luctuations of the conductivity and diamagnetic susceptibility in irty superconductors near the Anderson localization threshold

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Fluctuations of the conductivity and diamagnetic susceptibility in superconductors above T_c near the Anderson localization threshold are discussed. The present treatment is based on the self-consistent localization theory. It is shown that the resistive superconducting transition remains narrow near the localization threshold and becomes smeared only for $\sigma \lesssim \sigma_c (T_c/E_F)^{1/3}$. The fluctuation diamagnetic susceptibility is calculated in a wide temperature range both far from and within the critical region of the Anderson transition.

1. The properties of strongly disordered super-nductors are being studied theoretically and ex-erimentally. It is now clear that the existing clas-cal theory of dirty superconductors[1] should be odified for mean free paths ℓ close to k_F^{-1}, i.e., r mean free paths close to the interatomic dis-nce. The diffusion of electrons in three-dimension- compound is replaced by the localization of elec-ons, i.e., for mean free paths shorter than a criti-l value ℓ_c the system of electrons undergoes a ansition to the state of an Anderson insulator.[2] his transition manifests itself primarily by the drop the residual conductivity σ to zero when $\ell < \ell_c$. he transition occurs for $\approx \sigma_c = e^2 k_F/\pi^3\hbar = (2-5)\cdot 10^2$ $^{-1}\cdot cm^{-1}\cdot$(σ_c is the minimum Mott conductivity).

The effect of electron localization on the super-nducting properties of alloys was studied in Refs. -5 in the situation when ℓ decreases. It was found at T_c decreases with decreasing ℓ in a model which sumes that the density of states at the Fermi sur ce (N_F) and the parameter $\lambda_{e,ph}$ are independent ℓ. Such decrease of the critical temperature is due an increase in the parameter μ^* describing an ef-ctive Coulomb repulsion.[3] This occurs because the tardation of the Coulomb repulsion of electrons in-eases with decreasing diffusion coefficient. The mperature T_c begins to decrease in the region $\sigma >>$, but T_c drops rapidly only in the region $\sigma \approx \sigma_c$. laevskii and Sadovskii[4] and later Kotlyar and pitul'nik[5] used the self-consistent localization eory of Wollhardt and Wolfle[6] to calculate ξ in the gion $\sigma \lesssim \sigma_c$ and also in the regime of Anderson calization[4] ($\ell < \ell_c$). It was found that the cor-lation length at the localization threshold, where $\ell =$ and $\sigma = 0$, is given by $\xi = (\xi_0 \ell^2)^{1/3}$ (for order--magnitude estimates, we can replace ℓ in this ex-ession by k_F^{-1}). Although for dirty superconduc-rs (with $\sigma >> \sigma_c$) the quantity ξ^2 is proportional to σ, if $\ell \to \ell_c$ and $\sigma \to 0$, the value of ξ^2 remains nonzero both at the localization threshold and in the region of insulating behavior (the last statement is meagningful provided T_c does not vanish, as ℓ decreases, before the localization threshold is reached due to increasing μ^*).

It is well known that the region of strong thermodynamic fluctuations in a superconductor near its critical temperature widens with decreasing ξ. The width of such region is $\tau_G T_c$, where the Ginzburg parameter is given by $\tau_G = (E_F/T_c)^2 (\xi k_F)^{-6}$. The critical region for pure superconductors is quite narrow since $\tau_G = (T_c/E_F)^4$. The parameter τ_G increases with decreasing ℓ and is given in the region $\ell \leq \xi_0 = 0.18 v_F/T_c$ by $\tau_G = (T_c/E_F)(k_F l)^{-3}$. However, even for usual dirty superconductors $(k_F^{-1} \ll l \lesssim \xi_0)$ it remains quite small compared with unity.

It was noted in Ref. 5 that near the localization threshold where $\xi = (\xi_0 \ell^2)^{1/3}$ holds the parameter τ_G contains no longer a small formal parameter $(T_c/E_F)^n$ with $n > 0$ and can become of the order of unity. It follows that the superconducting transition near the localization threshold should be identical with the transition at the λ-point in helium and fluctuations shoud play very important role. For example, thermodynamic fluctuations of the superconducting order parameter or of the specific heat can be large in the whole region in which superconductivity occurs (provided the numerical value of τ_G does not happen to be small). It is also clear that the usual approach cannot be used to study fluctuation effects under such conditions since electrons, whose motion is originally of the diffusion type, become localized.

Experimentally, it is most interesting to study fluctuations of the conductivity and of the diamagnetic susceptibility near the localization threshold. Such fluctuations can be very easily measured and

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should be relatively large for the conductivity (since the normal one-electron conductivity is low at the localization threshold).

We shall, therefore, calculate the fluctuation corrections to the conductivity and diamagnetic susceptibility taking account of a frequency dependence of the diffusion coefficient near the localization threshold. Our calculation is based on the self-consistent theory of Wollhardt and Wolfle.[6] We shall neglect fluctuations of the superconducting order parameter due to disorder in the system. They are important near the localization threshold[7] and we shall discuss the relationship between these two types of fluctuation.

2. We shall first calculate the fluctuation propagator averaged over the positions of electron scattering centers. We shall average the fluctuation propagator using the self-consistent localization theory described in Ref. 6. The propagator in question is determined by the folowing expression in the ladder diagram approximation in the interaction in the Cooper channel:

$$L^{-1}(\mathbf{q}, \Omega_m) = \lambda^{-1} - \Pi(\mathbf{q}, \Omega_m). \tag{1}$$

Here, λ is the effective electron—electron interaction constant in the Cooper channel and $\Pi(\mathbf{q}, \Omega_m)$ is the polarization operator which is expressed in terms of the exact one-electron Green functions

$$\Pi(\mathbf{q}, \Omega_m) = T\sum_{\varepsilon_n}\int\frac{d\mathbf{p}}{(2\pi)^3}\int\frac{d\mathbf{p}'}{(2\pi)^3}\langle G(\mathbf{p}_+, \mathbf{p}'_+, \varepsilon_n+\Omega_m) \times G(\mathbf{p}'_-, \mathbf{p}_-, -\varepsilon_n)\rangle = -2\pi i T\sum_{\varepsilon_n}\Phi(\mathbf{q}, \omega_k = 2\varepsilon_n + \Omega_m), \tag{2}$$

where $\omega_k = 2\pi Tk$; $k = 0, \pm 1, \pm 2, \ldots$; the symbol $\langle\ldots\rangle$ indicates averaging with respect to the positions of scatterers. This polarization operator was evaluated for the system under study in Ref. 4 in the calculation of the coefficients in the Ginzburg—Landau equation in the special case $\omega_k = 2\varepsilon_n$ when one of the Green functions which appears in the definition of the function Φ is automatically a retarded Green function and the other an advanced Green function. It follows from Eq. (1) that we now require a more general expression for the function Φ corresponding to $\omega_k = 2\varepsilon_n + \Omega_m$. It can be easily shown that we can obtain the required generalization by introducing in the expression for Φ, derived in Refs. 4 and 6, an additional step function of the product of frequencies

$$\Phi(\mathbf{q}, \omega_k = 2\varepsilon_n + \Omega_m) = -\frac{N(E_F)\,\theta(\varepsilon_n(\varepsilon_n + \Omega_m))}{i\,|2\varepsilon_n + \Omega_m| + i\tilde{D}(|2\varepsilon_n + \Omega_m|)\,\mathbf{q}^2}. \tag{3}$$

The generalized frequency-dependent diffusion coefficient $\tilde{D}(\omega_k)$, which apears in this expression, is given within the self-consistent localization theory by

$$\tilde{D}(\omega_k) = \max\left\{\frac{D_0}{k_F R_l},\ D_0(\omega_k\tau)^{1/3}\right\}, \tag{4}$$

where $R_l = k_F^{-1} l_c |l - l_c|^{-1}$ is the localization radius; $D_0 = v_F \ell/3$ is the usual diffusion coefficient; $\tau = \ell/v_F$. Subsituting Eqs. (4) and (3) in Eq. (2) and performing the required summation over the fermion frequencies, we arrive at the following expressions for the polarization operator and the fluctuation propagator for small momenta and low frequencies:

$$\Pi^R(\mathbf{q}, \omega) = N(E_F)\left[\ln\frac{\omega_D}{4\pi T} + \frac{i\pi\omega}{8T} - \eta\mathbf{q}^2\right], \tag{5}$$

$$[L^R(\mathbf{q}, \omega)]^{-1} = -N(E_F)\left\{\ln T/T_c - \frac{i\pi\omega}{8T} + \eta\mathbf{q}^2\right].\tag{6}$$

However, in contrast to Ref. 8, we now have

$$\eta = \begin{cases}\dfrac{\pi D_0}{8T}\dfrac{1}{k_F R_l}, & R_l \ll \xi,\\[2mm] \dfrac{\tau^{1/3}D_0}{(4\pi T)^{2/3}}\zeta(5/3, 1/2), & R_l \geqslant \xi,\end{cases} \tag{7}$$

where $\xi = (\xi_0\ell^2)^{1/3}$ and $\zeta(\alpha, x)$ is the generalized Riemann zeta function.

Replacing summation in Eq. (2) by integration for temperatures far from T_c, we obtain the corresponding expressions for large momenta and high frequencies which are valid near the Anderson transition ($R_\ell \geq \xi$)

$$\Pi(\mathbf{q}, \Omega_m) = N(E_F)\left\{\ln\frac{\omega_D}{4\pi T} - \frac{3}{2}\ln\left[\left(\frac{\Omega_m}{4\pi T}\right)^{2/3} + \frac{\tau^{1/3}D_0\mathbf{q}^2}{(4\pi T)^{2/3}}\right]\right\}, \tag{8}$$

$$L^{-1}(\mathbf{q}, \Omega_m) = -N(E_F)\left\{\ln\frac{T}{T_c} + \frac{3}{2}\ln\left[\left(\frac{\Omega_m}{4\pi T}\right)^{2/3} + \frac{\tau^{1/3}D_0\mathbf{q}^2}{(4\pi T)^{2/3}}\right]\right\}. \tag{9}$$

3. We shall now calculate the fluctuation diamagnetic susceptibility χ_{fl} of a superconductor close to the Anderson transition at temperatures higher than T_c.

Far from the localization threshold when the inequality $R_\ell \leq \xi$ holds, we find from Eq. (4) that the renormalization of the diffusion coefficient reduces to the replacement by D_0 by $\tilde{D} = D_0/k_F R_\ell$. The susceptibility χ_{fl} can be obtained directly by dividing the result of Ref. 9 by $(k_F R_\ell)^{1/2}$. At temperatures close to T_c, we obtain

$$\chi_{fl} = -2.25\frac{|\chi_L|}{(k_F R_l)^{1/2}}\sqrt{T_c\tau}\left(\frac{T_c}{T - T_c}\right)^{1/2},\quad (T - T_c \ll T_c);$$

the temperature-dependent part of the fluctuation diamagnetic susceptibility far from the critical temperature is given by

$$\Delta\chi_{fl} = \chi(T) - \chi(T \sim T_c) = \frac{2|\chi_L|}{(k_F R_l)^{1/2}}\frac{\sqrt{T\tau}}{\ln T/T_c},\quad (T \gg T_c).$$

The situation in the vicinity of the localization threshold ($R_\ell \geq \xi$) is less trivial since the frequency dependence of the diffusion coefficient $\tilde{D}(\omega_k) = D_0(\omega_k\tau)^{1/3}$ is essential. As in Ref. 9, we can calculate the fluctuation diamagnetic susceptibility using the following general expression based on the exact one-electron Green functions:[10]

$$\chi_{fl}(T) = \frac{eT}{24\pi^2\hbar^2 c}\sum_{\Omega_m}\int_{-\infty}^{\infty} dq\,L(q, \Omega_m)\,\Pi'(q, \Omega_m)\,|_{H=0}, \tag{10}$$

However, in evaluating Eq. (10), we shall use the polarization operator and fluctuation propagator obtained by the method of the self-consistent theory of localization.

As usual, the main contribution to thermody-ic quantities, which is singular as a function of proximity to the superconducting transition tem-ature, is due to the term $\Omega_m = 0$ in Eq. (10). It lear that the complete dependence on the magnetic ld for weak fields is contained in the term $q^2+\frac{\pi H}{e\Phi_0}$ the polarization operator (Φ_0 is a quantum of the gnetic flux). It follows that

$$\Pi'(q,0)|_{H=0}=\frac{N(E_F)}{c\hbar}2\pi T\sum_{\varepsilon_n>0}\frac{\tilde{D}(2\varepsilon_n)}{[2\varepsilon_n+\tilde{D}(2\varepsilon_n)q^2]}. \tag{11}$$

forming the summation in Eq. (11) and then inte-ating in Eq. (10), we find that the fluctuation dia-gnetic susceptibility at the Anderson localization shold is given by

$$\chi_{fl}=-\left(\frac{\pi}{2}\right)^{2/3}\frac{\zeta^{1/3}(5/3,\ 1/2)}{3}|\chi_L'|(T_c\tau)^{2/3}\left(\frac{T_c}{T-T_c}\right)^{1/2}$$
$$=-0.24|\chi_L^1|(T_c\tau)^{2/3}\left(\frac{T_c}{T-T_c}\right)^{1/2}. \tag{12}$$

Far from the superconducting transition; terms her than $\Omega_m = 0$ have to be included in the sum in . (10). Using Eq. (8) corresponding to this uation, we find that the polarization operator is ven by

$$\Pi'(q,\Omega_m)|_{H=0}=-\frac{3}{2}N(E_F)\frac{(4\pi T\tau)^{1/3}\frac{D_0}{8\Phi_0 T}}{\left(\frac{\Omega_m}{4\pi T}\right)^{2/3}+\frac{\tau^{1/3}D_0q^2}{(4\pi T)^{2/3}}}. \tag{13}$$

bstituting Eqs. (13) and (9) in Eq. (10) and in-grating with respect to the momentum, we find at the remaining sum over the frequencies diverges rmally, which is usual in such thermodynamic cal-lations. Since all our expressions hold only at equencies $\Omega_m \lesssim \tau^{-1}$, we can introduce a cutoff remove the singularity which yields

$$\chi_{fl}(T\gg T_c)=-0.6|\chi_L|(T_c\tau)^{2/3}\left[\mathrm{li}(T_c\tau)^{-2/3}-\mathrm{li}\left(\frac{T}{T_c}\right)^{2/3}\right]. \tag{14}$$

Using the asymptotic expansion of the integral log-arithmic function, we find that the temperature-de-pendent part $\Delta\chi_{f\ell}$ is given by

$$\Delta\chi_{fl}=0.9|\chi_L|(T\tau)^{2/3}\frac{1}{\ln T/T_c}. \tag{15}$$

It can be seen that this result matches well the cor-responding expression for the metallic phase taken at the boundary of the critical region of the Anderson transition [where $R_\ell \sim \xi$ and, therefore $k_F R_\ell \sim (T\tau)^{-1/3}$].

All our results are given in Table I.

4. We shall now calculate the fluctuation con-ductivity of a disordered superconductor above T_c and at the threshold of the Anderson transition. As in the case of the fluctuation diamagnetic sus-ceptibility, the fluctuation conductivity of a super-conductor outside the critical region of the Anderson transition ($k_F^{-1} \ll R_l \ll \xi$) can be obtained by a trivial generalization of the usual expressions for dirty superconductors.[8,11] In the critical region ($R_\ell \gtrsim \xi$), it is necessary not only to include the frequency dependence of the diffusion coefficient, but also con-sider more carefully the averging of the required diagrams with respect to the position of scatterers. We shall consider only the paraconductivity,[8] i.e., the contribution to the current due to Cooper pairs created by fluctuations neglecting any change in the one-electron conductivity in the presence of such pairs (the Maki–Thompson contribution[12,13]). This is justified since the Maki–Thompson contribution is strongly suppressed by inelastic scattering of electrons which always takes place near the localiza-tion threshold.

Examples of diagrams describing the fluctuation conductivity including electron scattering from im-purities are shown in Fig. 1. It is clear that the

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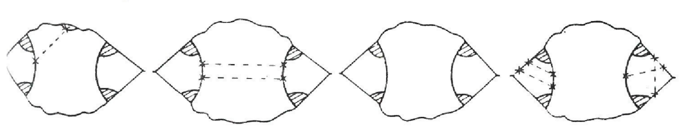
$k_F R_l$	$T-T_c \ll T_c$	$T \gg T_c$
1	$\chi_{fl}=-2.25\|\chi_L\|\sqrt{T_c\tau}\left(\frac{T_c}{T-T_c}\right)^{1/2}$	$\Delta\chi_{fl}=\frac{2\|\chi_L\|\sqrt{T\tau}}{\ln T/T_c}$
$\ll k_F R_l \leqslant (T\tau)^{-1/3}$	$\chi_{fl}=-\frac{2.25\|\chi_L\|}{(k_F R_l)^{1/2}}\sqrt{T_c\tau}\left(\frac{T_c}{T-T_c}\right)^{1/2}$	$\Delta\chi_{fl}=\frac{2\|\chi_L\|\sqrt{T\tau}}{(k_F R_l)^{1/2}\ln T/T_c}$
$k_F R_l \geqslant (T\tau)^{-1/3}$	$\chi_{fl}=-0.24\|\chi_L\|(T_c\tau)^{2/3}\left(\frac{T_c}{T-T_c}\right)^{1/2}$	$\Delta\chi_{fl}=\frac{0.9\|\chi_L\|(T\tau)^{2/3}}{\ln T/T_c}$

a b c d



IG. 1. Examples of diagrams contributing to the fluctuation conductivity of a superconductor at the Anderson localization threshold. he solid line denotes the exact one-electron Green functions, the wavy lines are the fluctuation propagators, and the dashed lines de-ote impurity scattering. The shaded three-line vertices denote the sum of diagrams with maximum intersection of impurity lines.

diagrams in Figs. 1a and 1b do not yield contributions divergent at T_c since at least one of the propagators carries a momentum of the order of k_F. It follows that blocks B consisting of three Green functions should be averaged independently with respect to the positions of scatterers. We shall carry out such averaging using the approximation of the self-consistent localization theory. Impurity scattering reduces, within the self-consistent localization theory, to a renormalization of three-line vertices including diagrams with maximum intersection of impurity lines (the diagram shown in Fig. 1c).[14] Using this method, we lose the contribution of the scattering effects represented by the diagram 1d.

We note in the evaluation of the remaining diagram that the block $B(q, \Omega_m, \omega_\nu)$ near T_c can be evaluated for small q and for zero external frequency ω_ν. It is easy to show that this block can be then written as the derivative of the polarization operator obtained earlier with respect to the momentum

$$B(q, \Omega_m, 0) = qC(\Omega_m) = -\frac{\partial}{\partial q}\Pi(q, \Omega_m). \qquad (16)$$

We can neglect the dependence of C on the frequency Ω_m near T_c. It follows that this quantity reduces to the well-known[4] coefficient of the gradient term in the Ginzburg–Landau equation: $C(0) = N(E_F)\xi^2$. The further calculation of the fluctuation conductivity near T_c is standard [8] and we shall omit calculation details. The fluctuation conductivity near T_c in the region of the Anderson transition is then given by the usual expression

$$\sigma_{AL} = \frac{1}{32}\frac{e^2}{\xi}\left(\frac{T_c}{T-T_c}\right)^{1/2}, \qquad (17)$$

but the correlation length is given by

$$\xi = \begin{cases} \dfrac{(\xi_0 l)^{1/2}}{k_F R_l}, & R_l \leqslant \xi, \\ 1.22\,(\xi_0 l^2)^{1/3}, & R_l \geqslant \xi. \end{cases} \qquad (18)$$

We find that the region of strong fluctuation superconductivity widens with decreasing ℓ since ξ decreases and it extends up to $\tau_G \sim 1$ for $\sigma \lesssim \sigma^* \simeq \sigma_c (T_c/E_F)^{1/3}$. We thus find that the resistive superconducting transition remains quite narrow even near the localization threshold but becomes smeared for the conductivities $\sigma \lesssim \sigma^*$. Spatial fluctuations of the superconducting order parameter caused by impurities [7] become also strong in this region ($\sigma \lesssim \sigma^*$). It follows that another method is required to treat fluctuations in this region.

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