

# LOCALIZATION EFFECTS IN HIGH-TEMPERATURE SUPERCONDUCTORS:

## THEORETICAL ASPECTS

*M. V. Sadovskii,*

*Institute for Electrophysics, USSR Academy of Sciences,  
Ural Branch, Sverdlovsk, 620219, USSR*

### I n t r o d u c t i o n

The basic idea of Anderson localization [1] and disorder induced metal-insulator transition has attracted great attention in recent years [2-6]. This disorder induced transition is due to a transformation of initially extended electronic states near the Fermi level of a metal to the spatially localized states of Anderson insulator at sufficiently large disorder.

Imagine that the initial metal is a superconductor, i.e. that due to some attractive interaction of electrons near the Fermi level they form Cooper pairs at low temperatures  $T < T_c$ . It is well known that the BCS-like ground state [7] is relatively insensitive to disordering conserving time-reversal invariance (normal, nonmagnetic impurities etc.) [8,9]. Then an important problem arises of the possible interplay of localization transition and superconductivity in a strongly-disordered metal, which is essentially a question of the possible coexistence of apparently opposite kinds of ground states (insulator versus superconductor). This problem is also very important from the experimental point of view, because in many cases superconducting transition is observed close to metal-insulator transition in highly disordered systems. Apparently high- $T_c$  oxides are quite relevant here being very close to metal-insulator transition which can be induced by only slight disordering.

The general aspects of the interplay of Anderson localization and superconductivity were analyzed in a number of papers [10-18].

---

To be presented at the International Workshop "Effects of Strong Disorder in HTSC", Sverdlovsk-Zarechny, USSR, June 1990.

Here we discuss some of the basic results as applied to high- $T_c$  systems, where many experimental results were also obtained, especially for the case of radiation disordering by fast-neutron irradiation [19-22]. This method is probably the purest way to investigate the effects of disordering on physical properties of high- $T_c$  superconductors due to the absence of any thermochemical effects (in case of low temperature irradiation). The growth of structural disorder leads to a number of drastic changes in the behaviour of both single-crystalline and ceramic samples [19-22]:

- (a) continuous metal-insulator transition at very slight disordering;
- (b) rapid degradation of  $T_c$ ;
- (c) apparent coexistence of hopping conductivity and superconductivity at intermediate disorder and anomalous (exponential) growth of resistivity with defect concentration;
- (d) approximate independence of the derivative of the upper-critical field  $H_{c2}(T)$  on the degree of disorder.

These anomalies were interpreted using the ideas of possible coexistence of Anderson localization and superconductivity and here we shall discuss mainly the theoretical aspects of this problem.

### 1. Why Localization Effects are Strong in High- $T_c$ Systems?

*(A) Two-Dimensionality.* All the known high- $T_c$  systems (with  $T_c > 30$  K) are strongly anisotropic or even quasi-two-dimensional conductors. For such systems it is natural to expect the strong enhancement of localization effects due to the known role of spatial dimensionality of  $d = 2$ : in purely two-dimensional case localization appears for infinitely small disorder [3-6].

For isotropic three-dimensional systems the typical conductivity scale at the Anderson transition is determined by the so called minimal metallic conductivity [2], which can be estimated just by the Drude conductivity for  $p_F l / h \approx 1$  (Ioffe-Regel criterion),

where  $p_F$  - is Fermi momentum,  $l$  - the mean free path, which gives

$$\sigma_c \approx e^2 p_F / 3\pi^2 h^2 \approx e^2 / 3\pi^2 h a \quad (1)$$

where the last estimate is valid for  $p_F \approx h/a$  ( $a$  - is the interatomic distance). For typical  $a \approx 3 \text{ \AA}$  this gives  $\sigma \approx 3 \cdot 10^2 \text{ Ohm}^{-1} \text{ cm}^{-1}$ . For quasi-two-dimensional system this conductivity scale is significantly enhanced. For the in-plane conductivity this scale can be estimated as [19,23]:

$$\sigma_c^{\parallel} \approx e^2 / \pi^2 h a_{\perp} \ln(E_F/w) \quad (2)$$

where  $a_{\perp}$  is the interplane distance and  $w$  is the interplane transfer integral. The logarithmic factor in (2) grows with diminishing overlap of electronic states on the nearest-neighbour planes. Thus for the strongly anisotropic (quasi-two-dimensional) systems, such as high- $T_c$  oxides, the value of in-plane minimal metallic conductivity may be larger than the usual estimate given by (1), and actually can exceed  $10^3 \text{ Ohm}^{-1} \text{ cm}^{-1}$  for typical values of  $E_F/w \geq 10$ . While due to continuous nature of Anderson transition [3-6], there is no rigorous meaning of minimal metallic conductivity, these estimates actually define the scale of conductivity near the metal-insulator transition caused by disorder. Then it is clear that most of the real samples of high- $T_c$  superconductors are quite close to Anderson transition and even the very slight disordering is sufficient to transform them into Anderson insulators.

However, we must stress that the enhancement of localization effects due to quasi-two-dimensionality is relatively weak [24], which is reflected by logarithmic only dependence on interplane transfer integral in (2).

(B) "Marginal" Fermi Liquid. There are serious reasons to believe that the importance of localization effects in high- $T_c$  oxides is actually due to much more fundamental cause [25]. These effects are strongly enhanced if we apply for such systems the concept of

"marginal" Fermi liquid [26]. This concept has been successfully used to describe the wide range of properties of high- $T_c$  oxides both in normal and superconducting state [26,27]. Basically the idea of "marginal" Fermi liquid is expressed by the following form of one-particle Green's function [26]:

$$G(Ep) = \frac{Z_p}{\varepsilon - \xi_p - ih/\tau_{QP}} + G_{incoh} \quad (3)$$

where  $\xi_p$  is renormalized quasi-particle energy,  $h/\tau_{QP}^{-1} \approx \text{Max}[\varepsilon, T]$  is anomalous (linear) decay-rate for these quasi-particles. The concept of "marginality" arises due to peculiar behaviour of quasiparticle residue

$$Z_p^{-1} \approx \ln \frac{h\omega_c}{|\xi_p|} \approx \ln \frac{h\omega_c}{|\varepsilon|} \quad (4)$$

where  $\omega_c$  is characteristic frequency scale of some kind of electronic excitations, which is the phenomenological parameter of the theory. From (4) it is clear that quasiparticle contribution to Green's function (3) vanishes precisely at the Fermi level, while exists close to it.

For disordered system we can estimate the impurity contribution to the scattering rate of quasi-particles in usual Born approximation as [25]:

$$\begin{aligned} h\tau_{QP}^{-1} &= 2n_i V^2 Z_p \text{Im} \sum_{\mathbf{q}} \Lambda^2(\mathbf{p}+\mathbf{q}, \mathbf{p}) G(\mathbf{p}+\mathbf{q}, \varepsilon) \approx \\ &\approx 2\pi n_i V^2 Z^2 \Lambda^2(\mathbf{q} \rightarrow 0) N(E_F) \approx Z \Lambda^2 h/\tau_0 \end{aligned} \quad (5)$$

where  $\Lambda$  represents the appropriate vertex part,  $V$  is impurity potential,  $n_i$  - impurity concentration,  $N(E_F) = Z^{-1} N_0(E_F)$  is the renormalized density of states at the Fermi level,  $N_0(E_F)$  -

same for noninteracting quasiparticles,  $\tau_0^{-1}$  - a scattering rate for noninteracting case. To get the last relation in (5) a weak dependence of vertices and self-energy on momentum was assumed. Now we can use the Ward identity for  $\Lambda(\mathbf{q} \rightarrow 0, \omega = 0)$  vertex of disordered Fermi-liquid theory [28-30]:

$$\Lambda(\mathbf{q} \rightarrow 0, \omega = 0) = (1 + F_0)^{-1} Z^{-1} \quad (6)$$

where  $F_0$  is the usual Landau parameter. As a result we can easily get a simple relation between the mean free paths of interacting and noninteracting quasiparticles [25]:

$$l_{QP} = (p_F/m^*)\tau_{QP} = (p_F/m)\tau_0/\Lambda^2(\mathbf{q} \rightarrow 0) = l_0(1 + F_0)^2 Z^2 \quad (7)$$

Here  $m^* = Z^{-1}m$  is the effective mass of the quasiparticle. Assuming  $F_0 \approx \text{const} < 1$  and using (4) we get at  $T = 0$ :

$$l_{QP} \approx l_0 / \left[ \ln \frac{h\omega_c}{|\varepsilon|} \right]^2 \quad (8)$$

Then from usual Ioffe-Regel criterion for localization  $p_F l_0/h \approx 1$ , we obtain that all quasiparticle states within the region of the of

$$|\varepsilon_c| \approx h\omega_c \exp\left[-\sqrt{p_F l_0/h}\right] \quad (9)$$

around the Fermi level in high- $T_c$  oxides are localized even for the case of weak impurity scattering  $p_F l_0/h \gg 1$ . For realistic estimates of  $h\omega_c \approx 0.1-0.2$  eV [26] and  $p_F l_0 < 5$  the width of this localized band may easily be of the order of hundreds of K, while for  $p_F l_0/h \approx 10$  and  $h\omega_c \approx 1000$  K we get  $|\varepsilon_c| \approx 40$  K. Obviously this band grows with disorder as the mean free path  $l_0$  drops. We can safely neglect this localization for  $T \gg |\varepsilon_c|$ , but for low enough temperatures localization effects become important

and all states are localized in the ground state.

The concept of localization of single-particle states in the ground state leads to a simple conjecture on the temperature behaviour of resistivity of single-crystalline samples of high- $T_C$  oxides. It is well known that experimentally there are two types of this behaviour. In ab plane resistivity always shows a linear-T behaviour in good samples ( with  $\rho_{ab} < 10^{-3}$  Ohm cm ). However along c axis situation is rather curious: most samples produce semiconductor-like behaviour  $\rho_c \approx 1/T$ , though some rare samples (more pure?) show metallic-like  $\rho_c \approx T$  (with strong anisotropy remaining:  $\rho_c/\rho_{ab} \approx 10^2$ ).

In case of localized states at the Fermi level and for finite temperatures it is important to compare localization length  $R_{loc}$  with diffusion length due to inelastic scattering  $L_D \approx \sqrt{D\tau_\phi}$ , where  $D$  is classical diffusion coefficient and  $\tau_\phi$  is phase coherence time determined by inelastic processes (typically  $\tau_\phi \approx T^{-p}$  where  $p$  is an integer) [6]. In case of  $L_D < R_{loc}$  electrons do not "feel" being localized and conductivity at high  $T$  may show metallic-like behaviour if  $R_{loc}$  is large enough. For localization to be important we must go to low enough temperatures, so that  $L_D > R_{loc}$ . In strongly anisotropic case we apparently have  $R_{loc}^{ab} \gg R_{loc}^c$  and both localization lengths diminish with disorder growth. Then we can have three types of behaviour:

Low T or	$L_D^{ab} \approx \sqrt{D_{ab}\tau_\phi} \gg R_{loc}^{ab}$	Semiconductor-
strong disorder	$L_D^c \approx \sqrt{D_c\tau_\phi} \gg R_{loc}^c$	like behaviour.

High T or	$L_D^{ab} < R_{loc}^{ab}$	Metallic behaviour in
low disorder	$L_D^C < R_{loc}^C$	both directions.

So in principle we can explain (at least qualitatively) all types of temperature behaviour of resistivity of high- $T_c$  oxides, though further analysis and quantitative estimates will strongly depend on the assumed mechanisms of inelastic scattering determining  $\tau_\phi$ .

## 2. Localization and Superconductivity: Anomalies in Normal and Superconducting State.

Let us consider now the problem of possible superconductivity in case of localized states at the Fermi level. Exact electronic states  $\phi_\nu(\mathbf{r})$  in a disordered system are defined as eigenvectors of Hamiltonian with random potential field. These states may be both extended or localized. Cooper pairing can be realized between the time-reversed partners  $\phi_\nu(\mathbf{r})$  and  $\phi_\nu^*(\mathbf{r})$  (with opposite spins) [9]. For the case of self-averaging order parameter (gap) it was shown by Anderson [9] that for the given value of pairing interaction  $T_c$  is essentially independent of the nature of these states: either extended or localized. However it is clear [10,11] that Cooper pairing can be achieved in the localized phase only for the electrons with centers of localization within the volume of characteristic size of the order of  $R_{loc}$ , because only these states have overlapping wave functions and can interact with each other. These states are splitted in energy on the scale of the order of  $[N(E_F)R_{loc}^3]^{-1}$  [2]. Obviously, to observe superconductivity we must demand that this splitting be smaller than the value of superconducting energy gap at  $T = 0$  [10,11]:

$$\Delta \gg [N(E_F)R_{loc}^3]^{-1} \quad (10)$$

or for strongly anisotropic high- $T_c$  systems:

$$\Delta \gg [N(E_F)R_{loc}^a R_{loc}^b R_{loc}^c]^{-1} \quad (11)$$

where we introduced localization lengths along the axes of orthorhombic lattice. These inequalities are equivalent to condition of rather large localization length, for isotropic case:

$$R_{loc} \gg [N(E_F)\Delta]^{-1/3} \approx (\xi_0 h^2 / p_F^2)^{1/3} \approx (\xi_0 a^2)^{1/3} \quad (12)$$

where  $\xi_0 \approx hv_F/\Delta$  is the coherence length of BCS theory ( $v_F$  - Fermi velocity). For high- $T_c$  oxides with rather large values of  $\Delta$  and small  $\xi_0$  this condition can be realized rather easily. Actually the physical meaning of (12) is very simple:  $R_{loc}$  must be much larger than the characteristic size of Cooper pair in strongly disordered system [10,11]. The physical picture of superconductivity in the region of localized phase can be illustrated by the following estimates. Consider two neighbouring regions of the size of the order of  $R_{loc}$  each. Transition probability of the electron transfer between these regions is given by:

$$P_T \approx \varphi^2 N(E_F) R_{loc}^3 \quad (13)$$

where  $\varphi$  is the amplitude of such transition. Anderson localization takes place if  $\varphi < [N(E_F)R_{loc}^3]^{-1}$ , i.e. for  $P_T < [N(E_F)R_{loc}^3]^{-1}$ , when there is no tunneling. However Cooper pairs can tunnel between these regions via Josephson-like coupling which gives the following estimate for transition probability:

$$E_J \approx [N(E_F)R_{loc}^3]^{2\varphi^2} \Delta \quad (14)$$

Then we can see that

$$E_J/P_T \approx \Delta N(E_F) R_{loc}^3 \gg 1 \quad (15)$$

in case inequality (10) is valid, so that Cooper pairs can tunnel

between regions of localized states and we can readily have  $E_J \gg [N(E_F)R_{loc}^3]^{-1}$ . So superconductivity is possible due to the fact of the absence of localization for Cooper pairs, while single-particle states are localized.

The existence of Meissner-like response to a vector-potential was explicitly demonstrated for localization region in [10,11] within the framework of microscopic derivation of Ginzburg-Landau (GL) theory for the system close to mobility edge (Cf. also [12,13]). For  $T = 0$  the same result was obtained later in [14].

It is important to stress here that in the vicinity of Anderson transition the superconducting order parameter  $\Delta$  actually becomes non self-averaging [15,17,18]. This leads to the important role of "statistical" (non-thermodynamic) fluctuations, leading to the incipient inhomogeneities of order parameter near  $T_c$  for system close to mobility edge ("superconducting drops" [17,18]). Roughly speaking this leads to a substantial smearing of superconducting transition. This problem deserves further studies, especially for quasi-two-dimensional systems.

GL coefficients, and most importantly that of the gradient term are significantly changed close to the Anderson transition, these changes are basically due to anomalous frequency dependence of diffusion coefficient close to the mobility edge [10,11]. We present here some results for quasi-two-dimensional case [19]. For the gradient term coefficient we have:

$$C_{\parallel, \perp} = N(E_F) \xi_{\parallel, \perp}^2 \quad (16)$$

where  $\xi_{\parallel}$  and  $\xi_{\perp}$  define in-plane and out-of-plane coherence lengths (size of a Cooper pair). Two important limits are determined by the condition:

$$w^2 \tau / 2\pi T_c \hbar \gg 1 \quad \text{i.e.} \quad \xi_{\perp} \approx \sqrt{\xi_{O\perp} l_{\perp}} \gg a_{\perp} \quad (17)$$

where  $\xi_{O\parallel} \approx \hbar v_F / \Delta$  and  $\xi_{O\perp} \approx w a_{\perp} / \Delta$  are BCS values of coherence

lengths,  $l_{\parallel} = v_F \tau$ ,  $l_{\perp} \approx wa_{\perp} \tau / h$  - are longitudinal and transverse mean free paths. Eq. (17) defines either anisotropic three-dimensional or "nearly two-dimensional" behaviour of superconductivity [19]. Real high- $T_C$  systems are somewhere in the middle.

We can define characteristic conductivity scale as:

$$\sigma^* \approx \sigma_c \parallel \xi_{O\parallel} / l_{\parallel} \left[ T_C / E_F w \right]^{2/3} \quad (18)$$

For  $w \approx E_F$  (18) reduces to isotropic result of Refs. 10,11:

$$\sigma^* \approx \sigma_c (p_F \xi_O / h)^{-1/3} \approx \sigma_c \left[ T_C / E_F \right]^{1/3} \quad (19)$$

The importance of  $\sigma^*$  is due to the fact that for  $\sigma > \sigma^*$  we have the usual behaviour of GL-coefficients, upper critical field  $H_{C2}^*$  etc., as in theory of "dirty superconductors" [8], while for  $\sigma < \sigma^*$  we have "localization regime" [15,16], where for example does not hold Gorkov's relation for  $H_{C2}'(T)$ . Here the characteristic sizes of Cooper pairs are estimated as:

$$\xi_{\parallel, \perp} \approx \xi_{O\parallel, \perp} \left[ T_C^2 / E_F w \right]^{1/3} \quad (20)$$

For isotropic case when  $w \approx E_F$  these expressions transform into that of Refs. 10,11, where close to the Anderson transition ( $\sigma < \sigma^*$ ) we obtained (Cf. (12)):

$$\xi \approx (\xi_O l^2)^{1/3} \approx (\xi_O a^2)^{1/3} \quad (21)$$

For high- $T_C$  oxides  $\xi_{O\parallel} \approx l_{\parallel}$ ,  $T_C \approx w \approx 0.1 E_F$  and actually  $\sigma^* \approx \sigma_c \parallel$  so that these systems are close to Anderson transition also in their superconducting behaviour.

The derivatives of the upper critical field in quasi-two-dimensional case are determined by ( $\phi_O = \pi ch/e$  is the magnetic flux quantum):

$$\left[ \frac{dH_{c2}^{\perp}}{dT} \right]_{T_c} = \frac{\phi_0}{2\pi\xi_{\parallel}^2 T_c} \quad (22)$$

$$\left[ \frac{dH_{c2}^{\parallel}}{dT} \right]_{T_c} = \frac{\phi_0}{2\pi\xi_{\parallel}\xi_{\perp} T_c}$$

with  $\xi_{\parallel, \perp}$  given above in (20), which leads to behaviour similar to that of Refs. 10,11. For  $\sigma_{\parallel} < \sigma^*$  there is no explicit dependence of derivatives of  $H_{c2}^{\parallel, \perp}(T)$  on conductivity and Gorkov's relation breaks down. The most important relation is given by:

$$\left( H_{c2}^{\parallel} \right)' / \left( H_{c2}^{\perp} \right)' \Big|_{T_c} = \xi_{\parallel} / \xi_{\perp} = hv_F / wa_{\perp} = v_F^{\parallel} / v_F^{\perp} \quad (23)$$

Thus, the anisotropy of  $H_{c2}'(T)$  is determined by the anisotropy of the Fermi velocity irrespective of the regime of superconductivity from the "pure" limit, through the usual "dirty" case, up to the vicinity of Anderson transition. Note also that for the system close to Anderson transition characteristic upward curvature of  $H_{c2}(T)$  dependence was theoretically demonstrated [10,11], which may well correlate with the known problem of similar behaviour in high- $T_c$  systems.

Experimental data obtained on radiationally disordered single-crystals of high- $T_c$  oxides [19-22] show the absence of direct correlation of resistivity and  $H_{c2}'(T)$  characteristic of "dirty superconductors (Gorkov's relation) which can be interpreted as due to the closeness to Anderson transition. The observed isotropisation of  $H_{c2}$  under disordering [21,22], i.e. isotropisation of Cooper pairs, may be interpreted via (23) as due to isotropisation of Fermi velocity. The remanent anisotropy of resistivities may be due to anisotropy of scattering mechanisms.

In the absence of any accepted microscopic theory for  $T_c$  in high- $T_c$  oxides it is rather difficult to discuss mechanisms of its degradation under disordering. However we can mention several general reasons for  $T_c$  drop apparently important for any BCS-like model of high-temperature superconductivity, all connected with the growth of disorder:

- (a) growth of Coulomb repulsion within Cooper pairs [11,32];
- (b) formation of correlation gap in the normal state electronic density of states [33,34];
- (c) growth of spin scattering effects due to the appearance of disorder induced local moments [19,33];
- (d) incipient inhomogeneities due to "statistical fluctuations" near Anderson transition [17,18].

The general discussion of these mechanisms can be found in the references given above. We should like only to stress now that correlation gap of Ref.32 is apparently not very much important here, because as it was shown in Ref.11 close to a mobility edge its value is of the order of  $[N(E_F)R_{loc}^3]^{-1}$  (due to a divergent  $\approx R_{loc}^2$  behaviour of dielectric constant), so that in case of our basic inequality (10), its effects can be safely neglected.

The experimentally observed peculiar exponential growth of electrical resistivity with defect concentration in radiationally disordered high- $T_c$  oxides was interpreted [19-22] as a manifestation of hopping-like conduction due to localization, which is probably present even in initial samples. From these data it is possible to analyze [19-22] the fluence dependence of  $R_{loc}$ . Then using the estimated values of  $N(E_F)$  and  $\Delta$  it can be shown that the basic inequality (10) determining the critical value of  $R_{loc}$  for the observation of superconductivity becomes invalid for fluences of fast neutrons  $\Phi > (5-7) \cdot 10^{18} \text{ cm}^{-2}$ . This is in surprisingly good agreement with experimental data. The  $T_c$  dependence on fluence (resistivity) can be satisfactorily described by theoretical relations [11], obtained for the localized phase as due to the growth of Coulomb repulsion within Cooper pairs.

## C o n c l u s i o n

Extreme sensitivity of high- $T_c$  oxides to disordering may have several explanations, among them based upon the idea of exotic types of pairing. However here we presented the point of view, that this instability and other anomalies can be explained by Anderson localization. Quasi-two-dimensional nature of high- $T_c$  systems leads to significant enhancement of localization effects at relatively weak disorder. These effects become even more important if the concept of "marginal" Fermi-liquid applies for high- $T_c$  oxides: the states at the Fermi level are localized for smallest possible disorder. This may help to realize rather exotic situation of superconducting transition in Anderson insulator. High- $T_c$  oxides are especially promising in this respect due to small size of Cooper pairs, so that there may be a wide region of parameters where localization length is larger than the Cooper pair. There is serious evidence that such situation is actually realized in radiationally disordered high- $T_c$  superconductors. Obviously much more work is needed to detalize specific predictions of the theory, as well as further experimental studies. In this respect especially important may be experiments on radiation disordering of isotropic oxides like  $Ba_{1-x}K_xBiO_3$  where different behaviour can be expected due to the absence of quasi-two-dimensionality.

## A c k n o w l e d g e m e n t s

The author is grateful to Prof. B.N.Goshchitskii and Dr. A.V.Mirmelstein for many useful discussions of experimental situation in radiationally disordered high- $T_c$  oxides. He is also grateful to Profs. C.Varma and G.Kotliar for information on their theoretical work prior to publication. Stimulating criticism and interest of Profs. A.Malozemoff and B.Batlogg were also important.

## R e f e r e n c e s

1. P.W.Anderson. Phys.Rev.109, 1492 (1958)
2. N.F.Mott. Metal-Insulator Transitions.Taylor&Francis,London,1974
3. Anderson Localization. Ed. by Y.Nagaoka & H.Fukuyama. Springer-Verlag, Berlin-NY, 1982
4. M.V.Sadovskii. Usp.Fiz.Nauk 133, 223 (1981)
5. M.V.Sadovskii. Soviet Sci.Reviews-Phys.Rev. Ed. by I.M.Khalatnikov. Vol.7, p.1, Harwood Academic, NY, 1986.
6. P.A.Lee, T.V.Ramakrishnan, Rev.Mod.Phys. 57, 287 (1985)
7. J.Bardeen, L.N.Cooper, J.R.Schrieffer, Phys.Rev. 106,162 (1957)
8. A.A.Abrikosov, L.P.Gorkov. JETP 35, 1158, (1958); 36, 319 (1959)  
L.P.Gorkov. JETP 37, 1407 (1959)
9. P.W.Anderson. J.Phys.Chem.Solids, 11, 26 (1959)
10. L.N.Bulaevskii, M.V.Sadovskii, JETP Lett. 39, 524 (1984)
11. L.N.Bulaevskii, M.V.Sadovskii, J.Low Temp.Phys. 59, 89 (1985)
12. A.Kapitulnik, G.Kotliar. Phys.Rev.Lett. 54, 473 (1985)
13. G.Kotliar, A.Kapitulnik. Phys.Rev. B33, 3146 (1986)
14. M.Ma, P.A.Lee. Phys.Rev. B32, 5658 (1985)
15. L.N.Bulaevskii, M.V.Sadovskii, JETP Lett. 43, 76 (1986)
16. L.N.Bulaevskii, A.A.Varlamov, M.V.Sadovskii. Fiz.Tverd.Tela, 28, 1799 (1986)
17. L.N.Bulaevskii, S.V.Panjukov, M.V.Sadovskii. JETP,92,672(1987)
18. L.N.Bulaevskii, S.V.Panjukov, M.V.Sadovskii. In "Problems of Theoretical Physics (Dedicated to 70th Anniversary of V.L.Ginzburg)", Nauka, Moscow, 1989
19. B.A.Aleksashin et al. JETP 95, 678 (1989)
20. S.A.Davydov et al. Int.J.Mod.Phys. B33, 87, (1989)
21. B.N.Goshchitskii et al.In "High Temperature Superconductivity (Int.Seminar on High-T<sub>c</sub> Superconductivity, Dubna, USSR.)".  
Ed by V.L.Aksenov, N.N.Bogoliubov and N.M.Plakida, World Scientific, Singapore, 1990
22. B.N.Goshchitskii et al. Proc.M<sup>2</sup>SHTSC,Stanford,USA, Physica C162-164, 1019 (1989)
23. V.N.Prigodin, Yu.A.Firsov. J.Phys.C17, L979 (1984)
24. Qiming Li, C.M.Soukoulis, E.N.Economou, G.S.Grest. Phys.Rev. B40, 2825 (1989)
25. G.Kotliar, C.M.Varma. Preprint (1990)
26. C.M.Varma, P.B.Littlewood, S.Schmitt-Rink, E.Abrahams, E.Ruckenstein. Rhys.Rev.Lett. 63, 1996 (1989)
27. Y.Kuroda, C.M.Varma. Preprint (1989)
28. O.Betbeder-Matibet, P.Nozieres. Ann.Phys.(NY), 37, 17 (1966)
29. C.Castellani, G.Kotliar, P.A.Lee, Phys.Rev.Lett.59, 323 (1987)
30. G.Kotliar. In "Anderson Localization", Ed. by T.Ando and H.Fukuyama. Springer Verlag, Berlin-NY, 1988
31. P.W.Anderson, K.A.Muttalib, T.V.Ramakrishnan, Phys.Rev.B28, 117 (1983)
34. N.F.Mott. Phil.Mag. 24, 935 (1971)