1+\frac{1}{\pi N_0}\int\frac{d^d\mathbf{q}}{(2\pi)^d}\frac{1}{-i\omega+D_{E_F}(\omega)\,\mathbf{q}^2}\right\}, \tag{5}$$

where $\gamma=\pi\rho V^2N_0$ is the Born rate of the scattering of electrons from point scatterers distributed at random with a density ρ in space and V is the amplitude of the scattering from a point potential. The most rigorous derivation of Eq. (5) which is based on exact diagrammatic analysis neglecting the terms less singular in the limit $\omega\to 0$ (for $d>2$!) is given in Ref. 15. Equation (5) is equivalent to an approach in which $U_{pp'}(q\omega)$ is given by the sum of the well-known sequence of maximum crossing diagrams ("cooperon") and the classical diffusion coefficient is replaced by the generalized coefficient defined by Eq. (4) (see Refs. 13-15). The solution of Eq. (5) for $2<d<4$ is given by[14-17]

$$M_{E_F}(\omega)=\frac{i}{\tau_{E_F}}-\frac{\omega_0^2(E_F)}{\omega}, \tag{6}$$

where $\omega_0^2(E_F)=-\lim_{\omega\to 0}\omega M_{E_F}(\omega)>0$ for $E_F<E_C$ and E_C defined by $\omega_0^2(E_C)=0$ is the mobility edge.

In the region $E_F<E_C$, we obtain from Eq. (3) the following result in the limit $\omega\to 0$:

$$\Phi^{RA}(\mathbf{q}\omega)=-N_0\frac{A_{E_F}(\mathbf{q})}{\omega+i\delta}, \tag{7}$$

$$A_{E_F}(\mathbf{q})=\frac{\omega_0^2(E_F)\tau_{E_F}}{\omega_0^2(E_F)\tau_{E_F}+D_{E_F}\mathbf{q}^2}=\frac{1}{1+R_{\mathrm{loc}}^2(E_F)\,\mathbf{q}^2}=\frac{1}{N_0}\chi_{\mathbf{q}}(E_F)\chi_{-\mathbf{q}}(E_F), \tag{8}$$

$$\chi_{\mathbf{q}}(E_F)=\chi^*_{-\mathbf{q}}(E_F)=\sqrt{N_0}\left[1\pm iqR_{\mathrm{loc}}(E_F)\right]^{-1}, \tag{9}$$

where

$$R_{\mathrm{loc}}(E_F)=\left[\omega_0^2(E_F)\tau_{E_F}\right]^{-1/2}D_{E_F}^{1/2}=\sqrt{\frac{2E_F}{dm}}\,\omega_0^{-1}(E_F) \tag{10}$$

plays the role of the localization radius and the quantity

$$D_{E_F}=\frac{2E_F}{dm}\tau_{E_F} \tag{11}$$

will be called the renormalized diffusion coefficient. It follows from Eqs. (8)-(10) that the results of the self-consistent localization theory are equivalent to the general localization criterion formulated in Ref. 18.

2. DENSITY OF STATES

We shall illustrate the effect of the interaction corrections for the one-electron density of states which is defined by the well-known expression

$$N(\varepsilon)=-\frac{1}{\pi}\int\frac{d^d\mathbf{p}}{(2\pi)^d}\operatorname{Im}G^R(\varepsilon\mathbf{p}), \tag{12}$$

where $\varepsilon=E-E_F$ is the energy measured from the Fermi level. We note that various definitions of the density of states can be given for a many-electron system[2,4-6,8] depending on the physical quantities which are to be defined. The density of states defined by Eq. (12) can be determined from tunneling experiments.[2,4]

For simplicity, we shall consider the electron–electron interaction described by a finite-range repulsive potential. Following Ref. 2, we shall calculate the simplest correction to the one-electron Green function described by the diagram in Fig. 2a, where the following vertex part defined by (Fig. 2b)

$$\gamma(\mathbf{q}\omega)=1+\int\frac{d^dp'}{(2\pi)^d}\Gamma_{\mathbf{pp}'}(\mathbf{q}\omega)\,G(\mathbf{p}'_+\varepsilon+\omega)\,G(\varepsilon\mathbf{p}'_-). \tag{13}$$

is used. The corresponding correction to the density of states is given by[3]

$$\frac{\delta N(\varepsilon)}{N_0}\approx$$
$$-\frac{1}{\pi N_0}\operatorname{Im}i\int\frac{d^d\mathbf{p}}{(2\pi)^d}\int\frac{d^d\mathbf{q}}{(2\pi)^d}\int_{\varepsilon}^{\infty}\frac{d\omega}{2\pi}v(\mathbf{q})\,\gamma^2_{RA}(\mathbf{q}\omega)\,G^A(\varepsilon-\omega\mathbf{p}-\mathbf{q})\left[G^R(\varepsilon\mathbf{p})\right]^2$$
$$=-\frac{1}{2\gamma^2}\operatorname{Im}\int\frac{d^d\mathbf{q}}{(2\pi)^d}\int_{\varepsilon}^{\infty}\frac{d\omega}{2\pi}\gamma^2_{RA}(\mathbf{q}\omega)\,v(\mathbf{q}). \tag{14}$$

It can be easily shown that

$$G^R(\varepsilon+\omega\mathbf{p}_+)\,G^A(\varepsilon\mathbf{p}_-)\,\gamma_{RA}(\mathbf{q}\omega)=-2\pi i\int\frac{d^d\mathbf{p}'}{(2\pi)^d}\Phi^{RA}_{\mathbf{pp}'}(\mathbf{q}\omega). \tag{15}$$

Using then the following result of the self-consistent theory employed in the derivation of Eq. (3) (see Ref. 13)

$$\int\frac{d^dp'}{(2\pi)^d}\Phi^{RA}_{\mathbf{pp}'}(\mathbf{q}\omega)\approx-\frac{1}{2\pi iN_0}\left[G^R(E_F+\omega\mathbf{p}_+)-G^A(E_F\mathbf{p}_-)\right]$$
$$\times\int\frac{d^dp'}{(2\pi)^d}\int\frac{d^dp''}{(2\pi)^d}\left\{1+\frac{d}{p_F^2q^2}(\mathbf{pq})(\mathbf{p}''\mathbf{q})\right\}\Phi^{RA}_{\mathbf{p}''\mathbf{p}'}(\mathbf{q}\omega), \tag{16}$$

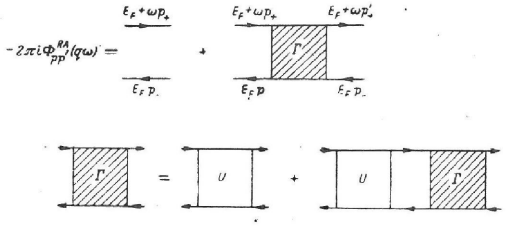

FIG. 1

we obtain from Eqs. (13) and (15) the required quantity in the self-consistent theory

$$\gamma_{RA}(\mathbf{q}\omega)=\left(\omega-\frac{1}{m}\,\mathbf{pq}+2i\gamma\right)\frac{\omega+M_{E_F}(\mathbf{q}\omega)-\frac{1}{m}\,\mathbf{pq}}{\omega^2+\omega M_{E_F}(\mathbf{q}\omega)-\frac{2E_F}{dm}\,\mathbf{q}^2}\approx\frac{2\gamma}{-i\omega+D_{E_F}(\mathbf{q}\omega)\,\mathbf{q}^2}, \tag{17}$$

Equation (17) holds for small ω and q. It can be seen that the quantity $\gamma(q\omega)$ determined within the self-consistent theory has the same form as for a "dirty" metal,[2-4] but the classical diffusion coefficient is now replaced by the generalized diffusion coefficient defined by Eq. (4). For example, Eqs. (17) and (6)-(8) yield in the localization region ($E_F < E_c$) for $\omega \to 0$

$$\gamma_{RA}(\mathbf{q}\omega)\approx\frac{2\gamma i}{\omega+i\delta}\,A_{E_F}(\mathbf{q}). \tag{18}$$

Equation (18) is quite general within the context of the localization criterion given in Ref. 18.

Equation (14) then assumes the form

$$\frac{\delta N(\varepsilon)}{N_0}=-\frac{1}{\pi}\,\mathrm{Im}\int\frac{d^d\mathbf{q}}{(2\pi)^d}\int_{\varepsilon}^{\infty}d\omega\,\frac{1}{\left[-i\omega+D_{E_F}(q\omega)\,\mathbf{q}^2\right]^2} \tag{19}$$

and, using the approximation defined by Eq. (6) ($q \to 0$), we obtain

$$\frac{\delta N(\varepsilon)}{N_0}\equiv\left(\frac{\delta N(\varepsilon)}{N_0}\right)_1+\left(\frac{\delta N(\varepsilon)}{N_0}\right)_2=-\frac{2}{\pi}\int_{\varepsilon}^{\infty}d\omega\int\frac{d^d\mathbf{q}}{(2\pi)^d}\,v(\mathbf{q})\,D_{E_F}\mathbf{q}^2\times\left\{\frac{\omega}{\left[\omega^2+\left(\omega_0^2(E_F)\,\tau_{E_F}+D_{E_F}\mathbf{q}^2\right)^2\right]^2}+\frac{\omega_0^2(E_F)\,\tau_{E_F}}{\omega}\,\frac{\omega_0^2(E_F)\,\tau_{E_F}+D_{E_F}\mathbf{q}^2}{\left[\omega^2+\left(\omega_0^2(E_F)\,\tau_{E_F}+D_{E_F}\mathbf{q}^2\right)^2\right]^2}\right\}. \tag{20}$$

Evaluating the integral with respect to ω, we find that

$$\left(\frac{\delta N(\varepsilon)}{N_0}\right)_1=-\frac{1}{\pi}\int\frac{d^d\mathbf{q}}{(2\pi)^d}\,v(\mathbf{q})\,\frac{D_{E_F}\mathbf{q}^2}{\varepsilon^2+\left[\omega_0^2(E_F)\,\tau_{E_F}+D_{E_F}\mathbf{q}^2\right]^2}, \tag{21}$$

$$\left(\frac{\delta N(\varepsilon)}{N_0}\right)_2=\frac{1}{\pi}\int\frac{d^d\mathbf{q}}{(2\pi)^d}\,v(\mathbf{q})\,D_{E_F}\mathbf{q}^2\left\{\frac{1}{\varepsilon^2+\left[\omega_0^2(E_F)\,\tau_{E_F}+D_{E_F}\mathbf{q}^2\right]^2}\,\frac{\omega_0^2(E_F)\,\tau_{E_F}}{\omega_0^2(E_F)\,\tau_{E_F}+D_{E_F}\mathbf{q}^2}+\frac{\omega_0^2(E_F)\,\tau_{E_F}}{\left[\omega_0^2(E_F)\,\tau_{E_F}+D_{E_F}\mathbf{q}^2\right]^3}\ln\frac{\varepsilon^2}{\varepsilon^2+\left[\omega_0^2(E_F)\,\tau_{E_F}+D_{E_F}\mathbf{q}^2\right]^2}\right\}_{|\varepsilon|\to 0} \tag{22}$$

$$\simeq\frac{2}{\pi}\int\frac{d^d\mathbf{q}}{(2\pi)^d}\,v(\mathbf{q})\,\frac{\omega_0^2(E_F)\,\tau_{E_F}}{\left[\omega_0^2(E_F)\,\tau_{E_F}+D_{E_F}\mathbf{q}^2\right]^3}\,D_{E_F}\mathbf{q}^2\ln\frac{|\varepsilon|}{\omega_0^2(E_F)\,\tau_{E_F}+D_{E_F}\mathbf{q}^2}.$$

For $\omega_0^2(E_F) = 0$, i.e., in the metallic phase, only the contribution defined by Eq. (21) is nonzero. For a contact interaction $v(q) = v_0$, we can use a simple estimate of the integral in Eq. (21) to obtain

$$\left(\frac{\delta N(\varepsilon)}{N_0}\right)_1=\frac{v_0}{\pi}\,\frac{S_d}{d-2}\begin{cases}|\varepsilon|^{\frac{d-2}{2}}-\tilde{E}^{\frac{d-2}{2}}, & |\varepsilon|\gg\omega_0^2(E_F)\,\tau_{E_F},\\ \omega_0^{d-2}(E_F)\,\tau_{E_F}^{\frac{d-2}{2}}-\tilde{E}^{\frac{d-2}{2}}, & |\varepsilon|\ll\omega_0^2(E_F)\,\tau_{E_F},\end{cases} \tag{23}$$

where $S_d = 2^{-(d-1)}\pi^{-d/2}/\Gamma(d/2)$. We have defined a characteristic energy $\tilde{E}$ related to the cutoff parameter for the upper integration limit in Eq. (21). Such a cutoff is required since the "diffusion" approximation in the integrand $[M_{E_F}(q\omega) \to M_{E_F}(\omega)]$ ceases to be valid for large momenta. Following the method of Refs. 16 and 17 where a similar cutoff in Eq. (5) was discussed, we shall choose the cutoff parameter to be of the order of the Fermi momentum $p_F = (2mE_F)^{1/2}$, which yields

$$\tilde{E}=D_{E_F}p_F^2. \tag{24}$$

Alternatively, we could choose the cutoff parameter to be equal to l^{-1}, i.e., equal to the reciprocal of the Born mean free path.[14,15] However, since the condition $l^{-1} \sim p_F$ is satisfied near the mobility edge, both definitions of the cutoff parameter are equivalent. It follows from scaling arguments of Ref. 8, which apply near the mobility edge in the limit $R_{loc}(E_F) \gg l, p_F^{-1}$, that the cutoff parameter satisfies $\sim R_{loc}^{-1}(E_F)$ but such a choice would contradict the self-consistent localization theory. The estimate defined by Eq. (23) holds provided

$$|\varepsilon|,\ \omega_0^2(E_F)\,\tau_{E_F}\ll\tilde{E}. \tag{25}$$

Equation (30) is replaced in the special case $d = 2$ by

$$\frac{\delta N(\varepsilon)}{N_0}=\frac{v_0}{2\pi D_{E_F}}\begin{cases}\ln\dfrac{|\varepsilon|}{\tilde{E}}, & |\varepsilon|\gg\omega_0^2(E_F)\,\tau_{E_F},\\ \ln\dfrac{\omega_0^2(E_F)\,\tau_{E_F}}{\tilde{E}}, & |\varepsilon|\ll\omega_0^2(E_F)\,\tau_{E_F}.\end{cases} \tag{26}$$

The contribution defined by Eq. (22) is dominant in an exponentially small neighborhood of the Fermi level and an estimate of the corresponding integral yields

$$\left(\frac{\delta N(\varepsilon)}{N_0}\right)_2=\frac{v_0}{\pi}\,\frac{S_d}{4-d}\,D_{E_F}^{-d/2}\left[\omega_0^2(E_F)\,\tau_{E_F}\right]^{\frac{d-2}{2}}\ln\frac{|\varepsilon|}{\omega_0^2(E_F)\,\tau_{E_F}}. \tag{27}$$

Equations (23) and (26) reduce in the metallic phase ($\omega_0^2 = 0$) to the well-known results of Refs. 2 and 3 including a renormalization of the diffusion coefficient. For $|\varepsilon| \gg \omega_0^2(E_F)\,\tau_{E_F}$, such a behavior is also obtained in the insulating phase. For $|\varepsilon| \ll \omega_0^2(E_F)\,\tau_{E_F}$, a kink in the density of states at the Fermi level, which occurs in the metallic phase, is rounded off and is replaced by a smooth minimum. However, a logarithmic singularity defined by Eq. (27) is obtained in an exponentially small neighborhood of the Fermi level. It is clear that our treatment applies only for $\delta N(\varepsilon)/N_0 \ll 1$.

We shall now quote explicit dependences which are obtained within the self-consistent localization theory in the insulating phase. For $2 < d < 4$, the solution of the

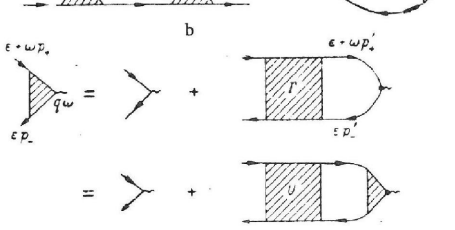

FIG. 2

system (5) is given by[16] (we omit unimportant constants)

$$\omega_0^2(E_F)\tau_{E_F}\sim\frac{\lambda}{4-d}\frac{E_F}{(p_FR_{1oc}(E_F))^d}\underset{E_F\leqslant E_c}{\sim}\frac{\lambda E_F}{4-d}\left|\frac{E_F-E_c}{E_c}\right|^{d\nu}, \quad (28)$$

$$\tilde{E}\sim E_F\left|\frac{d\omega_0^2(E_F)}{4E_F^2}\right|^{\frac{d-2}{2}}\sim E_F\left|\frac{1}{p_FR_{1oc}(E_F)}\right|^{d-2}\underset{E_F\leqslant E_c}{\sim}E_F\left|\frac{E_F-E_c}{E_c}\right|^{(d-2)\nu}, \quad (29)$$

$$D_{E_F}\sim\frac{1}{m}(p_FR_{1oc}(E_F))^{2-d}\underset{E_F\leqslant E_c}{\sim}\frac{1}{m}\left|\frac{E_F-E_c}{E_c}\right|^{(d-2)\nu}, \quad (30)$$

where $\lambda=\gamma/\pi E_F$ is a dimensionless perturbation parameter proportional to a random field and $\nu = 1/d-2$ is the critical index of the localization radius. The mobility edge E_c determined in a model of point scatterers distributed at random in space is given by[16]

$$E_c\sim\left\{\frac{d}{d-2}(2\pi)^{-d/2}\Gamma^{-1}(d/2)\right\}^{\frac{2}{4-d}}E_{sc}, \quad (31)$$

where $E_{sc}=m^{\frac{d}{4-d}}(\rho V^2)^{\frac{2}{4-d}}$ is the "strong coupling" energy (Refs. 1, 18) (for $E_F\sim E_{SC}$, we obtain $\lambda\sim 1$ and perturbation theory is no longer applicable). It follows from Eqs. (28) and (29) that the condition (25) can be easily satisfied. For $|\varepsilon|\ll\omega_0^2(E_F)\tau_{E_F}$, Eqs. (23) and (28)-(30) yield

$$\left(\frac{\delta N}{N_0}\right)_1\sim v_0\frac{4-d}{d-2}m^{\frac{d}{2}}E_F^{\frac{d}{2}-1}[1-(p_FR_{1oc}(E_F))^{d-2}]\underset{E_F\to E_c}{\sim}-v_0N_0\left|\frac{E_F-E_c}{E_c}\right|^{-(d-2)\nu}. \quad (32)$$

Similarly, we obtain from Eq. (27) the following result:

$$\left(\frac{\delta N(\varepsilon)}{N_0}\right)_2\sim-v_0m^{\frac{d}{2}}E_F^{\frac{d}{2}-1}\ln\frac{|\varepsilon|}{\lambda E_F}(p_FE_{1oc}(E_F))^{-d}\sim-v_0N_0\ln\frac{|\varepsilon|}{\lambda E_F}\left|\frac{E_F-E_c}{E_c}\right|^{d\nu}. \quad (33)$$

The divergence of the corrections to the density of states which occurs for $E_F\to E_c$ [a singularity analogous to that defined by Eq. (32) is also obtained in the metallic phase] indicates that our approximations break down in the vicinity of the mobility edge.

For d = 2, we obtain[13]

$$\omega_0^2(E_F)\tau_{E_F}\sim E_F\exp\left(-\frac{1}{\lambda}\right);\quad\tilde{E}\sim E_F\left[1-\exp\left(-\frac{1}{\lambda}\right)\right], \quad (34)$$

and, since the condition $\lambda\ll 1$ is satisfied, the contribution defined by Eq. (27) is dominant

$$\frac{\delta N(\varepsilon)}{N_0}\sim v_0N_0\begin{cases}\ln\dfrac{|\varepsilon|}{E_F}, & |\varepsilon|\gg E_F\exp\left(-\dfrac{1}{\lambda}\right),\\ \dfrac{1}{\lambda}\ln\dfrac{|\varepsilon|}{E_F}, & |\varepsilon|\ll E_F\exp\left(-\dfrac{1}{\lambda}\right),\end{cases} \quad (35)$$

where $N_0 = m/2\pi$ is the free-electron density of states in two dimensions.

3. COULOMB INTERACTION

We shall discuss in this section the Coulomb interaction and perform our calculations using the Matsubara method in three dimensions. The wavy line in Fig. 2a corresponds to a dynamically screened interaction

$$V(\mathbf{q}\omega_m)=\frac{4\pi e^2}{\mathbf{q}^2\varepsilon(\mathbf{q}\omega_m)},\quad\omega_m=2\pi mT, \quad (36)$$

where

$$\varepsilon(\mathbf{q}\omega_m)=1+\frac{4\pi e^2}{\mathbf{q}^2}\Pi(\mathbf{q}\omega_m) \quad (37)$$

is the permittivity and the polarization operator $\Pi(\mathbf{q}\omega_m)$ is defined by the diagram in Fig. 2c.

The "triangular" vertex in the Matsubara method is defined by

$$\gamma(\mathbf{q}\omega_m\varepsilon_n)=\theta(\varepsilon_n)\theta(\varepsilon_n+\omega_m)+\theta(-\varepsilon_n)\theta(-\varepsilon_n-\omega_m)+2\gamma\left\{\frac{\theta(\varepsilon_n)\theta(-\varepsilon_n-\omega_m)}{-\omega_m+D_{E_F}(-\omega_m)\mathbf{q}^2}+\frac{\theta(-\varepsilon_n)\theta(\varepsilon_n+\omega_m)}{\omega_m+D_{E_F}(\omega_m)\mathbf{q}^2}\right\},\quad\varepsilon_n=(2n+1)\pi T, \quad (38)$$

$$D_{E_F}(\omega_m)=\frac{2E_F}{dm}\frac{i}{M_{E_F}(\omega_m)},\quad M_{E_F}(\omega_m)=\frac{i}{\tau_{E_F}}-\frac{\omega_0^2(E_F)}{i\omega_m}. \quad (39)$$

The polarization operator is given by

$$\Pi(\mathbf{q}\omega_m)=N_0\left\{A_{E_F}(\mathbf{q})\delta_{\omega_m0}+\frac{D_{E_F}(\omega_m)\mathbf{q}^2}{\omega_m+D_{E_F}(\omega_m)\mathbf{q}^2}\theta(\omega_m)+\frac{D_{E_F}(\omega_m)\mathbf{q}^2}{-\omega_m+D_{E_F}(-\omega_m)\mathbf{q}^2}\theta(-\omega_m)\right\}\equiv\Pi_{1oc}(\mathbf{q}\omega_m)+\Pi_{reg}(\mathbf{q}\omega_m), \quad (40)$$

where $A_{E_F}(\mathbf{q})$ is defined by Eq. (8) and $\theta(\omega_m)=\begin{cases}1 & m\geqslant 0,\\ 0 & m<0.\end{cases}$

It can be seen that Eq. (40) contains a "localization" contribution

$$\Pi_{1oc}(\mathbf{q}\omega_m)=N_0A_{E_F}(\mathbf{q})\delta_{\omega_m0}. \quad (41)$$

Such a contribution reflects the well-known differences in the behavior of the static adiabatic and isothermal response in systems exhibiting nonergodic behavior.[19,20] The Matsubara response "senses" nonergodic behavior[20] and localization is a typical nonergodic phenomenon.[21-23] The polarization operator is proportional to the electron compressibility. The static isothermal compressibility is given by[20]

$$\varkappa^T(\mathbf{q}0)=\Pi(\mathbf{q}\omega_m=0); \quad (42)$$

the adiabatic compressibility is given by

$$\varkappa^A(\mathbf{q}0)=\Pi_{reg}(\mathbf{q}i\omega_m\to\omega+i\delta\to 0). \quad (43)$$

It then follows from Refs. 22 and 23 that

$$\varkappa^T(\mathbf{q}0)-\varkappa^A(\mathbf{q}0)=N_0A_{E_F}(\mathbf{q})=N_0\frac{1}{1+R_{1oc}^2(E_F)\mathbf{q}^2}. \quad (44)$$

The static isothermal polarization operator is given by

$$\Pi(\mathbf{q}\omega_m=0)=N_0\left\{\frac{1}{1+R_{1oc}^2(E_F)\mathbf{q}^2}+\frac{\mathbf{q}^2}{\mathbf{q}^2+R_{1oc}^{-2}(E_F)}\right\}=N_0. \quad (45)$$

The corresponding static adiabatic permittivity is given by

$$\varepsilon^A(\mathbf{q}\omega\to 0)=1+\frac{4\pi e^2}{\mathbf{q}^2}\Pi_{reg}(\mathbf{q}\omega\to 0)=\begin{cases}1+\dfrac{\varkappa_D^2}{\mathbf{q}^2}, & \mathbf{q}\gtrsim R_{1oc}^{-1}(E_F),\\ 1+\varkappa_D^2R_{1oc}^2(E_F), & \mathbf{q}\lesssim R_{1oc}^{-1}(E_F),\end{cases} \quad (46)$$

where $\varkappa_D^2 = 4\pi e^2N_0$. The static isothermal permittivity is given by

$$\varepsilon^T(\mathbf{q}0)=1+\frac{4\pi e^2}{\mathbf{q}^2}\Pi(\mathbf{q}\omega_m=0)=1+\frac{\varkappa_D^2}{\mathbf{q}^2}. \quad (47)$$

In fact, the permittivity defined by Eq. (47) governs screening of a static applied field under experimental conditions (Ref. 24). It follows from Eq. (47) that the static field is completely screened even in the localized phase. The physical interpretation of this result will be discussed elsewhere (see also Ref. 24). However, in the present context, the "localization" term defined by Eq. (41) is unimportant and its contribution to the diagram shown in Fig. 2a tends to zero in the limit $T \to 0$ and we can thus consider only the regular contribution $\Pi_{reg}(q\omega_m)$.

Using the vertex defined by Eq. (38) in the evaluation of the contribution shown in Fig. 2a, we obtain[2] (the additional factor of 2 takes account of spin degeneracy)

$$\delta N(\varepsilon_n) = \frac{2}{\pi}\int \frac{d^3q}{(2\pi)^3} T \sum_m V(\mathbf{q}\omega_m)\gamma^2(\mathbf{q}\omega_m\varepsilon_n)\int \frac{d^3p}{(2\pi)^3} G^2(\varepsilon_n\mathbf{p}) G(\varepsilon_n+\omega_m\mathbf{p}+\mathbf{q})$$

$$\approx -2iN_0T\sum_m \int \frac{d^3q}{(2\pi)^3} V(\mathbf{q}\omega_m)\left\{\frac{\theta(\varepsilon_n)\theta(-\varepsilon_n-\omega_m)}{\left[-\omega_m+D_{E_F}(-\omega_m)q^2\right]^2} - \frac{\theta(-\varepsilon_n)\theta(\varepsilon_n+\omega_m)}{\left[\omega_m+D_{E_F}(\omega_m)q^2\right]^2}\right\}, \quad (48)$$

$$V(\mathbf{q}\omega_m\to 0)\approx \frac{-\omega_m+D_{E_F}(-\omega_m)q^2}{N_0 D_{E_F}(-\omega_m)}\theta(-\omega_m)+\frac{\omega_m+D_{E_F}(\omega_m)q^2}{N_0 D_{E_F}(\omega_m)}\theta(\omega_m). \quad (49)$$

In the case $\varepsilon_n > 0$, it follows that

$$\delta N(\varepsilon_n>0) = -\frac{iT}{\pi^2}\sum_{\omega_m=-\infty}^{-\varepsilon_n} D_{E_F}^{-1}(\omega_m)\int_0^\infty dq\left[D_{E_F}(-\omega_m)q^2-\omega_m\right]^{-1} \quad (50)$$

$$= -\frac{iT}{\pi^2 D_{E_F}^{3/2}}\sum_{\omega_m=-\infty}^{-\varepsilon_n}\omega_m^{-2}\left[\omega_m-\omega_0^2(E_F)\tau_{E_F}\right]^2\int_0^\infty dx\left[x^2-\omega_m+\omega_0^2(E_F)\tau_{E_F}\right]^{-1}.$$

Using the analytic continuation and evaluating the imaginary part of the Green function, we arrive at the following correction to the density of states:

$$\delta N(\varepsilon) = \frac{1}{2\pi^2 D_{E_F}^{3/2}}\int_0^{\bar{E}} d\omega\,[f(\omega+\varepsilon)+f(\omega-\varepsilon)-1]\,\mathrm{Im}\,\Phi(\omega), \quad (51)$$

where $f(\varepsilon)$ is the Fermi distribution function and

$$\Phi(\omega) = \left[i\omega+\omega_0^2(E_F)\tau_{E_F}\right]^2\omega^{-2}\int_0^\infty dx\left[x^2+i\omega+\omega_0^2(E_F)\tau_{E_F}\right]^{-1}$$

$$\approx \begin{cases} \dfrac{\pi}{2\sqrt{2\omega}}, & \omega \gg \omega_0^2(E_F)\tau_{E_F}, \\ \dfrac{3\pi}{2\sqrt{2}}\omega^{-1}\left[\omega_0^2(E_F)\tau_{E_F}\right]^{1/2}, & \omega \ll \omega_0^2(E_F)\tau_{E_F}. \end{cases} \quad (52)$$

Substituting the asymptotic expression (52) in Eq. (51), we obtain

$$\delta N(\varepsilon) \approx \begin{cases} \dfrac{1}{2^{3/2}\pi^2 D_{E_F}^{3/2}}\left[T^{1/2}\varphi\left(\dfrac{|E|}{2T}\right)-\tilde{E}^{1/2}\right], & |\varepsilon|,\ T \gg \omega_0^2(E_F)\tau_{E_F}, \\ \dfrac{1}{2^{3/2}\pi^2 D_{E_F}^{3/2}}\left[\left(\omega_0^2(E_F)\tau_{E_F}\right)^{1/2}-\tilde{E}^{1/2}\right] \\ \quad +\dfrac{3\left(\omega_0^2(E_F)\tau_{E_F}\right)^{1/2}}{2^{3/2}\pi^2 D_{E_F}^{3/2}}\ln\dfrac{\mathrm{Max}\{|\varepsilon|,\ T\}}{\omega_0^2(E_F)\tau_{E_F}}, & |\varepsilon|,\ T \ll \omega_0^2(E_F)\tau_{E_F}, \end{cases} \quad (53)$$

where[2]

$$\varphi(x) = \frac{1}{\sqrt{2}}\int_0^\infty dy\sqrt{y}\left[\frac{1}{\mathrm{ch}^2(x-y)}+\frac{1}{\mathrm{ch}^2(x+y)}\right] \approx \begin{cases} 1.0, & x \ll 1, \\ \sqrt{2x}, & x \gg 1, \end{cases} \quad (54)$$

which represents a generalization of the results of Aronov and Al'tshuler[2] and is identical up to a constant with Eqs. (23) and (27) which apply to the zero-range interaction.

4. HARTREE AND OTHER DIAGRAMS

We have considered only the Fock correction described by the diagram in Fig. 2a. In fact, there are several other diagrams in the first order of perturbation theory with respect to the interaction and examples of some of these diagrams are shown in Fig. 3. We shall first consider the "Hartree" diagram in Fig. 3a. We can estimate its contribution within the self-consistent localization theory by the method of Refs. 3 and 4. In fact, the total vertex part in the self-consistent theory is given by (for small $\mathbf{q}$ and ω)

$$\Gamma_{pp'}^{RA}(\mathbf{q}\omega) \simeq \frac{2\rho V^2\gamma}{-i\omega+D_{E_F}(\omega)\mathbf{q}^2}. \quad (55)$$

Equation (55) is a generalization of the standard diffusion vertex obtained by summation of ladder diagrams. The result defined by Eq. (55) is almost self-evident and it can be easily shown that, when it is substituted in Eq. (13) it leads to Eq. (17). For a repulsive potential of finite range, we can follow Refs. 3 and 4 and use Eq. (55) to show that the ratio of the diagrams for the self-energy parts defined by Fig. 3a and Fig. 2a can be estimated as

$$F = \int d\Omega v(2p_F\sin\theta/2)\Big/\int d\Omega v(0) \quad (56)$$

(it is important to note that, in contrast to the Fock contribution, the frequency transfer which is equal to zero is important in the Hartree diagram rather than the small momenta transferred along the interaction line). The integrals with respect to the solid angle in Eq. (56) are taken from the Fourier transform of the interaction potential $v(q)$ and θ is the angle between two momenta on the Fermi surface. It can be easily shown that $F < 1$ for an interaction potential whose range is of the order of several interatomic distances. For example, for the screened Coulomb interaction in three dimensions,[3,4] we obtain

$$F = \frac{1}{2}\int_0^\pi d\theta\sin\theta\,\frac{1}{1+\dfrac{4p_F^2}{\varkappa_D^2}\sin^2\theta/2} = \frac{\varkappa_D^2}{4p_F^2}\ln\left[1+\frac{4p_F^2}{\varkappa_D^2}\right]. \quad (5$$

It follows that $F \ll 1$ for $\varkappa_D^2 \ll p_F^2$ but $F \to 1$ for $p_F^2 \ll$ For a contact interaction, $F = 1$ and the Hartree contribution in Fig. 3a including spin is twice as large as t Fock contribution defined in Fig. 2a. In this limit, all

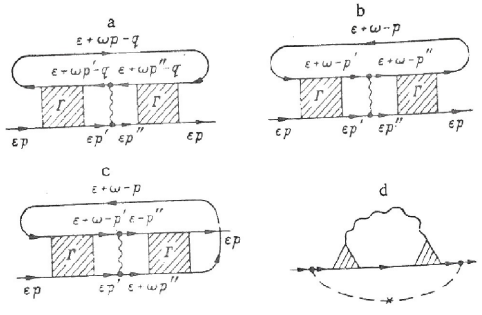

FIG. 3

the aforementioned corrections to the density of states simply change their signs. In the absence of interactions breaking invariance under time reversal (magnetic field, magnetic impurities, etc.), the contribution of the diagram shown in Fig. 3b which contains the interaction in the "Cooper" channel is equal to the contribution of the diagram shown in Fig. 3a and, therefore, is small of the order of F. A similar small parameter appears in the Fock diagram shown in Fig. 3c involving the interaction in the "Cooper" channel. As in the case of the diagram in Fig. 3a, the aforementioned diagram differs from the diagram shown in Fig. 2a since the momentum transferred along the interaction line is not small. The diagram shown in Fig. 3d can be easily estimated

$$\operatorname{Im}\Sigma(\varepsilon p)=\rho V^2\delta N(\varepsilon), \tag{58}$$

and we find that it contains an additional small parameter ρV^2. It follows that our treatment based on the diagram shown in Fig. 2a is valid for interaction potentials whose radius is not too small. The role of higher-order contributions in the interaction remains to be clarified.

The authors are grateful to D. I. Khomskii and M. I. Auslender for their discussions.

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Translated by D. Mathon