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Random Site Ising Model in Self-Avoiding Walk Approximation

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The free energy of random site Ising model with competing exchange interactions of nearest neighbours is studied with the use of high-temperature expansions in the self-avoiding walk approximation. Conditions for the instability of the paramagnetic state are determined by the convergence criterion of the random high-temperature series. The possibility of a concentration interval with the absence of the long-range magnetic order is studied for different 2d and 3d lattices.

Методами высокотемпературных разложений в приближении путей без пересечений исследуется свободная энергия модели Изинга со случайными узлами и конкурирующими обменными взаимодействиями ближайших соседей. Из определения порогов сходимости случайных высокотемпературных рядов найдены условия неустойчивости парамагнитного состояния. Для различных двумерных и трехмерных решеток исследуется возможность появления концентрационного интервала, в котором отсутствует дальний магнитный порядок.

1. Introduction

The random bond Ising model was extensively studied in recent years in connection with the spin-glass problem [1]. The attention to this model has grown considerably since the introduction of the frustration concept, which stresses the importance of the nontrivial disorder in such a system. At present it is generally accepted that for the temperature $T = 0$ on the phase diagram of such a model there exists a region of long-range ferromagnetic order for small concentrations of antiferromagnetic bonds, and a region of long-range antiferromagnetic order for large concentrations of such bonds, with a region of spin-glass phase for the intermediate range of concentrations. The existence of the spin-glass phase for $T \neq 0$ is still not clear and there are different opinions concerning this problem in the literature [3, 4]. Note, however, that the problem of determination of critical concentrations of ferromagnetic and antiferromagnetic bonds, separating the regions with long-range magnetic order from each other, or from a region with the absence of such order (irrespective of whether it is the region of the spin-glass phase, or of some unusual paramagnetic state with anomalously slow relaxation of spins), is of considerable interest by itself. This problem in the random bond model was analysed (both for $T = 0$, and for $T \neq 0$) by different approaches, beginning with the relatively simple variants of random mean-field theory, up to the methods of renormalization group and numerical simulations (for references see [5]). In a previous paper we have studied the free-energy of the random bond Ising model using the high-temperature expansions in the so-called self-avoiding walk approximation. This approximation while being very simple has demonstrated

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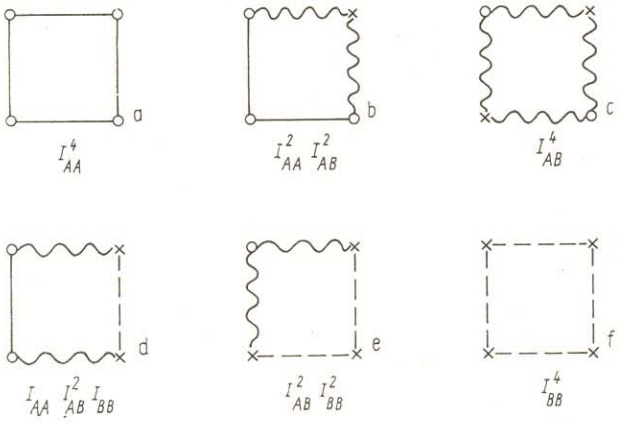

Fig. 1. Different distributions of A-atoms (circles) and B-atoms (crosses) over the corners of the plaquette and the appropriate frustration functions. Solid lines denote I_{AA}, wavy lines I_{AB}, and dashed lines I_{BB}

a rather high accuracy for the regular Ising model [6]. The analysis of the convergence criterion of the high-temperature expansion has shown the existence of critical concentrations c_1^* and c_2^* (analytical expressions have been found for $c_{1,2}^*$), at which cusps in the concentration dependences of the lines of Curie and Néel points have been obtained. These concentrations have been identified with the critical concentrations for the instability of long-range magnetic order, and their values for different lattices have been found to be very close to the results of much more refined methods [7].

In this paper we shall consider the random site Ising model in which the spins σ_A and σ_B are placed at the lattice sites with probabilities $c_A = 1 - c$ and $c_B = c$ and coupled via the three types of exchange interactions of nearest neighbours I_{AA}, $I_{AB} = I_{BA}$, and I_{BB}. For the appropriate choice of the signs of these interactions this model is also an example of a model with nontrivial magnetic disorder, analogous to the random bond case [1, 2]. Let us consider a square plaquette (an elementary closed path on the square or simple cubic lattice) and calculate the frustration function Φ_p [2], i.e. the product of exchange interactions around the plaquette, for different distributions of atoms of A and B types at the corners of the plaquette. According to [2] nontrivial disorder in a magnetic system (which cannot be gauged away by a local gauge transformation) appears when for some configuration we get $\Phi_p < 0$. From Fig. 1 we can see that $\Phi_p < 0$ occurs only for the configurations like in Fig. 1d with two B-atoms being nearest neighbours, if we choose opposite signs of $I_{AA} > 0$ and $I_{BB} < 0$. Thus we conclude that the nontrivial disorder in the random site Ising model appears only for $I_{AA} > 0$, $I_{AB} \gtrless 0$, $I_{BB} < 0$ or for $I_{AA} < 0$, $I_{AB} \gtrless 0$ and $I_{BB} > 0$.[2])

The problem of the appearance of the spin-glass phase was for the first time considered for the random site Ising model with $I_{AA} > 0$, $I_{AB} < 0$, $I_{BB} < 0$ in [8] in the mean-field approximation for $T = 0$ and later in [9] for all temperatures. It was shown that for $T = 0$ the spin-glass phase appears in honeycomb and square lattices, but does not appear in s.c. and b.c.c. lattices [8]. Below we study the conditions of stability for the paramagnetic state in this system using high-temperature expansion methods analogous to those used previously for the random bond model [5].

[2]) We consider only the lattices with an even number of sides of the elementary plaquette, i.e. we are not dealing with triangular and f.c.c. lattices.

2. High-Temperature Series for the Free Energy and Convergence Condition

The Hamiltonian for the random site Ising model is

$$H = -\sum_{\alpha\gamma} I_{\alpha\gamma} \sum_{\langle ij\rangle} p_{\alpha i} p_{\gamma j} \sigma_{\alpha i} \sigma_{\gamma j} , \tag{1}$$

where $\alpha, \gamma = \mathrm{A}, \mathrm{B}$ denote the types of atoms, $\sigma_{\alpha i} = \pm 1$ are Ising spins, and we choose $I_{\mathrm{AA}} > 0$, $I_{\mathrm{AB}} = I_{\mathrm{BA}} \gtrless 0$, $I_{\mathrm{BB}} < 0$. The projection operators $p_{\alpha i}$ are

$$p_{\alpha i} = \begin{cases} 1 & \text{if the lattice site } i \text{ is occupied by an } \alpha\text{-type atom,} \\ 0 & \text{in the opposite case,} \end{cases} \tag{2}$$

and their properties are defined by

$$\left.\begin{aligned} &\sum_{\alpha} p_{\alpha i} = 1 , \qquad p_{\alpha i}^{m} = p_{\alpha i} \qquad (m = 1, 2, \ldots) , \qquad p_{\mathrm{A}i} p_{\mathrm{B}i} = 0 , \\ &\langle p_{\alpha i} \rangle_{\mathrm{c}} = c_{\alpha} , \qquad \langle p_{\alpha i} p_{\gamma j} \rangle_{\mathrm{c}} = c_{\alpha} c_{\gamma} \qquad (i \neq j) . \end{aligned}\right\} \tag{3}$$

Here $\langle \ldots \rangle_{\mathrm{c}}$ denotes the concentration averaging and c_{α} the concentration of α-type atoms.

The partition function for the Hamiltonian (1) $Z\{\beta\}$ is reduced to a standard form [10]

$$Z\{\beta\} = \prod_{\langle ij\rangle} \cosh K_{ij} \sum_{\{\sigma\}} \left\{ \prod_{\langle ij\rangle} \left(1 + \sum_{\alpha\gamma} \sigma_{\alpha i} \sigma_{\gamma j} w_{ij}^{\alpha\gamma} \right) \right\} , \tag{4}$$

where

$$\left.\begin{aligned} &\cosh K_{ij} = \cosh \left(\sum_{\alpha\gamma} p_{\alpha i} p_{\gamma j} K^{\alpha\gamma} \right) = \sum_{\alpha\gamma} p_{\alpha i} p_{\gamma j} \cosh K^{\alpha\gamma} , \\ &w_{ij}^{\alpha\gamma} \equiv \tanh \left(p_{\alpha i} p_{\gamma j} K^{\alpha\gamma} \right) = p_{\alpha i} p_{\gamma j} \tanh K^{\alpha\gamma} = p_{\alpha i} p_{\gamma j} w^{\alpha\gamma} , \\ &K^{\alpha\gamma} = \beta I_{\alpha\gamma} . \end{aligned}\right\} \tag{5}$$

Calculating $\sum_{\{\sigma\}}$ in (4) one can see that due to $\sum_{\{\sigma\}} \sigma_{\alpha i} = 0$, $\sum_{\{\sigma\}} \sigma_{\alpha i}^2 = 1$ ($\sum_{\{\sigma\}}$ is normalized by division over $2N_{\mathrm{s}}$, where N_{s} is the total number of lattice sites [10]) the different contributions to $Z\{\beta\}$ can be represented by the closed paths on the lattice, both connected and unconnected.

Consider, for example, $\sum_{\{\sigma\}}$ over the product of $\sum_{\alpha\gamma} \sigma_{\alpha i} \sigma_{\gamma j} w_{ij}^{\alpha\gamma}$ around the plaquette defined by the sites $ijkl$. Using (3) we get

$$\sum_{\{\sigma\}} \left(\sigma_{\alpha i} \sigma_{\gamma i} p_{\alpha i} p_{\gamma i} \right) = \sum_{\{\sigma\}} \left(\sigma_{\alpha i}^2 p_{\alpha i} \right) \delta_{\alpha\gamma} = p_{\alpha i}^2 \delta_{\alpha\gamma} . \tag{6}$$

Then

$$\sum_{\{\sigma\}} \left(\sum_{\alpha\gamma} \sigma_{\alpha i} \sigma_{\gamma j} w_{ij}^{\alpha\gamma} \ldots \sum_{\delta\beta} \sigma_{\delta l} \sigma_{\beta i} w_{li}^{\delta\beta} \right) =$$

$$= \sum_{\alpha\alpha_1\alpha_2\alpha_3} w_{ij}^{\alpha\alpha_1} w_{jk}^{\alpha_1\alpha_2} w_{kl}^{\alpha_2\alpha_3} w_{li}^{\alpha_3\alpha} = \mathrm{Sp}_{\{\alpha\}} \left(\hat{w}_{ij} \hat{w}_{jk} \hat{w}_{kl} \hat{w}_{li} \right) . \tag{7}$$

Thus, each bond ij on the lattice is represented by the single matrix $\hat{w}_{ij}$ and at each lattice site an even number of bonds must meet. The contribution to $Z\{\beta\}$ from a given graph is equal to a $\mathrm{Sp}_{\{\alpha\}}$ of the product of $\hat{w}_{ij}$ over a closed path. It is easy to convince oneself that in the lowest orders in $\hat{w}$ most of these graphs are just closed self-avoiding walks (SAW) on the lattice.

The logarithm of the partition function,

$$\ln \tilde{Z}\{\beta\} \equiv \ln Z\{\beta\}/\sum_{\langle ij\rangle} \ln\cosh K_{ij} = \ln \sum_{\{\sigma\}} \prod_{\langle ij\rangle} \left(1 + \sum_{\alpha\gamma} \sigma_{\alpha i}\sigma_{\gamma j} w_{ij}^{\alpha\gamma}\right), \tag{8}$$

can also be represented graphically as an expansion over $\hat{w}_{ij}$. Only connected graphs will contribute to this expansion and they can be represented by the closed paths on a lattice, beginning and ending at a given site. Now every bond ij can be traced several times (cf. the expansion of $\ln(1+x) = x - \frac{1}{2}x^2 + \ldots$) while still only an even number of bonds can meet at a given vertex (site). If the bond ij is traced n times, we represent it by a factor $w_{ij}^{(n)} = p_{\alpha i} p_{\gamma i} (w^{\alpha\gamma})^n$. In the theory of high-temperature expansions these graphs are classified over their cyclomatic number: $C = l - p + 1$ [11], where l is the number of bonds on the lattice, p the number of vertices of the graph. Our approximation consists in the neglect of all the graphs with $C > 1$ (cf. [5]). As was shown by Domb [6], in a regular Ising model the use of only the $C = 1$ graphs to determine the temperature of a phase transition gives an error of the order of 3% for 3d lattices and of the order of 10% for 2d lattices, as compared to the exact results. This is due to a dominating role of $C = 1$ graphs in the high-temperature series. Graphs with $C > 1$ apparently effectively compensate each other. Combining the terms with the same number of bond passages we get

$$\begin{aligned}\ln \tilde{Z}\{\beta\} = {} & \sum_N \sum_i \sum_{\Gamma_N^i} \frac{1}{N} \operatorname{Sp}_{\{\alpha\}} (\hat{w}_{ij}\hat{w}_{jk} \ldots \hat{w}_{li}) - \\ & - \frac{1}{2} \sum_N \sum_i \sum_{\Gamma_N^i} \frac{1}{N} \operatorname{Sp}_{\{\alpha\}} (\hat{w}_{ij}^{(2)}\hat{w}_{jk}^{(2)} \ldots \hat{w}_{li}^{(2)}) + \\ & + \frac{1}{3} \sum_N \sum_i \sum_{\Gamma_N^i} \frac{1}{N} \operatorname{Sp}_{\{\alpha\}} (\hat{w}_{ij}^{(3)}\hat{w}_{jk}^{(3)} \ldots \hat{w}_{li}^{(3)}) - \ldots \end{aligned} \tag{9}$$

Here N is the number of lattice bonds (steps) of the closed SAW and the factor N^{-1} takes into account the equivalence of contributions differing by the choice of the initial site among N sites traced by the SAW Γ_N^i, beginning at the i-th site of the lattice.

Let us perform the concentration averaging of (9). For a SAW with N steps we get

$$\begin{aligned}\langle \operatorname{Sp}_{\{\alpha\}} (\hat{w}_{ij} \ldots \hat{w}_{li})\rangle_c &= \sum_{\alpha\alpha_1 \ldots \alpha_{N-1}} \langle p_{\alpha i} w^{\alpha\alpha_1} p_{\alpha_1 j} p_{\alpha_1 j} w^{\alpha_1\alpha_2} \ldots w^{\alpha_{N-1}} p_{\alpha i}\rangle_c = \\ &= \sum_{\alpha\alpha_1 \ldots \alpha_{N-1}} c_\alpha w^{\alpha\alpha_1} c_{\alpha_1} w^{\alpha_1\alpha_2} \ldots c_{\alpha_{N-1}} w^{\alpha_{N-1}\alpha} = \operatorname{Sp}_{\{\alpha\}} (\hat{d}_{(1)}^N) = \sum_m \lambda_{(1)m}^N , \end{aligned} \tag{10}$$

where $\hat{d}_{(1)}^{\alpha\gamma} \equiv c_\alpha w^{\alpha\gamma}$ and we use $p_{\alpha i}^2 = p_{\alpha i}$. In the last expression in (10) the summation goes over the eigenvalues $\lambda_{(1)m}$ of the matrix $\hat{d}_{(1)}$.

Analogously, if each step of the SAW Γ_N^i is traced n times, we get

$$\langle \operatorname{Sp}_{\{\alpha\}} (\hat{w}_{ij}^{(n)} \ldots \hat{w}_{li}^{(n)})\rangle_c = \operatorname{Sp}_{\{\alpha\}} (\hat{d}_{(n)}^N) = \sum_m \lambda_{(n)m}^N , \tag{11}$$

where $\hat{d}_{(n)}^{\alpha\gamma} = c_\alpha (\hat{w}^{\alpha\gamma})^n$ and $\lambda_{(n)m}$ is an eigenvalue of the matrix $\hat{d}_{(n)}$.

It is well known [6] that the number of the closed SAW's of N steps on the lattice, beginning and ending in a given site i, behaves as

$$U_N \approx N^{-h} K_c^N \qquad (N \gg 1) , \tag{12}$$

where $h > 0$ and K_c is the so-called connectivity constant of the lattice. Then we get

$$\langle \ln \tilde{Z} \{\beta\} \rangle \approx \sum_N \frac{1}{N} U_N \sum_m \left[\lambda^N_{(1)m} - \frac{1}{2} \lambda^N_{(2)m} + \frac{1}{3} \lambda^N_{(3)m} - \ldots \right]. \tag{13}$$

The series in (13) diverge if some of the eigenvalues $\lambda_{(n)m}$ satisfy the condition $K_c \lambda_{(n)m} = \pm 1$. Then it is clear that the divergence conditions of these series are determined by

$$\mathrm{Det} \, |\hat{1} \pm K_c \hat{d}_{(n)}| = 0 \, . \tag{14}$$

For $n = 1$, $c_A = 1$, $c_B = 0$, and $c_A = 0$, $c_B = 1$ (14) gives the divergence conditions for the regular Ising magnets (in SAW approximation),

$$1 - K_c w^{AA} = 0; \qquad 1 - K_c |w^{BB}| = 0 \, . \tag{15}$$

Note also that our analysis can be extended to the Heisenberg case with the classical spin S. In the regular case [6] it reduces to the replacement of $w = \tanh (I/T)$ by $w = L(IS^2/T)$, where $L(x) = \coth x - x^{-1}$ is the Langevin function. For the random site model this reduces to the replacement of $w^{\alpha\gamma} = \tanh (I_{\alpha\gamma}/T)$ by $w^{\alpha\gamma} = L(I_{\alpha\gamma} S_\alpha S_\gamma / T)$ in the formulae given above.

Finally, the analysis of the convergence of the averaged high-temperature series for $\langle \ln \tilde{Z} \{\beta\} \rangle_c$ reduces to the comparison of the solutions of two equations,

$$\begin{vmatrix} 1 \mp K_c c_A w^{AA} & \mp K_c c_A w^{AB} \\ \mp K_c c_B w^{BA} & 1 \mp K_c c_B w^{BB} \end{vmatrix} = 0 \, , \tag{16}$$

$$\begin{vmatrix} 1 - K_c c_A (w^{AA})^2 & -K_c c_A (w^{AB})^2 \\ -K_c c_B (w^{BA})^2 & 1 - K_c c_B (w^{BB})^2 \end{vmatrix} = 0 \, , \tag{17}$$

where $w^{\alpha\gamma}$ equals either $\tanh (I_{\alpha\gamma}/T)$ or $L(I_{\alpha\gamma} S_\alpha S_\gamma / T)$. We do not need to consider the determinants with terms of the order $(w^{\alpha\gamma})^3$, $(w^{\alpha\gamma})^4$ etc., because always $|w^{\alpha\gamma}|^3 < |w^{\alpha\gamma}|$, $(w^{\alpha\gamma})^4 < (w^{\alpha\gamma})^2$ if $0 < |w^{\alpha\gamma}| < 1$ etc., i.e. the respective series in (13) will diverge at lower temperatures than those determined by (16) and (17).

If in these equations we replace K_c by $Z - 1$ (Z is the number of the nearest neighbours), we get the equations determining the transition temperature in the so-called constant coupling approximation, while the replacement of K_c by Z and $\tanh x$ by x (or $L(x)$ by $x/3$) gives the results of the mean-field approximation [12]. Note, however, that here we get the conditions for the instability of the paramagnetic state without any assumptions about the order parameters in the condensed phases, which is most important for the possible spin-glass phase, where the Edwards-Anderson order parameter is under suspicion at present.

3. Phase Diagrams for Different Lattices

Consider now (16) and (17) for some special cases. Choose first $w^{AA} = |w^{AB}| = |w^{BB}| = w$ (i.e. $I_{AA} = |I_{AB}| = I_{BB}$). Then from (16) we get the divergence conditions as

$$1 - K_c (c_A - c_B) \, w_F - 2 K_c^2 c_A c_B w_F^2 = 0 \, , \tag{18}$$

$$1 + K_c (c_A - c_B) \, w_{AF} - 2 K_c^2 c_A c_B w_{AF}^2 = 0 \tag{19}$$

and from (17) we get the divergence criterion independent of concentration,

$$1 - K_c w_F^2 = 0 \, . \tag{20}$$

Table 1

	lattice			
	honeycomb	square	s.c.	b.c.c.
K_c	1.8484	2.6390	4.6826	6.5288
$K_c/\sqrt{2}$	1.3070	1.8661	3.3111	4.6166
$\sqrt{K_c}$	1.3596	1.6245	2.1639	2.5552
μ	1.7321	2.4142	4.5840	6.4032
$\mu/\sqrt{2}$	1.2248	1.7071	3.2414	4.5278
$\sqrt{\mu}$	1.3161	1.5538	2.1410	2.5305

The solutions of (18) and (19) coincide at the concentration $c_A^* = c_B^* = c^* = 0.5$, and

$$(w^*)^{-1} \equiv (w_F^*)^{-1} = (w_{AF}^*)^{-1} = K_c/\sqrt{2} \,. \tag{21}$$

The values of $(w^*)^{-1} = K_c/\sqrt{2}$ and $(w_f)^{-1} = \sqrt{K_c}$ are given in Table 1 for four different lattices. The values of K_c are taken from [13] we also give the estimates using instead of K_c the constant μ corresponding to the exact transition temperature for the regular Ising model, defined by the equation $1 - \mu w = 0$ [6]. The latter estimate allows to get an exact matching of our results with the regular case for $c = 0$ or $c = 1$. We hope this can be considered as a qualitative account of graphs with $c > 1$.

From Table 1 it is clear that for the square, s.c., and b.c.c. lattices $(w^*)^{-1} > (w_f)^{-1}$, but for the honeycomb lattice $(w_f)^{-1} > (w^*)^{-1}$. This means that for the square, s.c., and b.c.c. lattices the line of instability of the paramagnetic state is determined by (18) in the interval $0 < c < 0.5$ and by (19) in the interval $0.5 < c < 1$ (Fig. 2a). For the honeycomb lattice this line is determined by (20) in the interval $c_1^* = 0.471 < c < c_2^* = 0.529$ if we use K_c in our estimates ($c_1^* = 0.445$, $c_2^* = 0.555$ if we use the estimate via μ), and for the intervals $0 < c < c_1^*$ and $c_2^* < c < 1$ by (18) and (19), respectively (Fig. 2b). The interval $0 < c < c_1^*$ obviously can be identified as a region of transition from paramagnetic to ferromagnetic phase for the honeycomb lattice (this region reduces to $0 < c < c^*$ for the other lattices). Analogously in the region of $c_2^* < c < 1$ (or $c^* < c < 1$) we suppose the transition to the antiferromagnetic

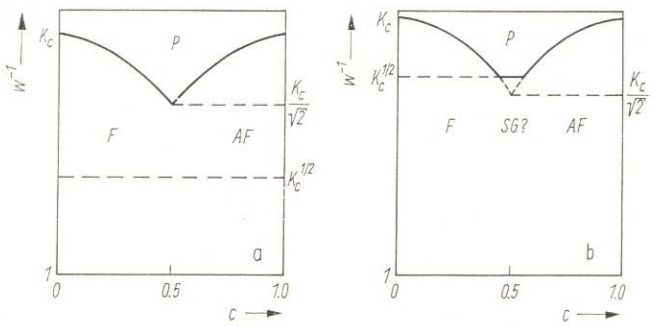

Fig. 2. Qualitative behaviour of the instability lines for the paramagnetic state (P) as a function of concentration c of B-atoms for a) square, s.c., and b.c.c. lattices and b) for honeycomb lattice for the case of $w^{AA} = |w^{AB}| = |w^{BB}| = w$ (i.e. $I_{AA} = |I_{AB}| = |I_{BB}|$)

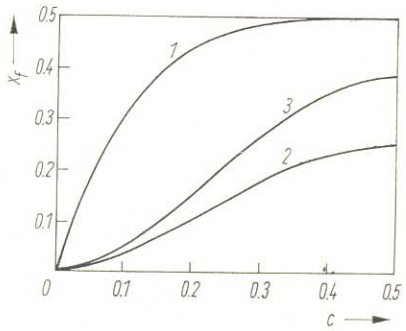

Fig. 3. Fraction of frustrated plaquettes x_f for the random bond model (1) as a function of concentration of antiferromagnetic bonds c, and for the random site model for square, s.c., and b.c.c. lattices (2) and for the honeycomb lattice (3), as a function of concentration c of B-atoms. The values of x_f for $c > 0.5$ are symmetric relative to the point $c = 0.5$

phase. Equation (20) after the appropriate replacements correspond to the condition of spin-glass instability obtained in the mean-field approximation [12].

Note that for the case of $w^{AA} = |w^{BB}|$ and $|w^{AB}| + w^{AA}$ (i.e. $I_{AA} = |I_{BB}| + |I_{AB}|$) no variation of the ratio $w^{AA}/|w^{AB}|$ can change qualitatively this picture for the lattices with square plaquettes (Fig. 3a) and for the honeycomb lattice (Fig. 3b).

Consider now the case of $w^{AA} = |w^{AB}| + |w^{BB}|$ (i.e. $I_{AA} = |I_{AB}| + |I_{BB}|$). Let us try to determine whether the change of I_{BB}, while the condition of $I_{AA} = |I_{AB}|$ is conserved, can lead to the equivalence of convergence conditions following from (16) and (17), for some concentration c^* and some ratio of $I_{AA}/|I_{BB}|$. In this case we have to satisfy simultaneously the following equations:

$$\left.\begin{aligned} &(1 - c^*)\, w^{AA} - c^*\, |w^{BB}| = 0\,, \\ &1 - K_c^2 c^* (1 - c^*)\, w^{AA}(w^{AA} + |w^{BB}|) = 0\,, \\ &1 - K_c[(1 - c^*)\, (w^{AA})^2 + c^* (w^{BB})^2] + K_c^2 c^* (1 - c^*)\, [(w^{AA})^2\, (w^{BB})^2 - (w^{AA})^4] = 0\,. \end{aligned}\right\} \tag{22}$$

This system has the following solutions:

$$\left.\begin{aligned} (w^{AA})^2 &= \frac{1}{2K_c^2}[K_c^2 - K_c + 2 + \sqrt{(K_c - 2)\,(K_c - 1)\,(K_c^2 + K_c + 2)}]\,, \\ |w^{BB}| &= [1 - (w^{AA})^2]/(K_c - 1)\, w^{AA}\,. \end{aligned}\right\} \tag{23}$$

It can be seen that for the honeycomb lattice $(w^{AA})^2$ is complex which means the incompatibility of (16) and (17), so that the intermediate region $c_1^* < c < c_2^*$ remains for any ratio of $I_{AA}/|I_{BB}|$. Similarly we find for the Ising case that for the ratios of $I_{AA}/|I_{BB}| > 5.576$ (4.699) for square lattice, $I_{AA}/|I_{BB}| > 28.470$ (26.768) for s.c., and for $I_{AA}/|I_{BB}| > 63.936$ (61.422) for b.c.c. lattice, the intermediate (spin-glass ?) region appears also for these lattices. (The numbers in parentheses above refer to the estimates via constant μ). From the above results we see that such situations for s.c. and b.c.c. lattices are apparently unrealistic.

Analogous results can be obtained for the case of $I_{AA} + |I_{AB}| = |I_{BB}|$.

4. Discussion

The above results for the random site model with competing exchange interactions are significantly different from the analogous results obtained previously for the random bond model [1, 2, 5, 7]. The main difference is the relative narrowness of the intermediate (spin-glass ?) region for the honeycomb lattice as well as its appearance in square, s.c., and b.c.c. lattices only for rather unrealistic values of exchange par-

ameters. Obviously, this difference in behaviour of two models can be linked with the difference in the concentration behaviour of the fraction of frustrated plaquettes x_f in both models. For the random bond model it is known [7] that $x_f^B = 4c(1-c) \times [(1-c)^2 + c^2]$. In the random site model frustrated square plaquettes appear only when two B-atoms are nearest neighbours, i.e. $x_{1f}^S = 4c^2(1-c)^2$. For the honeycomb lattice, the frustrated configurations appear only for some (not all!) particular distributions of two, three, and four atoms around the plaquette, and $x_{2f}^S = 6c^2(1-c)^4 + 12c^3(1-c)^3 + 6c^4(1-c)^2 = 6c^2(1-c)^2$. Then for the random site model we get Max $x_{1f}^S = 0.25$ and Max $x_{2f}^S = 0.375$ for $c = 0.5$. It is seen (cf. Fig. 3) that the fraction of frustrated plaquettes is significantly lower in the random site model, than in the random bond model (the honeycomb lattice is in a kind of intermediate position). Also the "halfwidth" of the maximum in the concentration dependence of x_f is considerably smaller for the random site model. Thus, while non-trivial disorder is possible in the random site model, its influence upon the physical properties of the system is considerably weaker as compared to the random bond case. Of course, the above discussion cannot be applied to the crystals with f.c.c. (and triangular) lattices.

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References

[1] S. F. Edwards and P. W. Anderson, J. Phys. F **5**, 965 (1975).
[2] G. Toulouse, Commun. Phys. **2**, 115 (1977).
[3] A. J. Bray and M. A. Moore, J. Phys. C **12**, 79 (1979).
[4] A. Khurana and J. A. Hertz, J. Phys. C **13**, 2715 (1980).
[5] M. V. Medvedev and M. V. Sadovskii, phys. stat. sol. (b) **109**, 49 (1982).
[6] C. Domb, J. Phys. C **3**, 256 (1970).
[7] S. Kirkpatrick, Phys. Rev. B **16**, 4630 (1977).
[8] M. V. Medvedev and E. L. Rumyantsev, phys. stat. sol. (b) **85**, 427 (1978).
[9] T. Oguchi and Y. Ueno, J. Phys. Soc. Japan **46**, 729 (1979).
[10] M. E. Fisher, in: Lectures in Theoretical Physics, Vol. VII c, Univ. Colorado Press, Boulder (Colorado) 1963.
[11] M. F. Sykes, J. W. Essam, B. R. Heap, and B. J. Hiley, J. Math. Phys. (New York) **7**, 1557 (1966).
[12] M. V. Medvedev and A. V. Zaborov, Fiz. Metallov i Metallovedenie **52**, 272, 472 (1981).
[13] V. K. S. Shante and S. Kirkpatrick, Adv. Phys. **20**, 325 (1971).

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