Attractive Hubbard model with disorder and the generalized Anderson theorem

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Using the generalized DMFT+Σ approach we have studied disorder influence on single–particle properties of the normal phase and superconducting transition temperature in attractive Hubbard model. The wide range of attractive potentials U was studied — from the weak coupling region, where both the instability of the normal phase and superconductivity are well described by BCS model, towards the strong coupling region, where superconducting transition is due to Bose – Einstein condensation (BEC) of compact Cooper pairs, formed at temperatures much higher than the temperature of superconducting transition. We have studied two typical models of conduction band with semi – elliptic and flat densities of states, appropriate for three-dimensional and two-dimensional systems respectively. For semi – elliptic density of states disorder influence on all single-particle properties (e.g. density of states) is universal for arbitrary strength of electronic correlations and disorder and is due only to the general disorder widening of conduction band. In the case of flat density of states universality is absent in general case, but still the disorder influence is due mainly to band widening and universal behavior is restored for large enough disorder. Using the combination of DMFT+Σ and Nozieres – Schmitt-Rink approximations we have studied disorder influence upon superconducting transition temperature T_c for the range of characteristic values of U and disorder, including the BCS-BEC crossover region and the limit of strong coupling. Disorder can either suppress T_c (in the weak coupling region) or significantly increase T_c (in strong coupling region). However in all cases the generalized Anderson theorem is valid and all changes of superconducting critical temperature are essentially due only to the general disorder widening of the conduction band.

I. INTRODUCTION

The problem of strong coupling superconductivity was studied for a long time, starting with pioneering papers by Eagles and Leggett [1,2]. Significant progress here was achieved by Nozieres and Schmitt-Rink [3], who suggested an effective method to study the transition temperature crossover from weak coupling BCS-like behavior towards Bose – Einstein condensation (BEC) scenario in the strong coupling region. Recent progress in experimental studies of quantum gases in magnetic and optical dipole traps, as well as in optical lattices, with controllable parameters, such as density and interaction strength (cf. reviews [4,5]), has increased the interest to superconductivity (superfluidity of Fermions) with strong pairing interaction, including the region of BCS – BEC crossover. One of the simplest models allowing the study of BCS – BEC crossover is the Hubbard model with attractive on site interaction. The most successive approach to the solution of Hubbard model, both in the case of repulsive interaction and for the studies of BCS – BEC crossover in case of attraction, is the dynamical mean field theory (DMFT) [6–8]. Attractive Hubbard model was studied within DMFT in a number of recent papers [9–13]. However, up to now there are only few studies of disorder influence on the properties of normal and superconducting phases in this model, especially in the region of BCS – BEC crossover. Qualitatively disorder effects in this region were analyzed in Ref. [14], where it was argued, that Anderson theorem remains valid in BCS – BEC crossover region in the case of s-wave pairing.

Diagrammatic approach to (weak) disorder effects upon superconducting transition temperature and properties of the normal phase in crossover region was developed recently in Ref. [15].

In recent years we have developed the generalized DMFT+Σ approach to Hubbard model [16–19], which is very convenient for the studies of different external (with respect to those taken into account in DMFT) interactions (such as pseudogap fluctuations [16–19], disorder [20,21], electron – phonon interaction [22]) etc. This approach is also well suited to analyze two–particle properties, such as optical (dynamic) conductivity [20,23]. In Ref. [13] we have used this approximation to calculate single – particle properties of the normal phase and optical conductivity in attractive Hubbard model. In a recent paper [24] DMFT+Σ approach was used by us to study disorder influence upon superconducting transition temperature, which was calculated in Nozieres – Schmitt-Rink approximation. In this paper for the case of semi – elliptic density of states of the "bare" conduction band, which is adequate for three – dimensional systems, we have numerically demonstrated the validity of the generalized Anderson theorem, so that all the changes of critical temperature are controlled only by general widening of the conduction band by disorder.

In this paper we present the analytic proof of such universal influence of disorder (in DMFT+Σ approximation) upon single – particle characteristics and temperature of superconducting transition for the case of semi – elliptic density of states, and also investigate disorder effects in the case of the "bare" band with flat density of states,

qualitatively appropriate for two – dimensional systems. It will be shown, that for the flat band model the universal dependence of single – particle properties and superconducting transition temperature is also realized for the case of strong enough disorder.

II.   DISORDERED HUBBARD MODEL WITHIN DMFT+Σ APPROACH

We consider the disordered nonmagnetic Hubbard model with attractive interaction with Hamiltonian:

$$H = -t \sum_{\langle ij \rangle \sigma} a_{i\sigma}^{\dagger} a_{j\sigma} + \sum_{i\sigma} \epsilon_i n_{i\sigma} - U \sum_i n_{i\uparrow} n_{i\downarrow}, \quad (1)$$

where $t > 0$ is transfer integral between nearest neighbors on the lattice, U represents Hubbard – like on site attraction, $a_{i\sigma}(a_{i\sigma}^{\dagger})$ is annihilation (creation) operator of an electron with spin σ, $n_{i\sigma} = a_{i\sigma}^{\dagger} a_{i\sigma}$ particle number operator on lattice site i, while local on site energies ϵ_i are assumed to be random variables (independent on different lattice sites). For the standard "impurity" diagram technique to be valid we take the Gaussian distribution of energy levels ϵ_i:

$$\mathcal{P}(\epsilon_i) = \frac{1}{\sqrt{2\pi}\Delta} \exp\left(-\frac{\epsilon_i^2}{2\Delta^2}\right) \quad (2)$$

Parameter Δ is the measure of disorder strength, while the Gaussian random field of random on site energy levels (independent on different sites – "white noise" correlation) induces "impurity" scattering, which is analyzed using the standard formalism of averaged Green's functions [25].

The generalized DMFT+Σ approach [16–19] extends the standard dynamical mean field theory (DMFT) [6–8] taking into account an additional "external" self-energy part $\Sigma_{\mathbf{p}}(\varepsilon)$ (in general case momentum dependent), which is due to some additional interaction outside DMFT, and gives an effective method to calculate both single – particle and two – particle properties [20,23]. The success of this generalized approach is based upon the choice of the single – particle Green's function in the following form:

$$G(\varepsilon, \mathbf{p}) = \frac{1}{\varepsilon + \mu - \varepsilon(\mathbf{p}) - \Sigma(\varepsilon) - \Sigma_{\mathbf{p}}(\varepsilon)}, \quad (3)$$

where $\varepsilon(\mathbf{p})$ is the "bare" electronic dispersion, while the complete self – energy is assumed to an additive sum of the local self – energy of DMFT and some "external" self – energy $\Sigma_{\mathbf{p}}(\varepsilon)$, due to neglect of the interference of Hubbard and "external" interactions. This allows the conservation of the standard form of self – consistent equations of the standard DMFT [6–8]. At the same time, at each step of DMFT iterations we consistently recalculate an "external" self – energy $\Sigma_{\mathbf{p}}(\varepsilon)$ using the appropriate approximate scheme, corresponding to the form of an additional interaction, while the local Green's function is also

"dressed" by $\Sigma_{\mathbf{p}}(\varepsilon)$ at each step of the standard DMFT procedure.

For "external" self – energy entering DMFT+Σ cycle for the problem of random scattering by disorder we use the simplest self – consistent Born approximation, neglecting diagrams with crossing "impurity" lines, which gives:

$$\Sigma_{\mathbf{p}}(\varepsilon) \to \tilde{\Sigma}(\varepsilon) = \Delta^2 \sum_{\mathbf{p}} G(\varepsilon, \mathbf{p}), \quad (4)$$

where $G(\varepsilon, \mathbf{p})$ is the single – electron Green's function (3) and Δ is the amplitude of site disorder.

To solve the effective single Anderson impurity problem of DMFT we used the numerical renormalization group approach (NRG) [26].

In the following we consider two models of "bare" conduction band. The first one is the band with semi – elliptic density of states (per unit cell and single spin projection):

$$N_0(\varepsilon) = \frac{2}{\pi D^2} \sqrt{D^2 - \varepsilon^2} \quad (5)$$

where D defined the band half-width. This model is appropriate for three – dimensional system. The second one is the model with the flat density of states, appropriate for two – dimensional case:

$$N_0(\varepsilon) = \begin{cases} \frac{1}{2D} & |\varepsilon| \le D \\ 0 & |\varepsilon| > D \end{cases}. \quad (6)$$

In principle, for two – dimensional systems we should take into account the presence of the weak (logarithmic) Van Hove singularity in the density of states. However, this singularity is effectively suppressed even by rather small disorder, so that the simple model of Eq. 6 is quite sufficient for our aims.

All calculations in this work has been done for the case of quarter – filled band (the number of electrons per lattice site n=0.5).

The temperature of superconducting transition in attractive model was analyzed in a number of papers [9,10,12], both from the condition of instability of the normal phase [9] (divergence of Cooper susceptibility) and from the condition of superconducting order parameter going to zero [10,12]. In a recent paper [13] we have determined the critical temperature from the condition of instability of the normal phase, reflected in the instability of DMFT iteration procedure. The results obtained in this way in fact coincided with those of Refs. [9,10,12]. Also in Ref. [13] to calculate T_c we have used the approach due to Nozieres and Schmitt-Rink [3], which allows the correct (though approximate) description of T_c in BCS – BEC crossover region. In a later work [24] we have used the combination of Nozieres and Schmitt-Rink and DMFT+Σ approximations for the detailed numerical studies of disorder dependence of T_c and the number of local pairs in the model with semi – elliptic density of states.

III. DISORDER INFLUENCE ON SINGLE – PARTICLE PROPERTIES FOR THE CASE OF SEMI–ELLIPTIC DENSITY OF STATES

In this section we shall analytically demonstrate, that in DMFT+Σ approximation disorder influence upon single – particle properties of disordered Hubbard model (both attractive or repulsive) with semi – elliptic "bare" conduction band is completely described by effects of general band widening by disorder scattering.

In the system of self – consistent equations DMFT+Σ equations [17,19,20] both information on the "bare" band and disorder scattering enter only on the stage of calculations of the local Green's function:

$$G_{ii} = \sum_{\mathbf{p}} G(\varepsilon, \mathbf{p}), \qquad (7)$$

where the full Green's function $G(\varepsilon, \mathbf{p})$ is determined by Eq. (3), while the self – energy due to disorder, in self – consistent Born approximation, is defined by Eq. (4). Thus, the local Green's function takes the form:

$$G_{ii} = \int_{-D}^{D} d\varepsilon' \frac{N_0(\varepsilon')}{\varepsilon + \mu - \varepsilon' - \Sigma(\varepsilon) - \Delta^2 G_{ii}} =$$
$$= \int_{-D}^{D} d\varepsilon' \frac{N_0(\varepsilon')}{E_t - \varepsilon'}, \qquad (8)$$

where we have introduced the notation $E_t = \varepsilon + \mu - \Sigma(\varepsilon) - \Delta^2 G_{ii}$. In the case of semi – elliptic density of states (5) this integral is easily calculated in analytic form, so that the local Green's function is written as:

$$G_{ii} = 2\frac{E_t - \sqrt{E_t^2 - D^2}}{D^2}. \qquad (9)$$

It is easily seen that Eq. (9) represents one of the roots of quadratic equation:

$$G_{ii}^{-1} = E_t - \frac{D^2}{4}G_{ii}, \qquad (10)$$

corresponding to the correct limit of $G_{ii} \to E_t^{-1}$ for infinitely narrow ($D \to 0$) band. Then

$$G_{ii}^{-1} = \varepsilon + \mu - \Sigma(\varepsilon) - \Delta^2 G_{ii} - \frac{D^2}{4}G_{ii} =$$
$$= \varepsilon + \mu - \Sigma(\varepsilon) - \frac{D_{eff}^2}{4}G_{ii}, \qquad (11)$$

where we have introduced D_{eff} – an effective half-width of the band (in the absence of electronic correlations, i.e. for $U = 0$) widened by disorder scattering:

$$D_{eff} = D\sqrt{1 + 4\frac{\Delta^2}{D^2}}. \qquad (12)$$

Eq. (10) was obtained from (8), thus comparing (11) and (10), we obtain:

$$G_{ii} = \int_{-D_{eff}}^{D_{eff}} d\varepsilon' \frac{\tilde{N}_0(\varepsilon')}{\varepsilon + \mu - \varepsilon' - \Sigma(\varepsilon)}, \qquad (13)$$

Here

$$\tilde{N}_0(\varepsilon) = \frac{2}{\pi D_{eff}^2}\sqrt{D_{eff}^2 - \varepsilon^2} \qquad (14)$$

represents the density of states in the absence of interaction U "dressed" by disorder. This density of states remains semi – elliptic in the presence of disorder, so that all effects of disorder scattering on single – particle properties of disordered Hubbard model in DMFT+Σ approximation are reduced only to disorder widening of conduction band, i.e. to the replacement $D \to D_{eff}$.

IV. DISORDER INFLUENCE ON SUPERCONDUCTING TRANSITION TEMPERATURE

Temperature of superconducting transition T_c is not a single – particle characteristic of the system. Cooper instability, determining T_c is related to divergence of two – particle loop in Cooper channel. In the weak coupling limit, when superconductivity is due to the appearance of Cooper pairs at T_c, disorder only slightly influences superconductivity with s-wave pairing [27,28]. The so called Anderson theorem is valid and changes of T_c are connected only with the relatively small changes of the density of states by disorder. Th standard derivation of Anderson theorem [27,28] uses the formalism of exact eigenstates of an electron in the random field of impurities. Here we present another derivation of Anderson theorem, using the exact Ward identity, which allows us to derive the equation for T_c, which will be used to calculate T_c in Nozieres – Schmitt-Rink approximation in disordered system.

In general, Nozieres – Schmitt-Rink approach [3] assumes, that corrections due to strong pairing attraction significantly change the chemical potential of the system, while possible correction due to this interaction to Cooper instability condition can be neglected, so that we can always use here the weak coupling (ladder) approximation. In such approximation the condition of Cooper instability in disordered Hubbard model takes the form:

$$1 = U\chi_0(q = 0, \omega_m = 0) \qquad (15)$$

where

$$\chi_0(q = 0, \omega_m = 0) = T\sum_{n}\sum_{\mathbf{pp'}} \Phi_{\mathbf{pp'}}(\varepsilon_n) \qquad (16)$$

represents the two – particle loop (susceptibility) in Cooper channel "dressed" only by disorder scattering, and $\Phi_{\mathbf{pp'}}(\varepsilon_n)$ is the averaged two – particle Green's function in Cooper channel ($\omega_m = 2\pi mT$ and $\varepsilon_n = \pi T(2n + 1)$ are the usual Boson and Fermion Matsubara frequencies).

To obtain $\sum_{\mathbf{pp'}} \Phi_{\mathbf{pp'}}(\varepsilon_n)$ we use the exact Ward identity, derived by us in Ref. [23]:

$$G(\varepsilon_n, \mathbf{p}) - G(-\varepsilon_n, -\mathbf{p}) =$$

$$= -\sum_{\mathbf{p}'} \Phi_{\mathbf{p}\mathbf{p}'}(\varepsilon_n)(G_0^{-1}(\varepsilon_n, \mathbf{p}') - G_0^{-1}(-\varepsilon_n, -\mathbf{p}')), \quad (17)$$

Here $G(\varepsilon_n, \mathbf{p})$ is the impurity averaged (but not containing Hubbard interaction corrections!) single – particle Green's function. Using the obvious symmetry $\varepsilon(\mathbf{p}) = \varepsilon(-\mathbf{p})$ and $G(\varepsilon_n, -\mathbf{p}) = G(\varepsilon_n, \mathbf{p})$, we obtain from the Ward identity (17):

$$\sum_{\mathbf{p}\mathbf{p}'} \Phi_{\mathbf{p}\mathbf{p}'}(\varepsilon_n) = -\frac{\sum_{\mathbf{p}} G(\varepsilon_n, \mathbf{p}) - \sum_{\mathbf{p}} G(-\varepsilon_n, \mathbf{p})}{2i\varepsilon_n}, \quad (18)$$

so that for Cooper susceptibility (16) we have:

$$\chi_0(q = 0, \omega_m = 0) =$$
$$= -T \sum_n \frac{\sum_{\mathbf{p}} G(\varepsilon_n, \mathbf{p}) - \sum_{\mathbf{p}} G(-\varepsilon_n, \mathbf{p})}{2i\varepsilon_n} =$$
$$= -T \sum_n \frac{\sum_{\mathbf{p}} G(\varepsilon_n, \mathbf{p})}{i\varepsilon_n}. \quad (19)$$

Performing now the standard summation over Matsubara frequencies [25], we obtain:

$$\chi_0(q = 0, \omega_m = 0) =$$
$$= -\frac{1}{4\pi i} \int_{-\infty}^{\infty} d\varepsilon \frac{\sum_{\mathbf{p}} G^R(\varepsilon, \mathbf{p}) - \sum_{\mathbf{p}} G^A(\varepsilon, \mathbf{p})}{\varepsilon} th\frac{\varepsilon}{2T} =$$
$$= \int_{-\infty}^{\infty} d\varepsilon \frac{\tilde{N}(\varepsilon)}{2\varepsilon} th\frac{\varepsilon}{2T}, \quad (20)$$

where $\tilde{N}(\varepsilon)$ is the density of states ($U = 0$) "dressed" by disorder scattering. In Eq. (20) the energy ε is reckoned from the chemical potential and if we reckon it from the center of conduction band we have to replace $\varepsilon \to \varepsilon - \mu$, so that the condition of Cooper instability (15) leads to the following equation for T_c:

$$1 = \frac{U}{2} \int_{-\infty}^{\infty} d\varepsilon \tilde{N}_0(\varepsilon) \frac{th\frac{\varepsilon - \mu}{2T_c}}{\varepsilon - \mu}, \quad (21)$$

where $\tilde{N}_0(\varepsilon)$ is again the density of states (calculated for $U = 0$) "dressed" by disorder scattering. At the same time, the chemical potential of the system at different values of U and Δ should be determined from DMFT+Σ calculations, i.e. from the standard equation for the number of electrons (band-filling), determined by Green's function given by Eq. (3), which allows us to find T_c for the wide range of model parameters, including the BCS-BEC crossover and strong coupling regions, as well as for different levels of disorder. This reflects the physical meaning of Nozieres – Schmitt-Rink approximation — in the weak coupling region transition temperature is controlled by the equation for Cooper instability (21), while in the limit of strong coupling it is determined as the the temperature of BEC, controlled by chemical potential. Thus, the joint solution of Eq. (21) and equation

for the chemical potential guarantees the correct interpolation for T_c through the region of BCS-BEC crossover. This approach gives the results for the critical temperature, which are quantitatively close to exact results, obtained by direct numerical DMFT calculations [13], but demands much less numerical efforts.

It should be stressed, that we have used the exact Ward identity, which allows the use of Eq. (21) also in the region of strong disorder, when the effects of Anderson localization may become relevant. Eq. (21) demonstrates, that the critical temperature depends on disorder only through the disorder dependence of the density of states $\tilde{N}(\varepsilon)$, which is the main statement of Anderson theorem. In the framework of Nozieres – Schmitt-Rink approach Eq. (21) is conserved also in the region of strong coupling, when the critical temperature is determined by BEC condition for compact Cooper pairs. In this case the chemical potential μ, entering Eq. (21), may significantly depend on disorder. However, in DMFT+Σ approximation this dependence of chemical potential (as well as any other single – particle characteristic) in the model with semi – elliptic density of states is only due to disorder widening of conduction band. Thus, both in BCS – BEC crossover and strong coupling regions the generalized Anderson theorem actually remains valid. Correspondingly, in the model of semi – elliptic band Eq. (21) leads to universal dependence of T_c on disorder, due to the change of $D \to D_{eff}$. Such universality is fully confirmed by numerical calculations of T_c in this model, performed in Ref. [24] (cf. also the results presented below).

V. MAIN RESULTS

Let us now discuss the main results of our numerical calculations, explicitly demonstrating the universal behavior of single – particle properties and superconducting transition temperature with disorder. We shall see, that all disorder effects are effectively controlled, in fact, only by the growth of half-width of conduction band, which for the case of semi – elliptic density of states are given by Eq. (12). In case of the band with flat density of states, the growth of disorder changes the shape of the density of states, making it semi – elliptic in the limit of strong enough disorder, while the effective half-width of the band is given by (cf. Appendix A):

$$\frac{D_{eff}}{D} = \sqrt{1 + \frac{\Delta^2}{D^2}} + \frac{1}{2}\frac{\Delta^2}{D^2} \ln \left(\frac{\sqrt{1 + \frac{\Delta^2}{D^2}} + 1}{\sqrt{1 + \frac{\Delta^2}{D^2}} - 1} \right). \quad (22)$$

As an example of the most important single – particle property we take the density of states. In Fig. 1 we show the evolution of the density of states with disorder in the model of semi – elliptic band [13]. We can see, that the growth of disorder smears the density of states and widens the band. This smearing somehow masks the peculiarities of the density of states due to correlation

effects. In particular, both quasiparticle peak and lower and upper Hubbard bands, observed in Fig. 1 in the absence of disorder are completely destroyed in the limit of strong enough disorder. However, we can easily convince ourselves, that this evolution is only due to the general widening of the band due to disorder (cf. (12), (22)), as all the data for the density of states belong to the same universal curve replotted in appropriate new variables, with all energies (and temperature) normalized by the effective bandwidth by replacing $D \rightarrow D_{eff}$, as shown in Fig. 2(a), in complete accordance with the general results, obtained above. In the case of conduction band with flat density of states, there is no complete universality, as is seen from Fig. 2(b) for low enough values of disorder. However, for large enough disorders the dashed curve shown in Fig. 2(b) practically coincides with universal curve for the density of states¡ shown in Fig. 2(a). This reflects the simple fact, that at large disorders the flat density of states effectively transforms into semi – elliptic (cf. Appendix A).

Going now to the analysis of superconducting transition temperature, in Fig. 3 we present the dependence of T_c (normalized by the critical temperature in the absence of disorder $T_{c0} = T_c(\Delta = 0)$) on disorder for different values of pairing interaction U for both models of initial "bare" density of states (semi – elliptic — Fig.3(a) and flat — Fig. 3(b)). Qualitatively the evolution of T_c with disorder is the same for both models. We can see, that in the weak coupling limit ($U/2D \ll 1$) disorder slightly suppresses T_c (curves 1). At intermediate couplings ($U/2D \sim 1$) weak disorder increases T_c, while the further growth of disorder suppresses the critical temperature (curves 3). In the strong coupling region ($U/2D \gg 1$) the growth of disorder leads to significant increase of the critical temperature (curves 5). However, we can easily see, that such complicated dependence of T_c on disorder is completely determined by disorder widening of the "bare" ($U = 0$) conduction band, demonstrating the validity of the generalized Anderson theorem for all values of U. In Fig. 4 curve with octagons show the dependence of the critical temperature $T_c/2D$ on coupling strength $U/2D$ in the absence of disorder ($\Delta = 0$) for both model of "bare" conduction bands (semi – elliptic — Fig. 4(a) and flat — Fig. 4(b)). We can see, that in both models in the weak coupling region superconducting transition temperature is well described by BCS model (in Fig. 4(a) the dashed curve represents the result of BCS model, with T_c defined by Eq. (21), with chemical potential independent of U and determined by quarter – filling of the "bare" band), while in the strong coupling region the critical temperature is determined by BEC condition for Cooper pairs and drops as t^2/U with the growth of U (inversely proportional to the effective mass of the pair), passing through the maximum at $U/2D_{eff} \sim 1$. The other symbols in Fig. 4(a) show the results for T_c obtained by combination of DMFT+Σ and Nozieres – Schmitt-Rink approximations for the case of semi – elliptic "bare" band. We can see, that all data (ex-pressed in normalized units of $U/2D_{eff}$ and $T_c/2D_{eff}$) ideally fit the universal curve, obtained in the absence of disorder. For the case of flat "bare" band, results of our calculations are shown in Fig. 4(b) and we do not observe the complete universality — data points, corresponding to different degrees of disorder somehow deviate from the curve, obtained in the absence of disorder. However, with the growth of disorder the form of the band becomes close to semi – elliptic and our data points move towards the universal curve, obtained for semi – elliptic case and shown by the dashed curve in Fig. 4(b), thus confirming the validity of the generalized Anderson theorem.

VI. CONCLUSION

In this paper, in the framework of DMFT+Σ generalization of dynamical mean field theory, we have studied disorder influence on single – particle properties (e.g. density of states) and temperature of superconducting transition in attractive Hubbard model. Calculations were made for a wide range of attractive interactions U, from the weak coupling region of $U/2D_{eff} \ll 1$, where both instability of the normal phase and superconductivity is well described by BCS model, up to the strong coupling limit of $U/2D_{eff} \gg 1$, where superconducting transition is determined by Bose – Einstein condensation of compact Cooper pairs, forming at temperatures much higher than the temperature of superconducting transition. We have shown analytically, that in the case of conduction band with semi – elliptic density of states, which is a good approximation for three – dimensional case, disorder influences all single – particle properties in a universal way — all changes of these properties are due only to disorder widening of the band. In the model of conduction band with flat density of states, which is appropriate for two – dimensional systems, there is no universality in the region of weak disorder. However, the main effects are again due to general widening of the band and complete universality is restored for high enough disorders, when the density of states effectively becomes semi – elliptic.

To study the superconducting transition temperature we have used the combination of DMFT+Σ approach and Nozieres — Schmitt-Rink approximation. For both models of conduction band density of states disordering may either suppress the critical temperature T_c (in the region of weak coupling) or significantly increase it (in the strong coupling region). However, in all these cases we have actually proven the validity of the generalized Anderson theorem. so that all the changes of transition temperature are, in fact, controlled only by the effects of general disorder widening of the conduction band. In case of initial semi – elliptic band disorder influence on T_c is completely universal, while in the case of initial flat band such universality is absent at weak disorder, but is completely restored for high enough disorder levels.

Finally we should like to present some additional comments on the methods and approximations used. Both DMFT+Σ and Nozieres – Schmitt-Rink approaches represent cetrain approximate interpolation schemes, strictly valid only in corresponding limiting cases (e.g. small disorder or small (large) U). However, both schemes demonstrate their effectiveness also in the case of intermediate values of U and intermediate (or even strong) disorder. Actually, the effectiveness of Nozieres – Schmitt-Rink (neglecting U corrections in Cooper channel) approximation was verified by comparison with direct DMFT calculations [13]. The use of DMFT+Σ to analyze the disorder effects in repulsive Hubbard model was shown to produce reasonable results for the phase diagram, as compared to exact numerical simulations of disorder in DMFT, including the region of large disorder (Anderson localized phase) [19–21]. However, the role of approximations made in DMFT+Σ, such as the neglect of the intrference of disorder scattering and correlation effects, deserves further studies.

Appendix A

For the band with flat density of states (at $U = 0$ and $\Delta = 0$) disorder leads both to widening of the band and to the change of the form of the density of states. Taking the density of states in the form given by Eq. (6) we calculate the local Green's function as:

$$G_{ii} = \frac{1}{2D} \int_{-D}^{D} d\varepsilon' \frac{1}{\varepsilon - \varepsilon' - \Delta^2 G_{ii}} = \frac{1}{2D} \ln \left(\frac{\varepsilon - \Delta^2 G_{ii} + D}{\varepsilon - \Delta^2 G_{ii} - D} \right), \quad (A1)$$

where energy ε is reckoned from the middle of the "bare" band. Let us introduce auxiliary notations, writing $G_{ii} = R - iI$. At the band edges $I \to 0$, so that expanding the r.h.s. of Eq. (A1) up to linear terms in I, we get:

$$R - iI \approx \frac{1}{2D} \ln \left(\frac{\varepsilon - \Delta^2 R + D}{\varepsilon - \Delta^2 R - D} \right) - iI \frac{\Delta^2}{(\varepsilon - \Delta^2 R)^2 - D^2} \quad (A2)$$

Equating the real parts in (A2) we obtain $R = \frac{1}{2D} \ln \left(\frac{\varepsilon - \Delta^2 R + D}{\varepsilon - \Delta^2 R - D} \right)$. Similarly, equating the imaginary parts at the band edges we get $\varepsilon - \Delta^2 R = \pm \sqrt{D^2 + \Delta^2}$,

and substituting this expression into logarithm in the previous expression, we find R and band edges positions at:

$$\varepsilon = \pm \left(\sqrt{D^2 + \Delta^2} + \frac{\Delta^2}{2D} \ln \left(\frac{\sqrt{D^2 + \Delta^2} + D}{\sqrt{D^2 + \Delta^2} - D} \right) \right) \quad (A3)$$

Thus, the half-width of the band D_{eff} widened by disorder in this model is determined by Eq. (22) used above.

We should note, that the Born approximation for disorder scattering used by us, though formally valid only for small disorder $\Delta \ll D$, the effects of Anderson localization at large disorders $\Delta \sim D$ do not qualitatively change the density of states [27], so that Born approximation gives qualitatively correct results also in the region of large disorder. Actually, this approximation neglects only the appearance exponentially small "tails" in the density of states, outside the "mean field" band edges [27] and gives more or less correct results inside such a band.

At large enough disorders almost any "bare" band width bandwidth $2D$ and arbitrary density of states $N_0(\varepsilon)$ acquires semi – elliptic density of states. In the limit of very large disorder $\Delta \gg D$ almost in the whole band, widened by disorder, we have $|\varepsilon - \Delta^2 R| \gg D$ and in the expression for the local Green's function we can neglect ε'-dependence in the denominator of the integrand:

$$R - iI = G_{ii} = \int_{-\infty}^{\infty} d\varepsilon' \frac{N_0(\varepsilon')}{\varepsilon - \varepsilon' - \Delta^2 G_{ii}} \approx \frac{1}{\varepsilon - \Delta^2 R + i\Delta^2 I} \quad (A4)$$

Then we immediately get:

$$\varepsilon - \Delta^2 R = \frac{\varepsilon}{2}; \qquad I = \frac{1}{2\Delta^2} \sqrt{4\Delta^2 - \varepsilon^2} \quad (A5)$$

so that the density of states "dressed" by disorder

$$N(\varepsilon) = -\frac{1}{\pi} Im G_{ii} = \frac{I}{\pi} = \frac{2}{\pi(2\Delta)^2} \sqrt{(2\Delta)^2 - \varepsilon^2} \quad (A6)$$

becomes semi – elliptic (5) with half-width $D_{eff} = 2\Delta$. Thus, at strong enough disorder any "bare" band becomes semi – elliptic, restoring universal dependence of single – particle properties on disorder discussed above. In this sense, the model of the "bare" band with semi – elliptic density of states is most appropriate for the studies of the effects of strong disorder.

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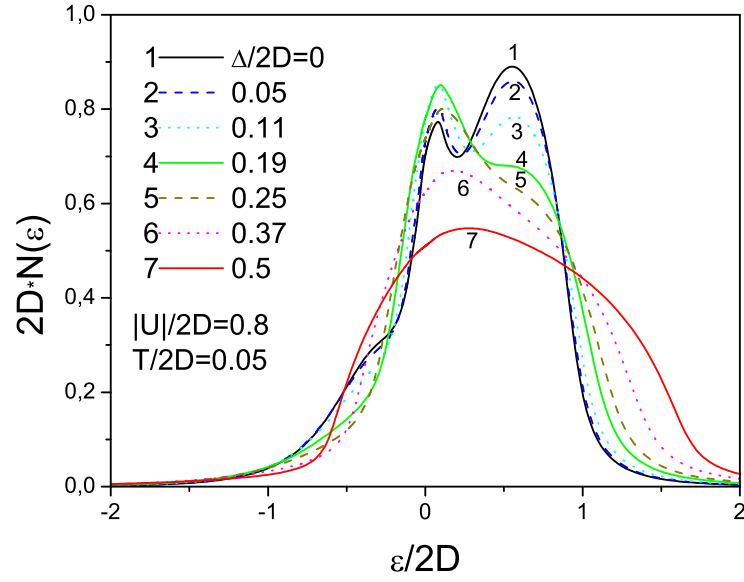
Figures

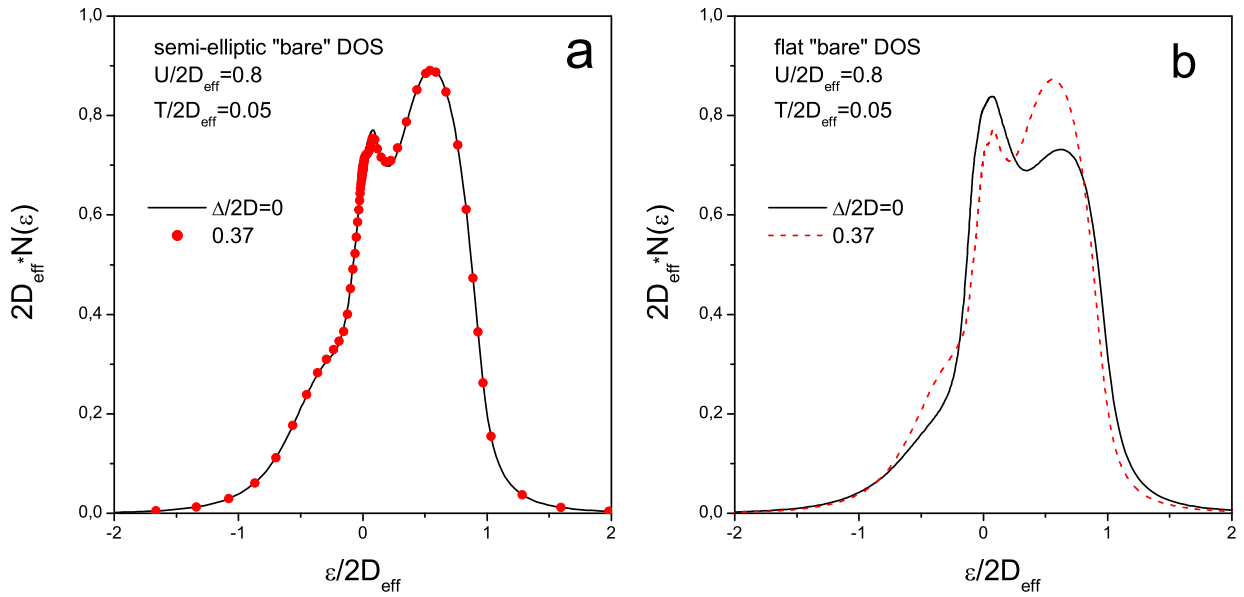FIG. 1: Dependence of the density of states on disorder in the model with semi − elliptic band.

FIG. 2: Universal dependence of the density of states on disorder: (a) — the model of semi − elliptic "bare" density of states; (b) — the model of flat "bare" density of states.
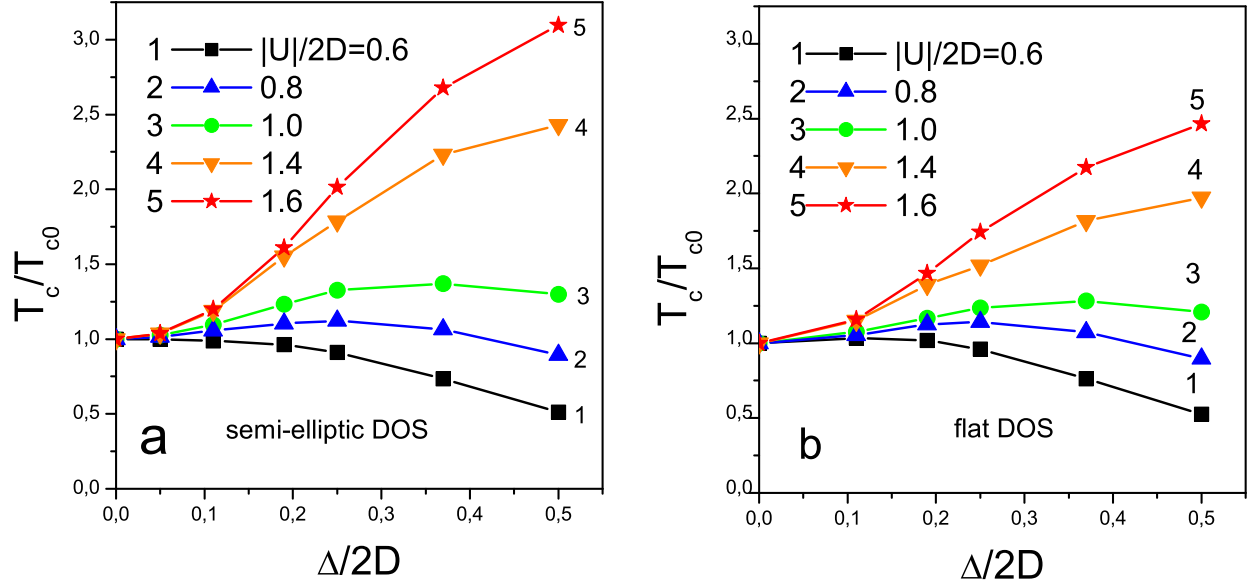
FIG. 3: Dependence of superconducting transition temperature on disorder for different values of Hubbard attraction U: (a) — semi – elliptic band; (b) — flat band.
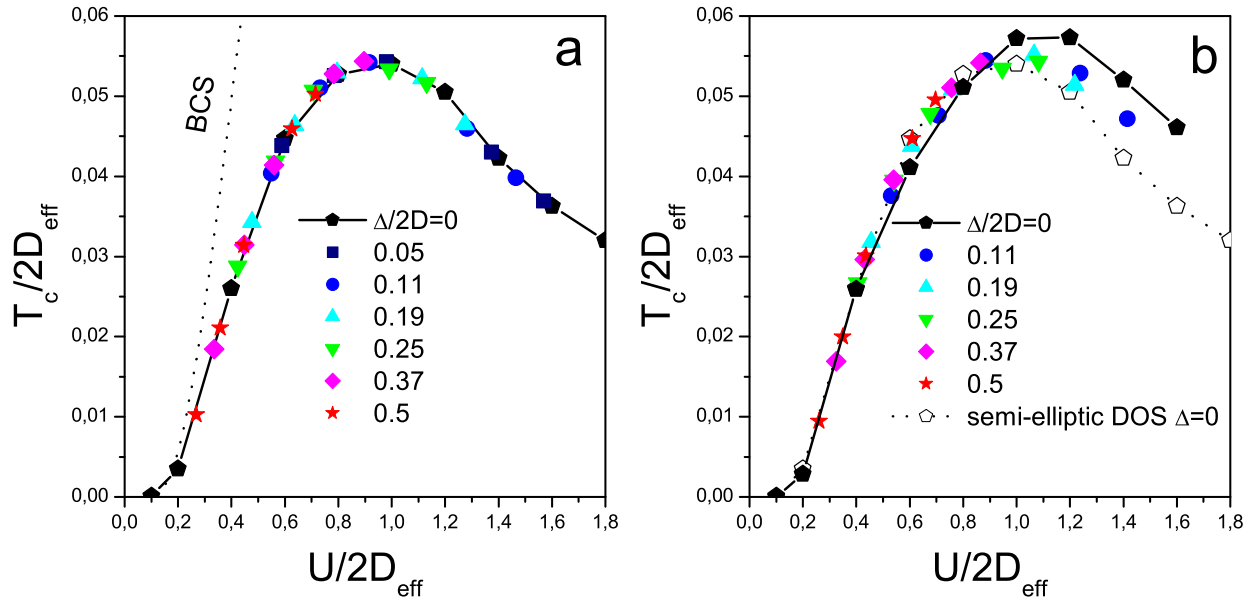

FIG. 4: Universal dependence of superconducting critical temperature on Hubbard attraction U for different disorder levels: (a) — semi – elliptic band. Dashed curve represent BCS dependence in the absence of disorder. (b) — flat band. Dashed line represents similar dependence for semi – elliptic band for $\Delta = 0$.