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Superconductivity in a Toy Model of the Pseudogap State

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We analyze superconducting pairing (s and d-wave) in a simple exactly solvable model of the pseudogap state induced by fluctuations of short–range order (e.g. antiferromagnetic), based on the model Fermi–surface with "hot"–patches. The average superconducting gap is found to be non zero in the temperature range above the mean-field T_c, where superconductivity persists apparently in separated "drops" due to fluctuations. We also calculate the spectral density and the density of states demonstrating that superconducting anomalies there also show up in the region of $T > T_c$, while at T_c itself there are no special features due to superconducting transition in a sample as a whole.

We consider a greatly simplified model of the pseudogap state, which is based on the idea of well – developed fluctuations of antiferromagnetic (AFM, SDW) short – range order which is qualitatively similar to the "hot spots" model of Ref. [1,2]. We assume that the Fermi surface of two – dimensional system of electrons to have nesting ("hot") patches of finite angular size α in $(0,\pi)$ and symmetric directions in the Brillouin zone, as shown in Fig.1 [3]. Similar Fermi surface was observed in a number of ARPES experiments on cuprate superconductors [4,5]. Fluctuations of short – range order are assumed to be static and Gaussian with the factorized Lorentzian correlator introduced in Ref. [3], with peaks centered at AFM scattering vectors:

$$S(\mathbf{q}) = \frac{1}{\pi^2}\frac{\xi^{-1}}{(q_x-Q_x)^2+\xi^{-2}}\frac{\xi^{-1}}{(q_y-Q_y)^2+\xi^{-2}} \quad (1)$$

where either $Q_x = \pm 2p_F$, $Q_y = 0$ or $Q_y = \pm 2p_F$, $Q_x = 0$ for incommensurate fluctuations, $\mathbf{Q} = (\pi/a, \pi/a)$ for commensurate case. Below we consider only incommensurate case. We shall assume that these fluctuations interact only with electrons from the "hot" (nesting) patches of the Fermi surface. Effective interaction of these electrons with fluctuations we shall model as $(2\pi)^2 W^2 S(\mathbf{q})$, where W is of dimensions of energy and defines the characteristic width of the pseudogap. This scattering is in fact of one – dimensional nature, allowing an exact solution.

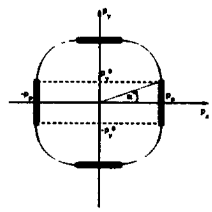

Figure 1. Model Fermi surface of two – dimensional system. "Hot" patches are shown by thick lines of the width of $\sim \xi^{-1}$.

In this work we consider only maximally simplified variant of this model with $\xi \to \infty$, when effective interaction with fluctuations (1) takes the simplest possible form:

$$(2\pi)^2 W^2 \left\{\delta(q_x \pm 2p_F)\delta(q_y) + \delta(q_y \pm 2p_F)\delta(q_x)\right\} \quad (2)$$

In this case we can easily sum all diagrams of the perturbation series for an electron scattered by these fluctuations [6] and obtain one – particle

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Green's function in the following form [3]:

$$G(\epsilon_n, p) = \int_0^\infty dD\mathcal{P}(D)\frac{i\epsilon_n + \xi_p}{(i\epsilon_n)^2 - \xi_p^2 - D(\phi)^2}, \quad (3)$$

where $\xi_p = v_F(|\mathbf{p}| - p_F)$ (v_F - Fermi velocity), $\epsilon_n = (2n+1)\pi T$, and fluctuating dielectric gap $D(\phi)$ which is different from zero only on the "hot" patches

$$D(\phi) = \begin{cases} D & ,0 \le \phi \le \alpha,\ \frac{\pi}{2} - \alpha \le \phi \le \frac{\pi}{2} \\ 0 & ,\alpha \le \phi \le \frac{\pi}{2} - \alpha \end{cases} \quad (4)$$

where ϕ – is polar angle, defining the direction of vector $\mathbf{p}$ in (p_x, p_y) – plane[2]. Distribution function of dielectric gap amplitude is given by [6]:

$$\mathcal{P}(D) = \frac{2D}{W^2}\exp\left(-\frac{D^2}{W^2}\right) \quad (5)$$

where W – is characteristic width of the pseudogap, which is nonzero only on "hot" patches.

Superconducting gap equations in this model [7] are very similar to those obtained in Ref. [8]. For fixed value of the dielectric gap these equations take the following form:

$$1 = \lambda\frac{4}{\pi}\int_0^{\omega_c} d\xi\left\{\int_0^\alpha d\phi e^2(\phi)\frac{th\frac{\sqrt{\xi^2+D^2+\Delta^2(D)e^2(\phi)}}{2T}}{\sqrt{\xi^2+D^2+\Delta^2(D)e^2(\phi)}} + \int_\alpha^{\pi/4} d\phi e^2(\phi)\frac{th\frac{\sqrt{\xi^2+\Delta^2(D)e^2(\phi)}}{2T}}{\sqrt{\xi^2+\Delta^2(D)e^2(\phi)}}\right\} \quad (6)$$

where λ – BCS coupling constant, $e(\phi)$ defines the angular dependence of superconducting gap:

$$e(\phi) = \begin{cases} 1 & (\ s\text{-wave pairing}) \\ \sqrt{2}\cos(2\phi) & (\ d\text{-wave pairing}) \end{cases}. \quad (7)$$

However, due to dielectric gap fluctuations we must perform additional averaging over dielectric gap fluctuations (5). The usual mean – field approach assumes self – averaging nature of the superconducting gap, i.e. its independence on fluctuations of dielectric gap. In this case we obtain

[2] For other values of ϕ the value of $D(\phi)$ is defined similarly to (4) by obvious symmetry considerations.

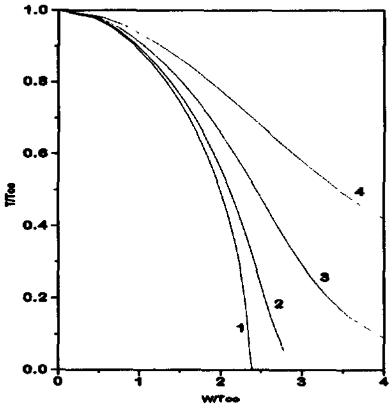

Figure 2. T_c/T_{c0} dependence on the effective width of the pseudogap W/T_{c0} for "hot patches" of different sizes (d-wave pairing). (1)—$\alpha = \pi/4$; (2)—$\alpha = \pi/6$; (3)—$\alpha = \pi/8$; (4)—$\alpha = \pi/12$.

equations for the mean – field Δ_{mf}:

$$1 = \lambda\frac{4}{\pi}\int_0^{\omega_c} d\xi\left\{\int_0^\infty dDDe^{-\frac{D^2}{W^2}}\int_0^\alpha d\phi e^2(\phi)\frac{th\frac{\sqrt{\xi^2+D^2+\Delta_{mf}^2e^2(\phi)}}{2T}}{\sqrt{\xi^2+D^2+\Delta_{mf}^2e^2(\phi)}} + \int_\alpha^{\pi/4} d\phi e^2(\phi)\frac{th\frac{\sqrt{\xi^2+\Delta_{mf}^2e^2(\phi)}}{2T}}{\sqrt{\xi^2+\Delta_{mf}^2e^2(\phi)}}\right\} \quad (8)$$

Equations for mean – field T_c are obtained from (8) by putting $\Delta_{mf} = 0$ and were studied in detail in Ref. [3], where we also derived Ginzburg-Landau expansion for this model. In Fig.2 we show T_c dependence on the effective width of the pseudogap for the case of d-wave pairing.

However, in our model we can directly calculate the average superconducting gap, taking the dependence on fluctuations of D into account explicitly:

$$<\Delta> = \int_0^\infty dD\mathcal{P}(D)\Delta(D) \quad (9)$$

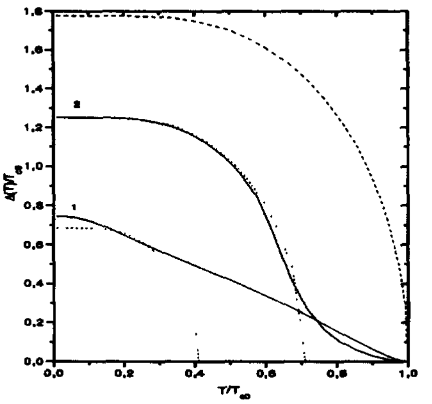

Figure 3. Temperature dependences of superconducting gaps Δ_{mf} (points), $< \Delta >$ (full lines) and Δ_0 (dashed line) in case of s-wave pairing. 1.—$\alpha = \pi/20$, $T_c/T_{c0} = 0.42$. 2.—$\alpha = \pi/6$, $T_c/T_{c0} = 0.71$.

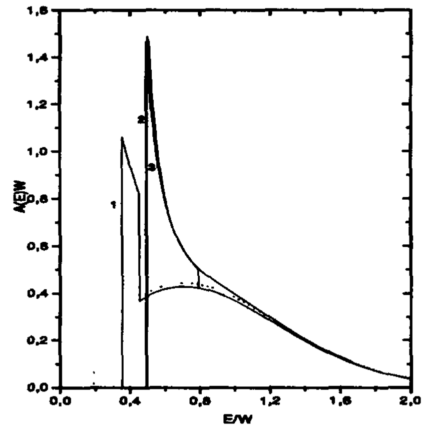

Figure 4. Spectral density on the Fermi surface in case of s-wave pairing for different values of T/T_{c0}: 1.–0.8; 2.–0.4; 3.–0.1. $\alpha = \pi/6$. Points: mean – field approximation for $A_{mf}(E)$ at $T/T_{c0} = 0.1$

with the immediate conclusion that the averaged gap (9) is in fact non zero up to temperature $T = T_{c0}$ – superconducting transition temperature in the absence of pseudogap (W=0). However, the mean – field transition temperature T_c, determined by standard approach, assuming self-averaging superconducting gap, for a superconductor with pseudogap is always lower than T_{c0} [3], as can be seen form Fig.2. Thus, apparently paradoxical, behavior of $< \Delta >$ signifies, probably, the appearance in the system of local regions with $\Delta \neq 0$ (superconducting "drops") induced by fluctuations of D for all temperatures $T_c < T < T_{c0}$, while coherent superconducting state appears in the sample only for $T < T_c$. Temperature dependences of average gap $< \Delta >$ and mean – field gap Δ_{mf}, obtained numerically from equations of our model for the case of s-wave pairing (d-wave curves are similar), are shown in Fig.3. Mean – field gap Δ_{mf} goes to zero at $T = T_c < T_{c0}$, while $< \Delta >$ is non zero up to $T = T_{c0}$, with unusual "tails" in temperature dependences of $< \Delta >$ in the region of

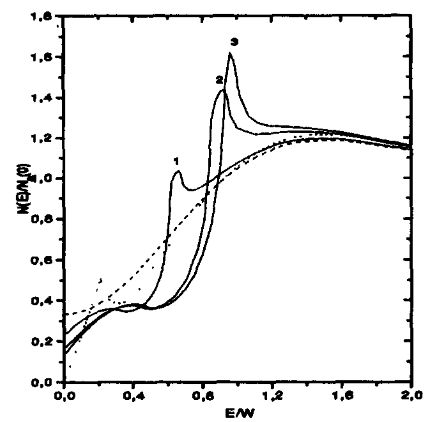

Figure 5. Density of states in case of d-wave pairing ($\alpha = \pi/6$). T/T_{c0} = 1.–0.8; 2.–0.48; 3.–0.1. Points: mean field density of states $N_{mf}(E)$ for $T/T_{c0} = 0.1$. Dashed line: pseudogap in the density of states for $T > T_{c0}$.

$T_c < T < T_{c0}$. Note that temperature dependences of $< \Delta(T) >$ shown in Fig.3 qualitatively resemble those observed in underdoped cuprates in ARPES [9] and specific – heat experiments [10].

We can also calculate one-electron spectral density in superconducting state for both s-wave and d-wave pairing [7]. It is determined by:

$$A(E,\xi_p) = \sum_i \frac{|E| + \xi_p SignE}{W^2} e^{-\frac{D_i^2}{W^2}} \frac{1}{\left|1 + \frac{d\Delta^2(D)}{dD^2}|_{D=D_i} e^2(\phi)\right|} \quad (10)$$

where D_i – are the positive roots of the equation $D^2 + \xi_p^2 + \Delta^2(D)e^2(\phi) - E^2 = 0$. Our results for the case of s-wave pairing are shown in Fig.4. Results for the d-wave case are rather similar [7]. These data demonstrate characteristic peaks and dips, similar to those observed in ARPES – experiments [9]. Of course all discontinuities on these curves will be smeared in case of fluctuations with finite correlation lengths ξ. The account of explicit dependence of superconducting gap on fluctuations of D leads to these anomalies appearing already in the temperature region $T_c < T < T_{c0}$. Nothing special happens with spectral densities at $T = T_c$.

Tunneling density of states in our model is given by:

$$\frac{N(E)}{N_0(0)} = \frac{4}{\pi}\frac{2}{W^2}\int_0^\infty dD D e^{-\frac{D^2}{W^2}} \left\{ \int_0^\alpha d\phi \frac{|E|}{\sqrt{E^2 - D^2 - \Delta^2(D)e^2(\phi)}} \theta(E^2 - \Delta^2(D)e^2(\phi) - D^2) + \int_\alpha^{\pi/4} d\phi \frac{|E|}{\sqrt{E^2 - \Delta^2(D)e^2(\phi)}} \theta(E^2 - \Delta^2(D)e^2(\phi)) \right\} \quad (11)$$

The behavior of the density of states for the case of d-wave pairing is shown in Fig.5. The exact density of states (which takes D fluctuations into account) does not "feel" superconducting transition in a sample as a whole which takes place at $T = T_c$. Characteristic width of the superconducting pseudogap in the density states is determined by Δ_0 (superconducting gap in the absense of AFM pseudogap), not by Δ_{mf}, as in mean – field approximation. Superconducting like features become observable in the density of states already for $T_c < T < T_{c0}$. This can, in principle, explain unusually high values of $2\Delta/T_c$ observed in tunnelling experiments in underdoped cuprates. Superconducting "drops" may also lead to additional diamagnetism above T_c.

The results obtained above show that the pseudogap state induced by AFM short – range order fluctuations (or similar CDW fluctuations) leads (in addition to the anomalies of the normal state [1,2]) also to rather unusual properties of superconducting state, related to partial dielectrization (non Fermi – liquid behavior) of electronic spectrum on the "hot" patches of the Fermi surface. These properties correlate well with a number of anomalies observed in the underdoped state of HTSC – cuprates. It is obvious that more serious comparison with experiments can only be performed in more realistic approach, taking into account, first of all, the effects of the finite correlation lengths ξ, which is relatively small in real systems. At low temperatures it is also important to take into account the dynamic nature of AFM fluctuations.

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