On the theory of superconductors with "odd" pairing

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We consider a model of superconductive pairing with a gap function that is odd in $k-k_F$. In this case superconductivity is possible even in the presence of arbitrarily strong point repulsion between electrons, which is an attractive feature from the viewpoint of the theory of high-T_c metal oxides. We suggest a model of pairing interaction in which the equations of the BCS theory can be solved exactly, which makes it possible to fully analyze the ranges of existence of ordinary ("even") and "odd" pairing in terms of the interaction parameters. We show that normal impurities (disorder) lead to extremely strong suppression of "odd" pairing, even stronger than magnetic impurities do in conventional superconductors.

1. INTRODUCTION

The conventional BCS theory of superconductivity[1] is based on the assumption that near the Fermi surface electrons with oppositely directed momenta and spins are effectively attracted to each other. It is also assumed that this attractive interaction in a certain sense exceeds the Coulomb repulsion of electrons at least in a fraction of the phase space, which is considered the necessary condition for the system to go into the superconducting state at low temperatures. Naturally, from this viewpoint the strong repulsion of electrons in models of the Hubbard type widely used in describing the electronic properties of metal oxides is a factor preventing superconductivity. There is a certain interest, therefore, in studying models in which this repulsion is strongly suppressed or entirely absent. A model of this type was suggested in a recent paper by Mila and Abrahams.[2] Their basic assumption is that the gap function of the BCS theory, $\Delta(\mathbf{k},\omega)$, depends only on $|\mathbf{k}|$, more precisely, on the quasiparticle energy $\xi_k=v_F(|\mathbf{k}|-k_F)$ measured from the Fermi level, and is an odd function of this independent variable. Here, in the BCS approximation, superconductivity proves possible with an arbitrarily strong point repulsion between electrons. From the physical standpoint it is evident that such a state can be realized at fairly strong repulsion, when the ordinary ("even") superconductivity is suppressed. References to earlier papers dealing with "odd" pairing can be found in Ref. 3.

The goal of this paper is to analyze in greater detail than is done in Ref. 2 the problem of "odd" pairing and its relation to the ordinary "even" case. We base our assumptions on a simple weak-binding approximation and employ a model pairing interaction that allows for an exact solution of the equations of the BCS theory. This makes it possible to consider in detail the basic ideas of the model investigated and to find analytical solutions that can easily be compared to the results obtained by Mila and Abrahams[2] by solving numerically the BCS integral equations for a more "realistic" interaction. We also examine the effect of normal impurities on the "odd" pairing of the type considered. The effect proves to be exceptionally strong,[4] with superconductivity suppressed even faster than when magnetic impurities are introduced into ordinary superconductors. The problem is solved both with the model interaction which allows for an exact solution, and numerically in the case of the "realistic" integration suggested in Ref. 2. Despite the obvious attractiveness of this type of pairing in explaining high-T_c superconductivity in metal oxides, the exceptionally high sensitivity to disorder makes it an unlikely mechanism when applied to high-T_c cuprites.

2. EQUATIONS FOR THE GAP AND THE TRANSITION TEMPERATURE

The model now to be considered is based on the fact[2] that the weak-binding equation in the BCS theory,[1]

$$\Delta(\xi)=-N(0)\int_{-\infty}^{\infty}d\xi'\;V(\xi,\xi')\frac{\Delta(\xi')}{2\sqrt{\xi'^2+\Delta^2(\xi')}}\times\tanh\frac{\sqrt{\xi'^2+\Delta^2(\xi')}}{2T},\qquad(1)$$

can have a nontrivial solution $\Delta(\xi)=-\Delta(-\xi)$ [i.e., odd in $k-k_F$, with $\xi=v_F(k-k_F)$], provided that $V(\xi,\xi')$ has an attractive term even in the presence of an (infinitely) strong point repulsion. Clearly,[2] for an odd $\Delta(\xi)$ the repulsive part of the interaction in (1) simply vanishes and the attractive term $V_2(\xi,\xi')$ can ensure pairing with nontrivial properties: the gap function $\Delta(\xi)$ vanishes at the Fermi surface, which leads to gapless superconductivity. It is worth noting that we are speaking of an isotropic model in which the gap vanishes everywhere at the Fermi surface, which distinguishes this model from anisotropic pairing, say, of the d-type.[3]

Thus, in what follows we assume that the interaction in Eq. (1) consists of two terms, $V(\xi,\xi')=V_1(\xi,\xi')+V_2(\xi,\xi')$, where

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$$V_1(\xi,\xi')=\begin{cases} U>0 & \text{if } |\xi|,|\xi'|<E_F, \\ 0 & \text{if } |\xi|,|\xi'|>E_F \end{cases} \qquad (2)$$

is the point repulsion of electrons, and $V_2(\xi,\xi')$ the effective pairing interaction (attraction), which is finite for $|\xi|,|\xi'|<\omega_c$ and $|\xi-\xi'|<\omega_c$ (the latter restriction is very important), with $\omega_c \ll E_F$ acting as a characteristic frequency of bosons whose exchange gives rise to pairing. The pairing "potential" $V_2(\xi,\xi')$ can be represented by different model functions e.g., say, a step function.[2] The following interaction was especially prominent in Ref. 2:

$$V_2(\xi,\xi') = \begin{cases} -V(|\xi-\xi'|/\omega_c)^{-2/3} \text{ if } |\xi|,|\xi'|, \ |\xi-\xi'|<\omega_c, \\ 0 \text{ if } |\xi|,|\xi'| \text{ or } |\xi-\xi'|>\omega_c, \end{cases} \qquad (3)$$

which was chosen exclusively in order to obtain a tunneling density of states in the superconductor $N_T(E)$ that behave like $|E|^2$ as $E\to 0$.

The BCS integral equation (1) with such a potential $V_2(\xi,\xi')$ was solved in Ref. 2 numerically, and some results obtained there qualitatively agree with the properties of high-T_c compounds.

In this paper we consider mainly model interaction of the form[4]

$$V_2(\xi,\xi')=\begin{cases} -V\cos\frac{\pi}{2}\frac{(\xi-\xi')}{\omega_c} \text{ if } |\xi|,|\xi'|,|\xi-\xi'|<\omega_c, \\ 0 \text{ if } |\xi|,|\xi'|, \text{ or } |\xi-\xi'|>\omega_c. \end{cases} \qquad (4)$$

The main advantage of this choice is that in the case at hand the integral equation for the gap is reduced to a transcendental equation and can easily be solved. In this respect Eq. (4) is not the only choice: several other "potentials" can be suggested that have the same properties. For instance, we could use the interaction potential $V_2(\xi,\xi')$ proportional to $\cosh(\xi-\xi')$ or $(\xi-\xi')^2$. However, the results obtained with (4) are in a certain sense the closest to those obtained via the "realistic" interaction (3) (say, for the density of states). But most qualitative conclusions are independent of the choice of "model" potential. The importance of the model potential (4) is, obviously, related also to the fact that practically any interaction that is an even function of $\xi-\xi'$ in the interval from $-\omega_c$ to ω_c can be represented by a Fourier cosine series. In this sense our discussion lays the foundation for analyzing the most general case.

It is worth noting that the realistic choice of $V(\xi,\xi')$ should actually have been carried out with the dielectric formalism of superconductivity theory,[5,6] where it is possible to arrive at extremely general expressions for the integral kernel in the BCS theory. But it is not clear how to derive the nontrivial dependence of the kernel on the independent variable $|\xi-\xi'|$, since a typical feature of the dielectric formalism is that $V(\xi,\xi')$ depends separately on ξ and ξ' (see Refs. 5 and 6).

The transition temperature of the superconductor is determined by the linearized equation

$$\Delta(\xi)=-N(0)\int_{-\infty}^{\infty} d\xi'\, V(\xi,\xi')\frac{\Delta(\xi')}{2\xi'}\tanh\left(\frac{\xi'}{2T_c}\right). \qquad (5)$$

Combining this with (2) and (4), we get

$$\Delta(\xi)=g\int_{-\omega_c}^{\omega_c} d\xi' \cos\left(\frac{\pi}{2}\frac{(\xi-\xi')}{\omega_c}\right)\frac{\Delta(\xi')}{2\xi'}\tanh\left(\frac{\xi'}{2T_c}\right) - \mu\int_{-E_F}^{E_F} d\xi'\frac{\Delta(\xi')}{2\xi'}\tanh\left(\frac{\xi'}{2T_c}\right) \qquad (6)$$

for $|\xi|<\omega_c$, and

$$\Delta(\xi)=-\mu\int_{-E_F}^{E_F} d\xi'\frac{\Delta(\xi')}{2\xi'}\tanh\frac{\xi'}{2T_c} \qquad (7)$$

for $\omega_c<|\xi|<E_F$, where we have introduced, as is common practice, dimensionless constants of pairing and repulsive coupling: $g=N(0)V$ and $\mu=N(0)U$.

The general solution to Eqs. (6) and (7) has the form

$$\Delta(\xi)=\begin{cases} \Delta_c\cos\frac{\pi}{2}\frac{\xi}{\omega_c}+\Delta_s\sin\frac{\pi}{2}\frac{\xi}{\omega_c}+\Delta \quad \text{if } |\xi|<\omega_c, \\ \Delta \quad \text{if } \omega_c<|\xi|<E_F, \end{cases} \qquad (8)$$

where Δ_c, Δ_s, and Δ are determined by solving the following system of algebraic equations:

$$\begin{cases} \Delta_c=gF_c\Delta_c+gF\Delta, \\ \Delta=-\mu F\Delta_c-\mu W'\Delta, \end{cases} \qquad (9)$$

$$\Delta_s=gF_s\Delta_s. \qquad (10)$$

with

$$F_c=\int_0^{\omega_c} d\xi\cos^2\left(\frac{\pi}{2}\frac{\xi}{\omega_c}\right)\frac{1}{\xi}\tanh\frac{\xi}{2T_c},$$

$$F_s=\int_0^{\omega_c} d\xi\sin^2\left(\frac{\pi}{2}\frac{\xi}{\omega_c}\right)\frac{1}{\xi}\tanh\frac{\xi}{2T_c},$$

$$F=\int_0^{\omega_c} d\xi\cos\left(\frac{\pi}{2}\frac{\xi}{\omega_c}\right)\frac{1}{\xi}\tanh\frac{\xi}{2T_c}, \qquad (11)$$

$$W=\int_0^{\omega_c} d\xi\frac{1}{\xi}\tanh\frac{\xi}{2T_c},$$

$$W'=\int_0^{E_F} d\xi\frac{1}{\xi}\tanh\frac{\xi}{2T_c}.$$

We see that Eq. (10), which determines T_c for "odd" pairing, is independent of the system of equations (9), which determines T_c for the "even" case. The repulsive interaction affects only "even" pairing, and from (9) we arrive at the following transcendental equation for T_c:

$$1 = gF_c - g\mu \frac{F^2}{1+\mu W'}, \qquad (12)$$

which can be written as

$$1 = gF_c - \mu^* W + \mu^* g(F_c W - F^2), \qquad (13)$$

where we have introduced the Coulomb pseudopotential

$$\mu^* = \frac{\mu}{1+\mu(W'-W)},$$

in which in the weak-binding range, when $T_c \ll \omega_c$, the difference $W'-W$ takes on the ordinary value of $\ln(E_F/\omega_c)$.

The temperature of transition to the "odd" state is determined from the equation

$$1 = gF_s = g\int_0^{\omega_c} d\xi \sin^2\left(\frac{\pi}{2}\frac{\xi}{\omega_c}\right)\frac{1}{\xi}\tanh\frac{\xi}{2T_c}. \qquad (14)$$

In the Appendix we derive Eqs. (12) and (14) starting from the problem of Cooper instability of the normal state.

Figure 1 depicts the results of solving Eqs. (12) and (14) numerically for different values of the coupling constants g and μ. We see that "even" pairing is predominant in the presence of a weak repulsive interaction: the temperature of the respective transition is higher than the temperature of the transition to the odd-gap state. As the repulsion grows, the situation changes, and at large values of g "odd" pairing becomes preferable. Note that in model (4) the pairing coupling constant has a critical value: "odd" pairing appears only for $g > g_c \approx 1.213$. Thus, in considering "odd" pairing formally we step outside the scope of the weak-binding approximation, for which the BCS equations were initially obtained.

In this sense the results represented in Fig. 1 in the region of large coupling constants are to a great extent nominal. For one thing, the practically linear increase of T_c with g, which Fig. 1 exhibits, has hardly any range of applicability and is due to the unreasonable extrapolation of the BCS equations obtained in the weak-binding approximation to the range of large g. Actually we should have proceeded more carefully in the spirit of Nozieres and Schmitt-Rink,[7] who consistently studied the transition from "loose" Cooper pairs in the weak-binding approximation to the compact bosons that emerge in the limit of an extremely strong pairing interaction. As is well known, with increasing g the transition is accompanied by saturation of T_c, whose value is determined (at large g values) by the well-known formula for the temperature of Bose condensation in a boson gas, in which there is practically no dependence on g.

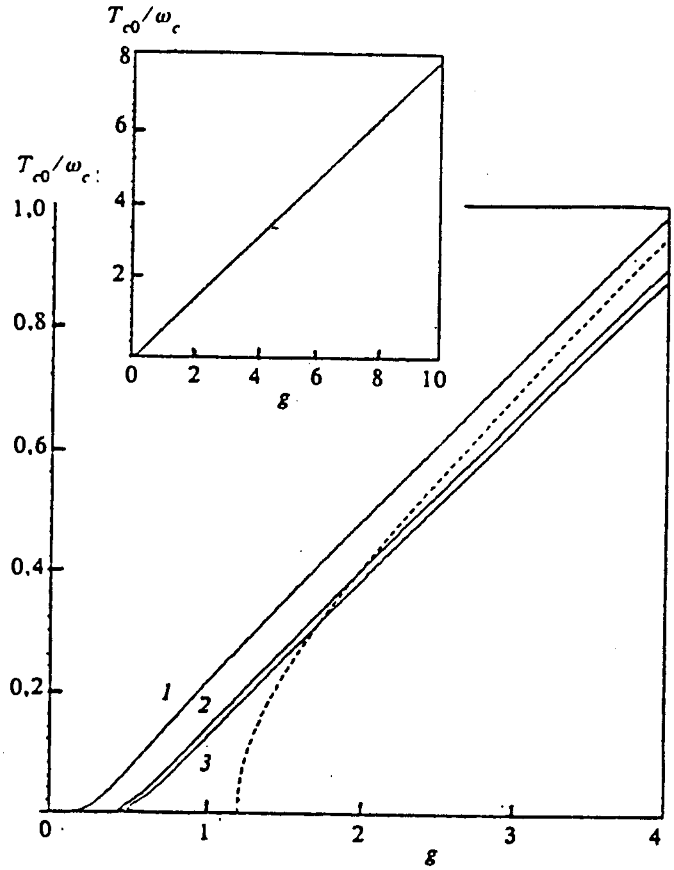

FIG. 1. T_{c0} vs the pairing coupling constant $g = N(0)V$ for "even" pairing (solid curves) and "odd" pairing (dashed curve). Curve *1* corresponds to $\mu = 0$, curve *2* to 1, and curve *3* to 10. It was assumed in calculations that $E_F/\omega_c = 50$. The inset presents a similar dependence for the "realistic" interaction (3).

Formally the critical value g_c of the coupling constant is absent from the "odd" pairing problem if the pairing interaction (3) is employed, which is obviously a result of this formula being divergent for $|\xi - \xi'| \to 0$. The respective dependence of T_c on $g = N(0)V$ for the problem of odd pairing, obtained by numerically solving Eq. (5) with potential (3), is depicted in the inset in Fig. 1. At the same time it is clear that in this case, too, "odd" pairing begins to dominate over "even" pairing only when there is fairly strong repulsion. Note that considering the limit of strong repulsion within the framework of the BCS theory is hardly justified since it allows only for the simplest Fock correction in the electron–electron interaction. Clearly, proceeding in this manner does not allow us to meaningfully consider the limit of $\mu \to \infty$. The above formal solution of the BCS equations, however, appears to correctly reflect the qualitative pattern of the transition from the common "even" pairing to "odd" pairing.

Let us now examine the temperature behavior of the gap function in the case of "odd" pairing in model (4). In accordance with Eq. (8), for the "odd" case we have

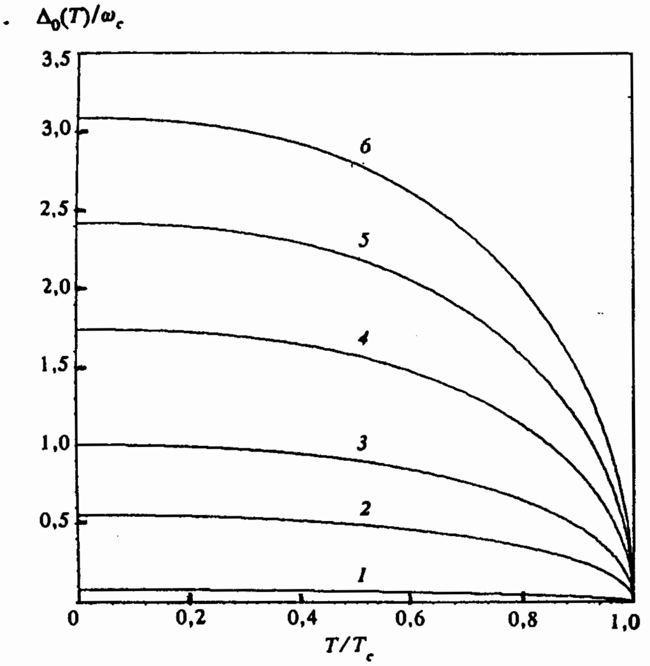

FIG. 2. Temperature dependence of $\Delta_0(T)$ in the case of "odd" pairing for several values of the pairing coupling constant. Curve *1* corresponds to $g=1.22$, curve *2* to 1.5, curve *3* to 2.0, curve *4* to 3.0, curve *5* to 4.0, and curve *6* to 5.0.

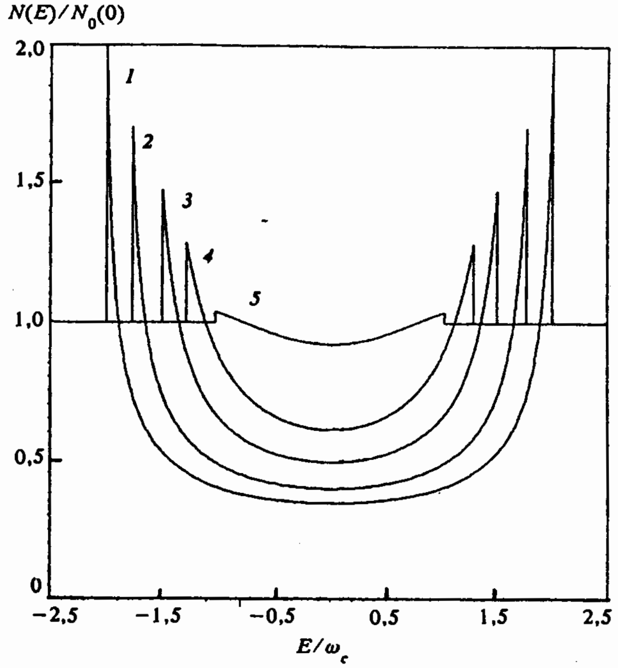

FIG. 3. Density of states in the "odd" pairing model for several characteristic temperature values. Curve *1* corresponds to $T/T_c=0$, curve *2* to 0.6, curve *3* to 0.8, curve *4* to 0.9, and curve 5 to 0.99. The value of the pairing constant g was set at three.

$$\Delta(\xi)=\begin{cases}\Delta_0(T)\sin\dfrac{\pi}{2}\dfrac{\xi}{\omega_c} & \text{if } |\xi|<\omega_c,\\ 0 \quad \text{if } |\xi|>\omega_c,\end{cases} \tag{15}$$

and the temperature dependence of Δ_0 is determined by the following equation, which follows from Eq. (1):

$$1=g\int_0^{\omega_c} d\xi' \sin^2\left(\frac{\pi}{2}\frac{\xi'}{\omega_c}\right) \times\frac{\tanh\left[\left(\dfrac{1}{2T}\right)\sqrt{\xi'^2+\Delta_0^2(T)\sin^2\left(\dfrac{\pi}{2}\dfrac{\xi}{\omega_c}\right)}\,\right]}{\sqrt{\xi'^2+\Delta_0^2(T)\sin^2\left(\dfrac{\pi}{2}\dfrac{\xi}{\omega_c}\right)}}, \tag{16}$$

whose solutions for several values of the pairing constant g are depicted in Fig. 2. The temperature dependence of $\Delta_0(T)$ resembles the one in the BCS theory but the two do not coincide. For one thing, at large pairing coupling constants, $g \gg g_c$, the value of $2\Delta_0(T=0)/T_c$ is approximately five, with a tendency toward decreasing with g.

The tunneling density of states can easily be calculated by the standard method.[2] Using (15), we get

$$\frac{N(E)}{N_0(0)}=\begin{cases}E\left[\varepsilon+\dfrac{\pi}{4}\dfrac{\Delta_0^2(T)}{\omega_c}\sin\left(\pi\dfrac{\varepsilon}{\omega_c}\right)\right]^{-1} & \text{if } |\varepsilon|<\omega_c,\\ 1 \quad \text{if} \quad \omega_c<|\varepsilon|,\end{cases} \tag{17}$$

where ε is determined from the equation

$$\varepsilon^2+\Delta_0^2(T)\sin^2\left(\frac{\pi}{2}\frac{\varepsilon}{\omega_c}\right)=E^2.$$

The respective curves for different temperatures are depicted in Fig. 3. The density of states is always gapless, and the "pseudogap" gets fuzzier as the temperature grows, with the positions of the peaks in the density of states depending rather weakly on temperature. Qualitatively these results are close to those obtained in Ref. 2 for the case of interaction (3) and can compare with the known features of the gap in high-T_c superconductors. If the ratio $2\Delta/T_c$ is determined from the positions of the peaks in the tunneling density of states, we get $2\Delta/T_c\approx 6$.

III. EFFECT OF NORMAL IMPURITIES

An interesting question is how normal (nonmagnetic) impurities (disorder) act on "odd" pairing. It is well known[1,8] that such disorder has practically no effect on the common "even" pairing. In the case at hand the equations for the normal and anomalous Green's functions have the standard form[8] valid in the weak-scattering limit:

$$G(\omega\xi)=-\frac{i\tilde{\omega}+\xi}{\tilde{\omega}^2+\xi^2+|\Delta(\xi)|^2},$$
$$F(\omega\xi)=\frac{\tilde{\Delta}^*(\xi)}{\tilde{\omega}^2+\xi^2+|\Delta(\xi)|^2}, \tag{18}$$

where $\omega=(2n+1)\pi T$,

$$\tilde{\omega}=\omega-\frac{\gamma}{\pi}\int_{-\infty}^{\infty}d\xi\,\frac{\tilde{\omega}}{\tilde{\omega}^2+\xi^2+|\Delta(\xi)|^2},$$

$$\tilde{\Delta}(\xi)=\Delta(\xi)+\frac{\gamma}{\pi}\int_{-\infty}^{\infty}d\xi\,\frac{\tilde{\Delta}^*(\xi)}{\tilde{\omega}^2+\xi^2+|\Delta(\xi)|^2}=\Delta(\xi). \tag{19}$$

Here $\gamma=\pi c V_0^2 N(0)$ is the rate of electron scattering by point impurities with potential V_0 randomly distributed with concentration c. The integral in the second equation in (19) vanishes because $\Delta(\xi)$ is an odd function and renormalization of the gap function owing to scattering on impurities is absent. It is because of this feature that impurities have such a strong effect on "odd" pairing. Note that the same situation occurs for anisotropic pairing, say of the d—type.[9,10]

The equation for the gap now has the form

$$\Delta(\xi)=-N(0)T\sum_{\omega_n}\int_{-\infty}^{\infty}d\xi' \times V_2(\xi,\xi')\frac{\Delta^*(\xi')}{\tilde{\omega}^2+\xi'^2+|\Delta^2(\xi')|^2}. \tag{20}$$

Near T_c this equation can be linearized, and we get

$$\Delta(\xi)=-N(0)T\sum_{\omega_n}\int_{-\infty}^{\infty}d\xi'\,V_2(\xi,\xi')\frac{\Delta(\xi')}{\tilde{\omega}^2+\xi'^2}, \tag{21}$$

where $\tilde{\omega}=\omega_n+\gamma\,\mathrm{sgn}\,\omega_n$.

The sum over Matsubara frequencies in (21) can be calculated in the ordinary way by going over to integration in the complex frequency plane. As a result the linearized equation for the gap can be written in several equivalent ways. One is

$$\Delta(\xi)=-N(0)\int_{-\infty}^{\infty}d\xi'\,V_2(\xi,\xi')\int_{-\infty}^{\infty}\frac{d\omega}{\pi}\tanh\left(\frac{\omega}{2T}\right) \times \mathrm{Re}\,G^R(-\omega\xi')\,\mathrm{Im}\,G^R(\omega\xi')\,\Delta(\xi'), \tag{22}$$

where $G^R(\omega\xi)=(\omega-\xi+i\gamma)^{-1}$is the retarded Green function of a normal metal with impurities. In another we have

$$\Delta(\xi)=-N(0)\int_{-\infty}^{\infty}d\xi'\,V_2(\xi,\xi')\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{1}{\xi'} \times\tanh\left(\frac{\omega+\xi'}{2T}\right)\frac{\gamma}{\omega^2+\gamma^2}\Delta(\xi'). \tag{23}$$

An equation of the form (22) was obtained for a superconductor with anisotropic d-coupling by Monthoux, Balatsky, and Pines;[10] in what follows we use Eq. (23).

For the model interaction (4) the gap again has the form of (15), and the equation for T_c follows directly from (23):

$$1=g\int_0^{\omega_c}\frac{d\xi'}{\xi'}\sin^2\left(\frac{\pi}{2}\frac{\xi'}{\omega_c}\right)\int_{-\infty}^{\infty}\frac{d\omega}{\pi}\tanh\left(\frac{\omega+\xi'}{2T_c}\right)\frac{\gamma}{\omega^2+\gamma^2}. \tag{24}$$

Figure 4 depicts T_c vs γ for several typical values of the

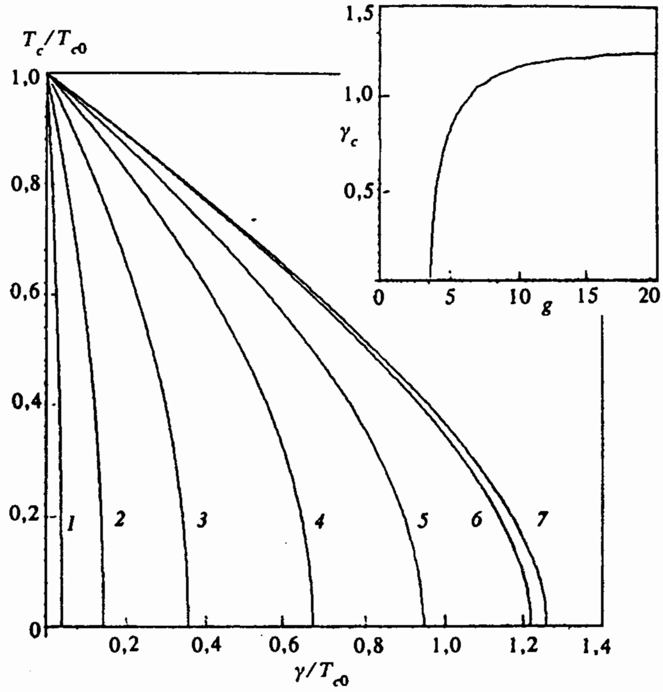

FIG. 4. T_c of "odd" pairing vs the scattering rate γ for different values of the pairing constant g. Curve *1* corresponds to $g=1.22$, curve *2* to 1.24, curve *3* to 1.30, curve *4* to 1.50, curve *5* to 2.0, curve *6* to 5.0, and curve *7* to 10.0. The inset presents the dependence of the critical scattering rate on the pairing coupling constant.

pairing constant g obtained by solving Eq. (24). We see that scattering on normal impurities strongly suppresses "odd" pairing. Superconductivity disappears at $\gamma\sim T_{c0}$, where T_{c0} is the transition temperature in the absence of scattering ($\gamma\rightarrow 0$) and is determined from Eq. (14). Destruction of superconductivity in this case proceeds even faster than if we were to introduce magnetic impurities into a common superconductor.[11] This manifests itself, for one thing, in the rapid decrease in the volume in which superconductivity exists on the "phase diagram" (Fig. 4) as $g\rightarrow g_c$ and in the absence of the universal T_c vs γ dependence characteristic of the case of magnetic impurities.

If the model interaction (3) is employed, T_c as a function of γ can be found by direct numerical solution of the linear integral equation (23). To calculate the minimum eigennumber determining the coupling constant g for a given temperature, we use the trace method and the Kellogg method.[12] In evaluating integrals of functions proportional to $|\xi-\xi'|^{-2/3}$ we use the methods for evaluating singular integrals on a segment[13] which make it possible to evaluate such integrals with an accuracy of the order of that of Gauss quadrature formulas. The procedure of calculating the minimum eigennumbers proves extremely sensitive to the accuracy with which symmetrized kernels are calculated. Satisfactory results are obtained by representing these kernels in terms of hypergeometric functions, which are calculated via summation of appropriate generating series to a given accuracy. While the Kellogg

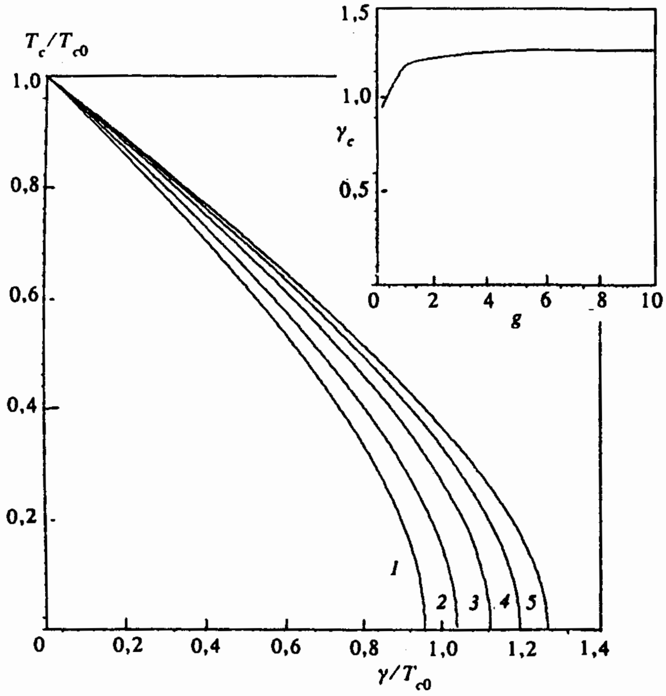

FIG. 5. T_c of "odd" pairing vs the scattering rate γ for different values of the pairing constant g in the model with the "realistic" interaction (3). Curve *1* corresponds to $g=0.17$, curve *2* to 0.25, curve *3* to 0.72, curve *4* to 1.15, and curve *5* to 6.41. The inset presents the dependence of the critical scattering rate on the pairing coupling constant.

method has a high rate of convergence in comparison to the trace method, at small coupling constants it has a tendency toward instability. The respective curves representing the T_c vs γ dependence are depicted in Fig. 5. We see that the qualitative picture obtained in the simpler model is retained completely. One can easily verify that the critical scattering rate γ_c corresponding to destruction of superconductivity ($T_c(\gamma \to \gamma_c) \to 0$) is determined from the following integral equation:

$$\Delta(\xi) = -N(0) \int_{-\infty}^{\infty} d\xi' \, V_2(\xi,\xi') \frac{1}{\pi\xi'} \operatorname{arctg}\left(\frac{\xi'}{\gamma_c}\right) \Delta(\xi'), \tag{25}$$

which for the interaction (4) is reduced to

$$1 = \frac{2}{\pi} g \int_0^{\omega_c} \frac{d\xi'}{\xi'} \sin^2\left(\frac{\pi}{2} \frac{\xi'}{\omega_c} \operatorname{arctg}\left(\frac{\xi'}{\gamma_c}\right)\right). \tag{26}$$

For $g \approx g_c$ this implies the dependence $\gamma_c \propto (g-g_c) \to 0$, which reflects the narrowing of the superconductivity region in Fig. 4. For $g \gg g_c$ (the tight-binding approximation) we have the universal result $\gamma_c/T_{c0} = 4/\pi \approx 1.273$. Actually, this result and T_c vs γ for $g \gg g_c$ are independent of the choice of model potential $V_2(\xi,\xi')$. For one thing, the universality of γ_c/T_{c0} follows from the fact that Eq. (14) for T_{c0} and Eq. (25) for γ_c assume the same form for $T_{c0} \gg \omega_c$ and $\gamma_c \gg \omega_c$ (i.e., $g \gg g_c$):

$$\Delta(\xi) = -\frac{N(0)}{4T_{c0}} \int_{-\infty}^{\infty} d\xi' \, V_2(\xi,\xi') \Delta(\xi'), \tag{27}$$

$$\Delta(\xi) = -\frac{N(0)}{\pi\gamma_c} \int_{-\infty}^{\infty} d\xi' \, V_2(\xi,\xi') \Delta(\xi'). \tag{28}$$

The equivalence of these equations makes it possible to take the constant factors equal, which leads to the above result for γ_c/T_{c0}. Similarly, it can easily be shown that for $T_c(\gamma) \gg \omega_c$ Eq. (23) is reduced to

$$\Delta(\xi) = -\frac{N(0)}{4T_c(\gamma)} f\left(\frac{\gamma}{T_c(\gamma)}\right) \int_{-\infty}^{\infty} d\xi' \, V_2(\xi,\xi') \Delta(\xi'), \tag{29}$$

where

$$f(z) = \int_{-\infty}^{\infty} \frac{d\omega}{\pi} \frac{1}{\cosh^2(\omega/2)} \frac{x}{x^2+\omega^2}.$$

Correspondingly, comparing (29) and (27), we see that in the tight-binding limit the dependence of T_c on γ is determined by the following universal equation:

$$\frac{T_c(\gamma)}{T_{c0} f[\gamma/T_c(\gamma)]} = 1. \tag{30}$$

It is worth noting, however, that the results for the tight-binding limit are fairly conditional and, as already noted, must be modified in the spirit of Ref. 7.

The results for γ_c in the case of the model interaction (4) and those obtained by numerically solving Eq. (25) with the model interaction (3) are depicted in the insets in Figs. 4 and 5, respectively.

IV. CONCLUSION

Here are the main results of our work. We have suggested a simple model of pairing interaction that makes it possible to obtain and fully investigate the exact solutions of the integral equation for the gap within the framework of the BCS theory both for the more or less common case of "even" (in $k-k_F$) pairing and for the exotic "odd" pairing. We show that "odd" pairing becomes preferable when there is a fairly strong repulsive force between the electrons and, in general, when the pairing interaction is fairly strong. This last feature (the tight binding) merits further, more rigorous, study of the transition from Cooper pairs to compact bosons. "Odd" pairing leads to a gapless pattern in superconductivity and to other divergences from the common BCS theory, such as the unusual evolution of the pseudogap in the density of states, the large value of $2\Delta_0/T_c$, etc., which are attractive from the standpoint of high-T_c superconductor theory.

At the same time normal impurities (disorder) strongly suppress "odd" pairing. Suppression is even stronger than in the case involving magnetic impurities in ordinary superconductors. Even in the tight-binding limit, superconductivity is destroyed as $\gamma \sim T_{c0}$, and as the pairing coupling constant decreases there is a sharp drop in the size of the region on the phase diagram where superconductivity exists.

High-T_c compounds are known to be fairly unstable under introduction of normal disorder.[14] But if we exclude the special cases, say, the introduction of Zn impu-

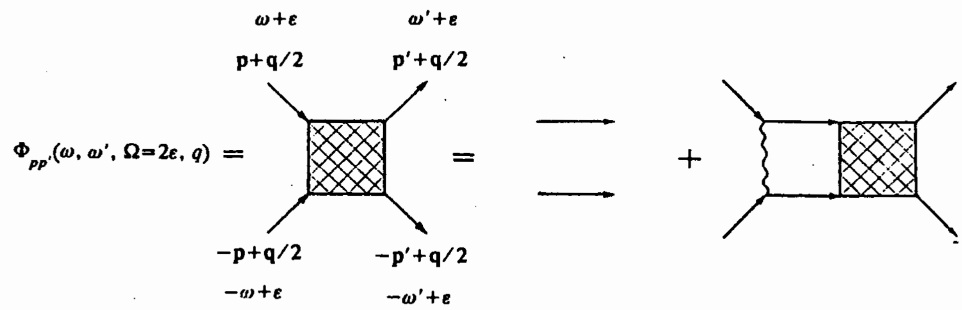

FIG. 6. Two-particle Green function in the Cooper channel.

rities, suppression of superconductivity in such compounds occurs fairly close to the disorder-induced metal–insulator transition, which is most likely related to Anderson localization of charge carriers.[14] This transition takes place at $\gamma\sim E_F \gg T_{c0}$, so that by this time "odd" superconductivity has been completely destroyed. This fact appears to make "odd" pairing an improbable mechanism for explaining high-T_c superconductivity in metal oxides. At the same time the possibility cannot be excluded that a number of effects in these oxides can be explained by the rapid suppression of the "odd" component of the superconducting order parameter under the process of disorder, while the "even" component is preserved (this component retains its stability under disorder). This aspect deserves further investigation.

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APPENDIX

Methodologically it is advisable to obtain the equations that determine T_c for "even" and "odd" pairing beginning with the normal phase, that is, as equations that determine the points of respective Cooper instabilities. Let us consider the two-particle Greens function in a Cooper channel, as shown in Fig. 6. It is convenient to link the instability of the normal state with the divergence of this function partially summed over the Matsubara frequencies,

$$\Phi_{pp'}(\Omega q)=-T\sum_{\omega\omega'}\Phi_{pp'}(\omega,\omega',\Omega,q), \tag{31}$$

at $q=\Omega=0$. We again consider the electron–electron interaction $V(\xi,\xi')$ as consisting of two parts defined in (2) and (4). In view of the isotropy of the system, $\Phi_{pp'}(0,0)$ can be represented as the function $\Phi(\xi,\xi')$, which is determined by the equation

$$\Phi(\xi,\xi')=Z(\xi)\delta_{\xi\xi'}+Z(\xi)N(0)\int_{-\infty}^{\infty}d\xi' \times V(\xi-\xi')\Phi(\xi,\xi'), \tag{32}$$

where

$$Z(\xi)=-T\sum_{\omega}G(\omega\xi)G(-\omega\xi) =-\frac{1}{2\xi}\tanh\left(\frac{\xi}{2T}\right). \tag{33}$$

Combining this with Eqs. (2) and (4) yields

$$\Phi(\xi,\xi')=Z(\xi)\delta_{\xi\xi'}+Z(\xi)\left\{\mu\int_{-E_F}^{E_F}d\xi'\,\Phi(\xi,\xi') -g\cos\left(\frac{\pi}{2}\frac{\xi}{\omega_c}\right)\int_{-\omega_c}^{\omega_c}d\xi'\cos\left(\frac{\pi}{2}\frac{\xi'}{\omega_c}\right)\Phi(\xi,\xi') -g\sin\left(\frac{\pi}{2}\frac{\xi}{\omega_c}\right)\int_{-\omega_c}^{\omega_c}d\xi'\sin\left(\frac{\pi}{2}\frac{\xi'}{\omega_c}\right)\Phi(\xi,\xi')\right\} \tag{34}$$

for $|\xi|,|\xi'|<\omega_c$, and, respectively,

$$\Phi(\xi,\xi')=Z(\xi)\delta_{\xi\xi'}+Z(\xi)\mu\int_{-E_F}^{E_F}d\xi'\,\Phi(\xi,\xi') \tag{35}$$

for $|\xi|$ or $|\xi'|>\omega_c$ and $|\xi|,|\xi'|<E_F$.

Let us introduce the following functions:

$$f_c(\xi)=\int_{-\omega_c}^{\omega_c}d\xi'\cos\left(\frac{\pi}{2}\frac{\xi'}{\omega_c}\right)\Phi(\xi,\ \xi'),$$
$$f_s(\xi)=\int_{-\omega_c}^{\omega_c}d\xi'\sin\left(\frac{\pi}{2}\frac{\xi'}{\omega_c}\right)\Phi(\xi,\ \xi'), \tag{36}$$
$$f(\xi)=\int_{-E_F}^{E_F}d\xi'\,\Phi(\xi,\xi'),$$

for which, as Eqs. (34) and (35) clearly show, the following system of equations:

$$f(\xi)=Z(\xi)-\mu W'f(\xi)+gFf_c(\xi),$$
$$f_c(\xi)=Z(\xi)\cos\left(\frac{\pi}{2}\frac{\xi}{\omega_c}\right)-\mu Ff(\xi)+gF_cf_c(\xi), \tag{37}$$
$$f_c(\xi)=Z(\xi)\sin\left(\frac{\pi}{2}\frac{\xi}{\omega_c}\right)+gF_sf_s(\xi),$$

where we have used the notation introduced in (11).

We see that the "even" and "odd" equations have separated. "Odd" pairing is due to the divergence of the function $f_s(\xi)$, and the respective instability condition has the form $1 = gF_s$, which coincides with (14). The first two equations in (37) determine the instability under "even" pairing. Clearly,

$$f_c(\xi) = Z(\xi)\left[\cos\left(\frac{\pi}{2}\frac{\xi}{\omega_c}\right) - \frac{\mu F}{1+\mu W'}\right]$$

$$\left[1 - gF_c + \frac{g\mu F^2}{1+\mu W'}\right]^{-1},$$

and the instability condition is

$$1 = gF_c - g\mu\frac{F^2}{1+\mu W'}, \quad (38)$$

which coincides with (12).

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