Многочастичная Локализация Андерсона

Борис Альтшулер
Колумбийский Университет

Летняя школа Фонда Дмитрия Зимина “Династия”
"Актуальные проблемы теории конденсированного состояния"
4 – 14 июля 2010г.

Фонд Дмитрия Зимина
Династия

Lecture 2.

0.Introduction

Previous Lecture:

1. Anderson Localization as Metal-Insulator Transition
   Anderson model.
   Localized and extended states. Mobility edges.

2. Spectral Statistics and Localization.
   Poisson versus Wigner-Dyson.
   Anderson transition as a transition between different types of spectra.
   Thouless conductance

3 Quantum Chaos and Integrability and Localization.
   Integrable ⟺ Poisson; Chaotic ⟺ Wigner-Dyson

Lecture 2.

1. Localization beyond real space

VOLUME 49 23 AUGUST 1982 NUMBER 8

Chaos, Quantum Recurrences, and Anderson Localization

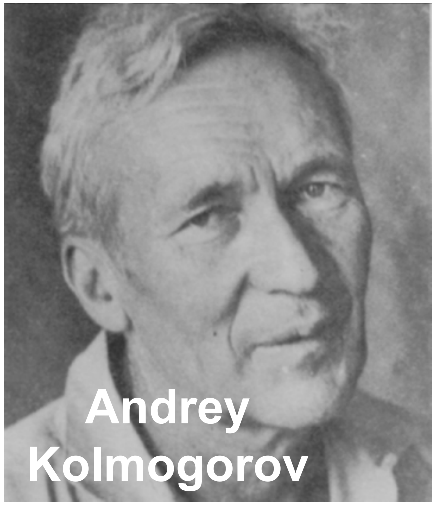
Shmuel Fishman, D. R. Grempel, and R. E. Prange
Department of Physics and Center for Theoretical Physics, University of Maryland, College Park, Maryland 20742
(Received 6 April 1982)

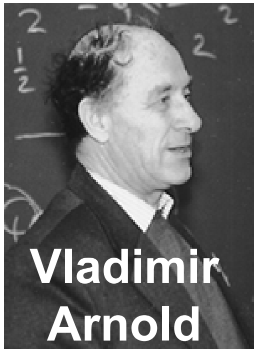
A periodically kicked quantum rotator is related to the Anderson problem of conduction in a one-dimensional disordered lattice. Classically the second model is always chaotic, while the first is chaotic for some values of the parameters. With use of the Anderson-model result that all states are localized, it is concluded that the *local* quasienergy spectrum of the rotator problem is discrete and that its wave function is almost periodic in time. This allows one to understand on physical grounds some numerical results recently obtained in the context of the rotator problem.

Localization in the angular momentum space

Kolmogorov – Arnold – Moser (KAM) theory

A.N. Kolmogorov,
Dokl. Akad. Nauk SSSR, 1954.
Proc. 1954 Int. Congress of Mathematics, North-Holland, 1957





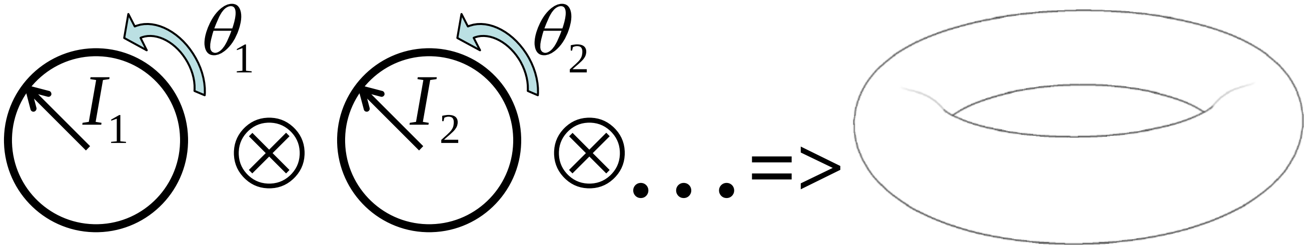
$$\hbar = 0$$

Integrable classical Hamiltonian $\hat{H}_0$, ***d>1***:

Separation of variables: ***d*** sets of action-angle variables

$$I_1, \theta_1 = 2\pi\omega_1 t; \ldots, I_2, \theta_2 = 2\pi\omega_2 t; ..$$

Quasiperiodic motion:
set of the frequencies, $\omega_1, \omega_2, .., \omega_d$ which are in general incommensurate. Actions I_i are integrals of motion $\partial I_i / \partial t = 0$



tori

Integrable dynamics:

Each classical trajectory is quasiperiodic and confined to a particular torus, which is determined by a set of the integrals of motion

space	Number of dimensions
real space	d
phase space: (x,p)	$2d$
energy shell	$2d-1$
tori	d

Each torus has measure zero on the energy shell !

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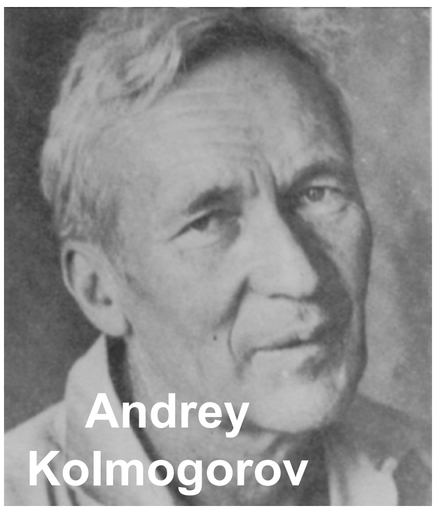

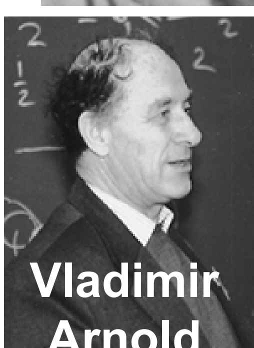

Integrable classical Hamiltonian $\hat{H}_0$, $d>1$:

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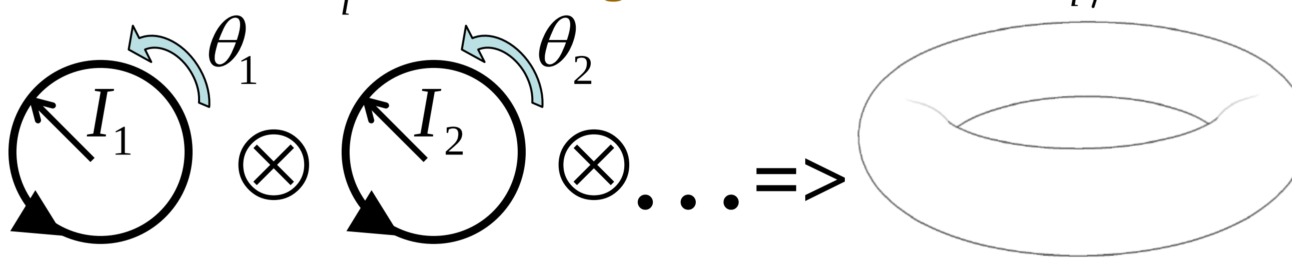

Q: Will an arbitrary weak perturbation $\hat{V}$ of the integrable Hamiltonian $\hat{H}_0$ destroy the tori and make the motion ergodic (when each point at the energy shell will be reached sooner or later) **?**

A: **Most of the tori survive weak and smooth enough perturbations**

KAM theorem

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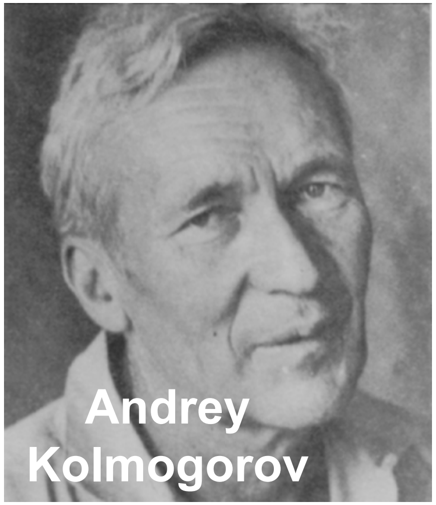
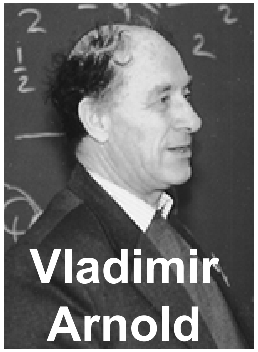
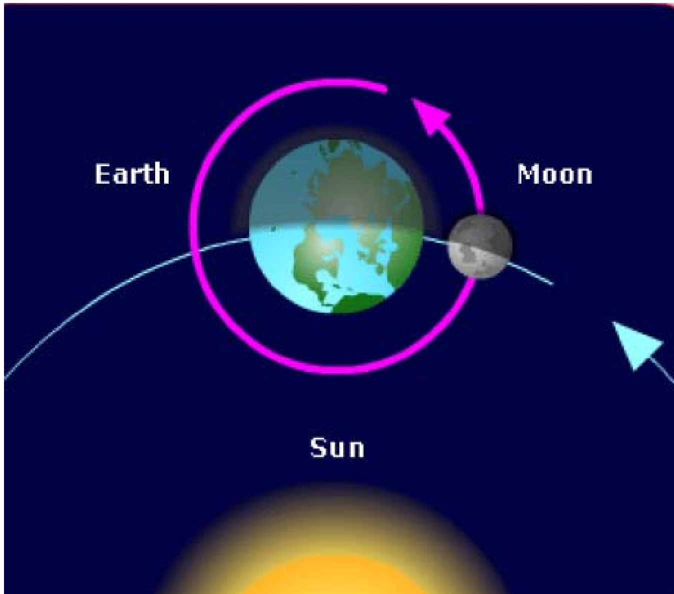
KAM theorem



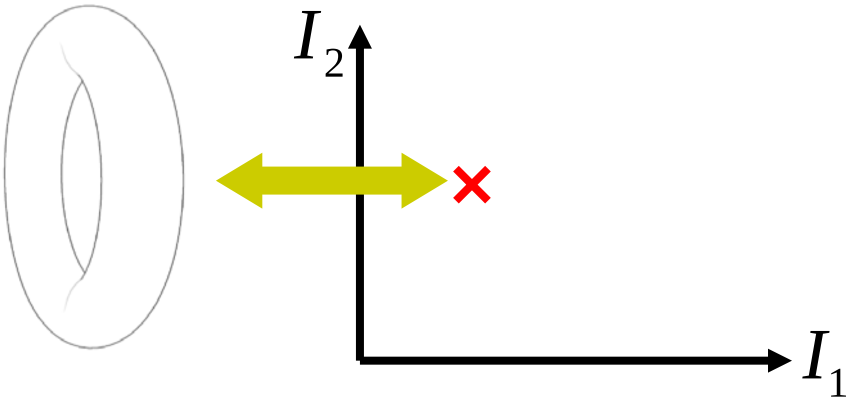


?

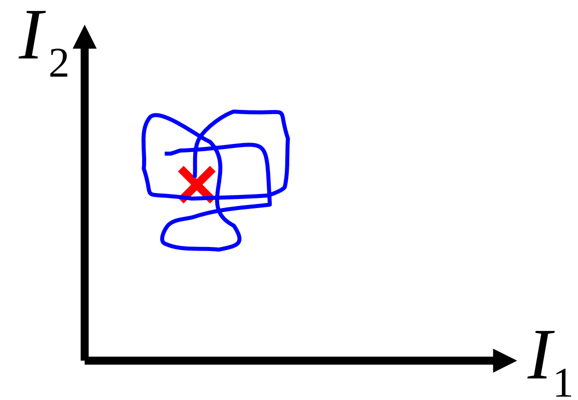


KAM theorem:

Most of the tori survive weak and smooth enough perturbations



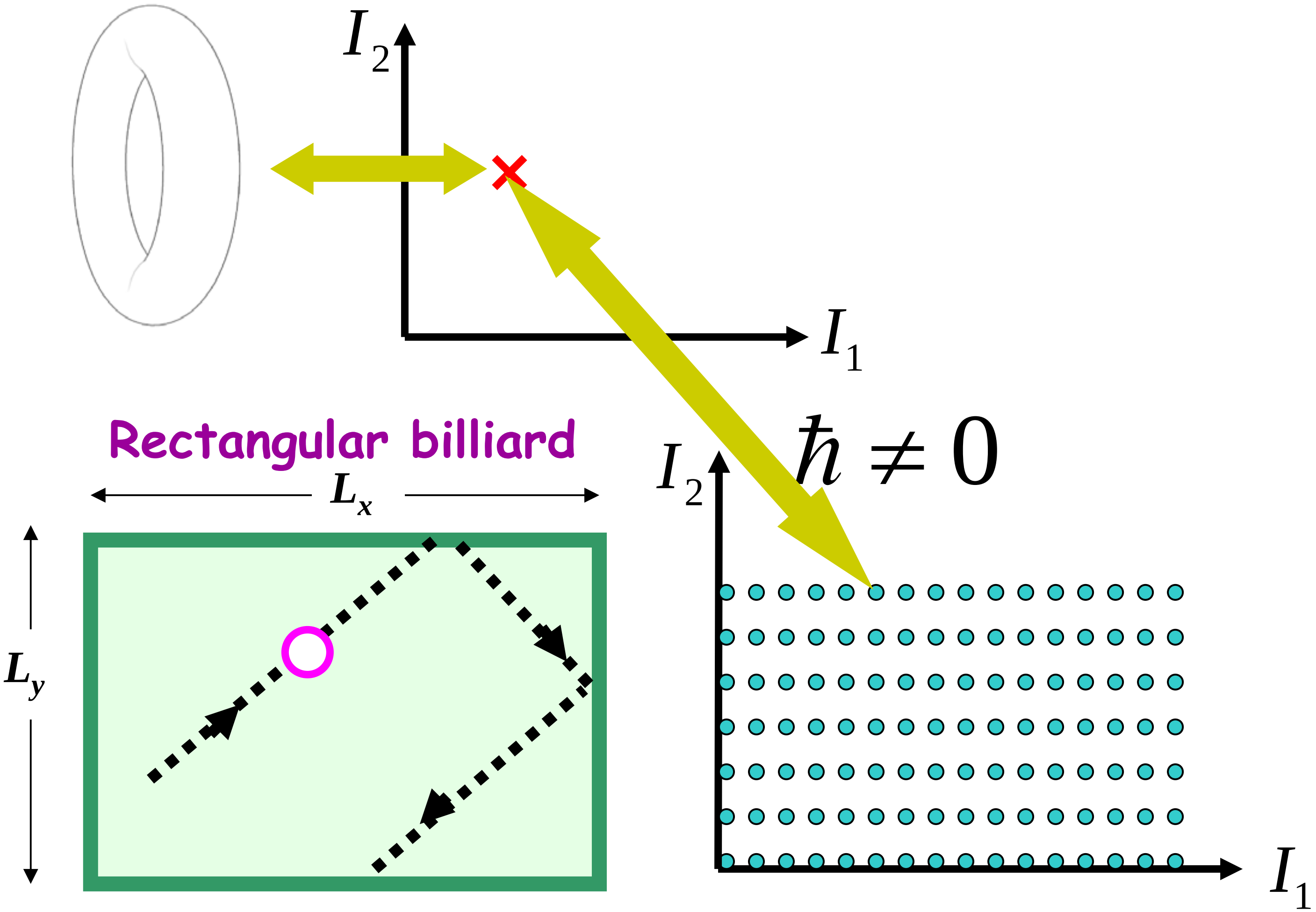
$$\hat{V} \neq 0$$



Each point in the space of the integrals of motion corresponds to a torus and vice versa

Finite motion.
Localization in the space of the integrals of motion?

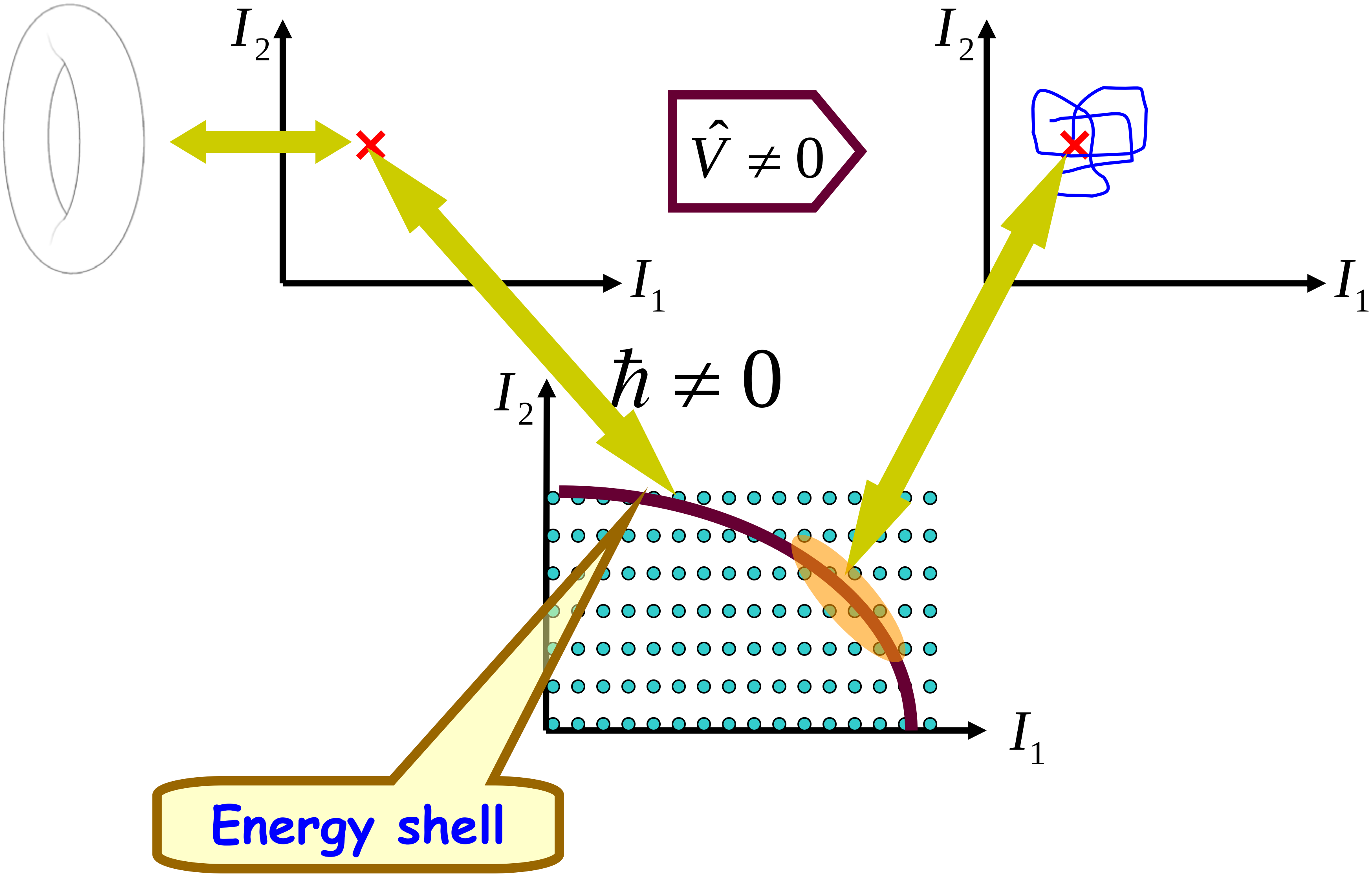
KAM theorem: Most of the tori survive weak and smooth enough perturbations

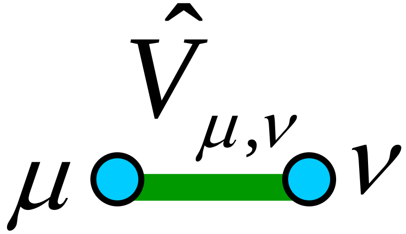


Two integrals of motion

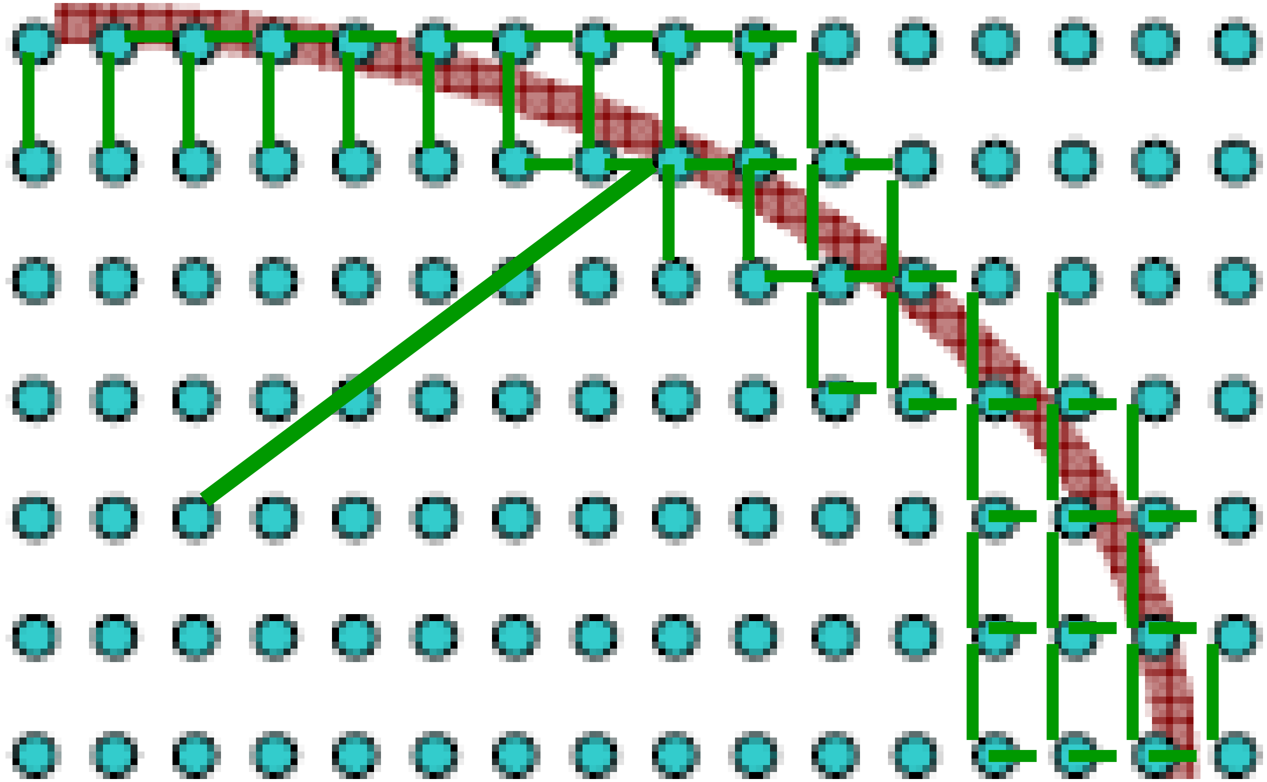
$$p_x = \frac{\pi n}{L_x}; \quad p_y = \frac{\pi m}{L_x}$$

KAM theorem: **Most of the tori survive weak and smooth enough perturbations**





Matrix element of the perturbation



$$\left|\mu\right\rangle = \left|\vec{I}^{(\mu)}\right\rangle$$

$$\vec{I}^{(\mu)} = \left\{I_1^{(\mu)}, \ldots, I_d^{(\mu)}\right\}$$

One can speak about localization provided that the perturbation is somewhat local in the space of quantum numbers of the original Hamiltonian

AL hops are local – one can distinguish "near" and "far"

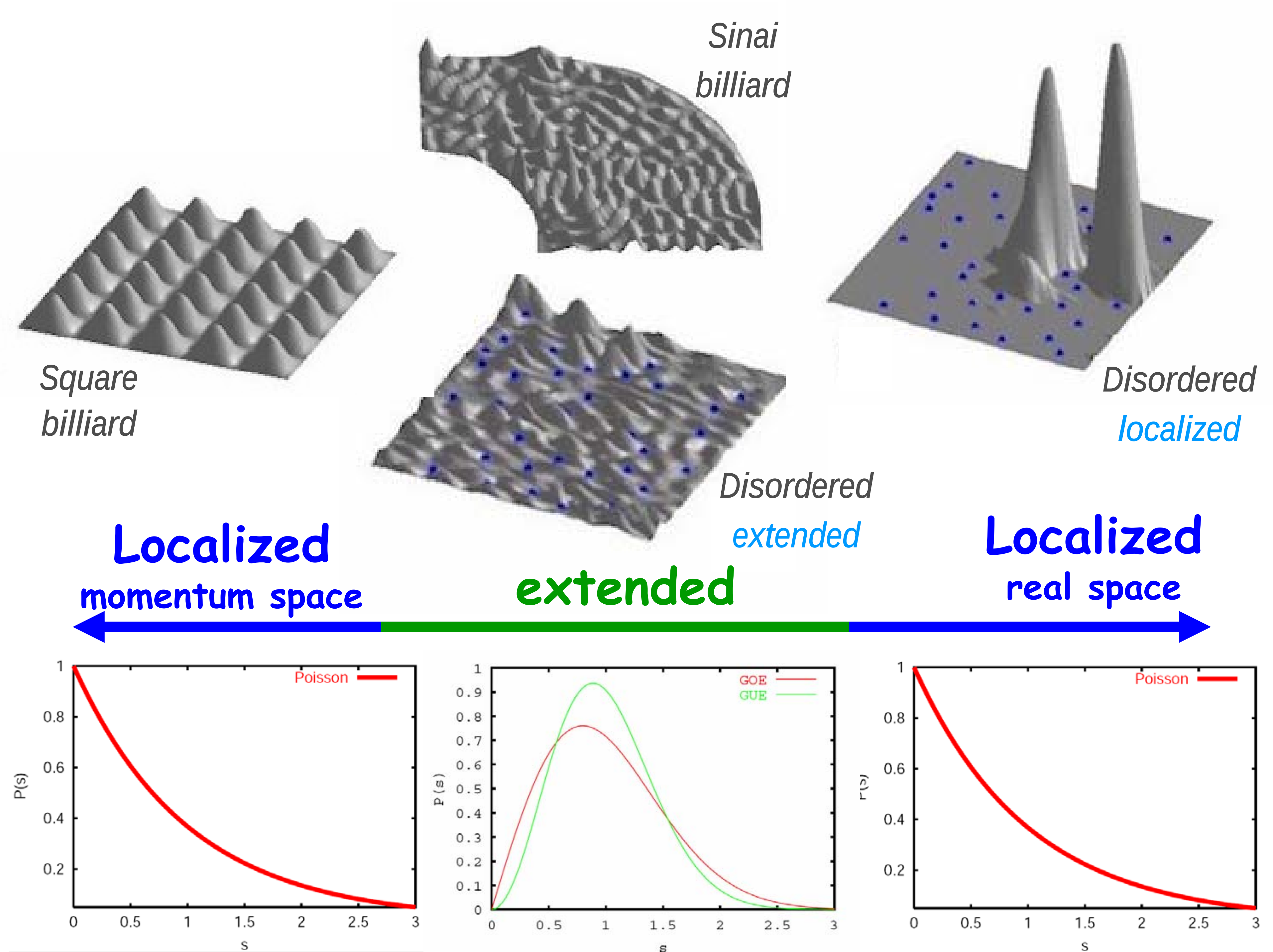
KAM perturbation is smooth enough

Consider an integrable system.
Each state is characterized by a set of quantum numbers.

It can be viewed as a point in the space of quantum numbers. The whole set of the states forms a lattice in this space.

A perturbation that violates the integrability provides matrix elements of the hopping between different sites (Anderson model !?)

Weak enough hopping:
    Localization - Poisson
Strong hopping:
    transition to Wigner-Dyson


Sinai
billiard
Square
billiard
Disordered
extended
Disordered
localized
Localized
momentum space
extended
Localized
real space
Poisson
P(s)
s
0 0.5 1 1.5 2 2.5 3
0.2 0.4 0.6 0.8 1
GOE
GUE
P(s)
s
0 0.5 1 1.5 2 2.5 3
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Poisson
s
0 0.5 1 1.5 2 2.5 3
0.2 0.4 0.6 0.8 1

Strong disorder	localized
Weak disorder	extended

Strong disorder	localized
Moderate disorder	extended
No disorder chaotic	extended
No disorder integrable	localized
Too weak disorder int.	localized

Glossary

Classical	Quantum
Integrable $H_0 = H_0(\vec{I})$	Integrable $\hat{H}_0 = \sum_{\mu} E_{\mu} \lvert \mu \rangle \langle \mu \rvert, \quad \lvert \mu \rangle = \lvert \vec{I} \rangle$
KAM	Localized
Ergodic – distributed all over the energy shell Chaotic	Extended ?

Glossary

Classical	Quantum
Integrable $H_0 = H_0(\vec{I})$	**Integrable** $\hat{H}_0 = \sum_\mu E_\mu \|\mu\rangle\langle\mu\|, \quad \|\mu\rangle = \|\vec{I}\rangle$
KAM	Localized
Ergodic (chaotic)	Extended ?

Q: Any Hamiltonian can be diagonalized.

$$\hat{H}_0 = \sum_\mu E_\mu |\mu\rangle\langle\mu| \quad ?$$

A: Yes, but second condition is crucial.

$$|\mu\rangle = |\vec{I}\rangle \Rightarrow$$ Poisson spectral statistics

Extended states: Level repulsion, anticrossings, Wigner-Dyson spectral statistics

Extended states: Level repulsion, anticrossings, Wigner-Dyson spectral statistics

Localized states: Poisson spectral statistics

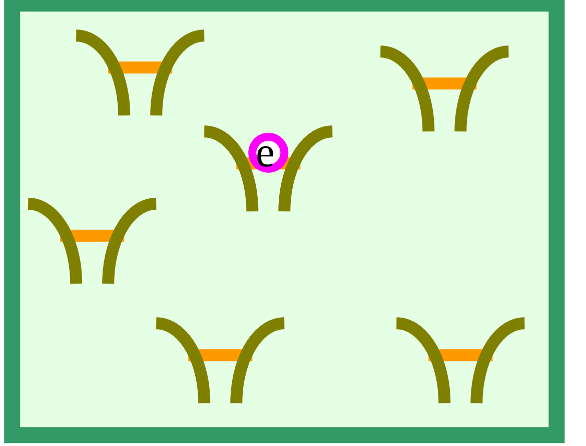
Invariant (basis independent) definition

Example 1

Doped semiconductor

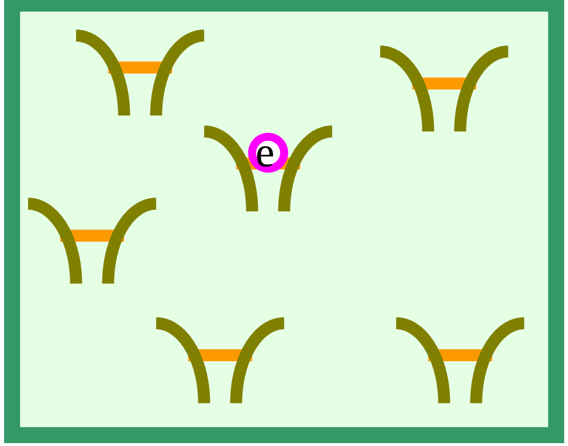
Low concentration of donors ⇒ Electrons are localized on donors ⇨ **Poisson**

Higher donor concentration ⇒ Electronic states are extended ⇨ **Wigner-Dyson**



Example 1

Doped semiconductor



Low concentration of donors ⇨ Electrons are localized on donors ⇨ **Poisson**

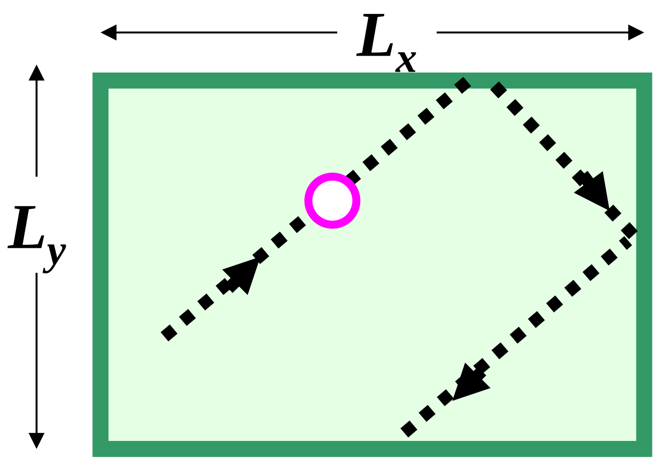
Higher donor concentration ⇨ Electronic states are extended ⇨ **Wigner-Dyson**

Example 2

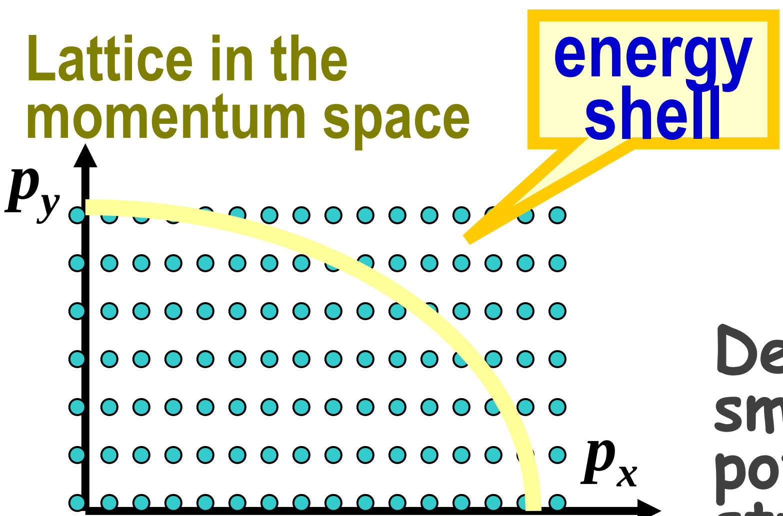
Rectangular billiard

Two integrals of motion

$$p_x = \frac{\pi n}{L_x}; \quad p_y = \frac{\pi m}{L_y}$$



Lattice in the momentum space

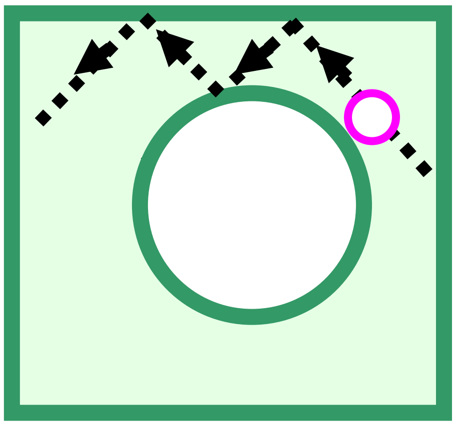


Ideal billiard – localization in the momentum space ⇨ **Poisson**

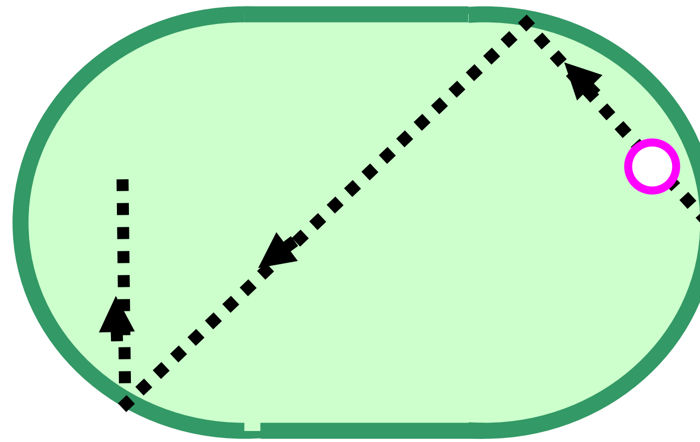
Deformation or smooth random potential. If strong enough – delocalization in the momentum space ⇨ **Wigner-Dyson**

Example 3

Chaotic Systems - proven



Sinai billiard



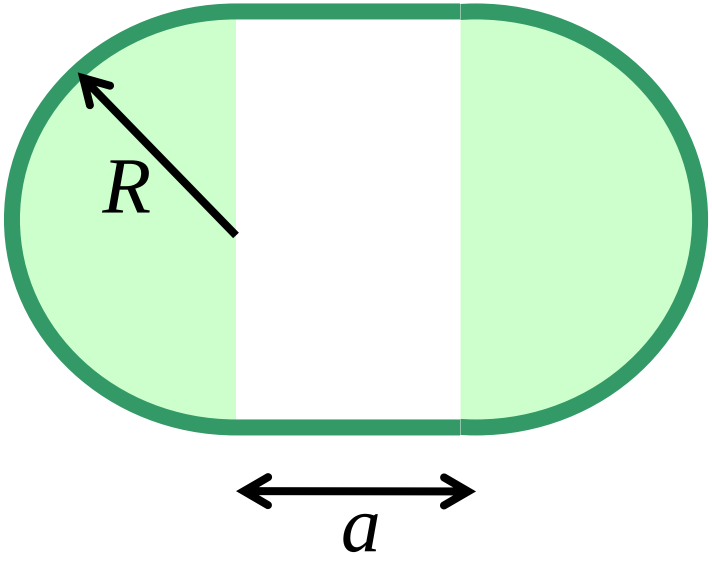
Stadium

Yakov Sinai

Leonid Bunimovich

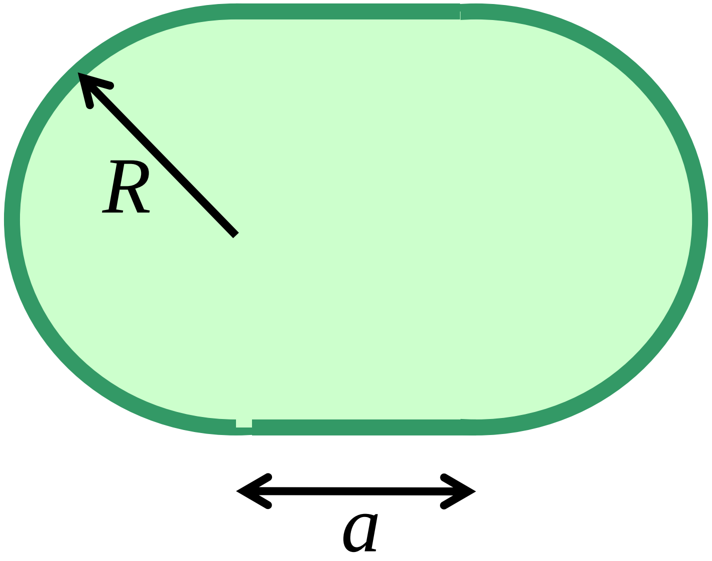
Example 3

Stadium



Example 3

Stadium



$$\varepsilon \equiv \frac{a}{R}$$ - parameter

VOLUME 77, NUMBER 23 PHYSICAL REVIEW LETTERS 2 DECEMBER 1996

Localization and diffusion in the angular momentum space

Example 3

Diffusion and Localization in Chaotic Billiards

Fausto Borgonovi,[1,3,4] Giulio Casati,[2,3,5] and Baowen Li[6,7]
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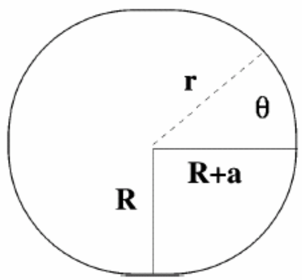
…Istituto Nazionale di Fisica Nucleare, Sezione di Milano, Milano, Italy
[6]*Department of Physics and Centre for Nonlinear and Complex Systems, Hong Kong Baptist University, Hong Kong*
[a7]*Center for Applied Mathematics and Theoretical Physics, University of Maribor, Krekova 2, 2000 Maribor, Slovenia*
(Received 29 July 1996)

$$\varepsilon \equiv \frac{a}{R}$$



$\varepsilon > 0$ **Chaotic stadium**

$\varepsilon \to 0$ **Integrable circular billiard**

Angular momentum is the integral of motion

$$\hbar = 0; \quad \varepsilon << 1$$

Angular momentum is not conserved

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Localization and diffusion in the angular momentum space

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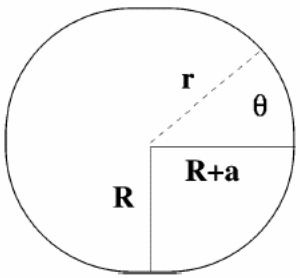


$\varepsilon > 0$ **Chaotic stadium**

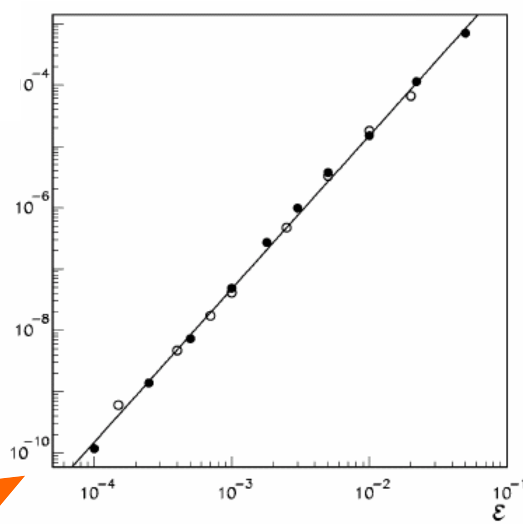
$\varepsilon \to 0$ **Integrable circular billiard**

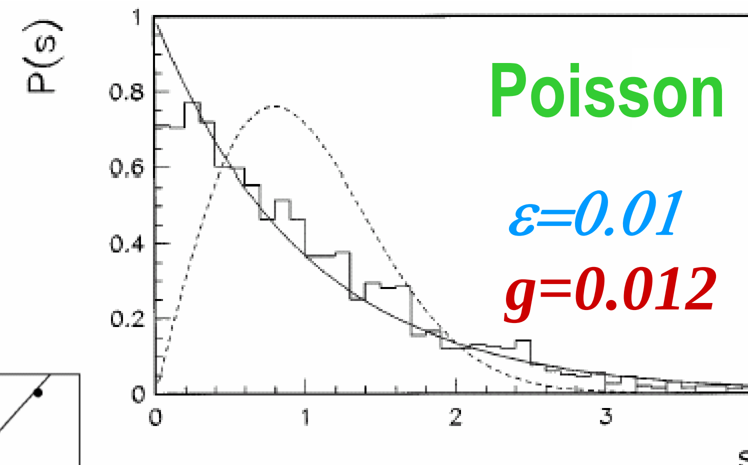
Angular momentum is the integral of motion

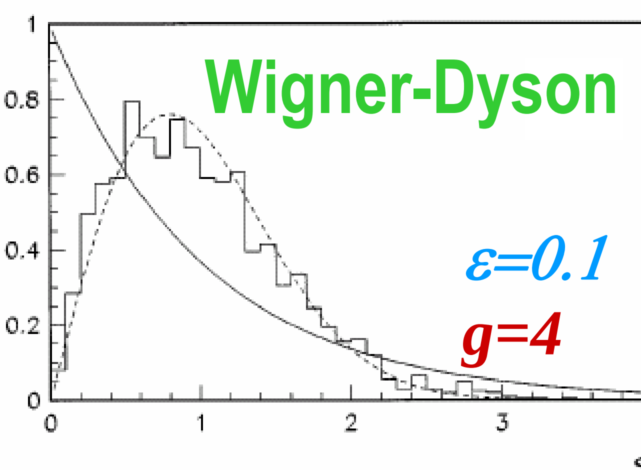
$$\hbar = 0; \quad \varepsilon << 1$$

Diffusion in the angular momentum space

$$D \propto \varepsilon^{5/2}$$







Example 4 D.Poilblanc, T.Ziman, J.Bellisard, F.Mila & G.Montambaux

Europhysics Letters, v.22, p.537, 1993

1D Hubbard Model on a periodic chain

$$H = t\sum_{i,\sigma}\left(c^{+}_{i,\sigma}c_{i+1,\sigma} + c^{+}_{i+1,\sigma}c_{i,\sigma}\right) + U\sum_{i,\sigma} n_{i,\sigma}n_{i,-\sigma} + V\sum_{i,\sigma,\sigma'} n_{i,\sigma}n_{i+1,\sigma'}$$

Onsite interaction

n. neighbors interaction

$V = 0$ Hubbard model **integrable**

$V \neq 0$ extended Hubbard model **nonintegrable**

Example 4

D.Poilblanc, T.Ziman, J.Bellisard, F.Mila & G.Montambaux
Europhysics Letters, v.22, p.537, 1993

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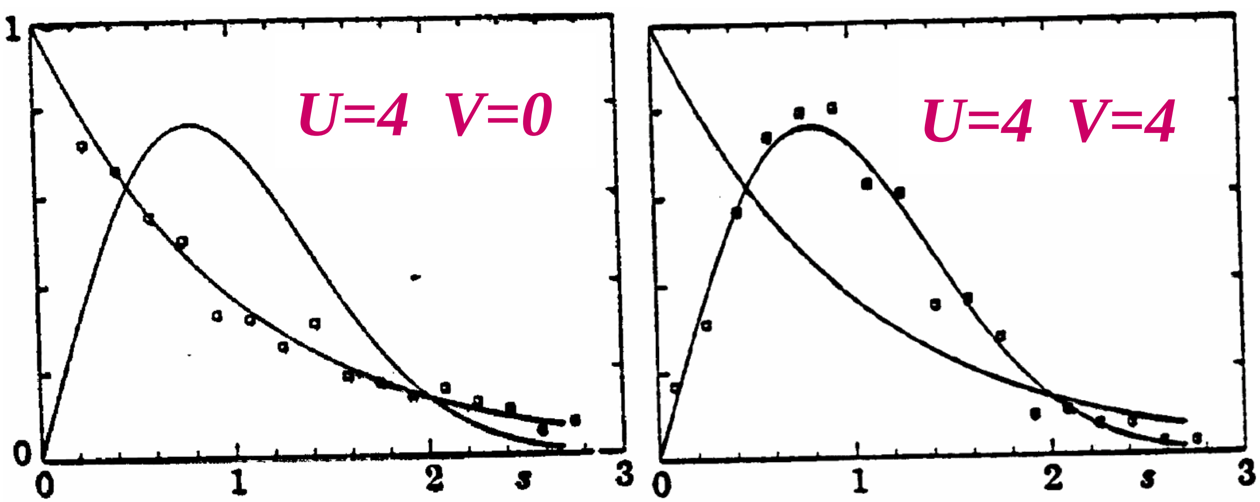
Onsite interaction

n. neighbors interaction

$V = 0$ Hubbard model **integrable**

$V \neq 0$ extended Hubbard model **nonintegrable**

12 sites
3 particles
Total spin 1/2
Total momentum $\pi/6$



Example 5

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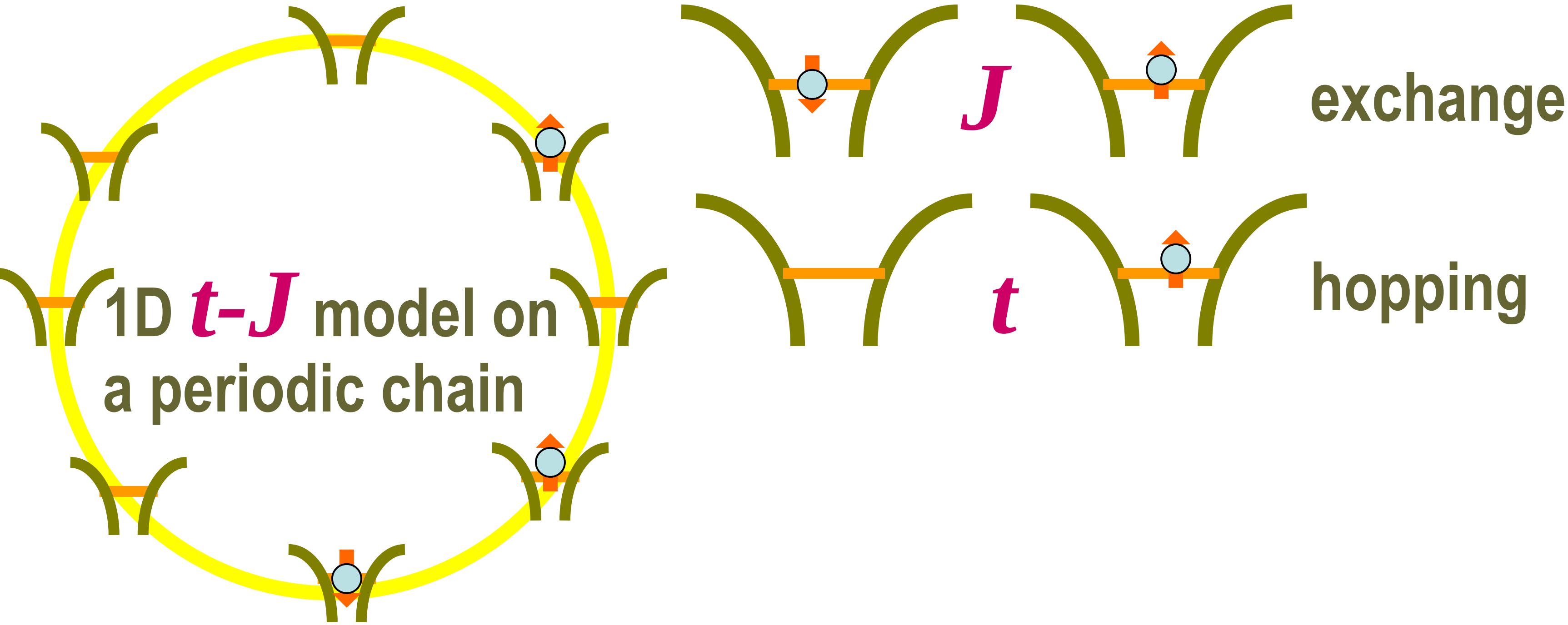
Europhysics Letters, v.22, p.537, 1993



Example 5

D.Poilblanc, T.Ziman, J.Bellisard, F.Mila & G.Montambaux

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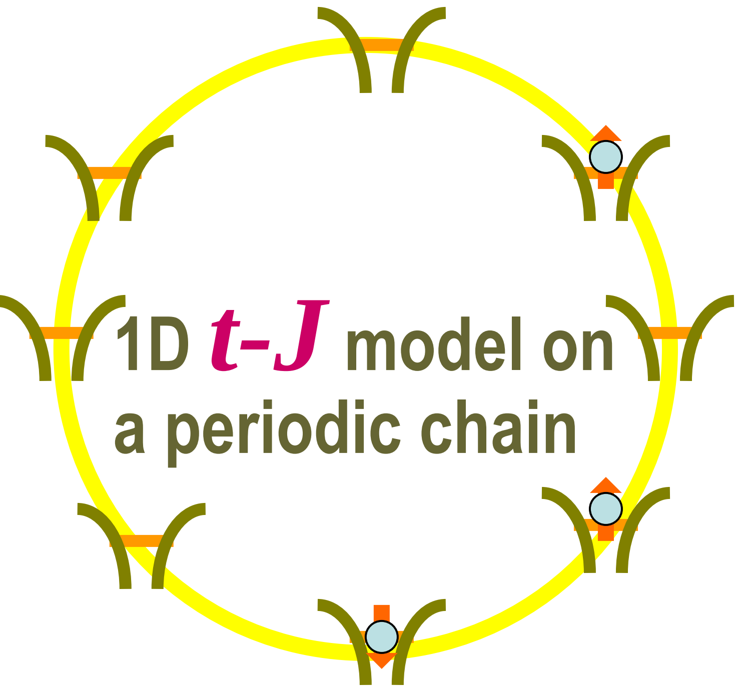

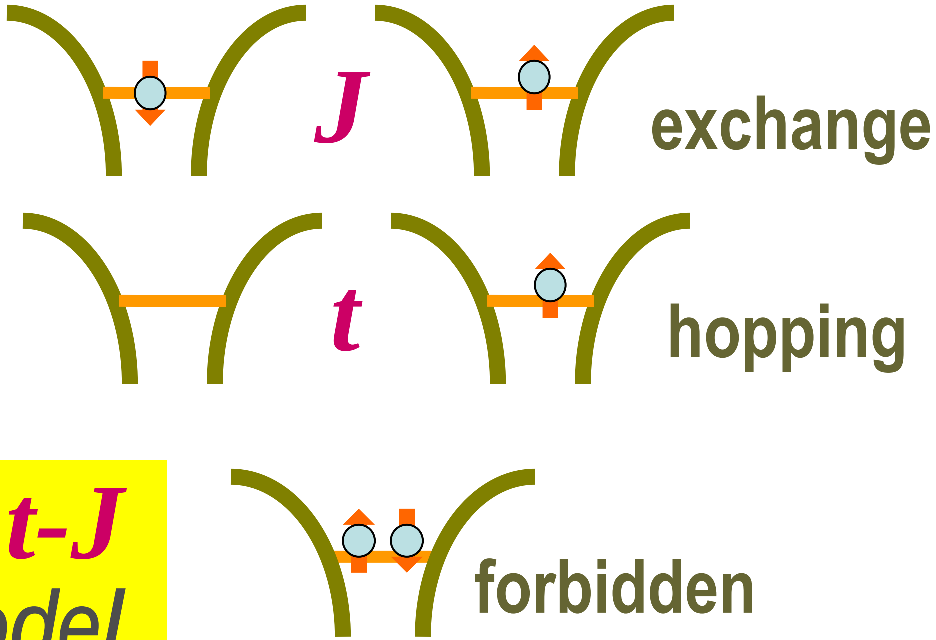

1d t-J
model

Example 5

D.Poilblanc, T.Ziman, J.Bellisard, F.Mila & G.Montambaux

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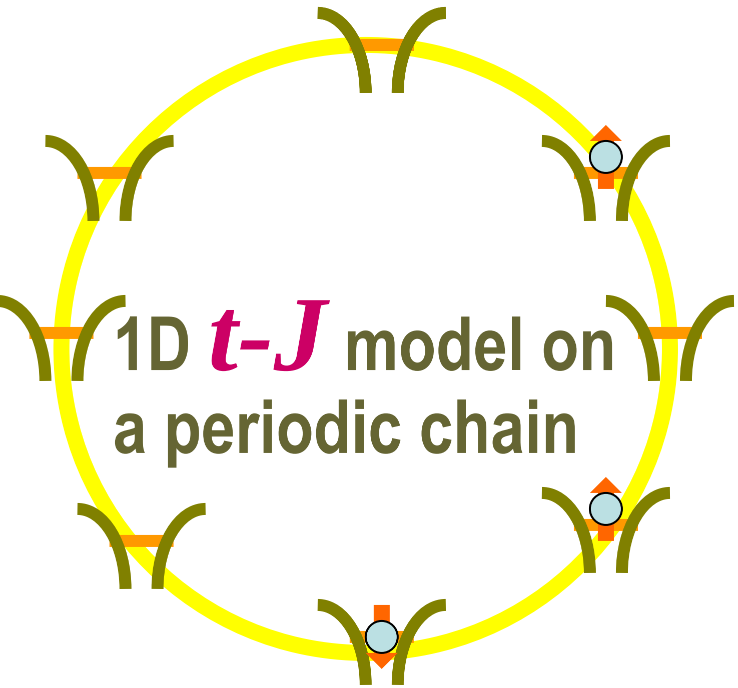
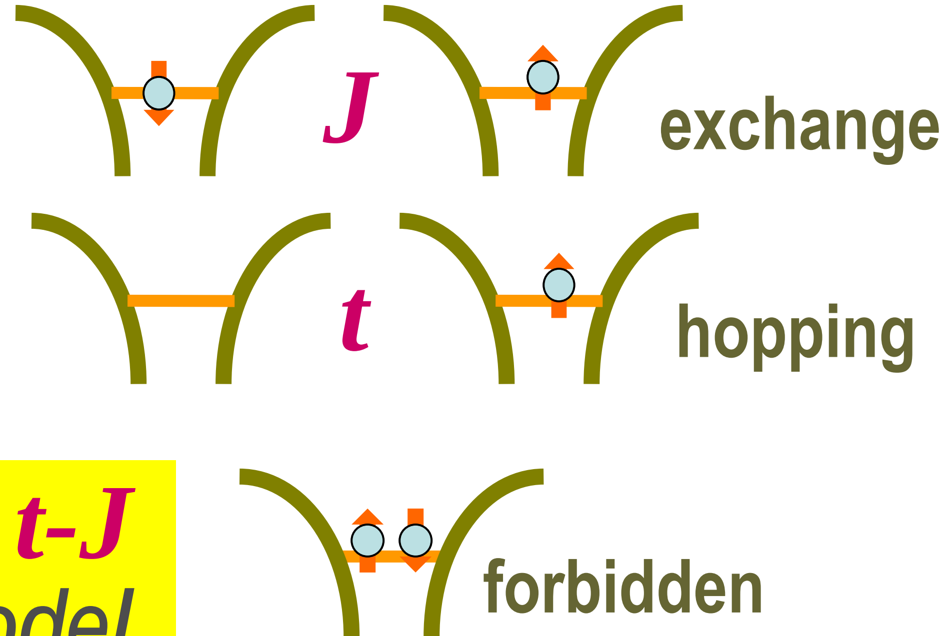
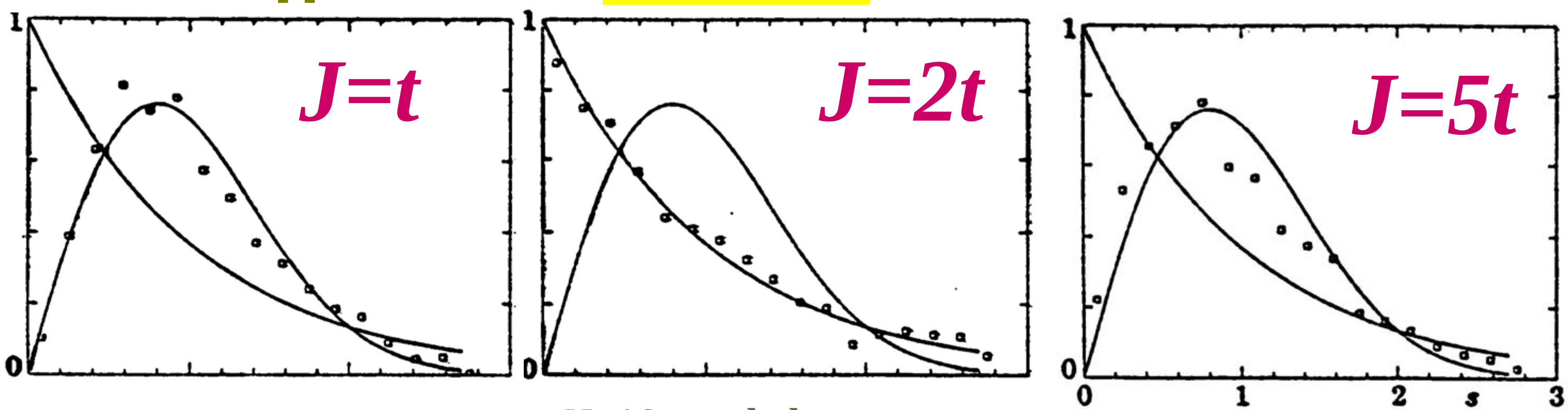




1d *t-J* model



N=16; one hole

Wigner-Dyson random matrix statistics follows from the delocalization.

Q: ***Why the random matrix theory (RMT) works so well for nuclear spectra*** **?**

Many-Body excitations are delocalized !

What does it mean ?

Consider a finite system of quantum particles, e.g., fermions. Let the one-particle spectra be chaotic (Wigner-Dyson).

Q: What is the statistics of the many-body spectra?

a) The particles do not interact with each other ⟹ Poisson: individual energies are conserving quantum numbers.

b) The particles do interact ????

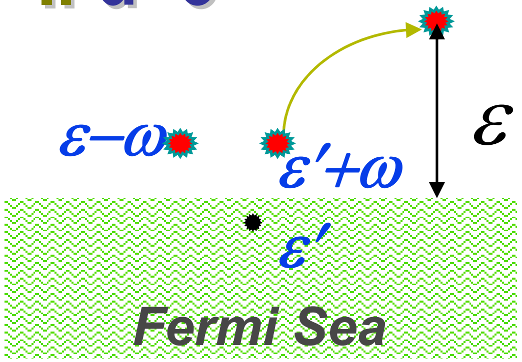
Lecture 2.

2. Many-Body excitation in finite systems

Decay of a quasiparticle with an energy $\mathcal{E}$ in Landau Fermi liquid

Quasiparticle decay rate at $T = 0$ in a clean Fermi Liquid.

I. d=3



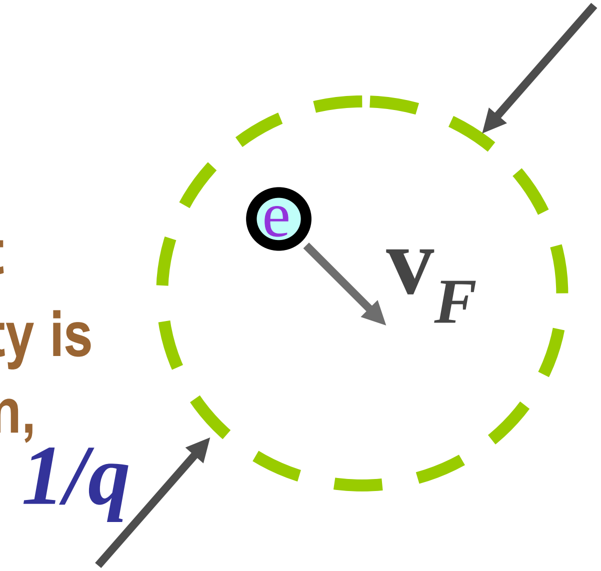
$$\frac{\hbar}{\tau_{e-e}(\varepsilon)} \propto \left(\text{coupling constant}\right)^2 \frac{\varepsilon^2}{\varepsilon_F} \qquad d = 3$$

Reasons:

- At small ε the energy transfer, ω, is small and the integration over ε' and ω gives the factor ε^2.

- The momentum transfer, q, is large and thus the scattering probability at given ε' and ω does not depend on ε', ω or ε

Quasiparticle decay rate at $T = 0$ in a clean Fermi Liquid.

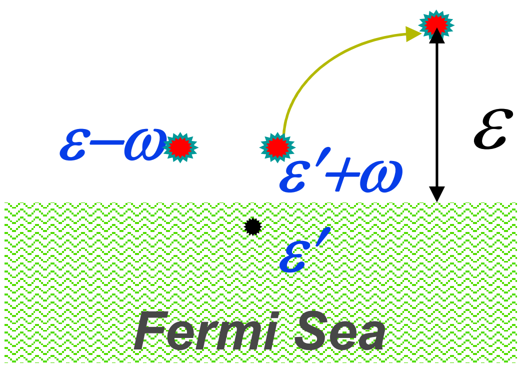


II. Low dimensions

Small moments transfer, q , become important at low dimensions because the scattering probability is proportional to the squared time of the interaction, $(q\mathrm{v}_F)^{-2}$

$$\frac{\hbar}{\tau_{e-e}(\varepsilon)} \propto \begin{cases} \varepsilon^2/\varepsilon_F & d=3 \\ \left(\varepsilon^2/\varepsilon_F\right)\log\left(\varepsilon_F/\varepsilon\right) & d=2 \\ \varepsilon & d=1 \end{cases}$$

Quasiparticle decay rate at $T = 0$ in a clean Fermi Liquid.



$$\frac{\hbar}{\tau_{e-e}(\varepsilon)} \propto \begin{cases} \varepsilon^2/\varepsilon_F & d=3 \\ \left(\varepsilon^2/\varepsilon_F\right)\log\left(\varepsilon_F/\varepsilon\right) & d=2 \\ \varepsilon & d=1 \end{cases}$$

Conclusions:

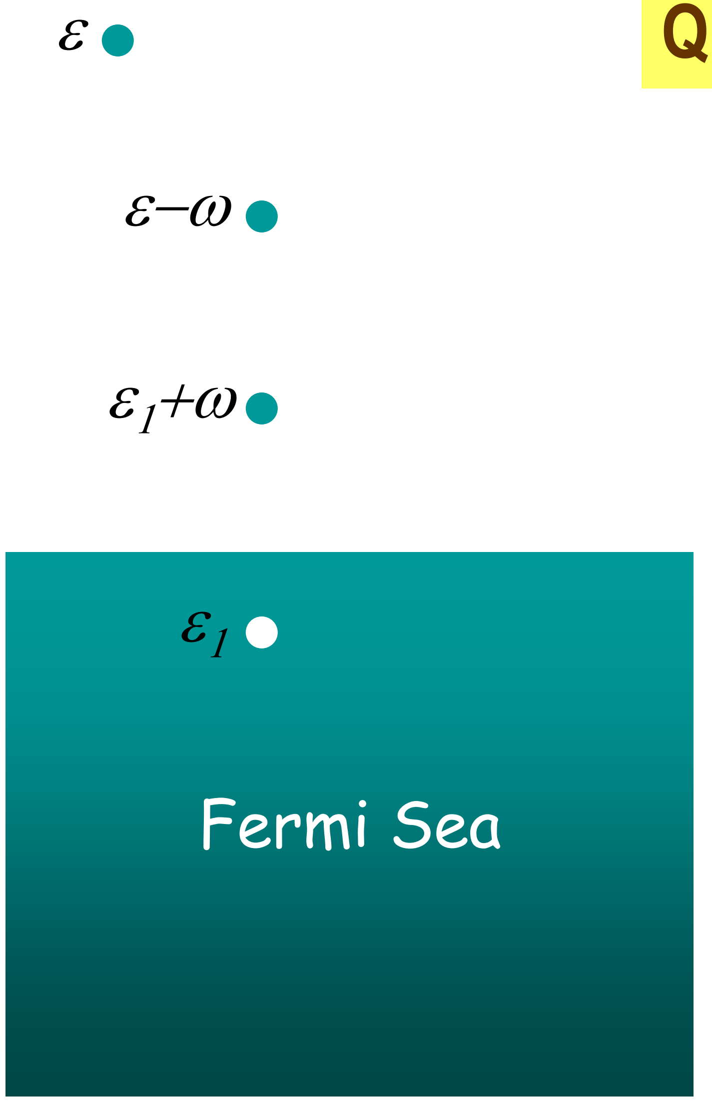
1. For d=3,2 from $\varepsilon \ll \varepsilon_F$ it follows that $\varepsilon\tau_{e-e} \gg \eta$, i.e., that the **qusiparticles** are well determined and the Fermi-liquid approach is applicable.

2. For d=1 $\varepsilon\tau_{e-e}$ is of the order of η, i.e., that the Fermi-liquid approach is not valid for ***1d*** systems of interacting fermions.

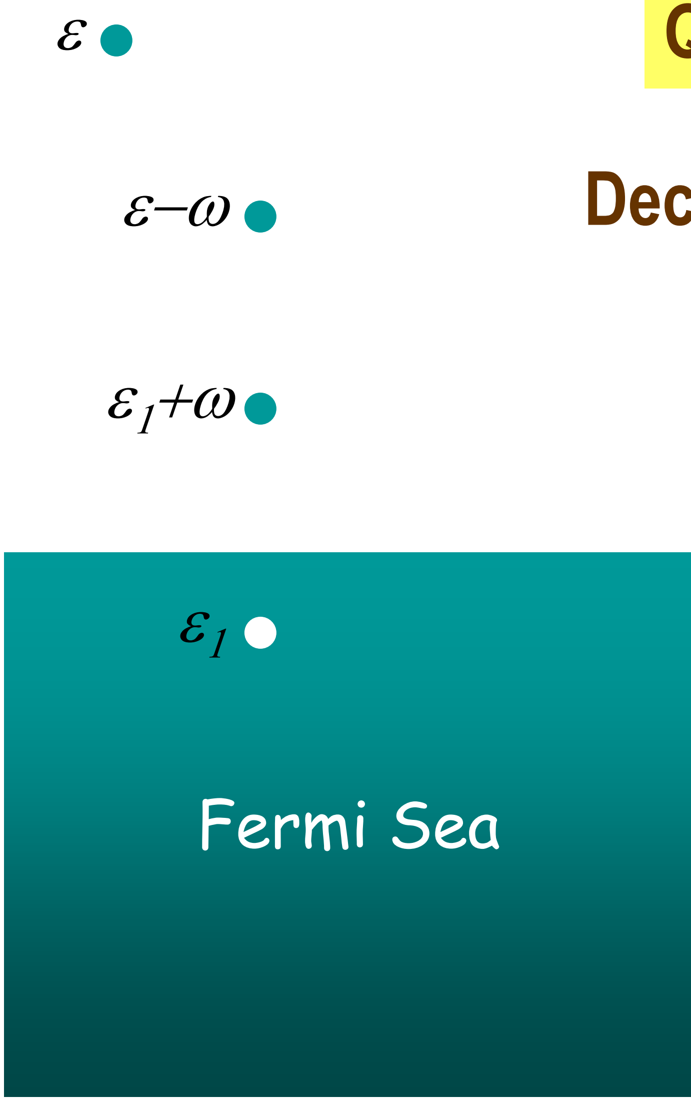
Luttinger liquids

Decay of a quasiparticle with an energy ε in Landau Fermi liquid

Quantum dot – zero-dimensional case ?



Decay of a quasiparticle with an energy ε in Landau Fermi liquid



Quantum dot – zero-dimensional case ?

Decay rate of a quasiparticle with energy ε

(U.Sivan, Y.Imry & A.Aronov,1994)

Fermi Golden rule:

$$\gamma(\varepsilon) \propto \delta_1 \left(\frac{\varepsilon}{E_T} \right)^2$$

Mean level spacing (δ_1)

Thouless energy (E_T)

Decay rate of a quasiparticle with energy ε in 0d.

(U.Sivan, Y.Imry & A.Aronov,1994)

Fermi Golden rule:

$$\gamma(\varepsilon) \propto \delta_1 \left(\frac{\varepsilon}{E_T} \right)^2$$

Mean level spacing

Thouless energy

Recall: $\frac{E_T}{\delta_1} \equiv g$

Thouless conductanc e

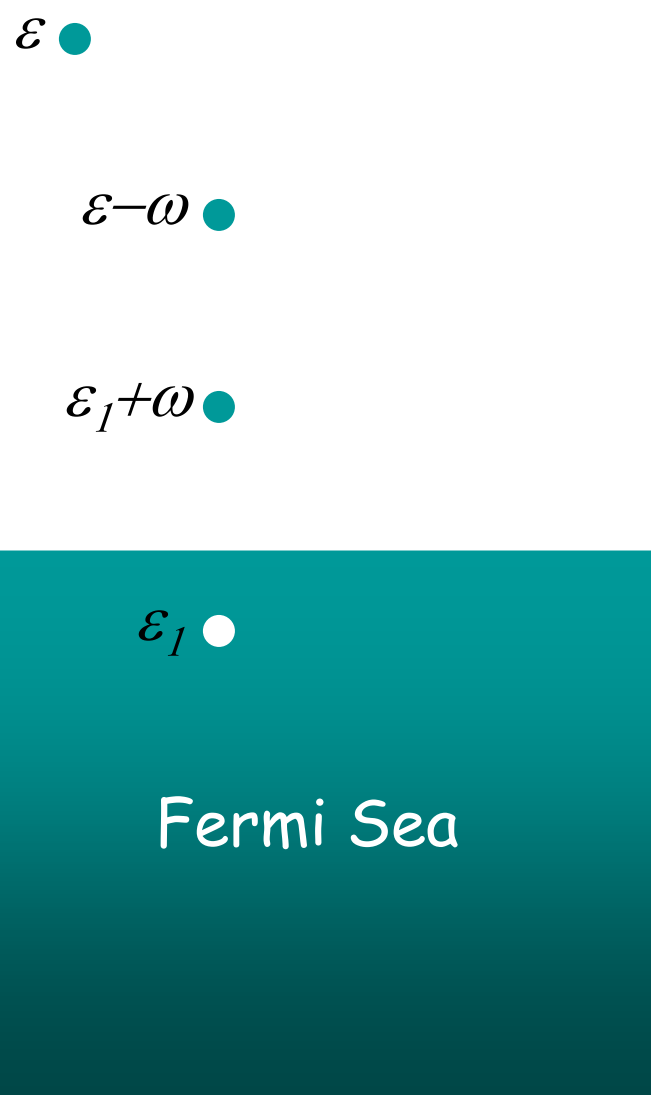
Def: Zero dimensional system

$$E_T >> \varepsilon >> \delta_1 \quad \Rightarrow \quad g >> 1$$

One particle states are extended all over the system

Decay rate of a quasiparticle with energy ε in 0d.

Problem:



zero-dimensional case

↓

one-particle spectrum is discrete

↓

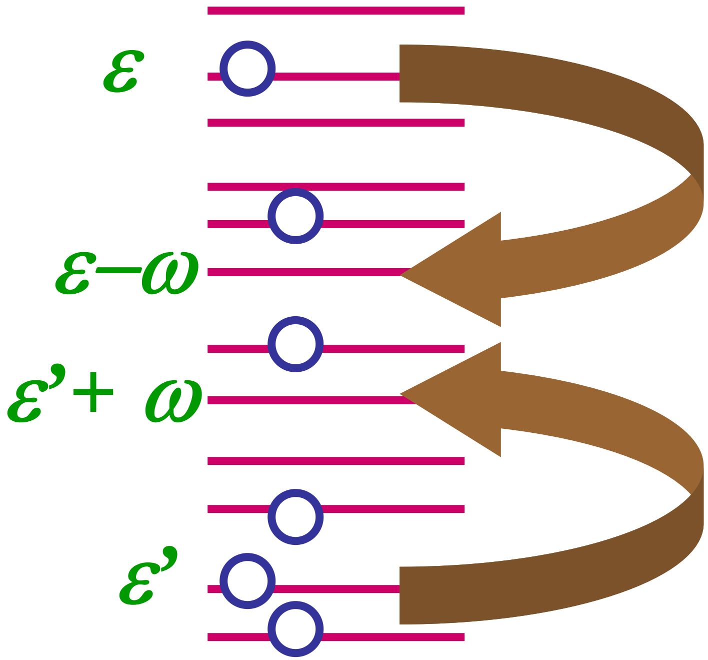
equation

$$\varepsilon_1+\varepsilon_2 = \varepsilon'_1+\varepsilon'_2$$

can not be satisfied exactly

Recall: in the Anderson model the site-to-site hopping **does not** conserve the energy

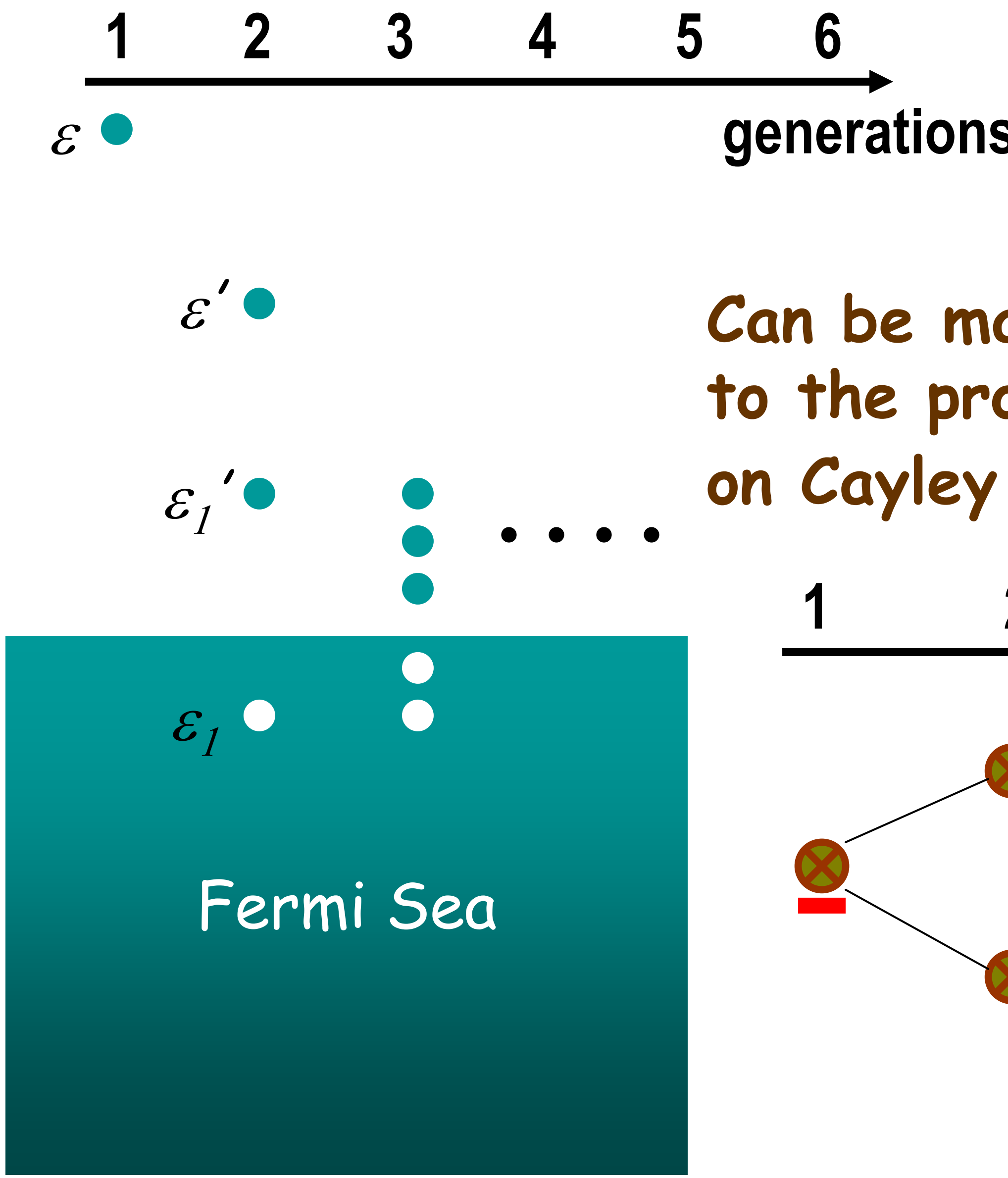
Decay rate of a quasiparticle with energy ε in 0d.



Offdiagonal matrix element

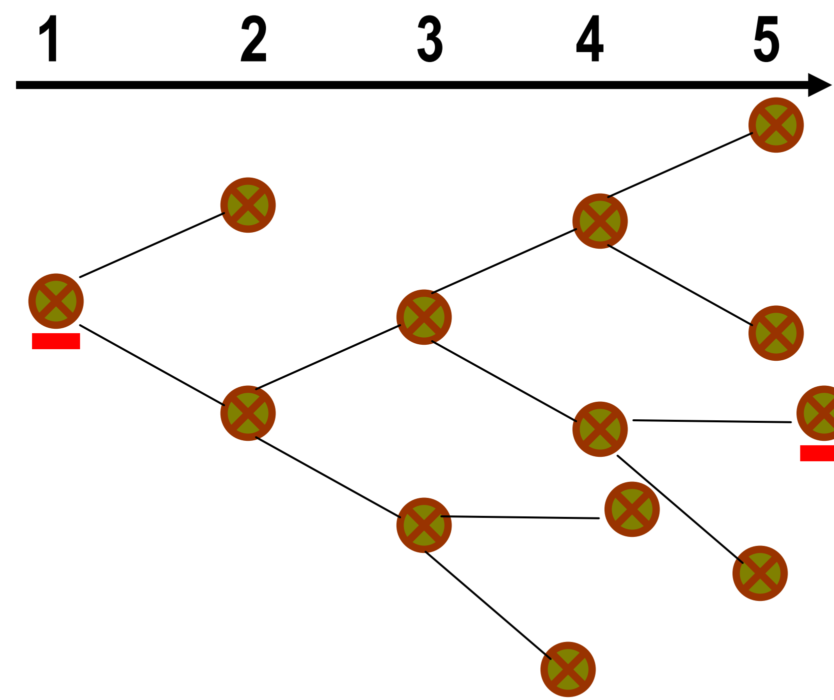
$$M\left(\omega,\varepsilon,\varepsilon'\right)\propto\frac{\delta_1}{g}<<\delta_1$$

Chaos in Nuclei - Delocalization?



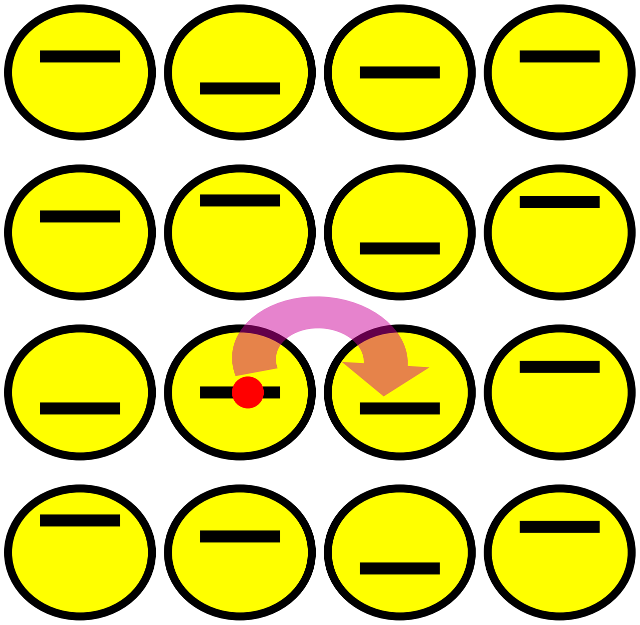
Delocalization in Fock space

Can be mapped (approximately) to the problem of localization on Cayley tree



Conventional Anderson Model

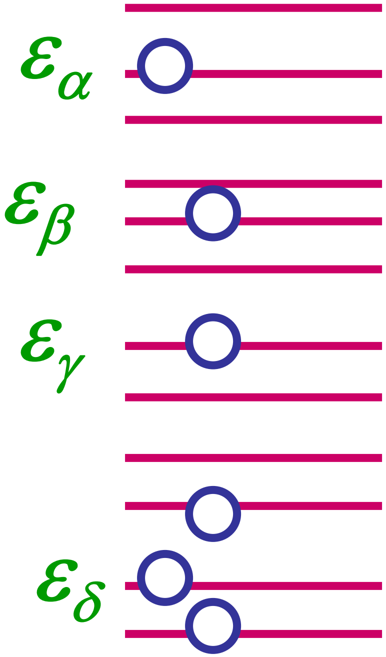
- one particle,
- one level per site,
- onsite disorder
- nearest neighbor hoping



Basis: $|i\rangle$, i **labels sites**

Hamiltonian: $\widehat{H} = \hat{H}_0 + \hat{V}$

$$\hat{H}_0 = \sum_i \varepsilon_i |i\rangle\langle i| \qquad \hat{V} = \sum_{i,j=n.n.} I|i\rangle\langle j|$$



0d system; no interactions

many (N) particles **no interaction:**
Individual energies ε_α and thus occupation numbers $n^{(\alpha)}$ are conserved

↓

N conservation laws
"integrable system"

↓

integrable system

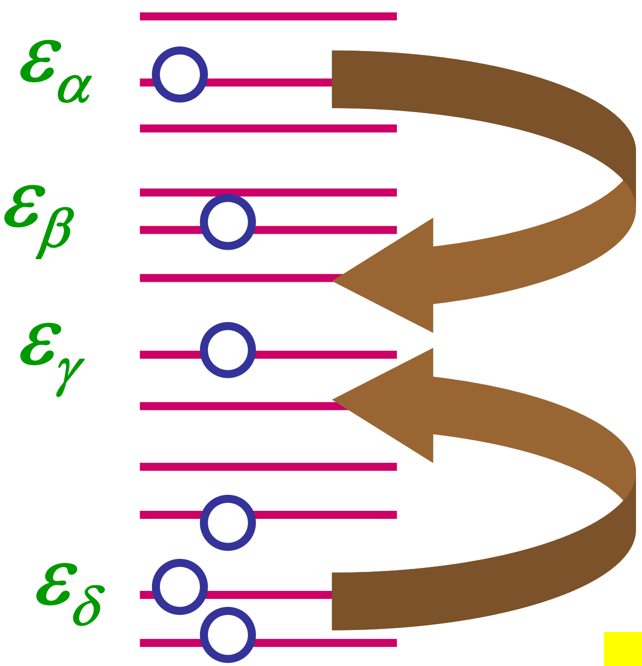
↓

$$\hat{H} = \sum_\mu E_\mu |\mu\rangle\langle\mu|$$

$$\mu = \{n^{(\alpha)}\}$$

$$E_\mu = \sum_\alpha n^{(\alpha)} \varepsilon_\alpha$$

0d system with interactions



Basis: $\left|\mu\right\rangle$ $\qquad \mu = \left\{ n^\alpha \right\}$

$n^\alpha = 0,1$ occupation numbers $\qquad \alpha$ labels levels

Hamiltonian: $\hat{H} = \hat{H}_0 + \hat{V}$

$$\hat{H}_0 = \sum_\mu E_\mu \left|\mu\right\rangle\left\langle\mu\right| \qquad \hat{V} = \sum_{\mu,\eta(\mu)} I \left|\mu\right\rangle\left\langle\nu(\mu)\right|$$

$$\left|\nu(\mu)\right\rangle = \left|..,n^\alpha - 1,..,n^\beta - 1,..,n^\gamma + 1,..,n^\delta + 1,..\right\rangle$$

Conventional Anderson Model

Basis: $\left| i \right\rangle$

i labels sites

$$\hat{H} = \sum_i \varepsilon_i \left| i \right\rangle \left\langle i \right| + \sum_{i,j=n.n.} I \left| i \right\rangle \left\langle j \right|$$

Many body Anderson-like Model

Basis: $\left| \mu \right\rangle, \quad \mu = \left\{ n^\alpha \right\}$

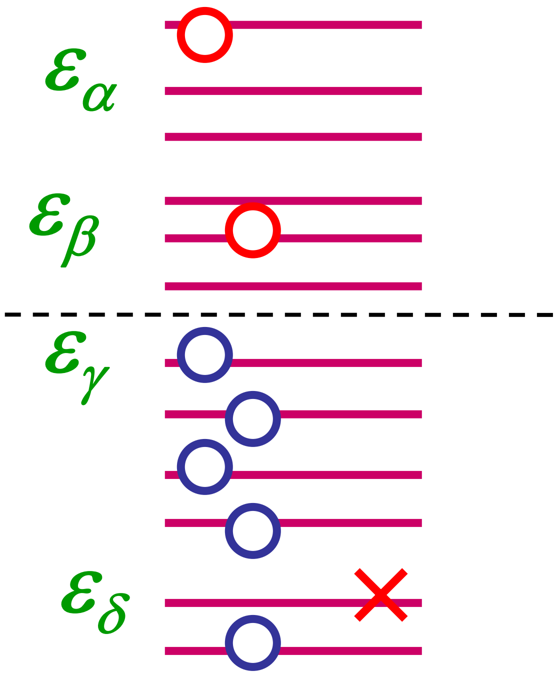
α labels levels

$n^\alpha = 0,1$ occupation numbers

$$\hat{H} = \sum_\mu E_\mu \left| \mu \right\rangle \left\langle \mu \right| + \sum_{\mu,\nu(\mu)} I \left| \mu \right\rangle \left\langle \nu(\mu) \right|$$

"nearest neighbors": $\left| \nu(\mu) \right\rangle = \left| .., n^\alpha - 1, .., n^\beta - 1, .., n^\gamma + 1, .., n^\delta + 1, .. \right\rangle$

0d system with interactions



Basis: $|\mu\rangle$ $\quad \mu = \{n^\alpha\}$

$n^\alpha = 0,1$ occupation numbers $\quad \alpha$ labels levels

Hamiltonian: $\hat{H} = \hat{H}_0 + \hat{V}$

$$\hat{H}_0 = \sum_\mu E_\mu |\mu\rangle\langle\mu|$$

$$\hat{V} = \sum_{\mu,\eta(\mu)} I |\mu\rangle\langle\nu(\mu)|$$

$$|\nu(\mu)\rangle = |.., n^\alpha - 1, .., n^\beta - 1, .., n^\gamma + 1, .., n^\delta + 1, ..\rangle$$

Few excitations ⟹ no recombination ⟹ Cayley tree

Isolated quantum dot – 0d system of fermions

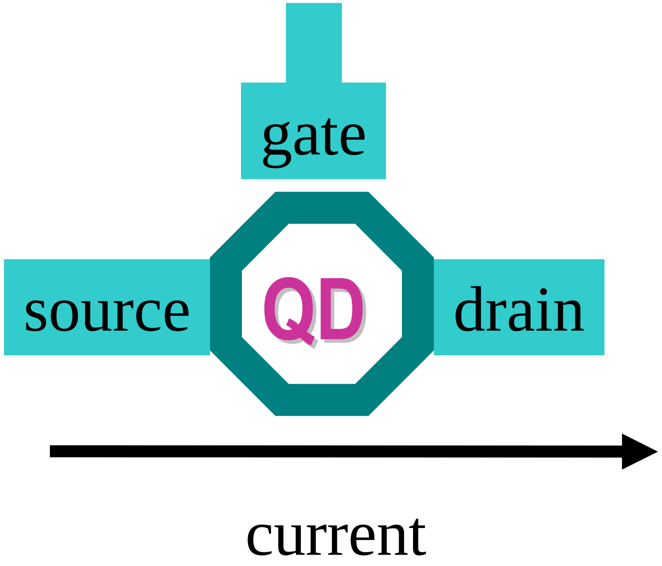
Exact many-body states:
Ground state, excited states

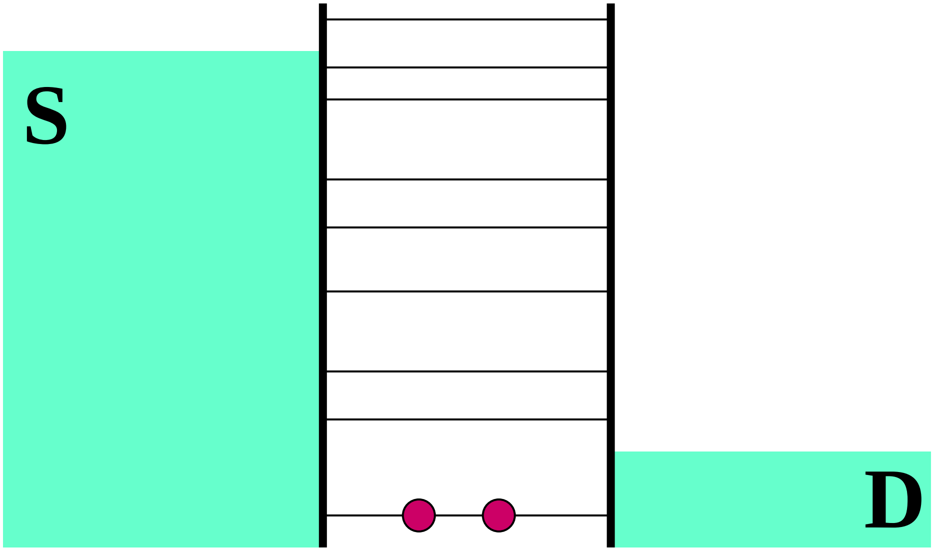
Exact means that the imaginary part of the energy is zero!

Quasiparticle excitations:

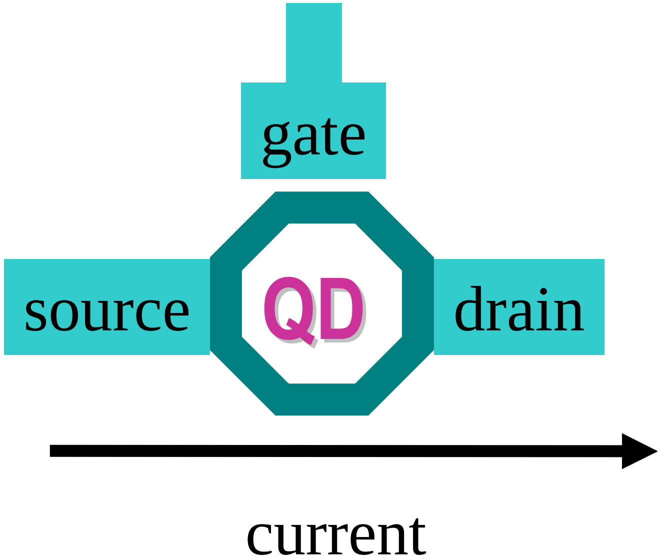
Finite decay rate

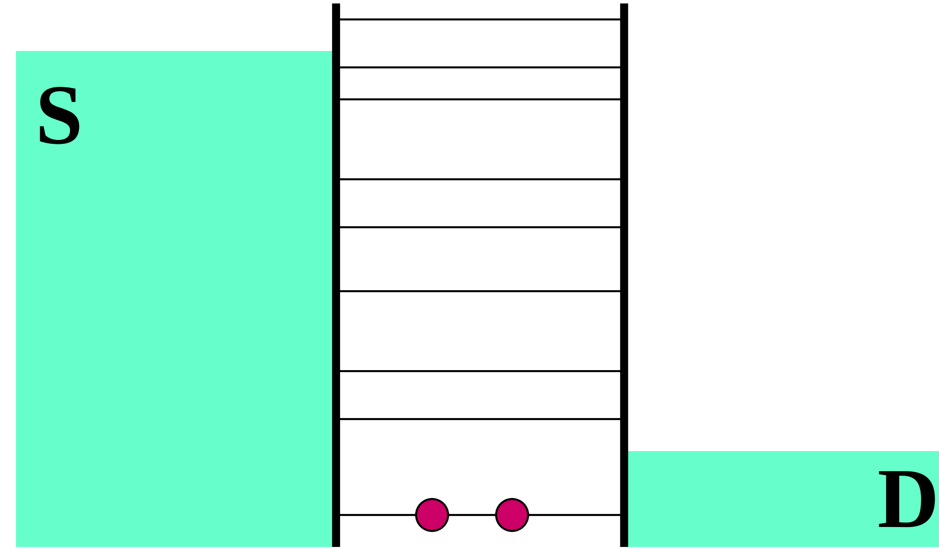
Q: What is the connection ?

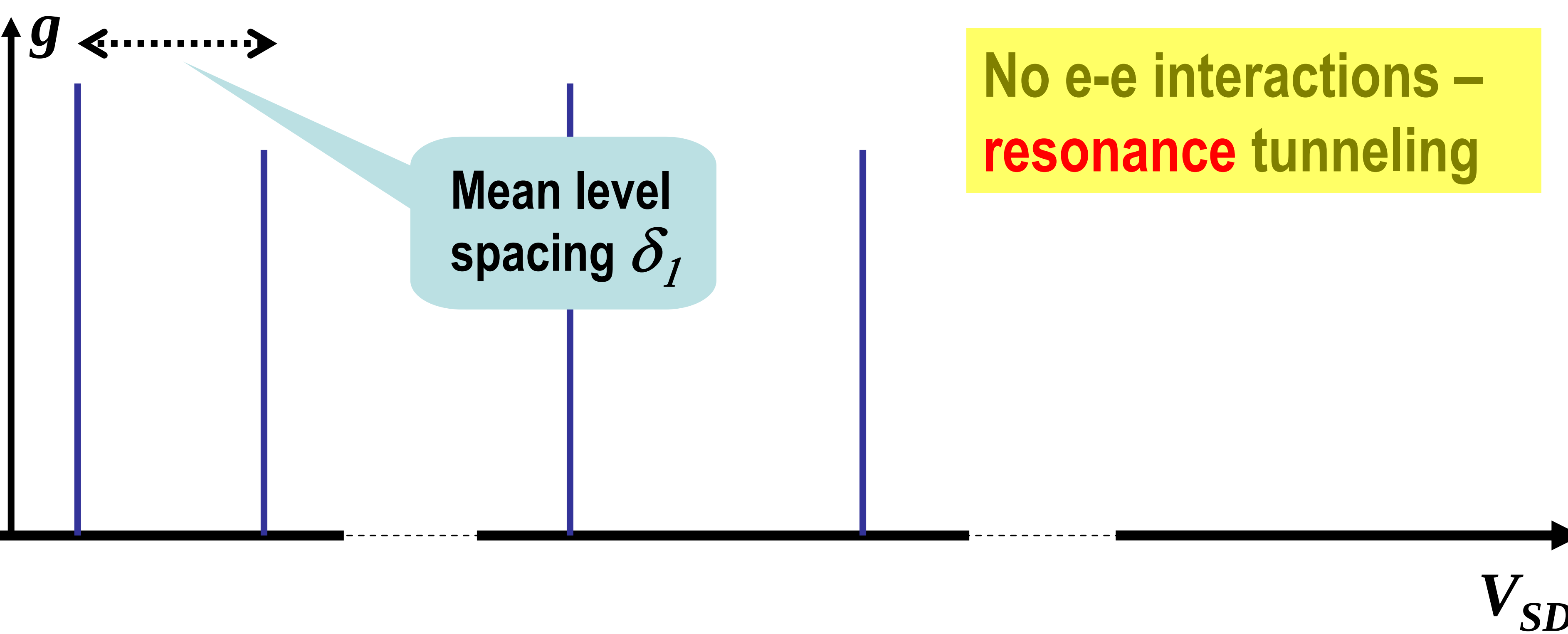




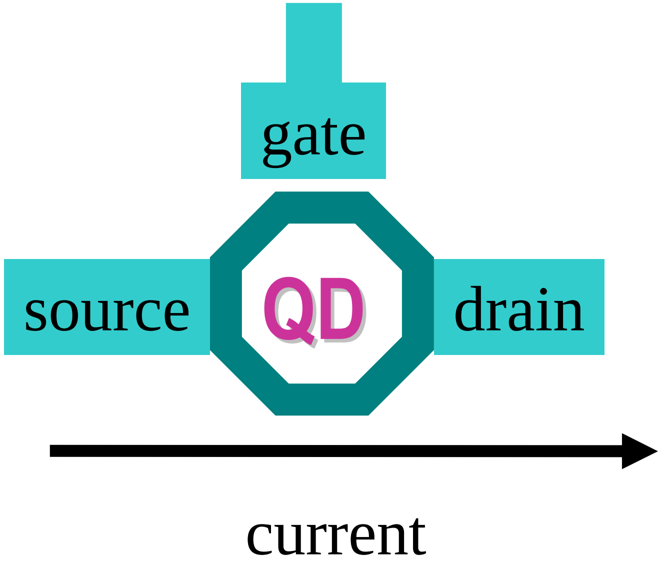
No e-e interactions – resonance tunneling

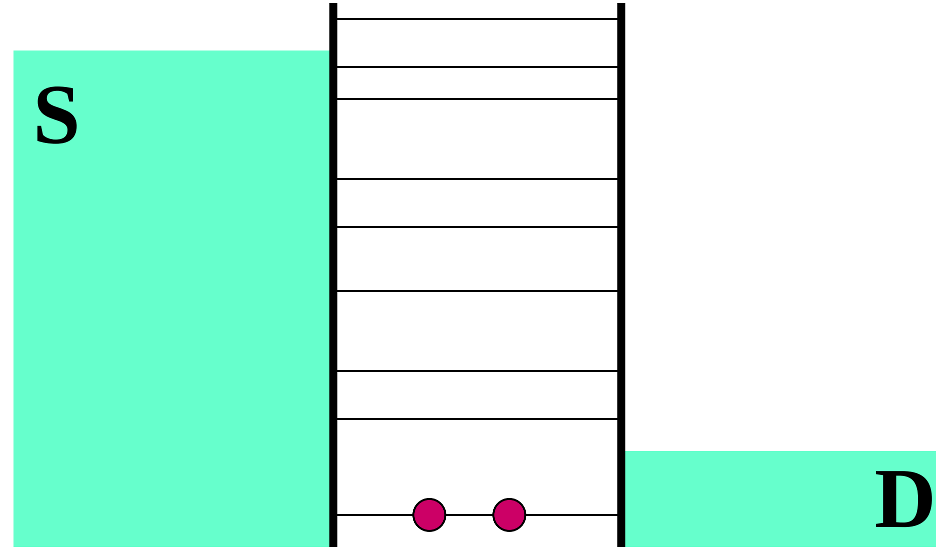




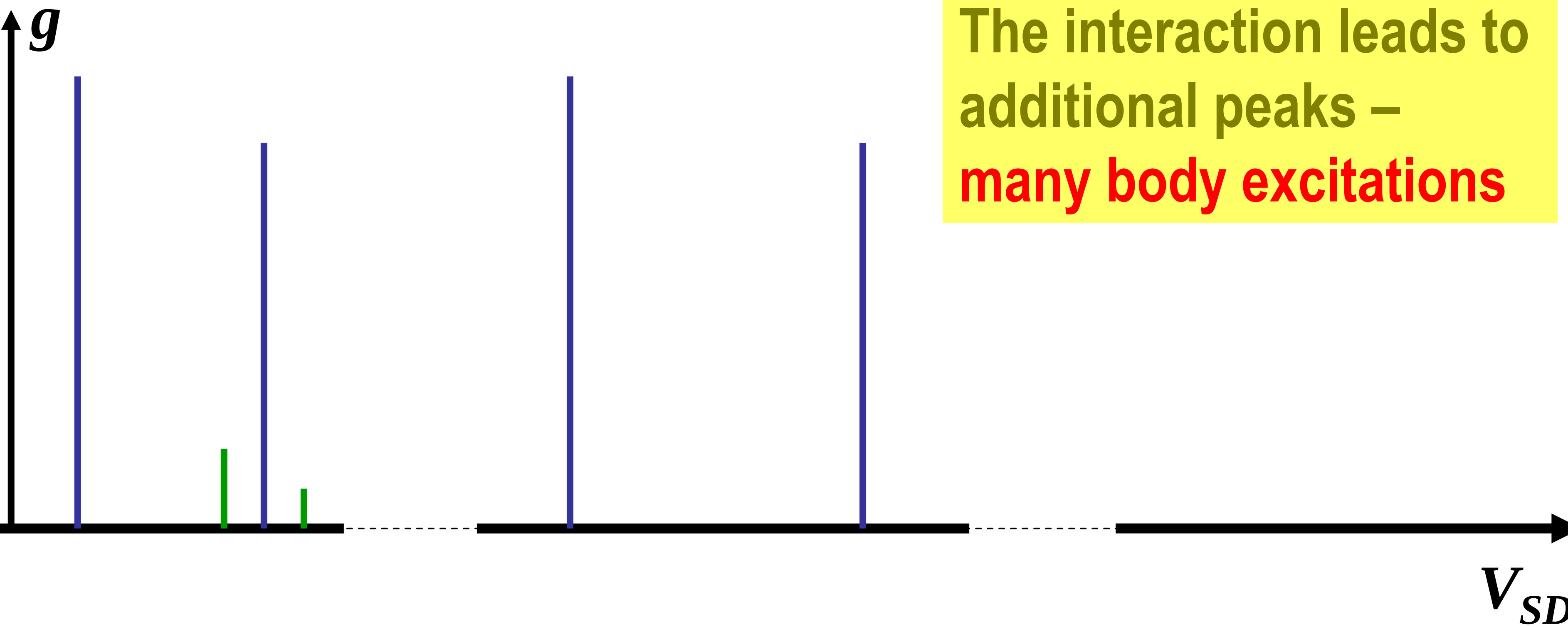


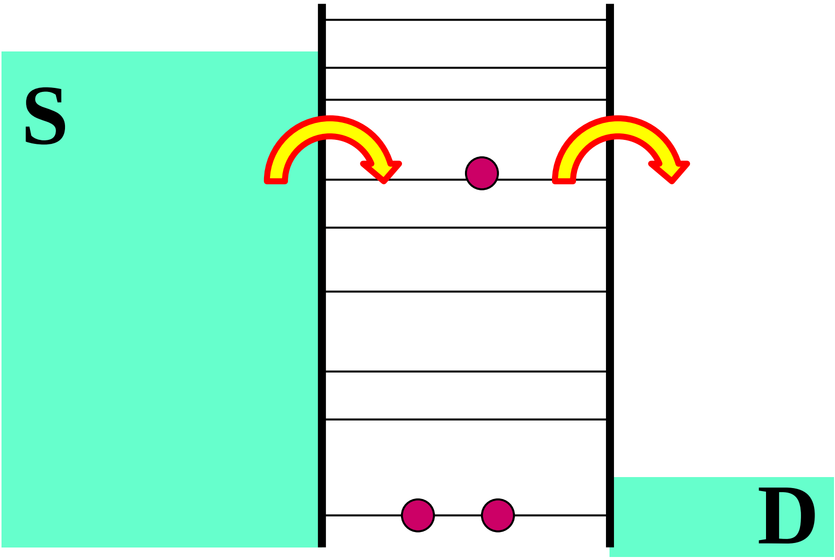
No e-e interactions – resonance tunneling



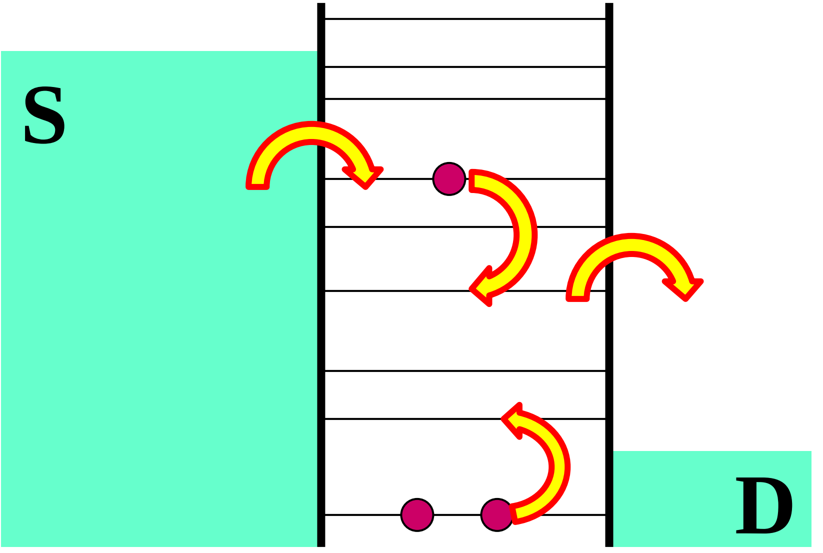


The interaction leads to additional peaks – **many body excitations**

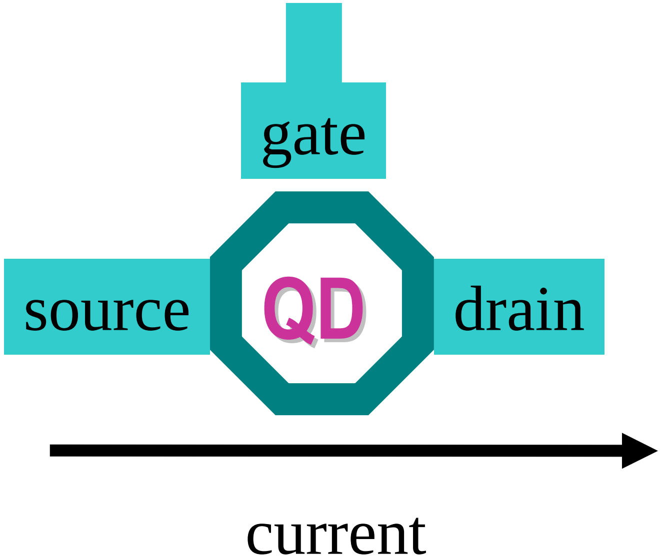


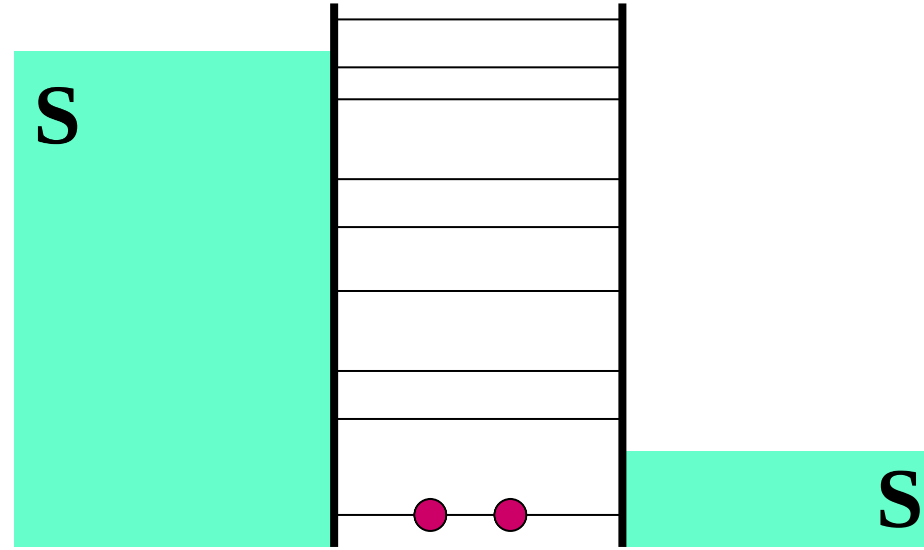


Resonance tunneling
Peaks

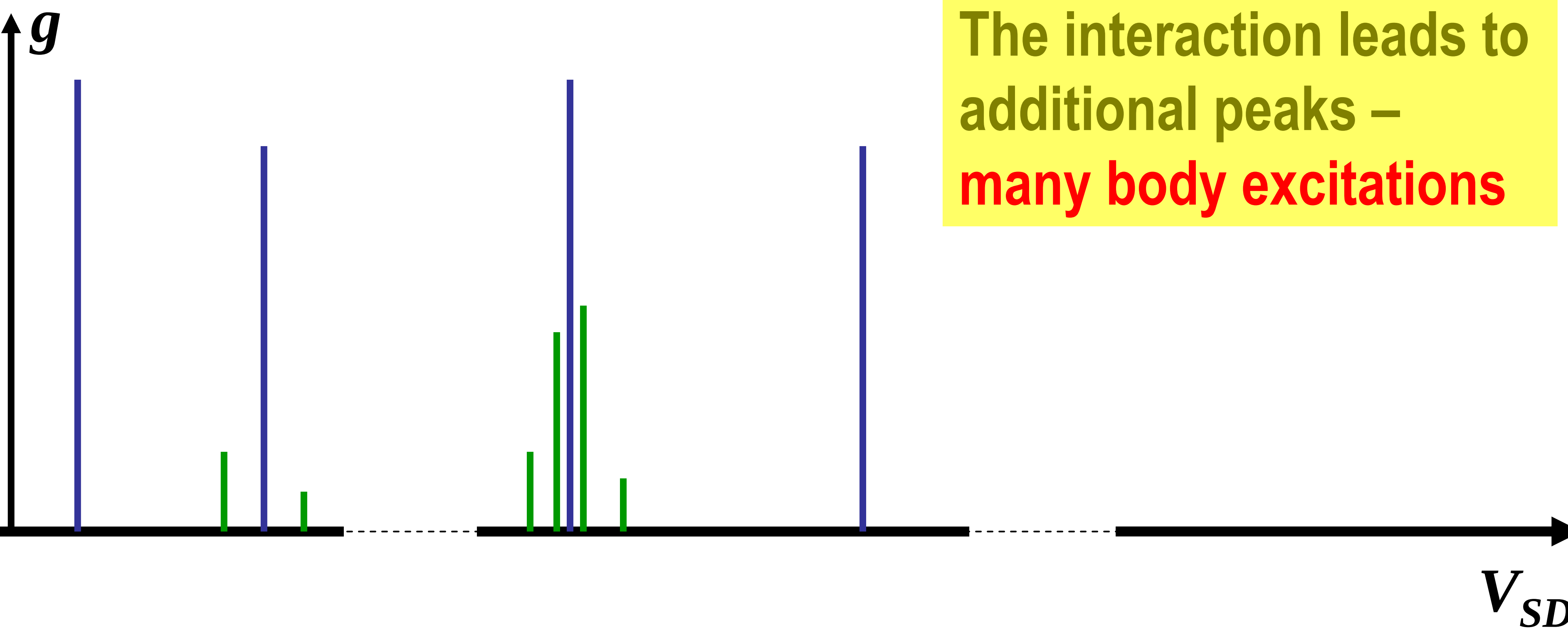


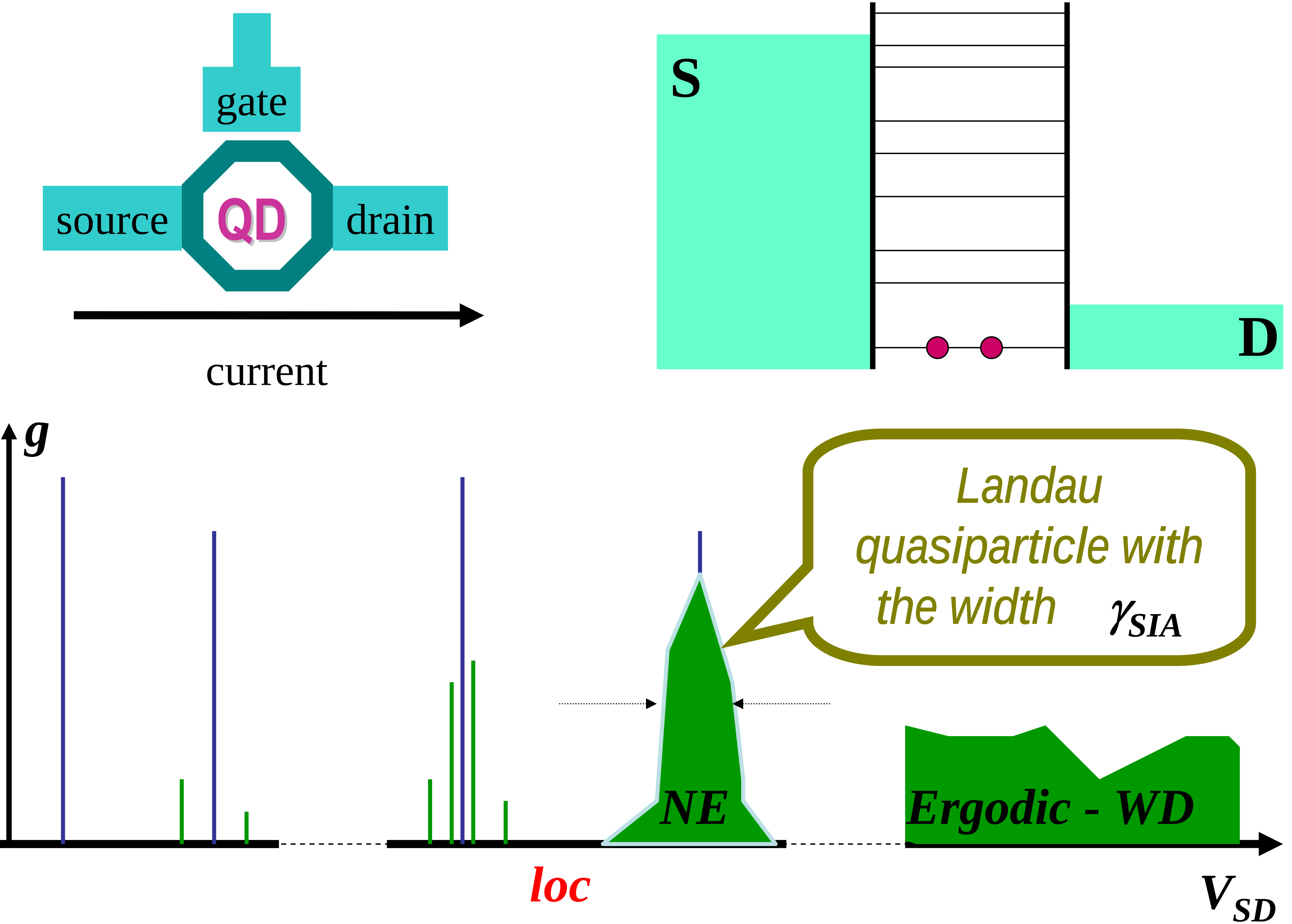
Inelastic cotunneling
Additional peak



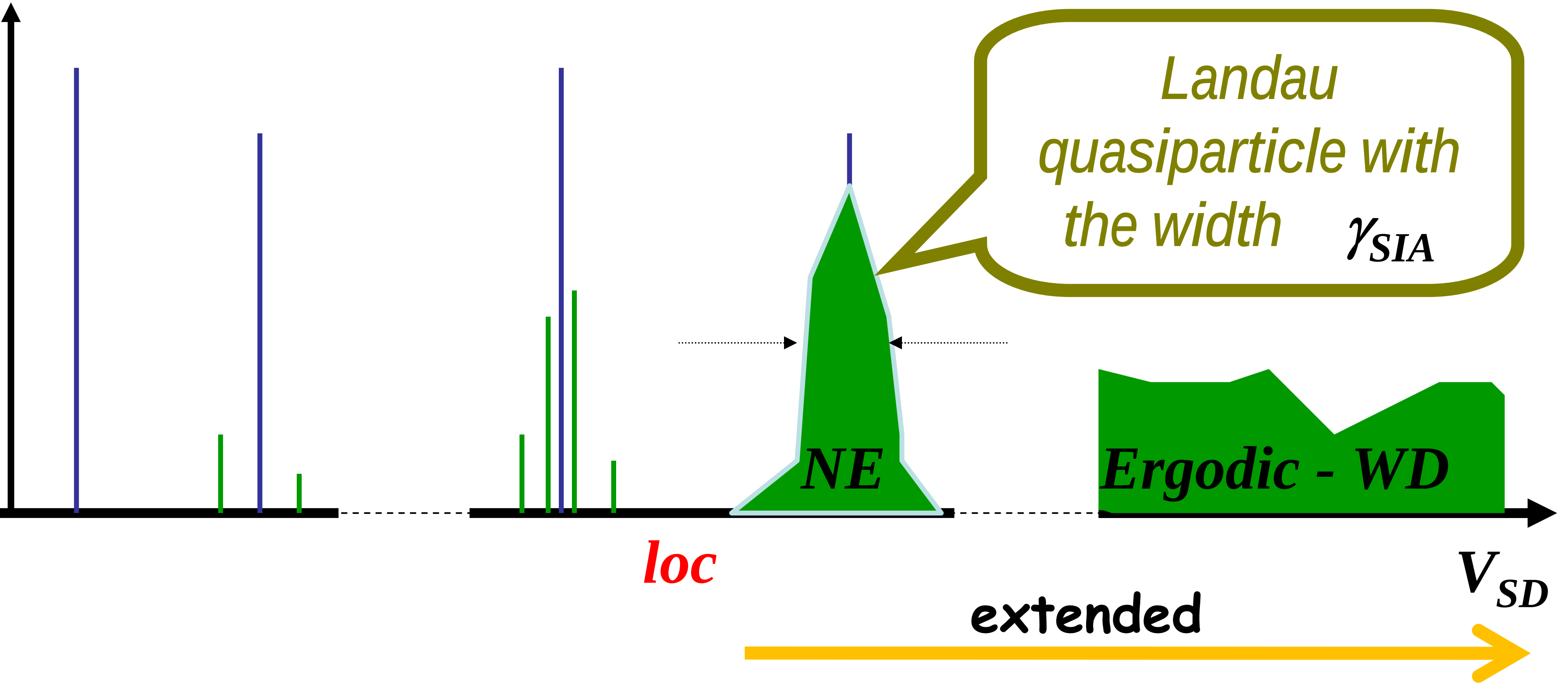


The interaction leads to additional peaks –
many body excitations





gate
source
QD
drain
current
S
D
g
Landau quasiparticle with the width γ_{SIA}
NE
Ergodic - WD
loc
V_{SD}



Localized - finite # of the satelites

Extended - infinite # of the satelites

(for finite ε the number of the satelites is always finite)

Ergodic – nonergodic crossover!

J. Phys. C: Solid State Phys., Vol. 6, 1973. Printed in Great Britain. © 1973

Anderson Model on a Cayley tree

A selfconsistent theory of localization

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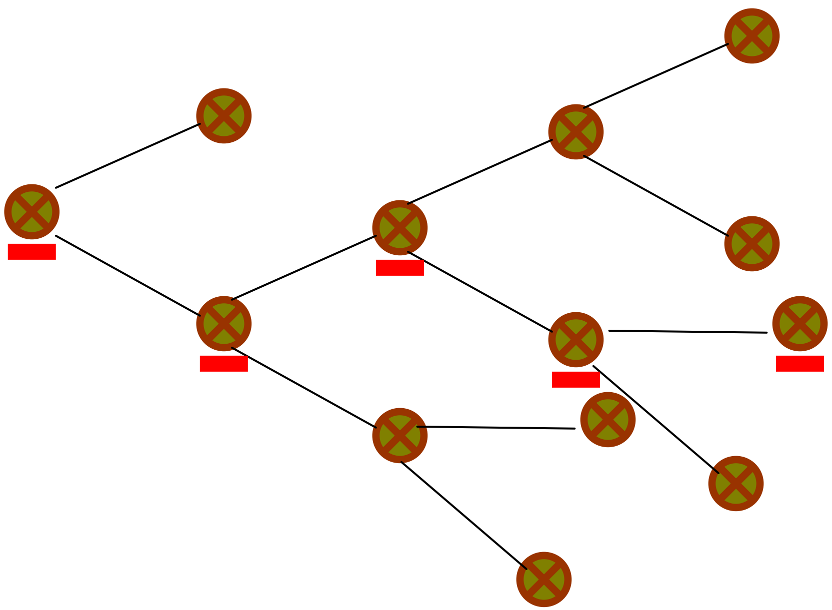
Received 12 January 1973

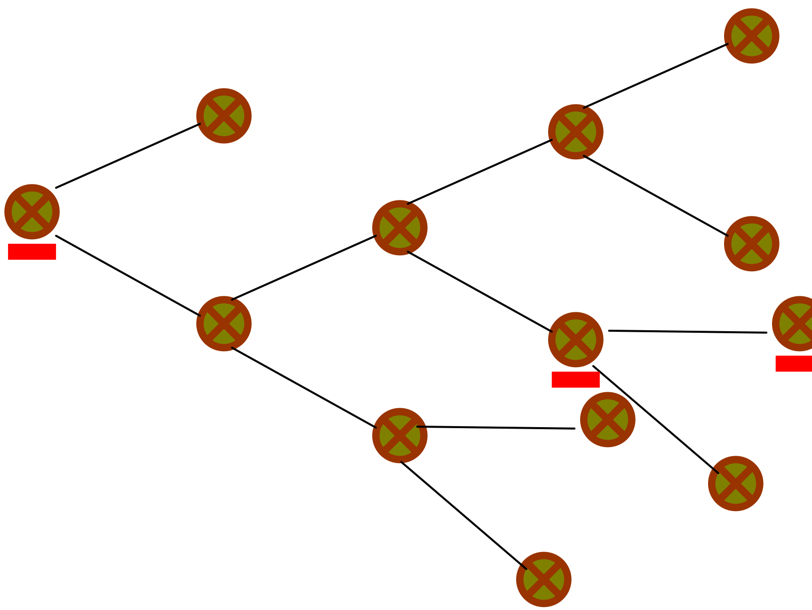
Abstract. A new basis has been found for the theory of localization of electrons in disordered systems. The method is based on a selfconsistent solution of the equation for the self energy in second order perturbation theory, whose solution may be purely real almost everywhere (localized states) or complex everywhere (nonlocalized states). The equations used are exact for a Bethe lattice. The selfconsistency condition gives a nonlinear integral equation in two variables for the probability distribution of the real and imaginary parts of the self energy. A simple approximation for the stability limit of localized states gives Anderson's 'upper limit approximation'. Exact solution of the stability problem in a special case gives results very close to Anderson's best estimate. A general and simple formula for the stability limit is derived; this formula should be valid for smooth distribution of site energies away from the band edge. Results of Monte Carlo calculations of the selfconsistency problem are described which confirm and go beyond the analytical results. The relation of this theory to the old Anderson theory is examined, and it is concluded that the present theory is similar but better.

Anderson Model on a Cayley tree

I, W ***K*** – branching number

$$I_c \propto \frac{W}{K}\frac{1}{\ln K}$$





$$I = \frac{W}{K} +$$

Resonance at every generation

$$\frac{W}{K \ln K} < I < \frac{W}{K}$$

Sparse resonances

Definition: We will call a quantum state $|\mu\rangle$ ergodic if it occupies the number of sites N_μ on the Anderson lattice, which is proportional to the total number of sites N:

$$\frac{N_\mu}{N} \xrightarrow[N\to\infty]{} 0$$

nonergodic

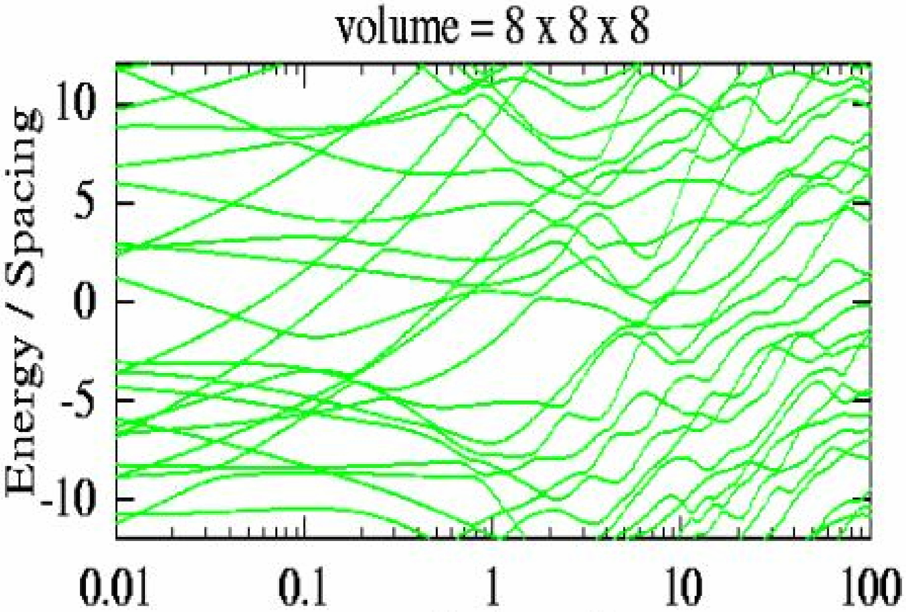
$$\frac{N_\mu}{N} \xrightarrow[N\to\infty]{} const > 0$$

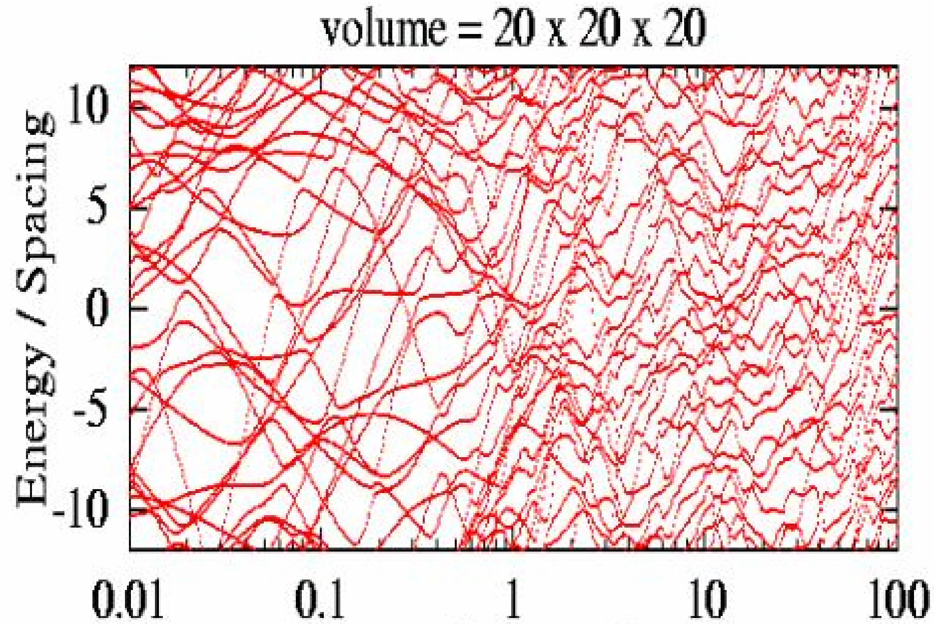
ergodic

Localized states are obviously not ergodic: $N_\mu \xrightarrow[N\to\infty]{} const$

Q: Is each of the extended state ergodic ?

A: In 3D probably yes

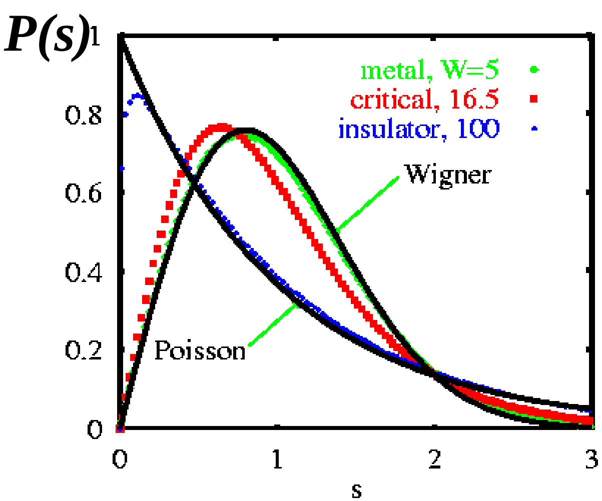


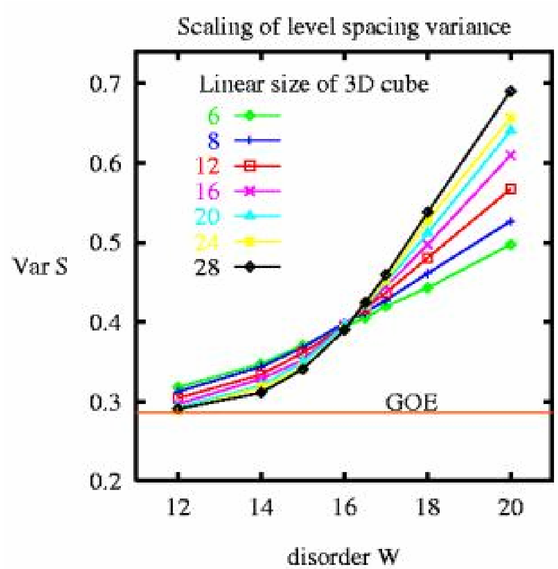


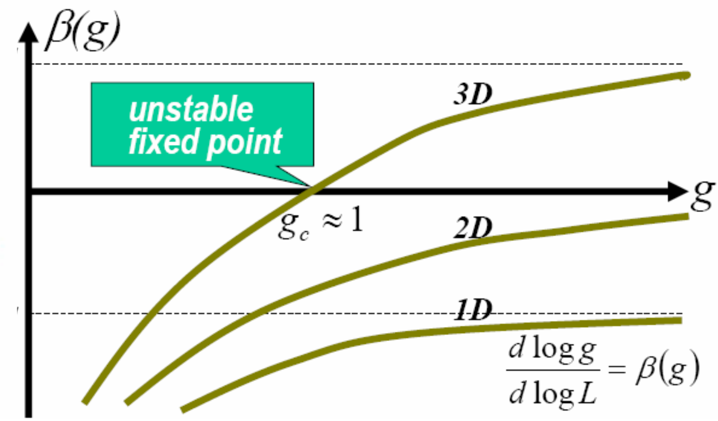
Thouless conductance ***g***

The bigger the system the sharper the transition

In 3D –only critical point. Metal is ergodic Corresponds to the scaling theory







In 3D – the transition is sharp in the limit, when system size tends to infinity, only critical point. Extended states are always ergodic states. This follows from the scaling theory.

This is doubtful already in 4D : variance of the mesoscopic fluctuations

$$\left\langle \left(\delta\sigma\right)^2 \right\rangle \propto \int \frac{d\vec{q}}{q^4}$$

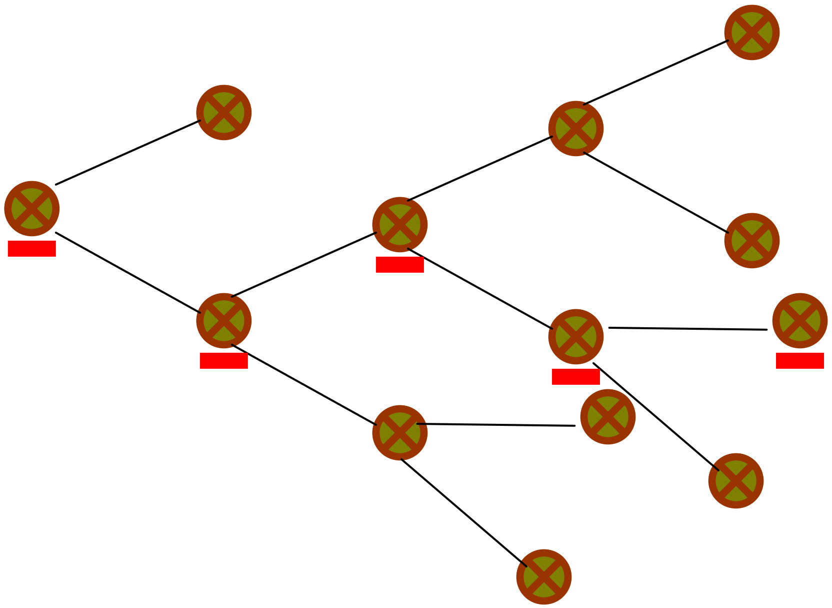
shows ultraviolet divergence.

For very high dimensions close to the transition the extended states are almost for sure nonergodic !

nonergodic states

Such a state occupies infinitely many sites of the Anderson model but still negligible fraction of the total number of sites

Example of nonergodicity: Anderson Model Cayley tree:



transition

K – branching number

$$I_c = \frac{W}{K \ln K}$$

ergodicity

$$n \propto \ln N$$

$$I_{erg} \sim W$$

crossover

$$I < W/(K \ln K)$$

Resonance is typically far $\quad n = const \quad$ localized

$$W/K > I > W/(K \ln K)$$

Resonance is typically far $\quad n \sim \ln N \quad$ nonergodic

$$W > I > W/K$$

Typically there is a resonance at every step $\quad n \sim \ln N \quad$ nonergodic

$$I > W$$

Typically each pair of nearest neighbors is at resonance $\quad n \sim N \quad$ ergodic

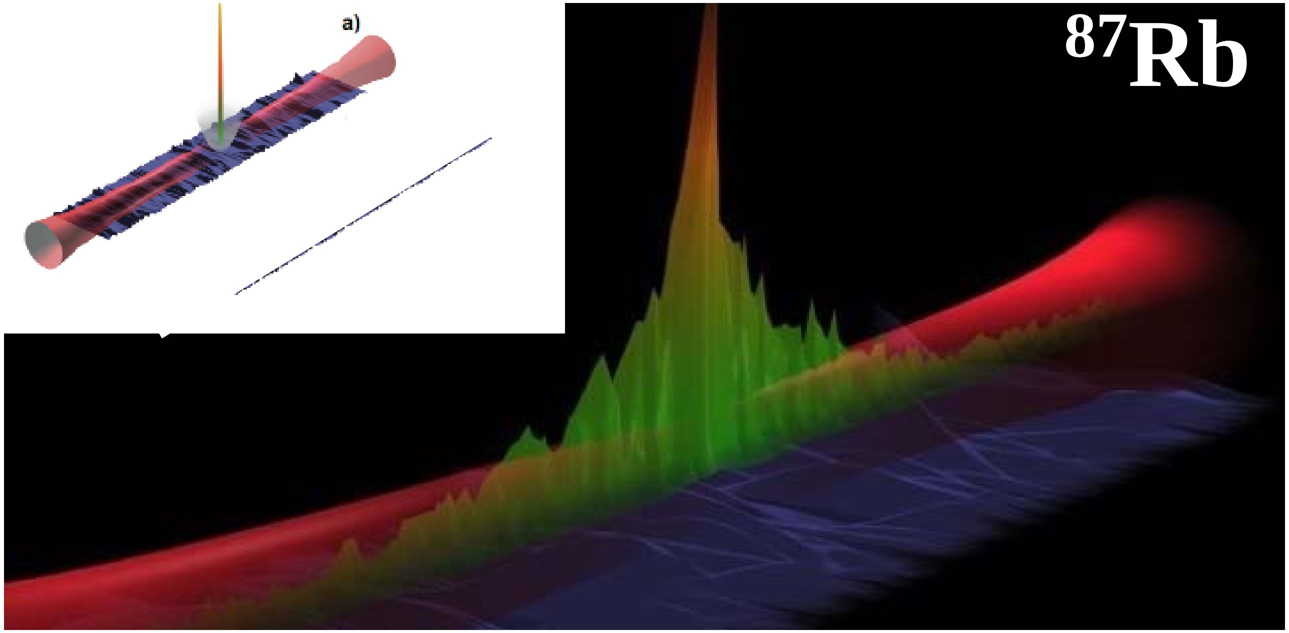
Lecture 2.

3. Many-Body localization

Experiment

Cold Atoms

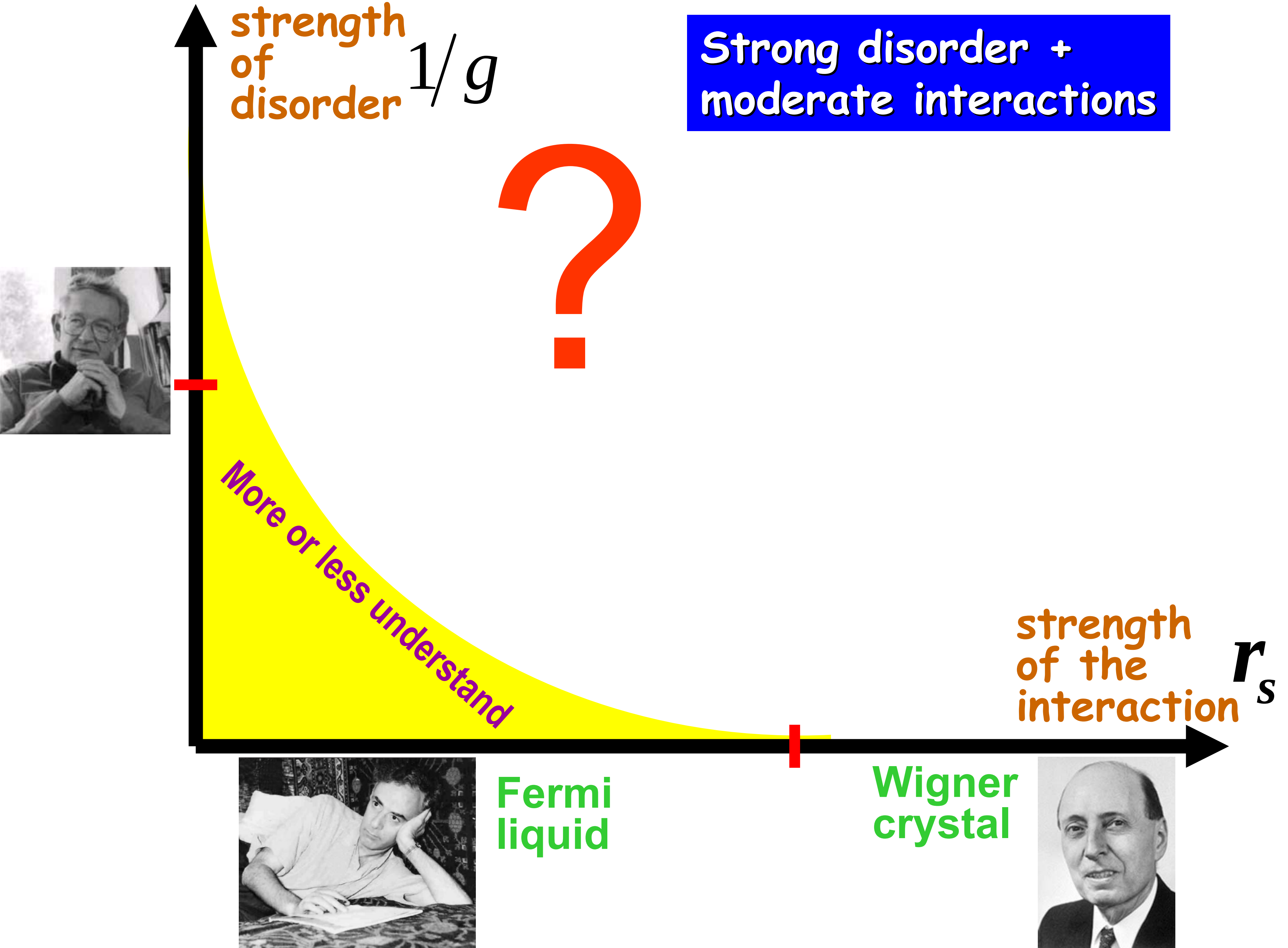
J. Billy, V. Josse, Z. Zuo, A. Bernard, B. Hambrecht, P. Lugan1, D.Clément, L.Sanchez-Palencia, P. Bouyer & A. Aspect, "Direct observation of Anderson localization of matter-waves in a controlled Disorder" ***Nature*** **453, 891-894 (12 June 2008)**



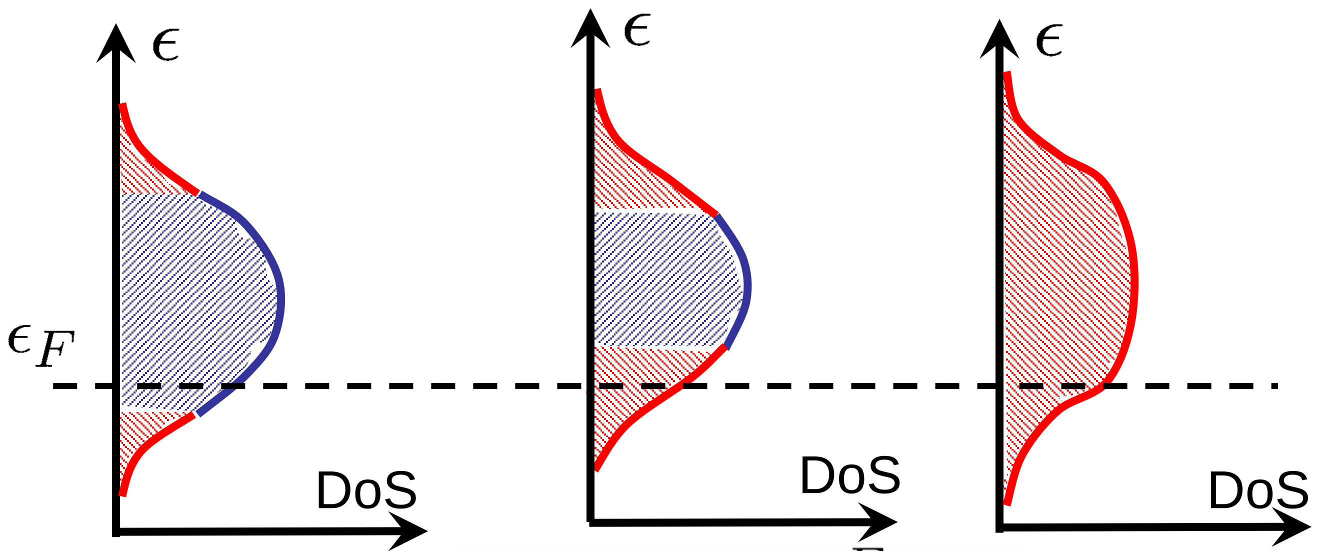
L. Fallani, C. Fort, M. Inguscio: "Bose-Einstein condensates in disordered potentials" arXiv:0804.2888

Q: What about electrons ?

A: Yes,… but electrons interact with each other


strength of disorder $1/g$
Strong disorder + moderate interactions
?
More or less understand
strength of the interaction r_s
Fermi liquid
Wigner crystal

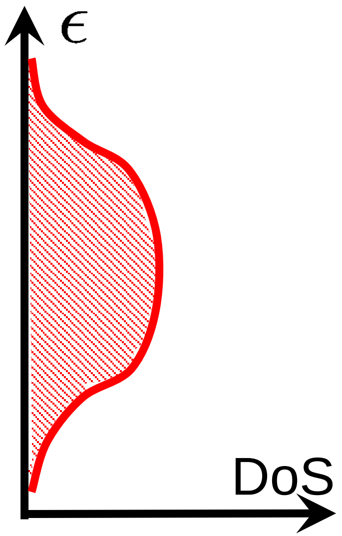
Temperature dependence of the conductivity of noninteracting electrons



$$\sigma(T \to 0) > 0 \qquad \sigma(T) \propto e^{-\frac{E_c - \epsilon_F}{T}} \qquad \sigma(T) = 0$$

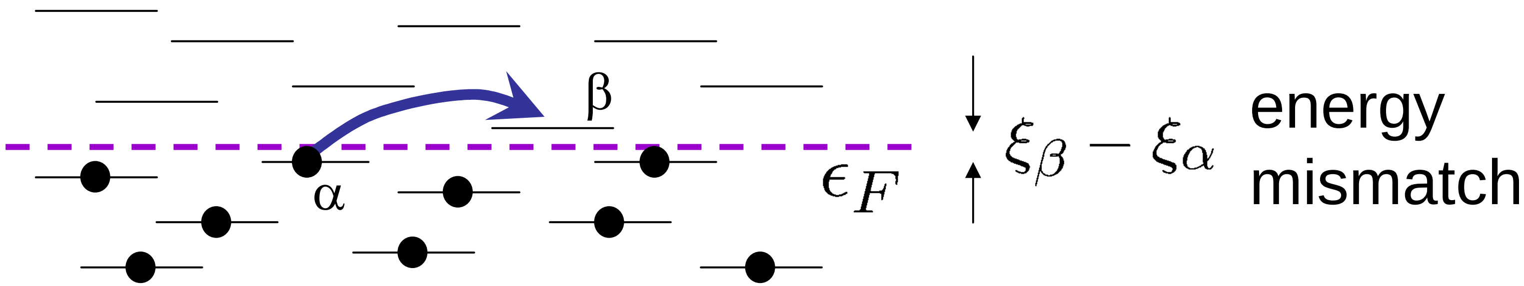
Temperature dependence of the conductivity
one-electron picture

Assume that all the states are localized



$$\sigma(T) = 0 \quad \forall T$$

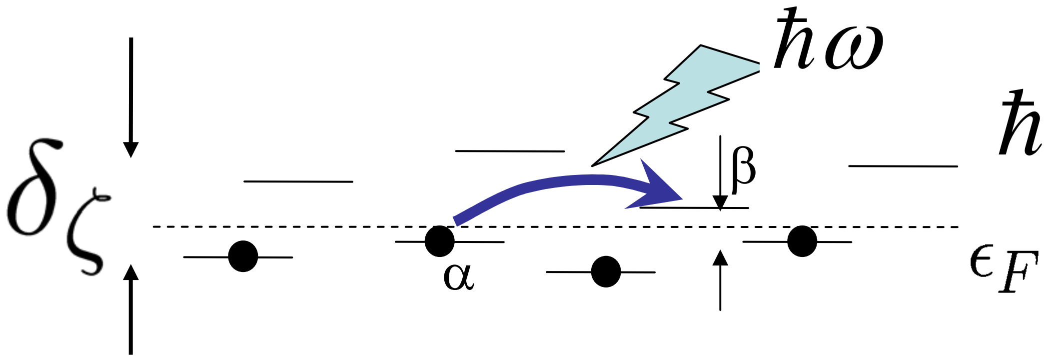
Inelastic processes
transitions between localized states



$$T = 0 \quad \Rightarrow \quad \sigma = 0$$

$$T > 0 \quad \Rightarrow \quad \sigma = ?$$

Phonon-assisted hopping



$$\hbar\omega = \varepsilon_\alpha - \varepsilon_\beta$$

Variable Range Hopping

N.F. Mott (1968)

$$\sigma(T) \propto T^{\gamma} \exp\left[-\left(\frac{\delta_\zeta}{T}\right)^{\frac{1}{d+1}}\right]$$

Mechanism-dependent prefactor

Optimized phase volume

Any bath with a continuous spectrum of delocalized excitations down to $\omega = 0$ will give the same exponential

Common belief: Anderson Insulator weak e-e interactions ⇒ Phonon assisted hopping transport

Can hopping conductivity exist **without phonons** ?

Given:

1. All one-electron states are localized
2. Electrons interact with each other
3. The system is closed (no phonons)
4. Temperature is low but finite

Find: DC conductivity $\sigma(T, \omega = 0)$ (zero or finite?)

Q: Can e-h pairs lead to phonon-less variable range hopping in the same way as phonons do ?

Q: Can e-h pairs lead to phonon-less variable range hopping in the same way as phonons do ?

A#1: Sure

1. Recall phonon-less *AC conductivity:*
   N.F. Mott (1970)

$$\sigma(\omega) = \frac{e^2 \zeta_{loc}^{d-2}}{\hbar} \left(\frac{\hbar\omega}{\delta_\zeta}\right)^2 \ln^{d+1}\left|\frac{\delta_\zeta}{\hbar\omega}\right|$$

2. FDT: there should be Nyquist noise

3. Use this noise as a bath instead of phonons

4. Self-consistency (whatever it means)

Q: Can e-h pairs lead to phonon-less variable range hopping in the same way as phonons do ?

A#1: **Sure**

A#2: **No way** (L. Fleishman. P.W. Anderson (1980))

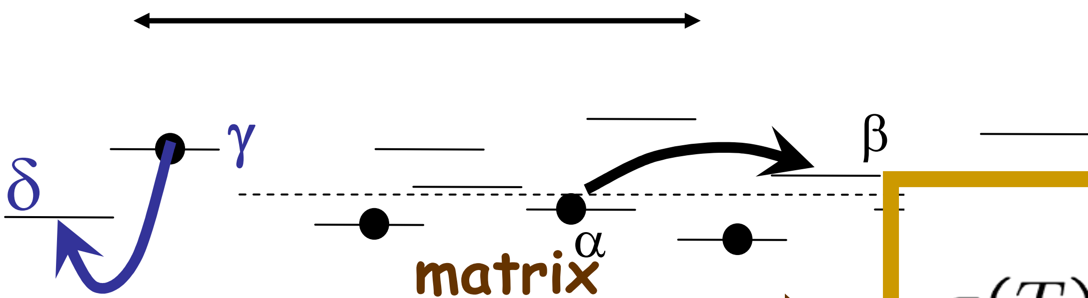
Except maybe Coulomb interaction in 3D

$$\sigma(\omega) \simeq \frac{e^2 \zeta_{loc}^{d-2}}{\hbar}\left(\frac{\hbar\omega}{\delta_\zeta}\right)^2 \ln^{d+1}\left|\frac{\delta_\zeta}{\hbar\omega}\right|$$

is contributed by rare resonances

$R \to \infty$

$$\omega = \xi_\beta - \xi_\alpha = \xi_\gamma - \xi_\delta$$



$R \to \infty \Rightarrow$ matrix element vanishes

$$\sigma(T) \propto 0 \times \exp\left[-\left(\frac{\delta_\zeta}{T}\right)^{\frac{1}{d+1}}\right]$$

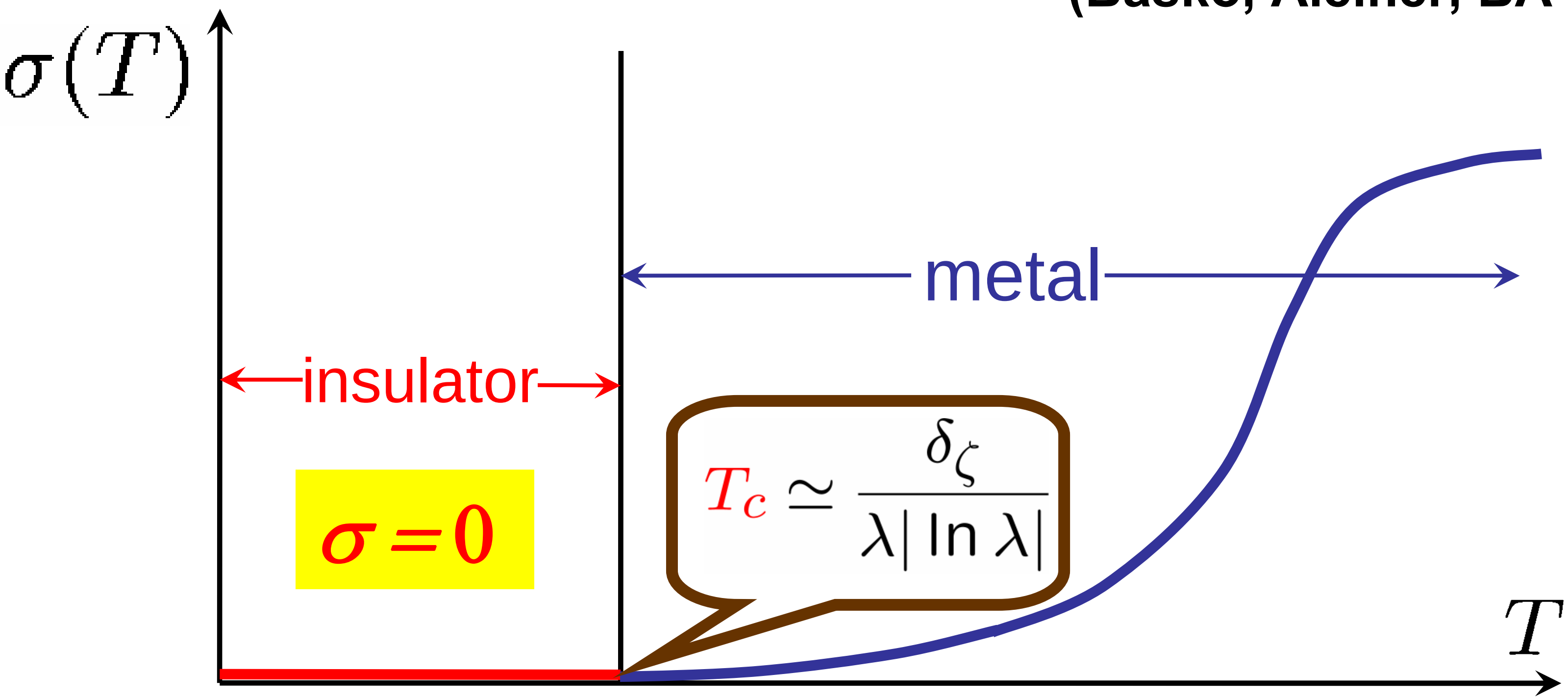
Q: Can e-h pairs lead to **phonon-less** variable range hopping in the same way as phonons do ?

A#1: Sure

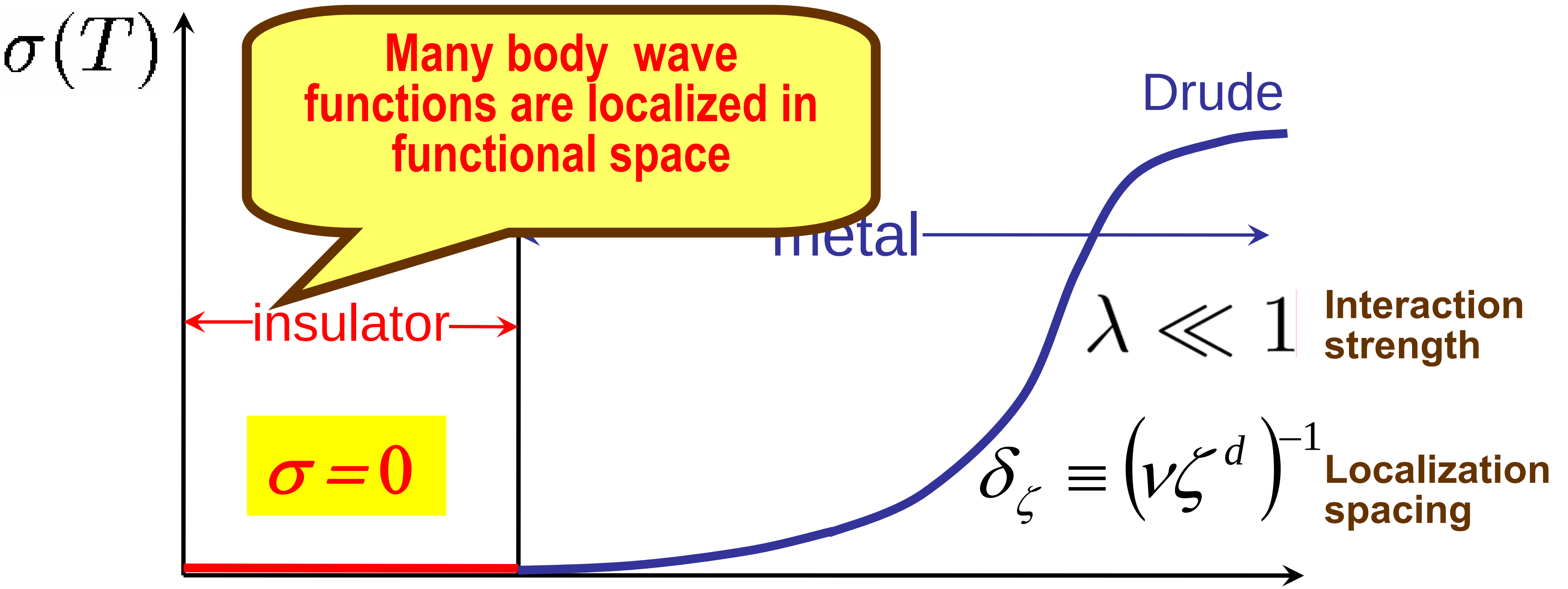
A#2: No way **(L. Fleishman. P.W. Anderson (1980))**

A#3: Finite temperature Metal-Insulator Transition

(Basko, Aleiner, BA (2006))



Finite temperature Metal-Insulator Transition



$$T_c \simeq \frac{\delta_\zeta}{\lambda |\ln \lambda|}$$

D.M. Basko, I.L. Aleiner & BA,
Annals of Phys. 321, 1126 (2006)
cond-mat/0506617 v1 23 Jun 2005

Main postulate of the Gibbs Statistical Mechanics – equipartition (microcanonical distribution):

In the equilibrium all states with the same energy are realized with the same probability.

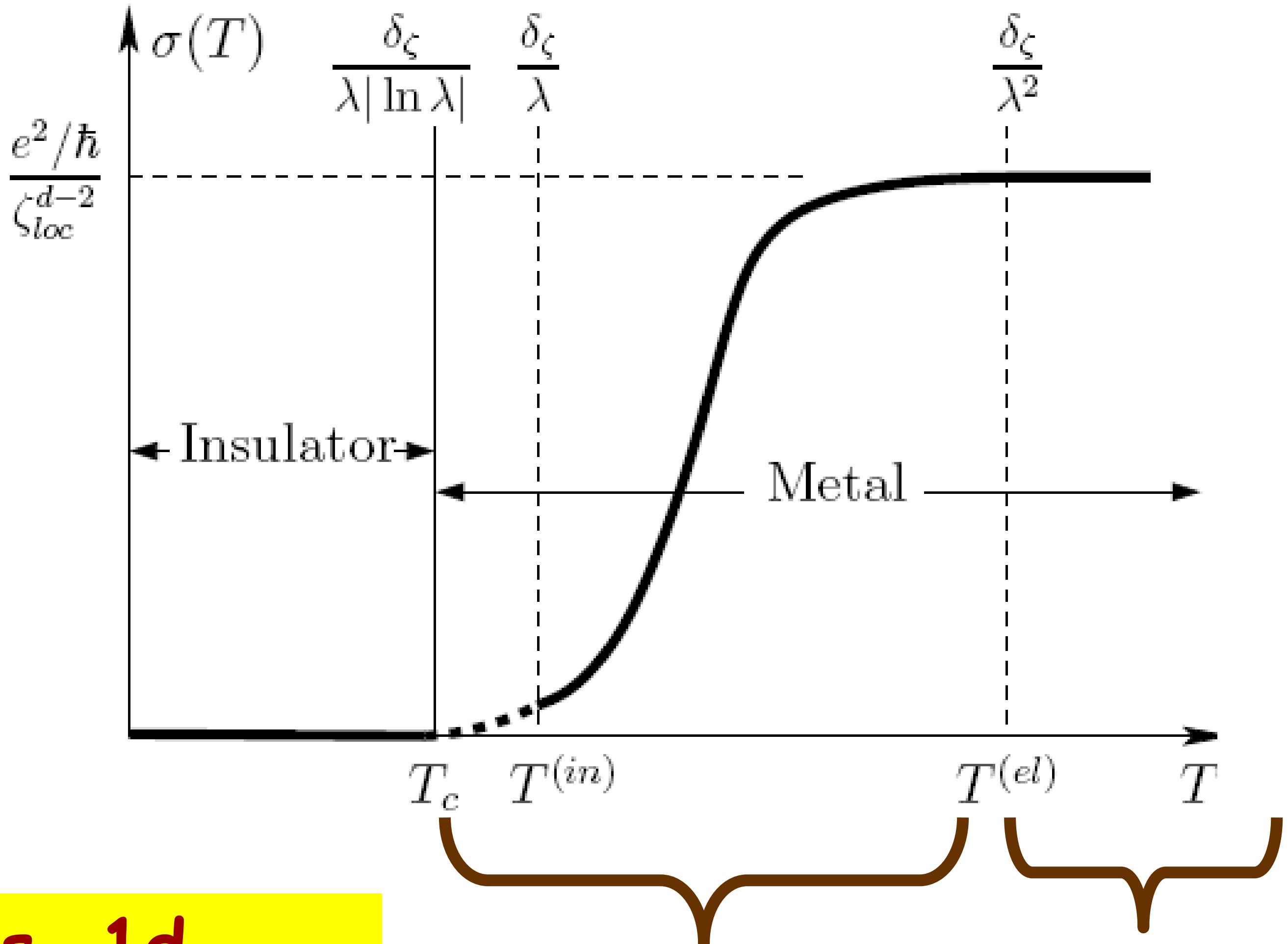
Without interaction between particles the equilibrium would never be reached – each one-particle energy is conserved.

Common believe: Even weak interaction should drive the system to the equilibrium.
Is it always true?

Many-Body Localization:

1. It is not localization in a real space!
2. There is **no relaxation** in the localized state in the same way as wave packets of localized wave functions do not spread.

Finite temperature Metal-Insulator Transition



Includes, 1d case, although is not limited by it.

Bad metal

Good (Drude) metal

There can be no finite temperature phase transitions in one dimension!

This is a dogma.

Justification:

1. Another dogma: every phase transition is connected with the appearance (disappearance) of a long range order

2. Thermal fluctuations in 1d systems destroy any long range order, lead to exponential decay of all spatial correlation functions and thus make phase transitions impossible

There can be no finite temperature phase transitions connected to any long range order in one dimension!

Neither metal nor Insulator are characterized by any type of long range order or long range correlations.

Nevertheless these two phases are distinct and the transition takes place at finite temperature.