Normal impurities in superconductors with an "odd" pairing

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(Submitted 23 March 1993)
Pis'ma Zh. Eksp. Teor. Fiz. **57**, No. 8, 494–497 (25 April 1993)

Effect of normal impurities in superconductors whose gap function is odd in $k-k_F$ are analyzed. A superconductivity is possible in this case even in the presence of an arbitrarily strong point repulsion between electrons. This circumstance is attractive from the standpoint of a theory for the high-T_c superconductivity of metal oxides. Impurities lead to an extremely strong suppression of this pairing—stronger than in the case of magnetic impurities in conventional superconductors.

Mila and Abrahams[1] have recently proposed an interesting model which allows the existence of a superconducting pairing no matter how strong the point repulsion of the electrons.[1] A model of this sort would naturally be of great interest for explaining the high-T_c superconductivity of metal oxides. This model is based on a demonstration of the fact that the equation for the gap in BCS theory,

$$\Delta(\xi)=-N(0)\int_{-\omega_c}^{\omega_c} d\xi' V(\xi,\xi')\frac{\Delta(\xi')}{2[\xi'^2+\Delta^2(\xi')]^{1/2}}\tanh\frac{[\xi'^2+\Delta^2(\xi')]^{1/2}}{2T}, \quad (1)$$

can have a nontrivial solution $\Delta(\xi)=-\Delta(-\xi)$[i.e., one which is odd in $k-k_F$; here $\xi=v_F(k-k_F)$] if $V(\xi,\xi')$ contains an attractive interaction $-V_2(\xi,\xi')<0$ (which is nonzero for $|\xi|, |\xi'|<\omega_c$, where $|\xi-\xi'|<\omega_c$), even in the presence of a strong (even infinitely strong) point repulsion $V_1(\xi,\xi')=U>0$ for $|\xi|, |\xi'|<E_F$. In the case of an odd $\Delta(\xi)$, the repulsive part of the interaction drops out of (1), while the attraction $V_2(\xi,\xi')$ can support a pairing with nontrivial properties (the gap function vanishes at the Fermi surface, leading to a gapless superconductivity).

In a case in which there are normal (i.e., nonmagnetic) impurities, the equations for the normal and anomalous Green's functions take the standard form[2] which is valid in the weak-scattering limit:

$$G(\omega\xi)=-\frac{i\tilde{\omega}+\xi}{\tilde{\omega}^2+\xi^2+|\Delta(\xi)|^2}, \quad F(\omega\xi)=\frac{\tilde{\Delta}^*(\xi)}{\tilde{\omega}^2+\xi^2+|\Delta(\xi)|^2}, \quad (2)$$

where $\omega=(2n+1)\pi T$,

$$\tilde{\omega}=\omega-\frac{\gamma}{\pi}\int_{-\infty}^{\infty} d\xi\frac{\tilde{\omega}}{\tilde{\omega}^2+\xi^2+|\Delta(\xi)|^2},$$

$$\widetilde{\Delta}(\xi)=\Delta(\xi)+\frac{\gamma}{\pi}\int_{-\infty}^{\infty}d\xi\frac{\widetilde{\Delta}(\xi)^*}{\widetilde{\omega}^2+\xi^2+|\Delta(\xi)|^2}=\Delta(\xi). \tag{3}$$

Here $\gamma = \pi c V_0^2 N(0)$ is the frequency at which electrons are scattered by point impurities with a potential V_0. These impurities are distributed at random in a concentration c. The integral in the second equation vanishes because $\Delta(\xi)$ is odd, and there is no renormalization of the gap function due to scattering by impurities. This circumstance is the reason for the strong effect of impurities on "odd" pairing. The same situation prevails in the case of an anisotropic pairing, e.g., of the d type.[3,4]

The equation for the gap now takes the form

$$\Delta(\xi)=N(0)T\sum_{\omega_n}\int_{-\infty}^{\infty}d\xi' V_2(\xi,\xi')\frac{\widetilde{\Delta}^*(\xi')}{\widetilde{\omega}^2+\xi'^2+|\Delta^2(\xi')|^2}. \tag{4}$$

Near the transition temperature T_c, Eqs. (3) and (4) can be linearized in terms of $\Delta(\xi)$, in such a way that we find, after the standard calculations, the following linear equation for the gap, which determines T_c:

$$\Delta(\xi)=N(0)\int_{-\infty}^{\infty}d\xi' V_2(\xi,\xi')\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{1}{\xi'}\tanh\left(\frac{\omega+\xi'}{2T}\right)\frac{\gamma}{\omega^2+\gamma^2}\Delta(\xi'). \tag{5}$$

In the analysis below we use the model interaction

$$V_2(\xi,\xi')=\begin{cases} V\left[\cos\frac{\pi}{2}\frac{\xi-\xi'}{\omega_c}+1\right] & \text{for } |\xi|,\ |\xi'|<\omega_c,\quad |\xi-\xi'|<\omega_c, \\ 0 \quad \text{for } |\xi|,\ |\xi'|>\omega_c, |\xi-\xi'|>\omega_c. \end{cases} \tag{6}$$

The basic advantage of this choice is that in this case the integral equations for the gap reduce to transcendental equations and can be solved easily. The model interactions used in Ref. 1 do not allow such a simple analysis, and they basically have no other serious advantages. The basic qualitative results discussed below do not depend on the choice of model interaction.

The gap function in the case we are considering is

$$\Delta(\xi)=\Delta_0(T)\sin\left(\frac{\pi}{2}\frac{\xi}{\omega_c}\right)\quad \text{for } |\xi|<\omega_c \tag{7}$$

and $\Delta(\xi) = 0$ for $|\xi| > \omega_c$. The equation for T_c reduces in this case to

$$1=N(0)V\int_0^{\omega_c}\frac{d\xi'}{\xi'}\sin^2\left(\frac{\pi}{2}\frac{\xi'}{2\omega_c}\right)\int_{-\infty}^{\infty}\frac{d\omega}{\pi}\tanh\left(\frac{\omega+\xi'}{2T_c}\right)\frac{\gamma}{\omega^2+\gamma^2}. \tag{8}$$

In the "pure" limit ($\gamma \to 0$) we find the dependence of T_c on the pairing coupling constant $g=N(0)V$, shown in Fig. 1. Pairing occurs at $g > g_c = 1.213$. Figure 2 shows T_c versus γ for several characteristic values of the pairing constant g. The normal impurities strongly suppress the "odd" pairing. The superconductivity disappears at $\gamma \sim T_{c0}$, and its suppression is even more pronounced than in the case of magnetic impurities in conventional superconductors.[5] This circumstance is seen, in particular,

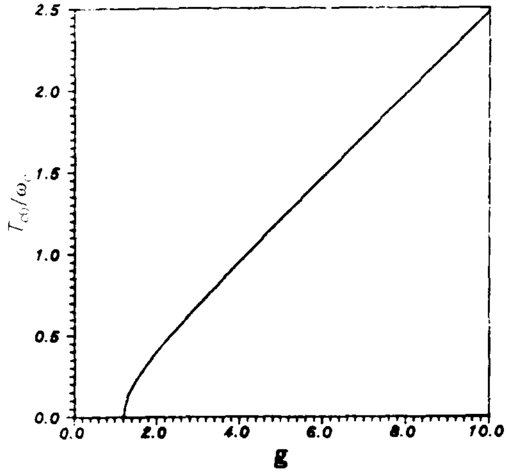

FIG. 1. T_{c0} versus the pairing coupling constant $g=N(0)V$ in interaction model (6).

in the fact that in the limit $g \to g_c$ the region in which a superconducting state exists on the "phase diagram" in Fig. 2 disappears, and we do not see the universal behavior characteristic of the case of magnetic impurities.

The critical scattering index γ_c, corresponding to $T_c(\gamma \to \gamma_c) \to 0$, is found from (5) by means of an equation of the type

$$\Delta(\xi)=N(0)\int_{-\infty}^{\infty} d\xi' V_2(\xi,\xi')\frac{1}{\pi\xi'} \arctan\left(\frac{\xi'}{\gamma_c}\right)\Delta(\xi'). \tag{9}$$

For interaction (6), this equation reduces to

$$1=\frac{2}{\pi} N(0)V\int_0^{\omega_c}\frac{d\xi'}{\xi'} \sin\left(\frac{\pi}{2}\frac{\xi'}{\omega_c}\right) \arctan\left(\frac{\xi'}{\gamma_c}\right). \tag{10}$$

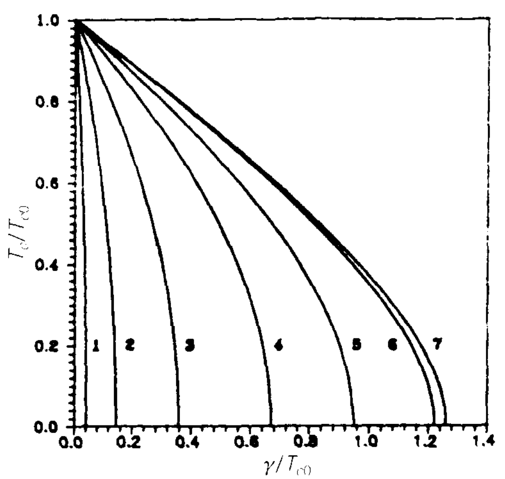

FIG. 2. T_c versus the scattering frequency γ for various values of the pairing constant g: *1*—g=1.22; *2*—1.24; *3*—1.30; *4*—1.5; *5*—2.0; *6*—5.0; *7*—10.0.

It is now a straightforward matter to show that at $g \gg g_c$ we have a universal result: $\gamma_c/T_{c0} = 4/\pi \approx 1.273$. It is not difficult to verify that this result, like the shape of the $T_c(\gamma)$ at $g \gg g_c$, is independent of the choice of model potential $V_2(\xi, \xi')$. In the limit $g \to g_c$, we always have the behavior $\gamma_c \propto (g - g_c) \to 0$. The corresponding behavior can be seen clearly in Fig. 2.

As we mentioned earlier, this model is attractive for explaining the high-T_c superconductivity of metal oxides.[1] The high-T_c superconductors of these systems are known to be extremely sensitive to a structural disorder.[6] It follows from existing experimental data[6] that the superconductivity of the metal oxides is destroyed near a metal–insulator transition caused by a disorder, i.e., at $\gamma \sim E_F$, and by no means at $\gamma \sim T_{c0} \ll E_F$. We believe that this circumstance makes this model an unlikely candidate for explaining the high-T_c superconductivity of the cuprates.

We wish to thank M. A. Érkabaev for assistance in the numerical calculations. This work was supported by the Scientific Council on the Problem of High-Temperature Superconductivity and was carried out as part of Project 90135 of the State Program of Research on Superconductivity. This work was also supported in part by a grant from the Soros fund, awarded by the American Physical Society.

[1] F. Mila and E. Abrahams, Phys. Rev. Lett. **67**, 2379 (1991).
[2] A. A. Abrikosov and L. P. Gor'kov, Zh. Eksp. Teor. Fiz. **34**, 1158 (1958) [Sov. Phys. JETP **7**, 801 (1958); **36**, 319 (1959); **9**, 220 (1959)].
[3] Y. Suzumura and H. J. Schulz, Phys. Rev. B **39**, 11398 (1989).
[4] P. Monthoux, A. V. Balatsky, and D. Pines, Preprint (1992).
[5] A. A. Abrikosov and L. P. Gor'kov, Zh. Eksp. Teor. Fiz. **39**, 1781 (1960) [Sov. Phys. JETP **12**, 1243 (1961)].
[6] B. A. Aleksashin, A. I. Voronin, S. V. Verkhovskiĭ *et al.*, Zh. Eksp. Teor. Fiz. **95**, 678 (1989) [Sov. Phys. JETP **68**, 382 (1989)].

Translated by D. Parsons