Coexistence of superconductivity and magnetism at the nanoscale

A. Buzdin

Institut Universitaire de France, Paris

and Condensed Matter Theory Group, University of Bordeaux

in collaboration with L. Bulaevskii, J. P. Brison, J. Cayssol, D. Denisov, M. Houzet, A. Melnikov, N. Pugach, V. Ryazanov, A. Samokhvalov…

Kourovka, 2012

Outline I

- Recall on magnetism and superconductivity coexistence.
- Ferromagnetic superconductors.Non-uniform magnetic structures. Re-entrant superconductivity.
- Origin and the main peculiarities of the proximity effect in superconductor-ferromagnet systems.
- Josephson π-junction.
- Spin-valve effet. Long ranged triplet correlations. Domain wall superconductivity.
- φ-junction. Direct coupling between ferromagnetism and superconductivity.
- Perspectives and possible applications.

Outline II

1. **Larkin-Ovchinnikov-Fulde-Ferrell state.**
   LOFF or FFLO states.
   The origin of the non-uniform modulated superconducting state.

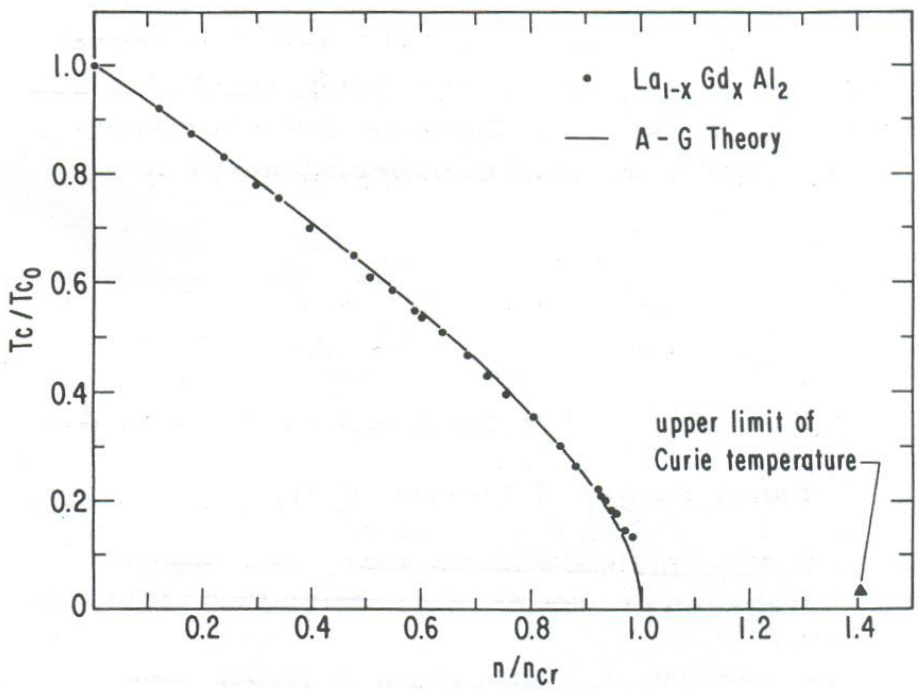
2. **Experimental evidences of FFLO state.**

3. **Exactly solvable models of FFLO state.**

4. **Vortices in FFLO state. Role of the crystal structure.**

5. **Supefluid ultracold Fermi gases with imbalanced state populations: one more candidate for FFLO state?**

Magnetism and Superconductivity Coexistence



Ginzburg, 1956 – in the framework of the model of electromagnetic interaction.

$$\frac{dT_c}{dx} \approx -\Theta_m x$$

(Abrikosov and Gorkov, 1960)

The critical temperature variation versus the concentration n of the Gd atoms in $La_{1-x}Gd_xAl_2$ alloys (Maple, 1968). T_{c0}=3.24 K and n_{cr}=0.590 atomic percent Gd.

The earlier experiments (Matthias *et al*., 1958) demonstrated that the presence of the magnetic atoms is very harmful for superconductivity.

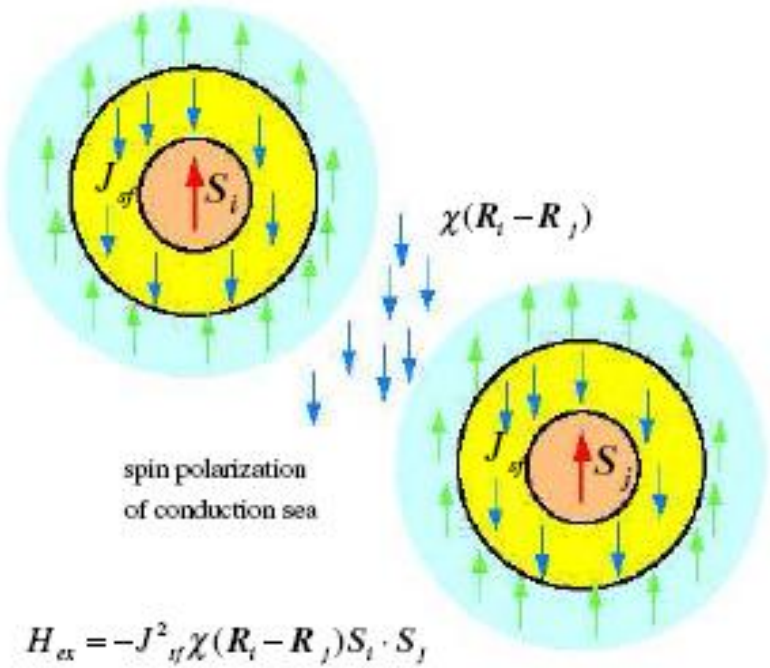
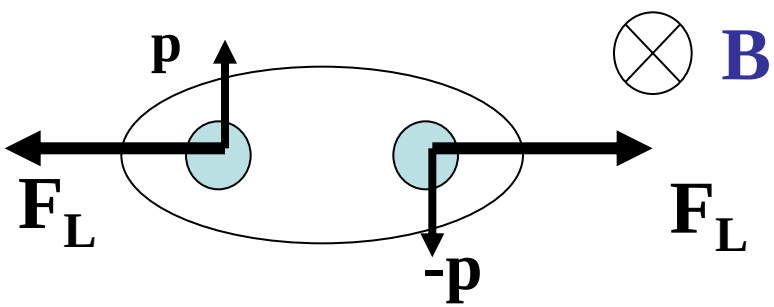


FIG. 3: Schematic mechanism of the indirect RKKY exchange (EX) interaction H_{RKKY} in rare earth superconductors. The spin $\mathbf{S}_i$ at $\mathbf{R}_i$ polarizes spins of conduction electrons with t spin susceptibility $\chi(\mathbf{R}_i - \mathbf{R}_j)\mathbf{S}_{\mathbf{R}_i}$. The polarization is transferred to the spin $\mathbf{S}_j$ at $\mathbf{R}_j$.

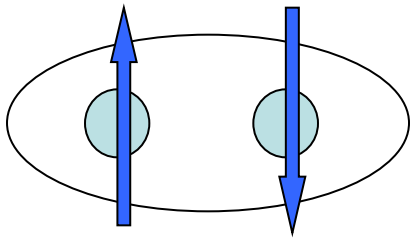
Antagonism of magnetism (ferromagnetism) and superconductivity

- Orbital effect (Lorentz force)



Electromagnetic mechanism (breakdown of Cooper pairs by magnetic field induced by magnetic moment)

- Paramagnetic effect (singlet pair)



S_z=+1/2 S_z=-1/2

$\mu_B H \sim \Delta \sim T_c$

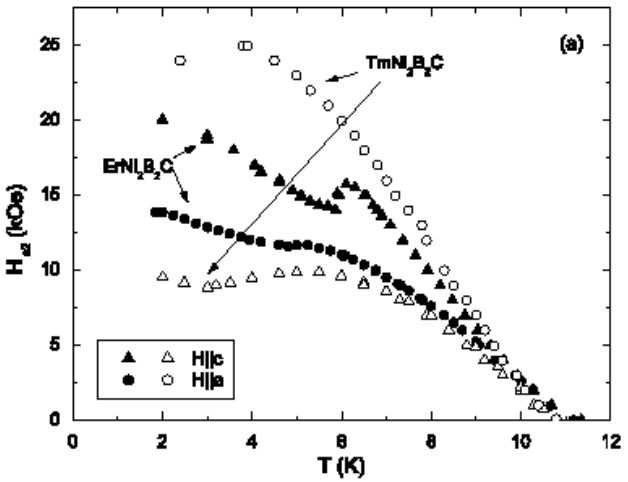
$$I\,(\vec{S}\cdot\vec{s}) \approx T_c$$

Exchange interaction

No antagonism between antiferromagnetism and superconductivity

	T_c (K)	T_N (K)
$NdRh_4B_4$	5.3	1.31
$SmRh_4B_4$	2.7	0.87
$TmRh_4B_4$	9.8	0.4
$GdMo_6S_8$	1.4	0.84
$TbMo_6S_8$	2.05	1.05
$DyMo_6S_8$	2.05	0.4
$ErMo_6S_8$	2.2	0.2
$GdMo_6Se_8$	5.6	0.75
$ErMo_6Se_8$	6.0	1.1
$DyNi_2B_2C$	6.2	11
$ErNi_2B_2C$	10.5	6.8
$TmNi_2B_2C$	11	1.5
$HoNi_2B_2C$	8	5

Usually $T_c > T_N$

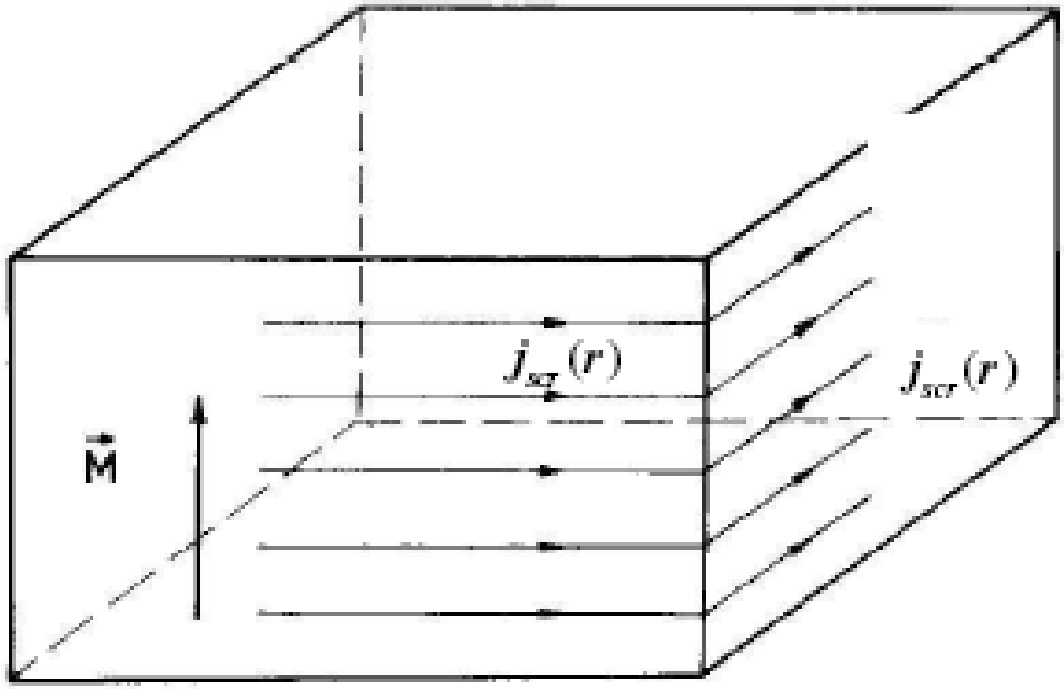


Characteristic energy for magnetism (per one atom/electron)

$$E_{mag} \sim \Theta \sim I^2/E_F$$

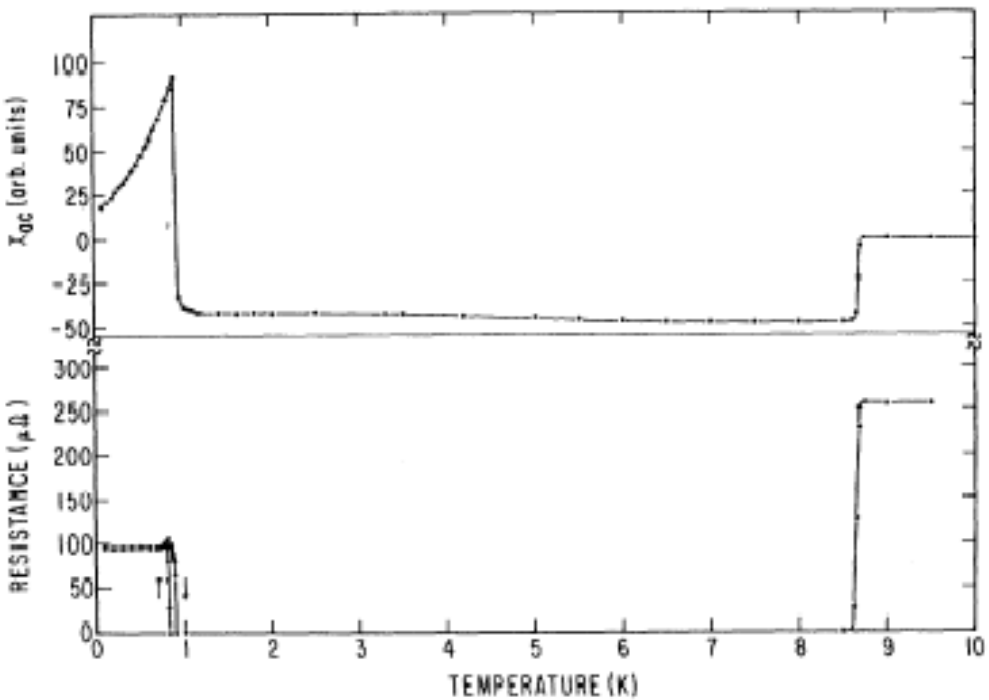
Characteristic energy for superconductivity (per one electron)

$$E_s \sim T_c(T_c/E_F) << T_c$$

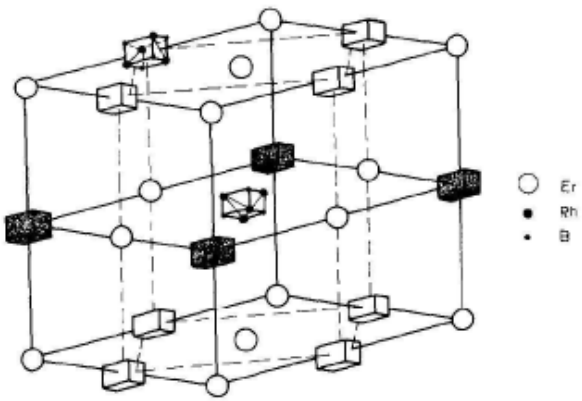


$$M(r) = n_M \mu S \;\Rightarrow\; B_{dp}(r) \;\Rightarrow\; j_{sq}(r) \;\Rightarrow\; B_{sq}(r)$$
$$\Rightarrow\; B(r) = B_{dp}(r) + B_{sq}(r) = \nabla \times A(r)$$

FERROMAGNETIC CONVENTIONAL (SINGLET) SUPERCONDUCTORS



A. C. susceptibility and resistance versus temperature in $ErRh_4B_4$ (Fertig *et al.*,1977).



RE-ENTRANT SUPERCONDUCTIVITY in $ErRh_4B_4$!

Electromagnetic interaction

$$F_s = F_n + \int \left\{ a|\psi|^2 + \frac{b}{2}|\psi|^4 + \frac{\hbar^2}{4m}\left|\left(\nabla - \frac{2ie}{\hbar c}\boldsymbol{A}\right)\psi\right|^2 + \frac{\boldsymbol{B}^2}{8\pi} - \frac{\boldsymbol{BH}}{4\pi}\right\} dV$$

$$\delta F_s(\boldsymbol{A}) = \int \left\{ \frac{e^2|\psi|^2}{mc^2}(\boldsymbol{A})^2 \right\} dV = \frac{e^2|\psi|^2}{mc^2}\sum_q |\boldsymbol{A}_q|^2 = \frac{1}{8\pi\lambda^2}\sum_q |\boldsymbol{A}_q|^2$$

$$F_m = \int \left\{ A\boldsymbol{M}^2 + \frac{B}{2}\boldsymbol{M}^4 + a^2(\nabla \boldsymbol{M})^2 \right\} dV$$

A ~ (T-θ)

$$\delta F_s(\boldsymbol{A}) + F_m = \sum_q (A + q^2a^2)|\boldsymbol{M}_q|^2 + \frac{1}{8\pi\lambda^2}\sum_q |\boldsymbol{A}_q|^2$$

$$if\ q\lambda \gg 1, \qquad then\ \boldsymbol{B} = 4\pi\boldsymbol{M}, \qquad \boldsymbol{B} = rot\boldsymbol{A},\ |\boldsymbol{B}_q|^2 = q^2|\boldsymbol{A}_q|^2 = (4\pi)^2\ |\boldsymbol{M}_q|^2$$

$$\delta F = (q^2a^2)|\boldsymbol{M}_q|^2 + \frac{2\pi}{\lambda^2 q^2}\ |\boldsymbol{M}_q|^2$$

$$q_0 = \frac{1}{\sqrt{a\lambda}}$$

d=2 /q

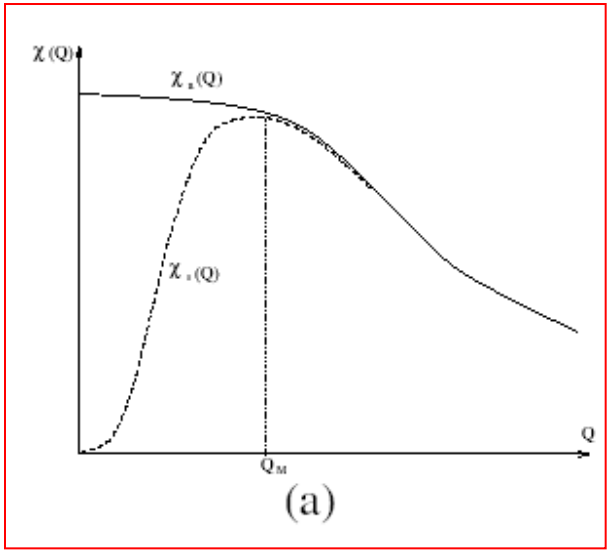
a<<d <<

Coexistence phase

$$E_m = -\sum_Q \frac{\chi(Q)}{2}\left|h_Q\right|^2$$

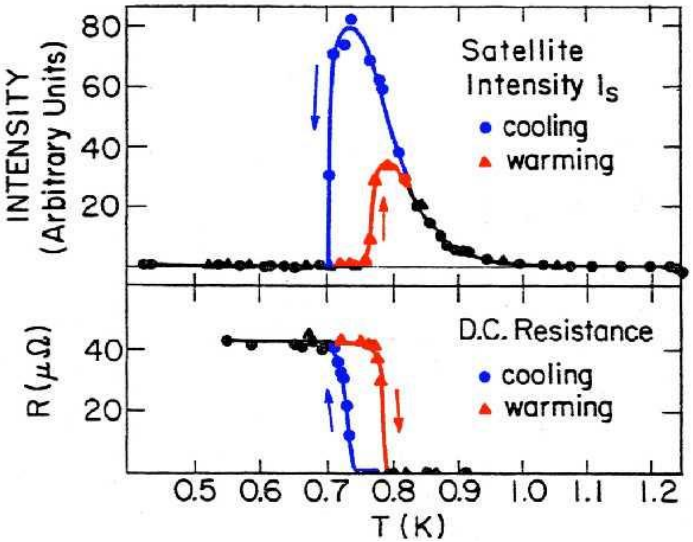
h=I<S>

At T=0 and $Q\xi_0$>>1 following (**Anderson and Suhl, 1959**)

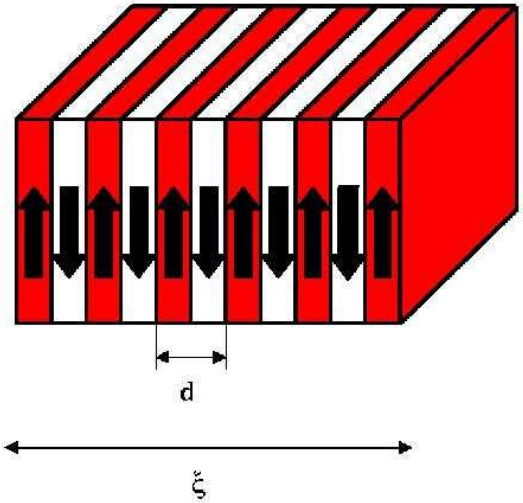


$$\frac{\chi_s(Q)-\chi(Q)}{\chi(0)} = \frac{\pi}{2Q\xi_0}$$

$d \sim (\xi a^2)^{1/3}$



Intensity of the neutron Bragg scattering and resistance as a function of temperature in an $ErRh_4B_4$ (**Sinha *et al.*,1982**). The satellite position corresponds to the wavelength of the modulated magnetic structure around 92 Å.



$d \sim (\xi a)^{1/2}$

$a \ll d \ll \xi$

$HoMo_6S_8$

T_c=1.8 K
T_m=0.74 K
T_c=0.7 K

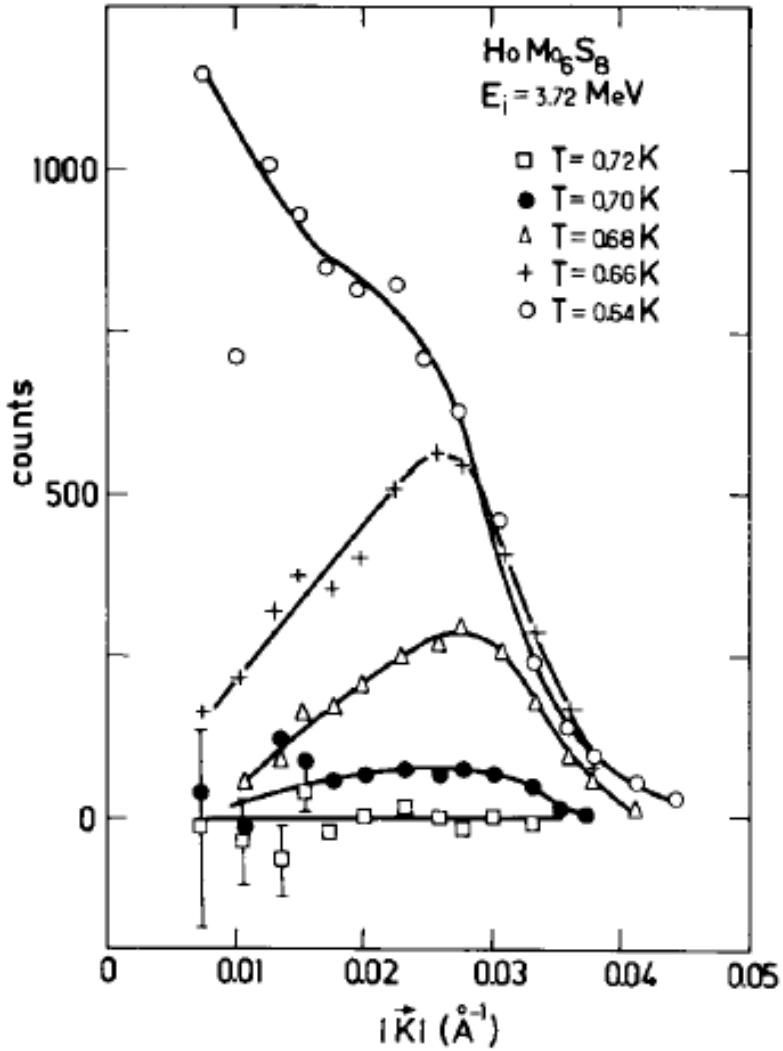
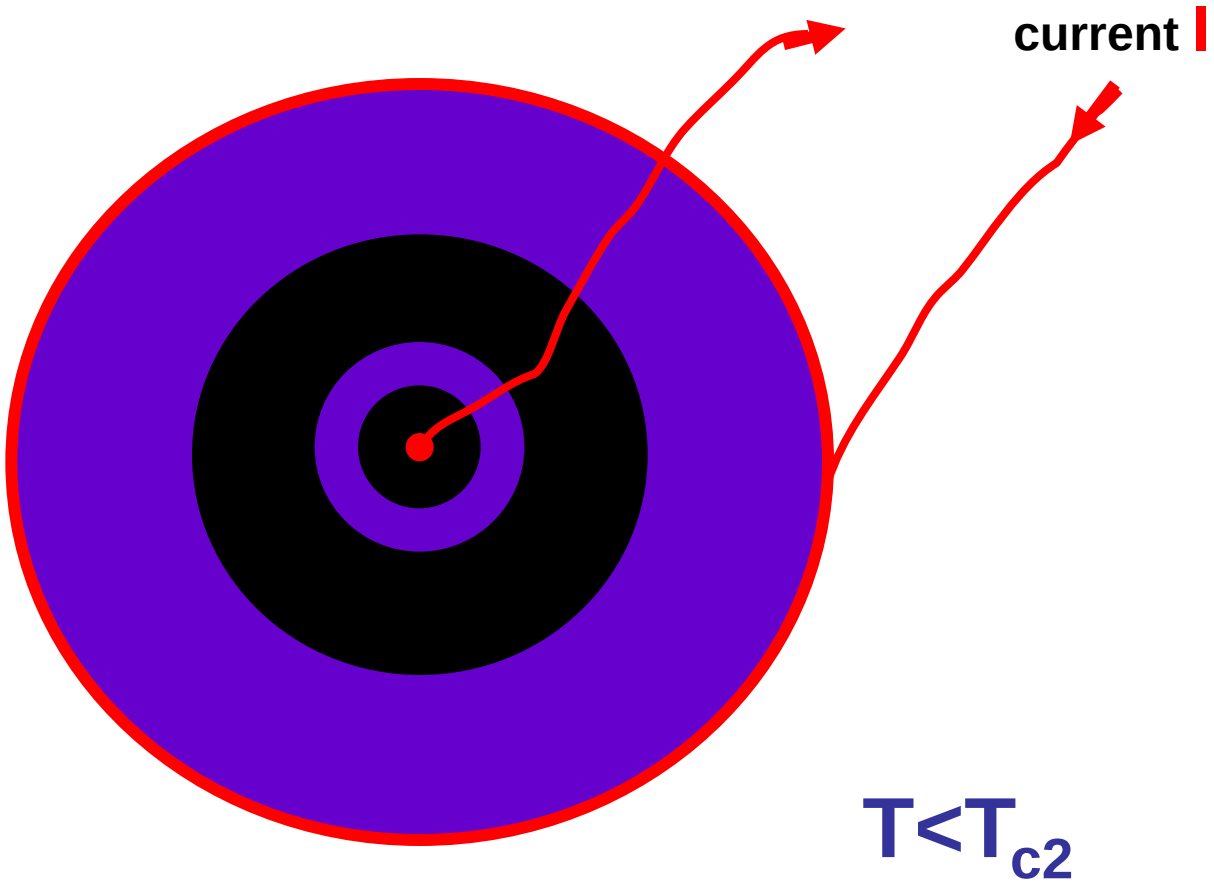
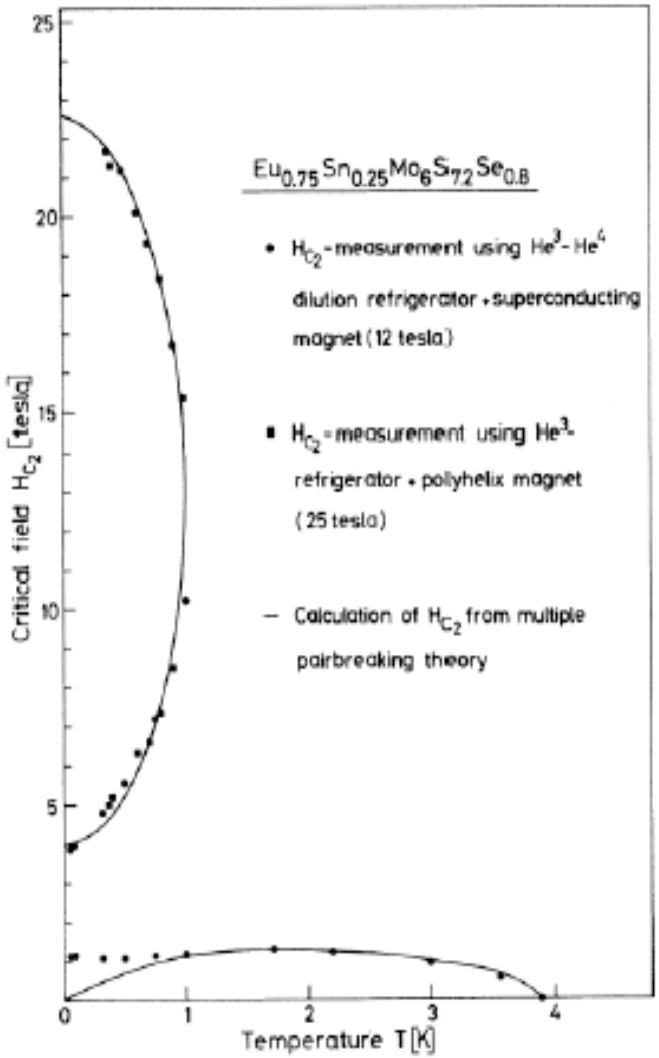


Figure 21. Measurements of small-angle scattering in the visinity of the re-entrant transition in $HoMo_6S_8$. The peak at $|\mathbf{k}| = 0{\cdot}027$ Å develops upon cooling and disappears below 0·64 K [53].

Auto-waves in reentrant superconductors?



magnetic superconductors: re-entrant phase



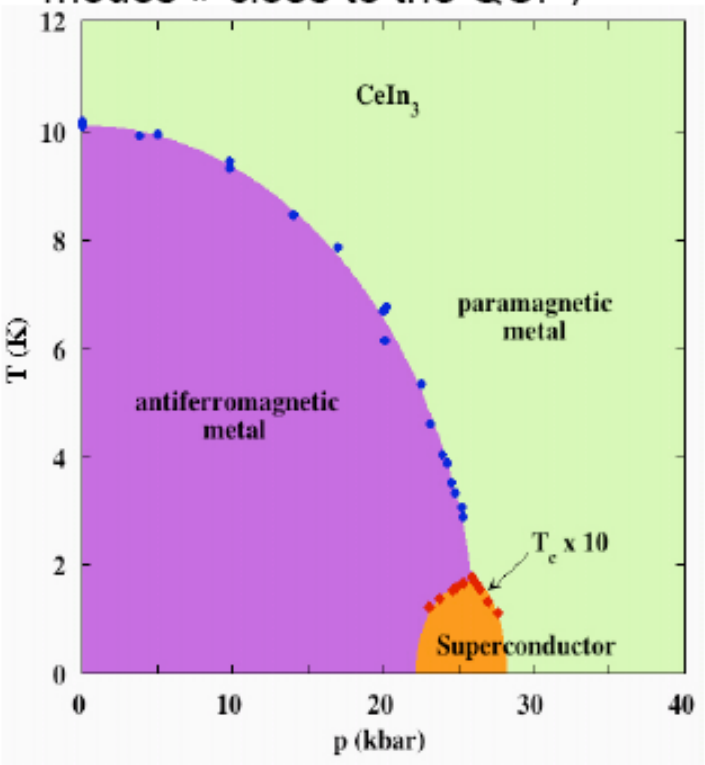
Classical explanation : Jaccarino-Peter effect

- strong paramagnetic limitation in low field
- compensated by external field at higher fields
- J<0 (magnetic ions/conduction electrons)

H.W. Meul et al., PRL **53** (1984) 497

INTRODUCTION: Superconductivity in heavy fermion systems

Unconventional pairing mechanism
(role of correlations, magnetic « soft modes » close to the QCP)



Data: G. Knebel et al

Mathur, N. D. *et al.*, *Nature* 394, 39–43 (1998).

Unconventional superconducting state
(symmetry breaking: driven by the pairing mechanism)

s-wave (A_{1g})
$\Delta_k = \Delta_0$

d-wave (d_{x2-y2}, B_{1g})
$\Delta_k = \Delta_0(k_x^2 - k_y^2)$

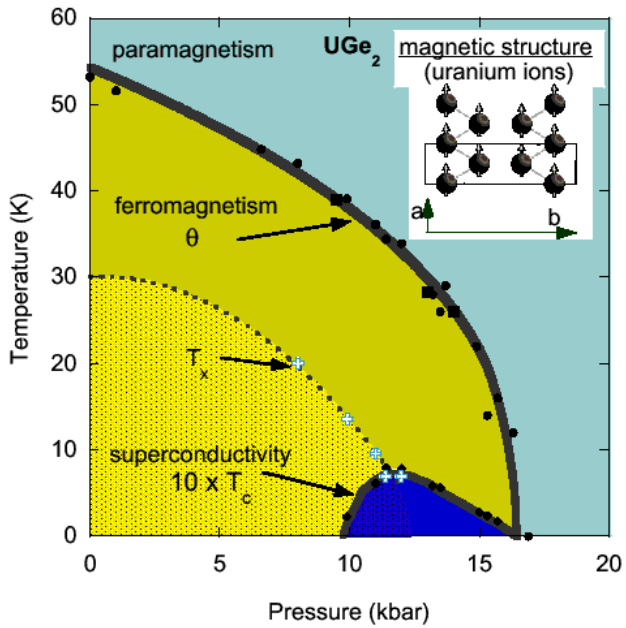
f-wave (E_{2u})
$\mathbf{d_k} = \mathbf{e}_z \Delta_0 k_z (k_x + i k_y)^2$

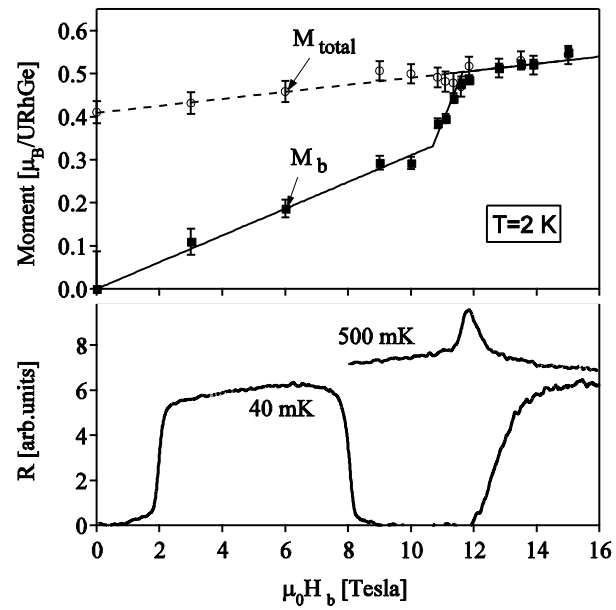
FERROMAGNETIC UNCONVENTIONAL (TRIPLET) SUPERCONDUCTORS

UGe_2 (Saxena *et al.*, 2000) and URhGe (Aoki *et al.*, 2001)

Triplet pairing

UGe_2

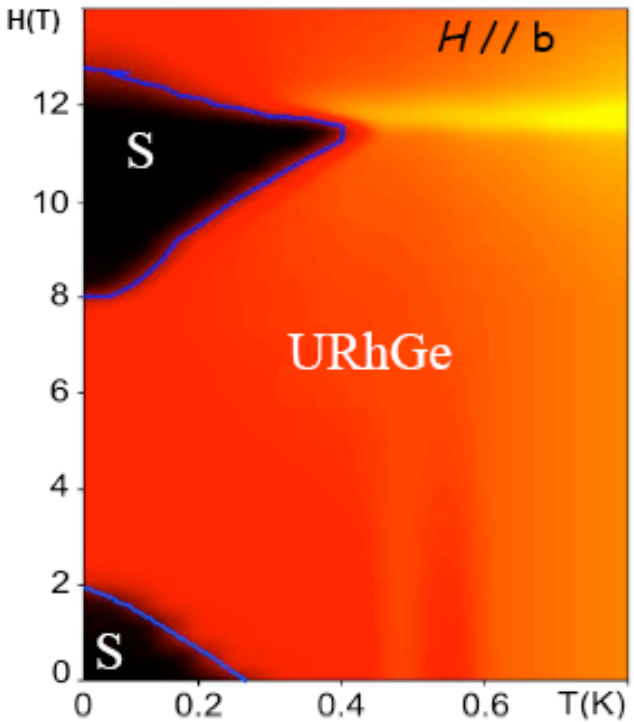




URhGe (a) The total magnetic moment M total and the component M_b measured for H// to the b axis . In (b), variation of the resistance at 40 mK and 500 mK with the field re-entrance of SC between 8-12 T (Levy et al 2005).

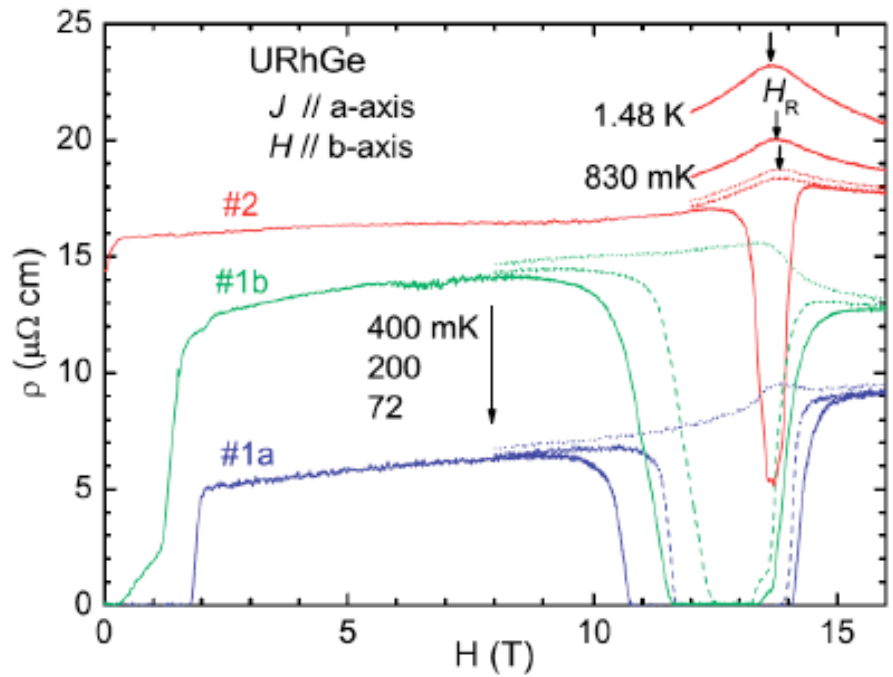
Very recently (2007): UCoGe θ=3K; T_c=0.8K

Heavy fermion ferromagnetic superconductors: re-entrant phase



F. Lévy, I. Sheikin, B. Grenier, A. D. Huxley, **Science 309** (2005) 1343–1346

- no paramagnetic limitation (p-wave)
- same electrons => J>0 (ferromagnetic)
- new phase is very robust:
  - higher T_c
  - higher orbital limit



URhGe: T_{Curie} = 9.5 K
m_s = 0.4μ_B /URhGe, M_s~ 500G

A. Miyake, D. Aoki and J. Flouquet JPSJ **77**, (2008) 094709

Heavy fermion superconductors: ferromagnetic superconductors

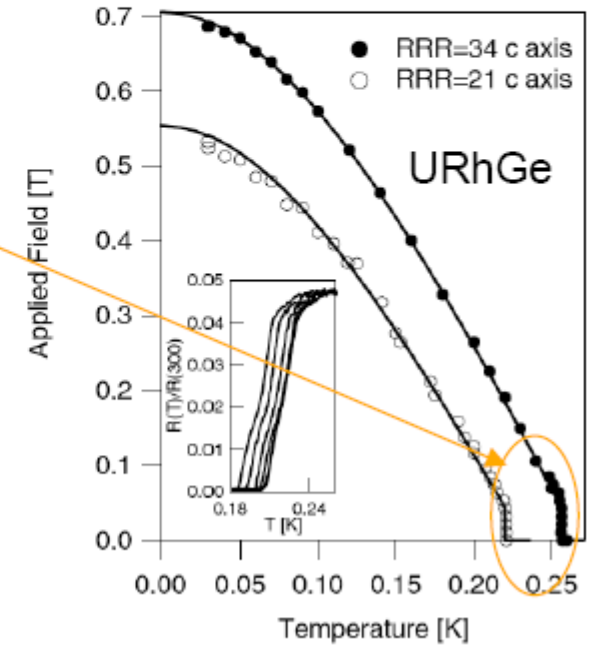
- Spontaneous vortex state ? need [1] $M_s > B_{c1}\left(\frac{\lambda}{w}\right)^{2/3}$

  not met in URhGe, maybe in UCoGe [2]

- Survives when single domain !
  - superconducting if $H_{c2}(T)>\alpha M_s$
  - internal field :

$$B_{eff} = \mu_0 H_{int} + \alpha M_s$$
$$\mu_0 H_{int} = 0 \text{ for } M < M_s$$
$$\mu_0 H_{int} = B_0 - \mu_0 N M_s \text{ for } M = M_s$$

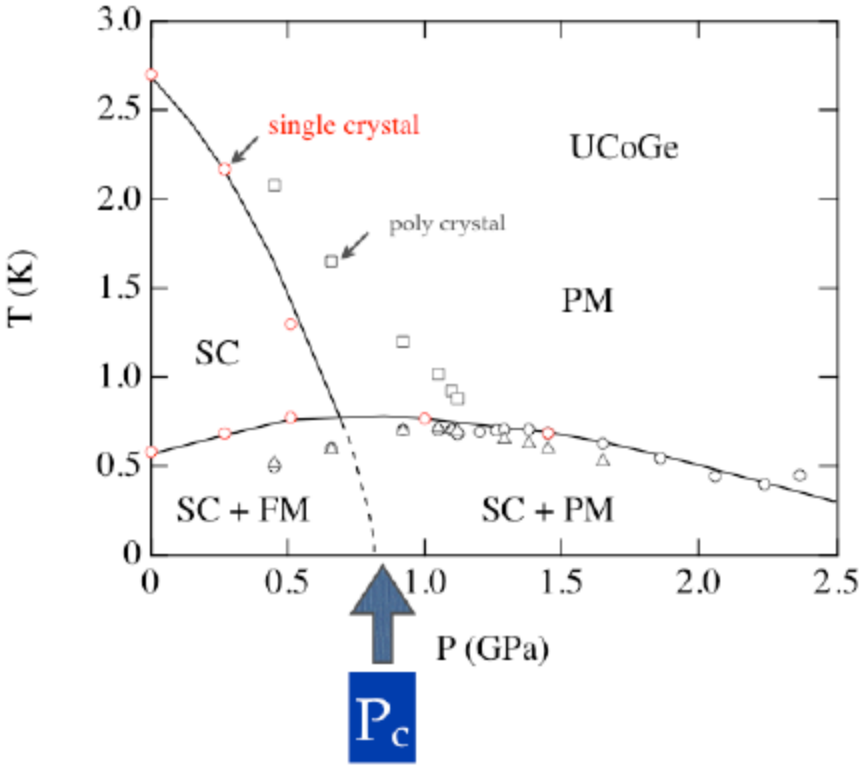
URhGe: T_{Curie} = 9.5 K
m_s = 0.4μ_B /URhGe, M_s~ 500G



Hardy & Huxley PRL94, 247006 (2005)

[1] M. Faure and A. I. Buzdin, PRL 94, 187202 (2005)
[2] T. Ohta et al., JPSJ, 77 (2008) 023707

UCoGe: a newcomer



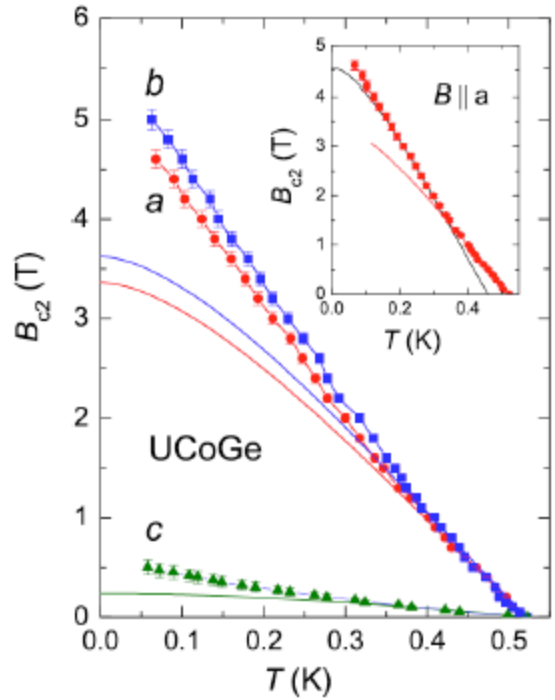
E.Hassinger , D. Aoki, G. Knebel and J. Flouquet,
JPSJ 77 No. 7, July 2008, 073703

V. Mineev, JPSJ 77, October, 2008, 103702

Discovered by A. de Visser et al.:

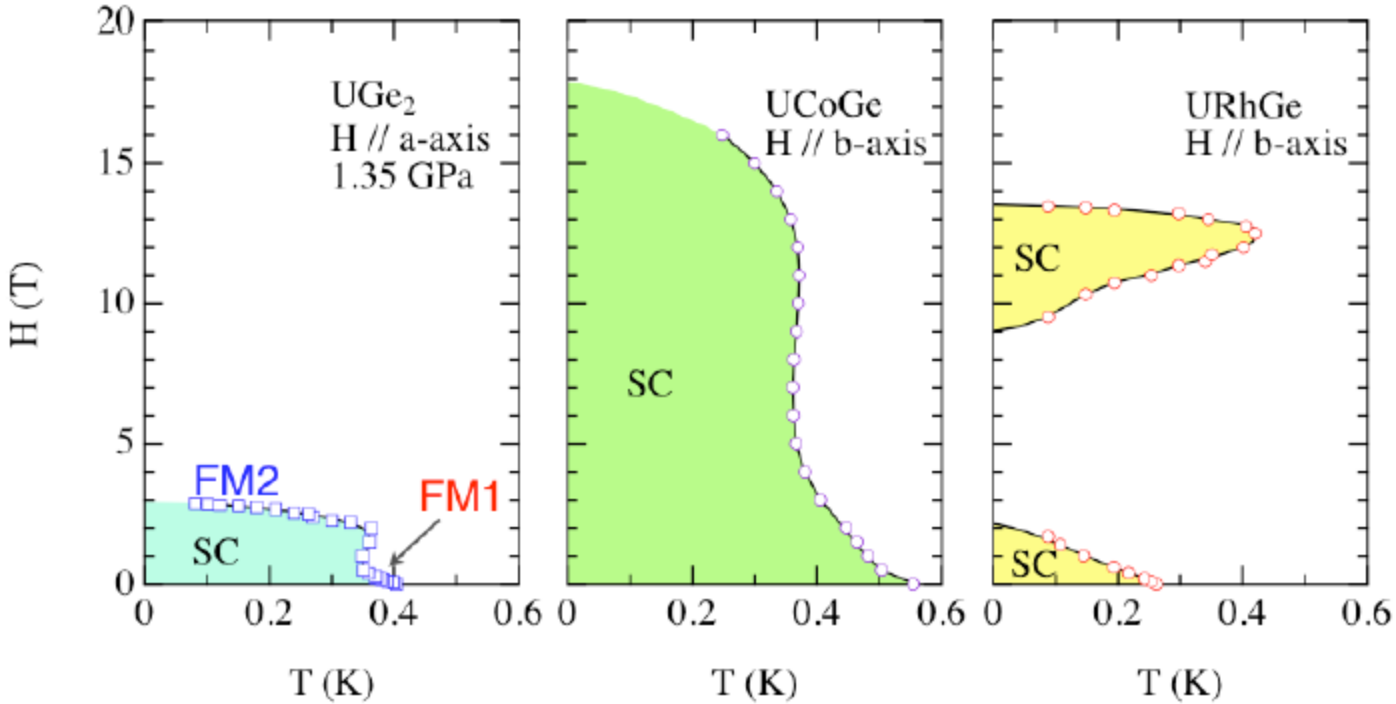
N.T. Huy et al.: Phys. Rev. Lett. 99 (2007) 067006

T_{curie}~2.5K, T_c~0.5K, m_s~0.07μ_B



N. T. Huy, D. E. de Nijs, Y. K. Huang, and A. de Visser
PRL 100, 077002 (2008)

UCoGe: new crystals… and re-entrant phase uncovered



D. Aoki et al.

UGe_2:T_{c_max}~0.8K, T_{curie}~35K, at 12kbars ms~1.4μ_B at 0kbar.

UCoGe:T_c~0. 5K, T_{curie}~2.5K, m_s~0.07μ_B

URhGe:T_c~0.25K, T_{curie}~9.5K, m_s~0.4μ_B

Superconducting order parameter behavior in ferromagnet

Standard Ginzburg-Landau functional:

$$F = a|\Psi|^2 + \frac{1}{4m}|\nabla\Psi|^2 + \frac{b}{2}|\Psi|^4$$

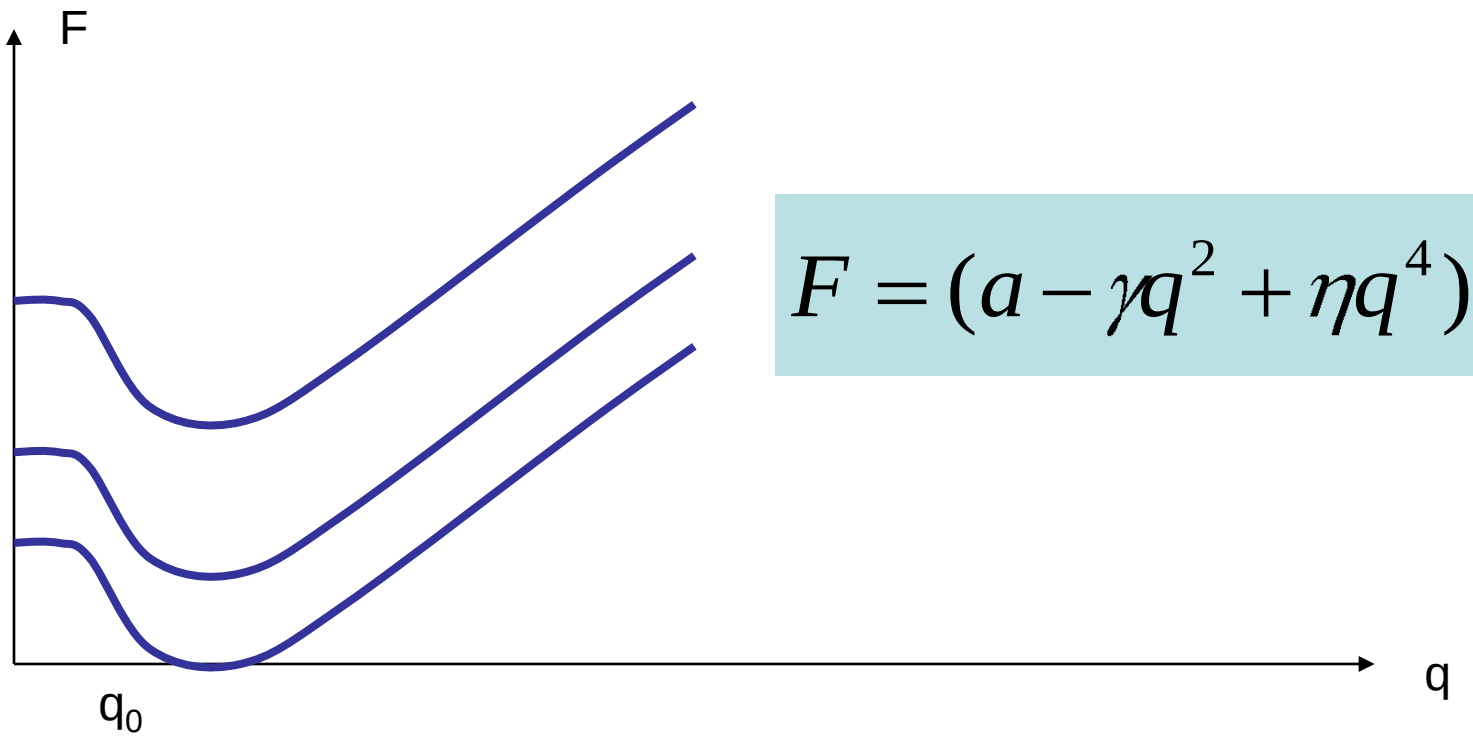
The minimum energy corresponds to Ψ=const

The coefficients of GL functional are functions of internal exchange field h !

Modified Ginzburg-Landau functional ! :

$$F = a|\Psi|^2 - \gamma|\nabla\Psi|^2 + \eta|\nabla^2\Psi|^2 + \ldots$$

The **non-uniform** state Ψ~exp(iqr) will correspond to minimum energy and higher transition temperature



$$F = (a - \gamma q^2 + \eta q^4)\left|\Psi_q\right|^2$$

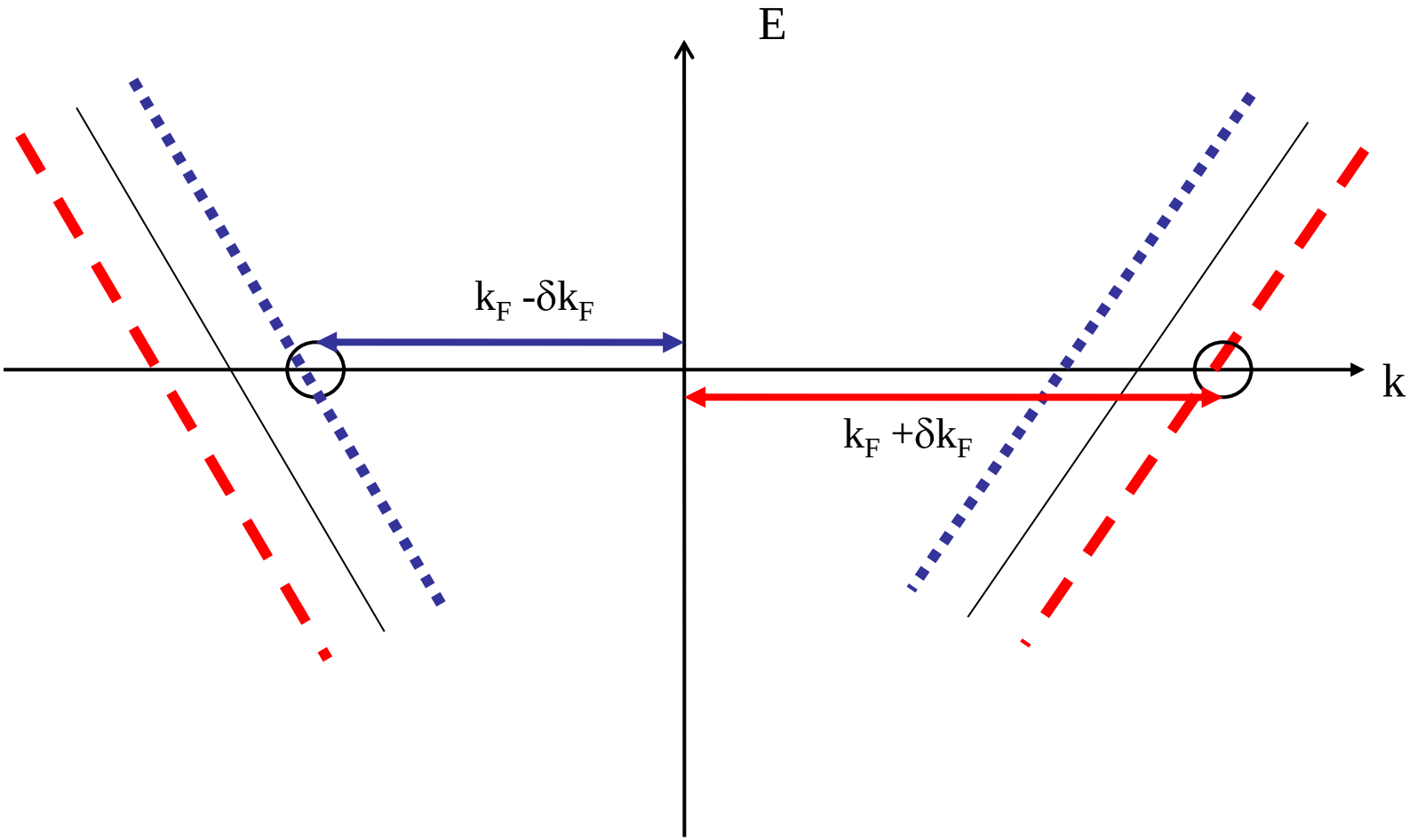
$\Psi \sim \exp(iqr)$ - **Fulde-Ferrell-Larkin-Ovchinnikov state (1964).**

Only in pure superconductors and in the very narrow region.

Proximity effect in ferromagnet ?

In the usual case (normal metal):

$$a\Psi - \frac{1}{4m}\nabla^2\Psi = 0, \text{ and solution for } T > T_c \text{ is } \Psi \propto e^{-qx}, \text{ where } q = \sqrt{4ma}$$

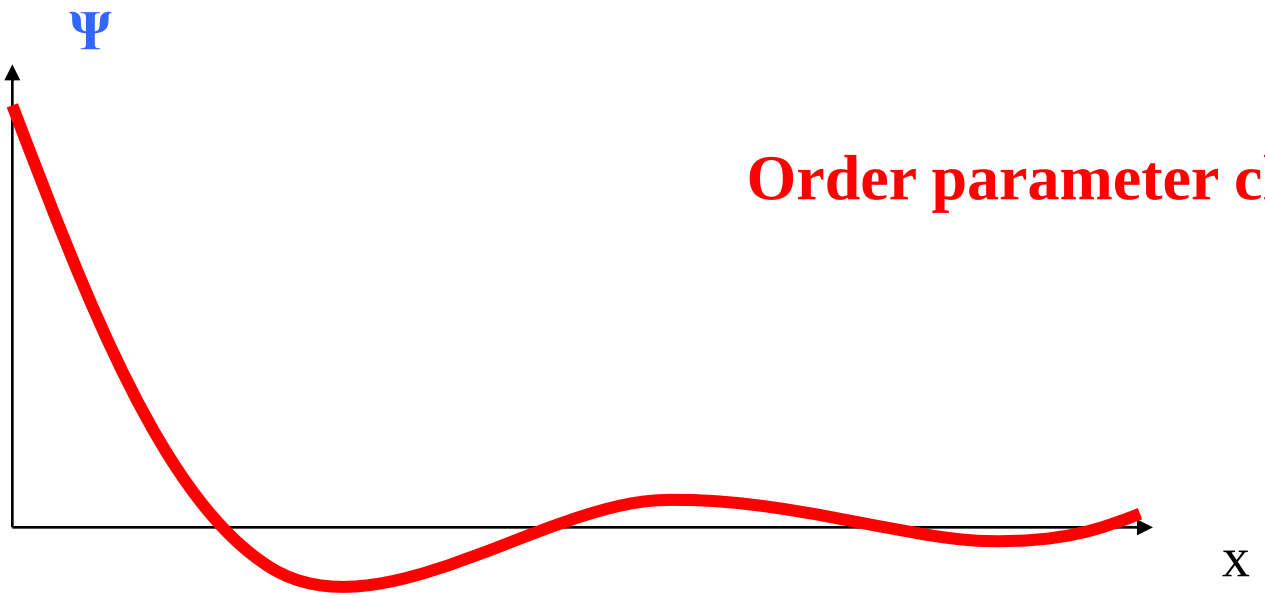


The total momentum of the Cooper pair is
$-(k_F - \delta k_F) + (k_F - \delta k_F) = 2\,\delta k_F$

In ferromagnet (in presence of exchange field) the equation for superconducting order parameter is different

$$a\Psi + \gamma\nabla^2\Psi - \eta\nabla^4\Psi = 0$$

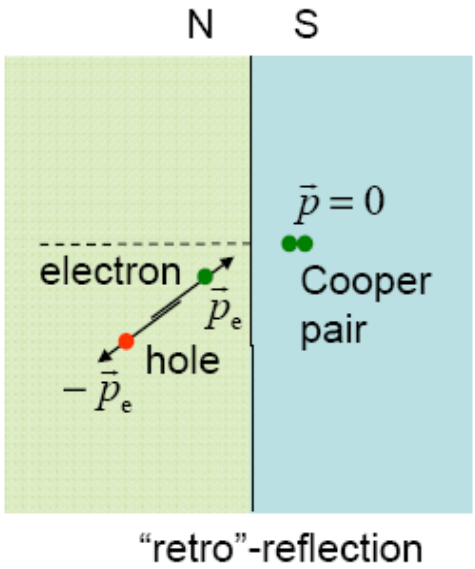
Its solution corresponds to the order parameter which decays with oscillations! $\Psi \sim \exp[-(q_1 + iq_2)x]$



Order parameter changes its sign!

Proximity effect as Andreev reflection

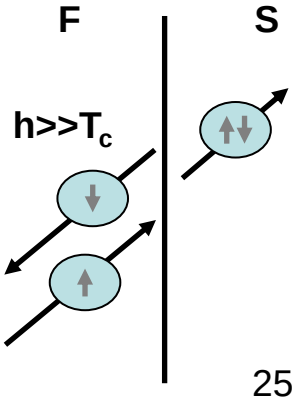
p_F



Classical Andreev reflection

Quantum Andreev reflection

$$p_{F\uparrow} \neq p_{F\downarrow}$$



Eilenberger equations

$$\left(\omega + ih(x) + \frac{1}{2\tau}G(x,\omega)\right)f(x,\theta,\omega) + \frac{1}{2}v_F \cos\theta \frac{\partial f(x,\theta,\omega)}{\partial x}$$

$$= \left(\Delta(x) + \frac{1}{2\tau}F(x,\omega)\right)g(x,\theta,\omega),$$

$$G(x,\omega) = \int \frac{d\Omega}{4\pi} g(x,\theta,\omega), \quad F(x,\omega) = \int \frac{d\Omega}{4\pi} f(x,\theta,\omega),$$

$$f(x,\theta,\omega)f^{+}(x,\theta,\omega) + g^{2}(x,\theta,\omega) = 1, \qquad \text{(A3)}$$

Theory of S-F systems in dirty limit

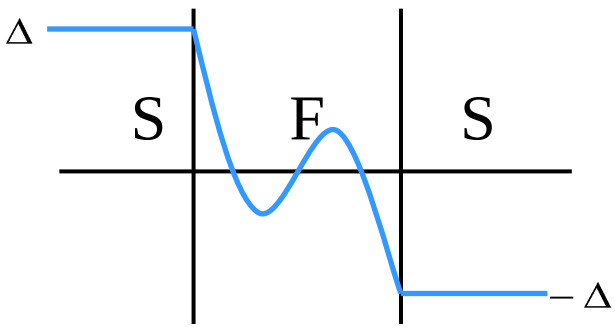
Analysis on the basis of the Usadel equations

$$-\frac{D_f}{2}\vec{\nabla}^2 F_f(x,\omega,h) + (\omega + ih)F_f(x,\omega,h) = 0$$

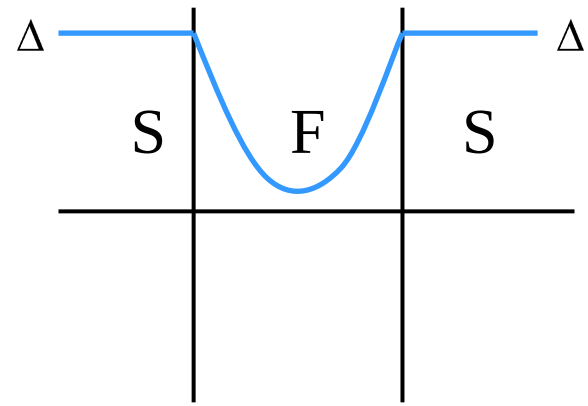
$$G_f^2(x,\omega,h) + F_f(x,\omega,h)F_f^*(x,\omega,-h) = 1$$

leads to the prediction of the oscillatory - like dependence of the critical current on the exchange field h and/or thickness of ferromagnetic layer.

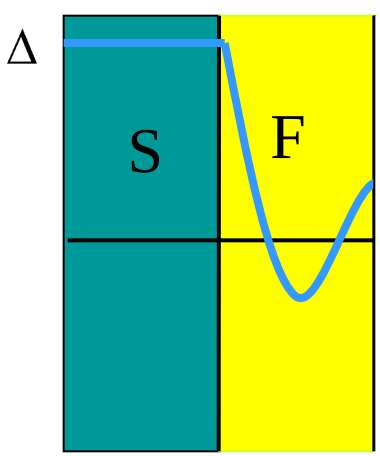
Remarkable effects come from the possible shift of sign of the wave function in the ferromagnet, allowing the possibility of a « π-coupling » between the two superconductors (π-phase difference instead of the usual zero-phase difference)



« π phase »



« 0 phase »



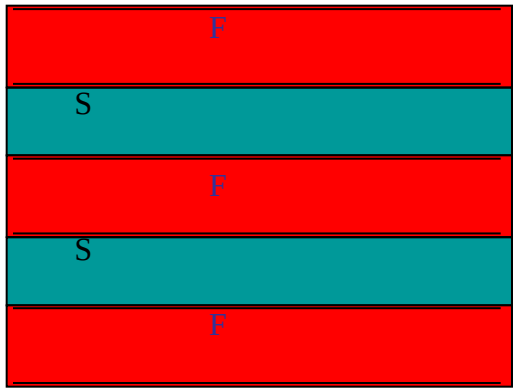
S/F bilayer

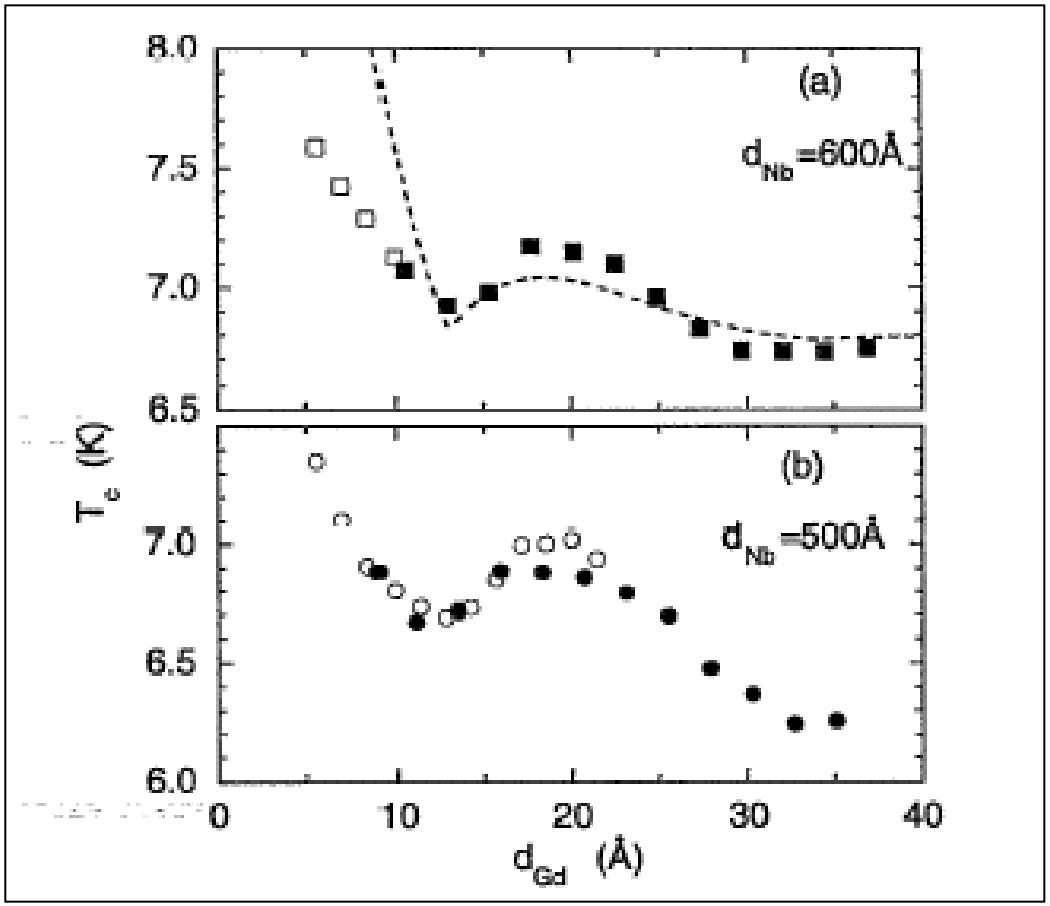
$$\xi_f = \sqrt{D_f / h}$$

h-exchange field,

D_f - diffusion constant

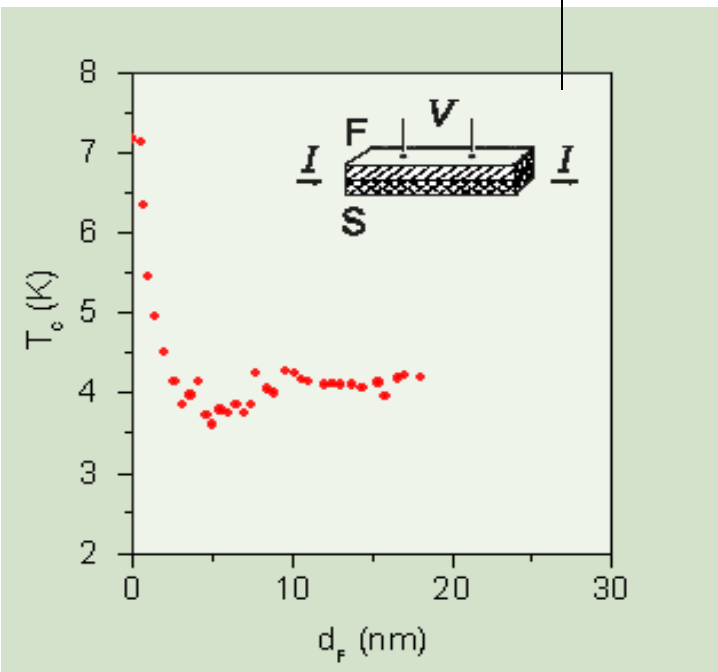
The oscillations of the critical temperature as a function of the thickness of the ferromagnetic layer in S/F multilayers has been predicted in 1990 and later observed on experiment by Jiang et al. PRL, **1995**, in Nb/Gd multilayers



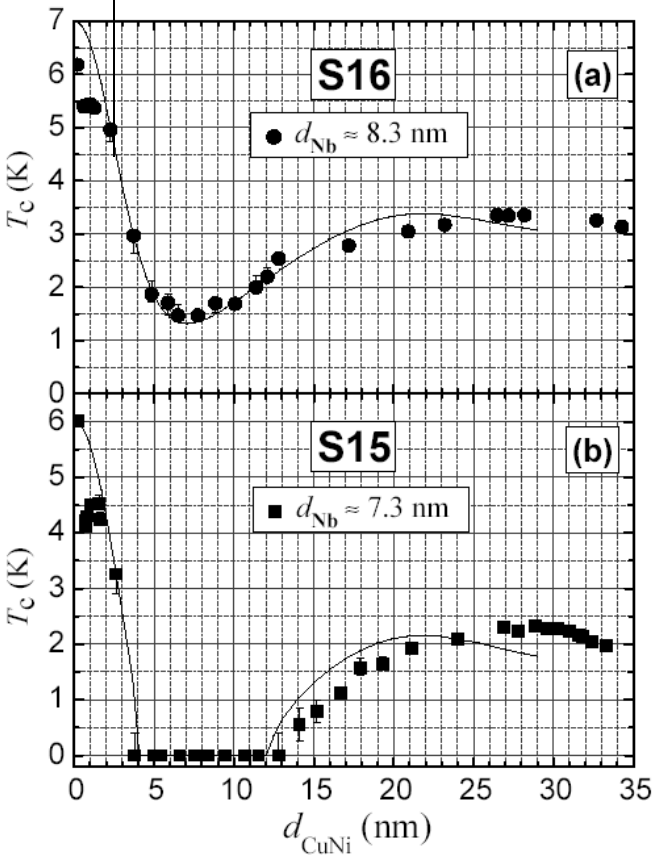


SF-bilayer T_c-oscillations

Ryazanov et al. JETP Lett. 77, 39 (2003) Nb-$Cu_{0.43}Ni_{0.57}$



V. Zdravkov, A. Sidorenko et al
cond-mat/0602448 (2006)
Nb-$Cu_{0.41}Ni_{0.59}$



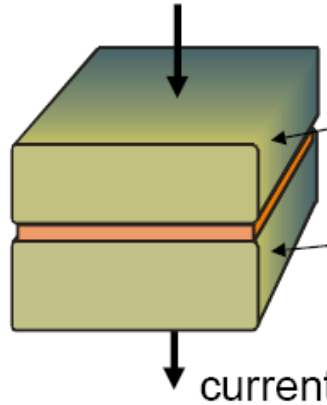
$\mathbf{d_{Fmin} = (1/4)\ \lambda_{ex}}$ largest T_c-suppression

Josephson effect

superconductor

tunnel barrier

superconductor



$$\Psi_1 = |\Psi_1| \exp(i\theta_1)$$

$$\Psi_2 = |\Psi_2| \exp(i\theta_2)$$

$$|\Psi_1| = |\Psi_2|$$

superconducting phase difference: $\varphi = \theta_1 - \theta_2$

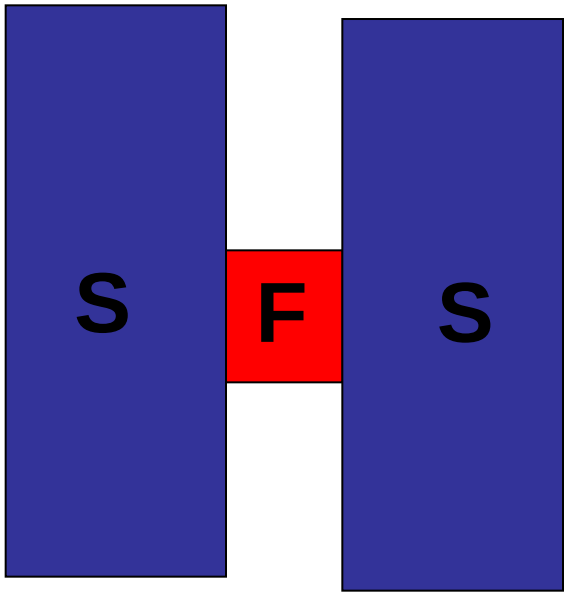
Josephson relations

$$\begin{cases} I_s = I_c \sin\varphi \\ V = \dfrac{\hbar}{2e}\dfrac{d\varphi}{dt} \end{cases}$$

Electromagnetic radiation at the frequency f

$$f = \frac{V}{\Phi_0}$$

S-F-S Josephson junction in the clean/dirty limit

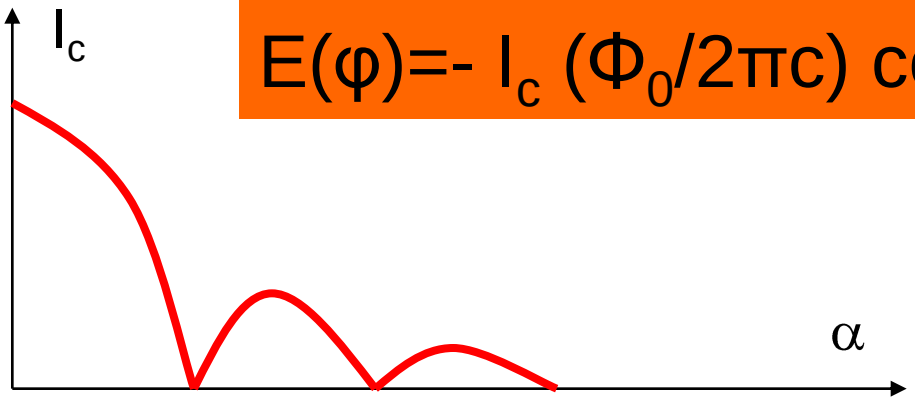


Damping oscillating dependence of the critical current I_c as the function of the parameter $\alpha = hd_F / v_F$ has been predicted.

(Buzdin, Bulaevskii and Panjukov, JETP Lett. 81)

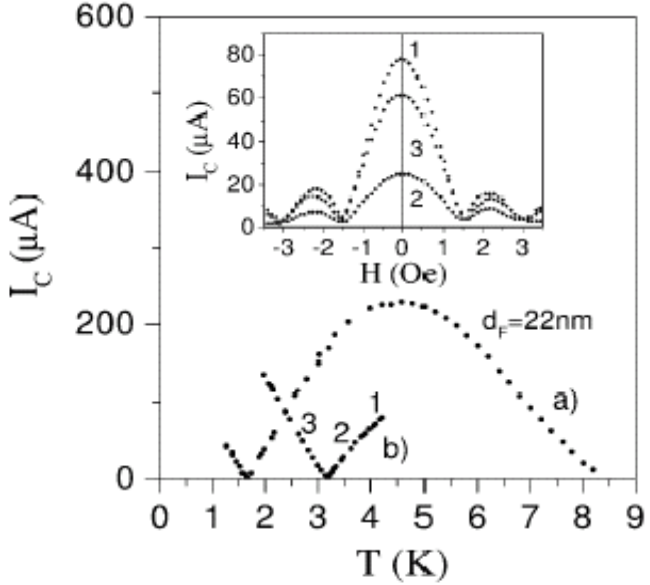
h- exchange field in the ferromagnet, d_F - its thickness

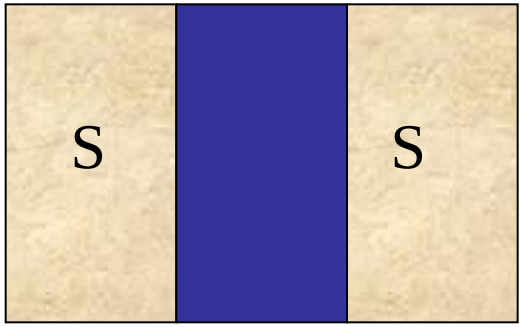
$E(\varphi) = - I_c (\Phi_0 / 2\pi c) \cos\varphi$

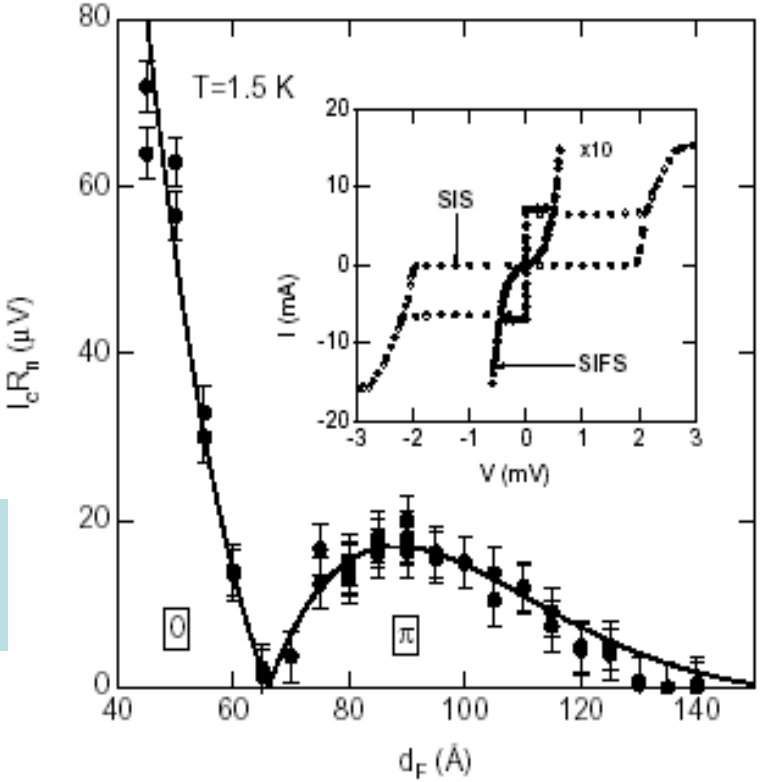


$J(\varphi) = I_c \sin\varphi$

The oscillations of the critical current as a function of temperature (for different thickness of the ferromagnet) in S/F/S trilayers have been observed on experiment by Ryazanov et al. 2000, PRL







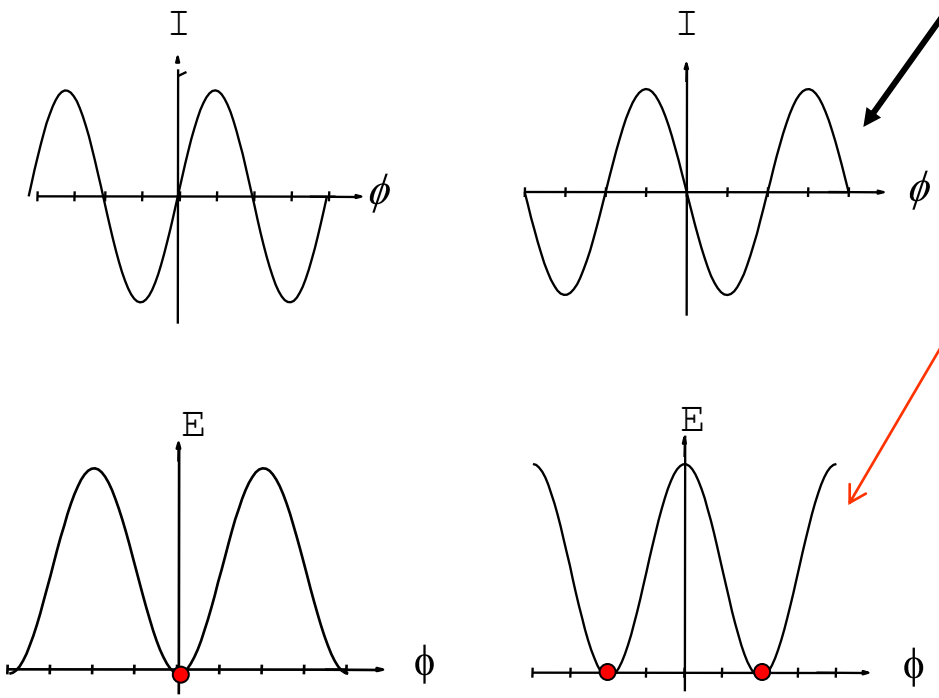
and as a function of a ferromagnetic layer thickness by Kontos et al. 2002, PRL

Phase-sensitive experiments

π-junction in one-contact interferometer

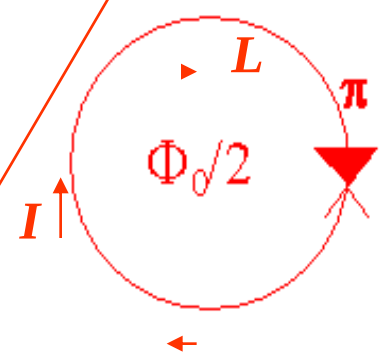
0-junction
minimum energy at 0

π-junction
minimum energy at π



$I = I_c \sin(\pi+\phi) = -I_c \sin\phi$

$E = E_J[1-\cos(\pi+\phi)] = E_J[1+\cos\phi]$



$2\pi L I_c > \Phi_0/2$

$\phi = \pi = (2\pi/\Phi_0)\int A\,dl = 2\pi\,\Phi/\Phi_0$

Spontaneous circulating current in a closed superconducting loop when $\beta_L > 1$ with NO applied flux

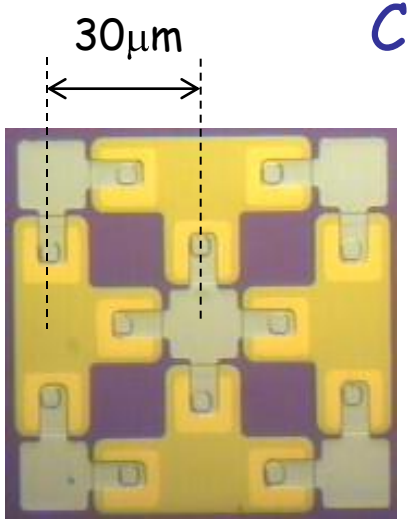
$\beta_L = \Phi_0/(4\pi L I_c)$

$\Phi = \Phi_0/2$

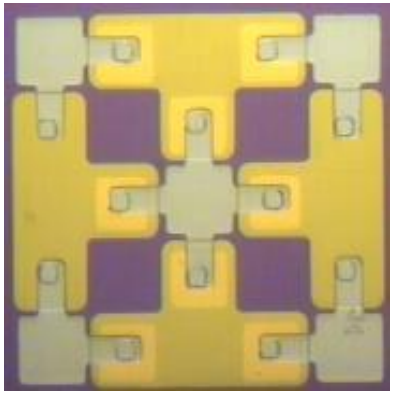
Bulaevsky, Kuzii, Sobyanin, JETP Lett. 1977

Cluster Designs (Ryazanov et al.)

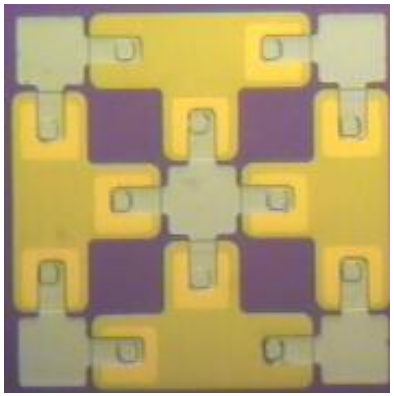
2 x 2



unfrustrated

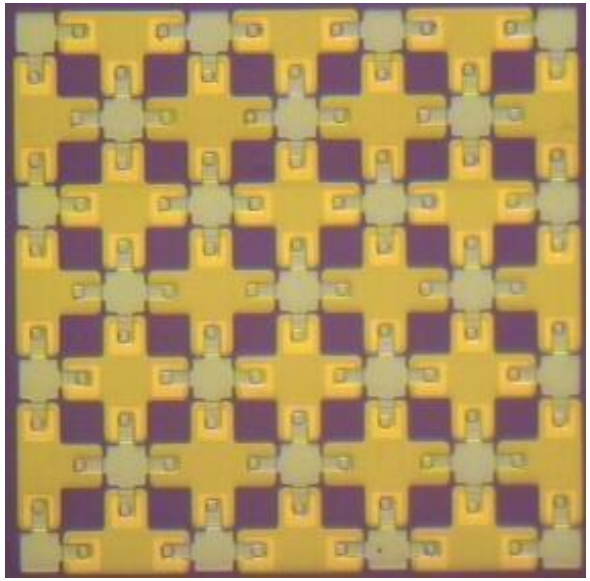


fully-frustrated

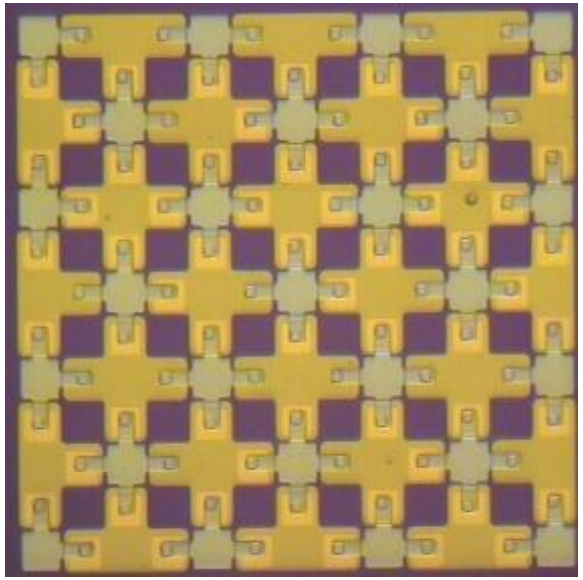


checkerboard-frustrated

6 x 6



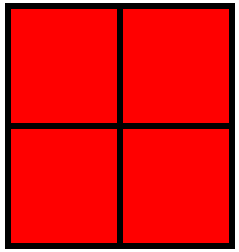
fully-frustrated

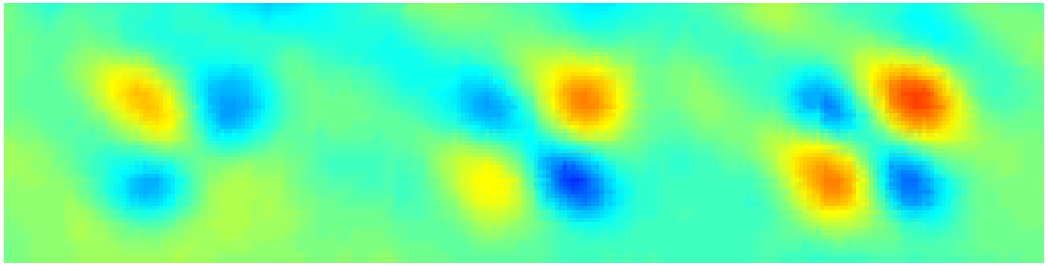


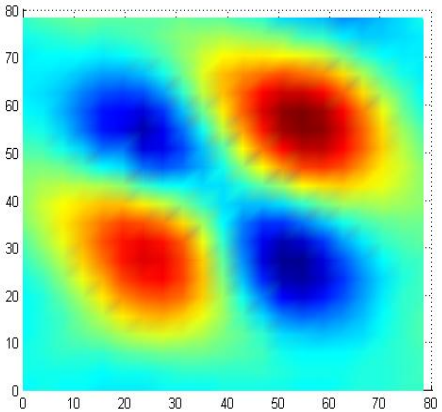
checkerboard-frustrated

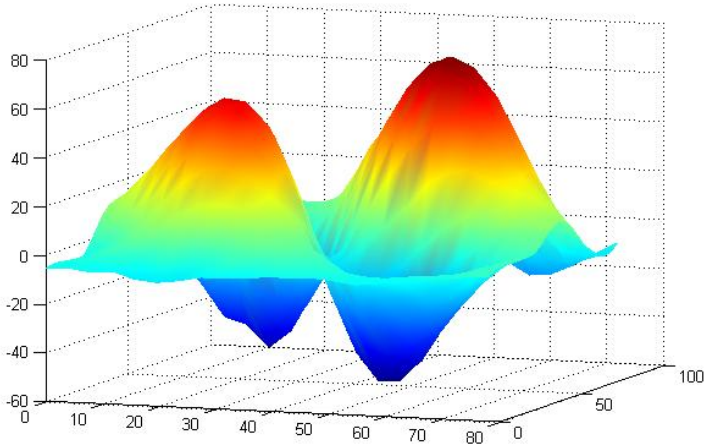
2 x 2 arrays: spontaneous vortices

Fully frustrated

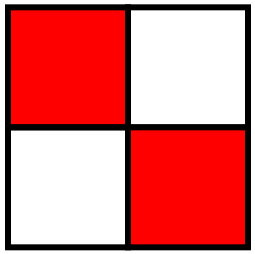


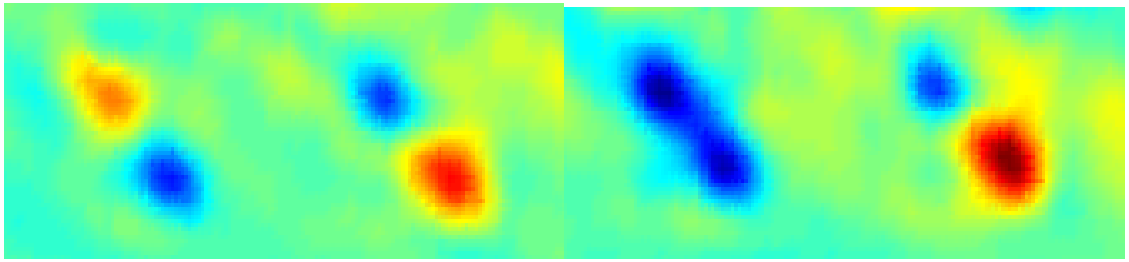






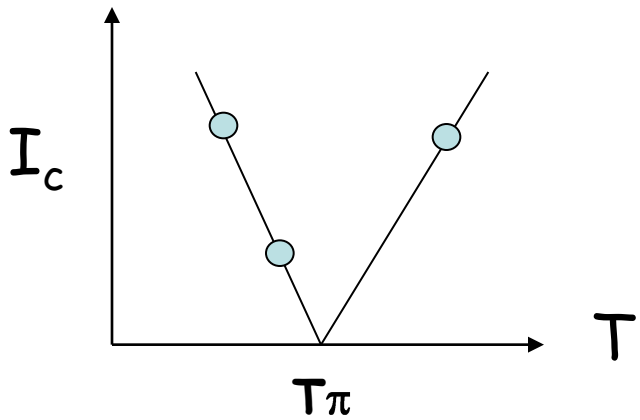
Checkerboard frustrated



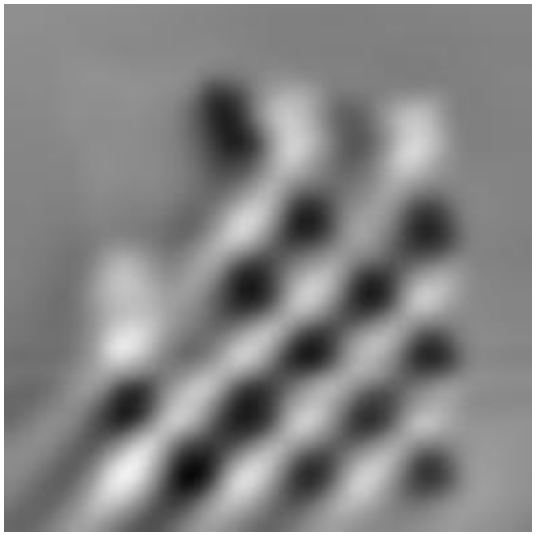


Scanning SQUID Microscope images

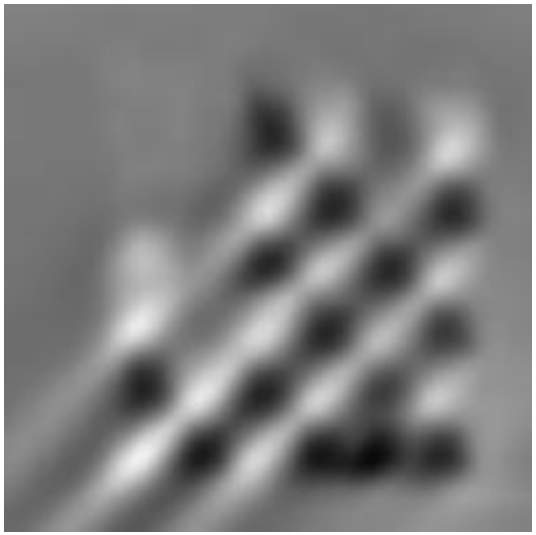
(Ryazanov et al., Nature Physics, 2008))



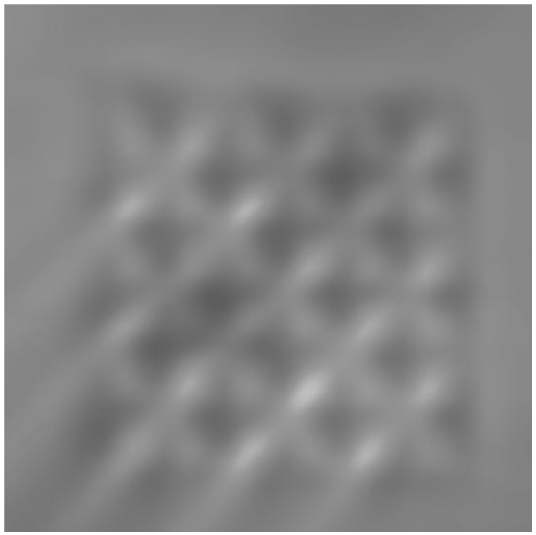
T = 1.7K



T = 2.75K

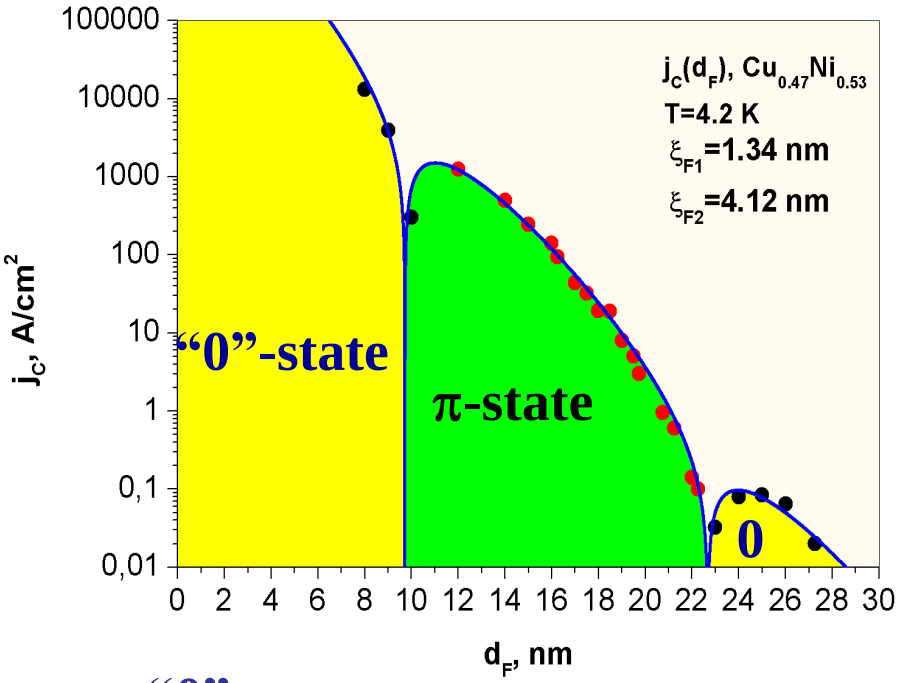


T = 4.2K



Critical current density vs. F-layer thickness (V.A.Oboznov et al., PRL, 2006)

$$I_c = I_{c0}\exp(-d_F/\xi_{F1})\,|\cos(d_F/\xi_{F2}) + \sin(d_F/\xi_{F2})|$$



$$d_F >> \xi_{F1}$$

Spin-flip scattering decreases the decaying length and increases the oscillation period.

$$\xi_{F2} > \xi_{F1}$$

$Nb\text{-}Cu_{0.47}Ni_{0.53}\text{-}Nb$

"0"-state

$$I = I_c \sin\varphi$$

π-state

$$I = I_c \sin(\varphi + \pi) = -I_c \sin(\varphi)$$

Critical current vs. temperature

Nb-$Cu_{0.47}Ni_{0.53}$-Nb
d_F=9-24 nm
$h=E_{ex}$ ~ 850 K (T_{Curie}= 70 K)

"Temperature dependent" spin-flip scattering

$$\frac{1}{\xi_{F1}} = \frac{1}{\xi_F}\sqrt{\sqrt{1+\left(\frac{1}{h\tau_s}\right)^2}+\left(\frac{1}{h\tau_s}\right)},$$

$$\frac{1}{\xi_{F2}} = \frac{1}{\xi_F}\sqrt{\sqrt{1+\left(\frac{1}{h\tau_s}\right)^2}-\left(\frac{1}{h\tau_s}\right)}$$

$$\xi_{F2} > \xi_{F1}$$

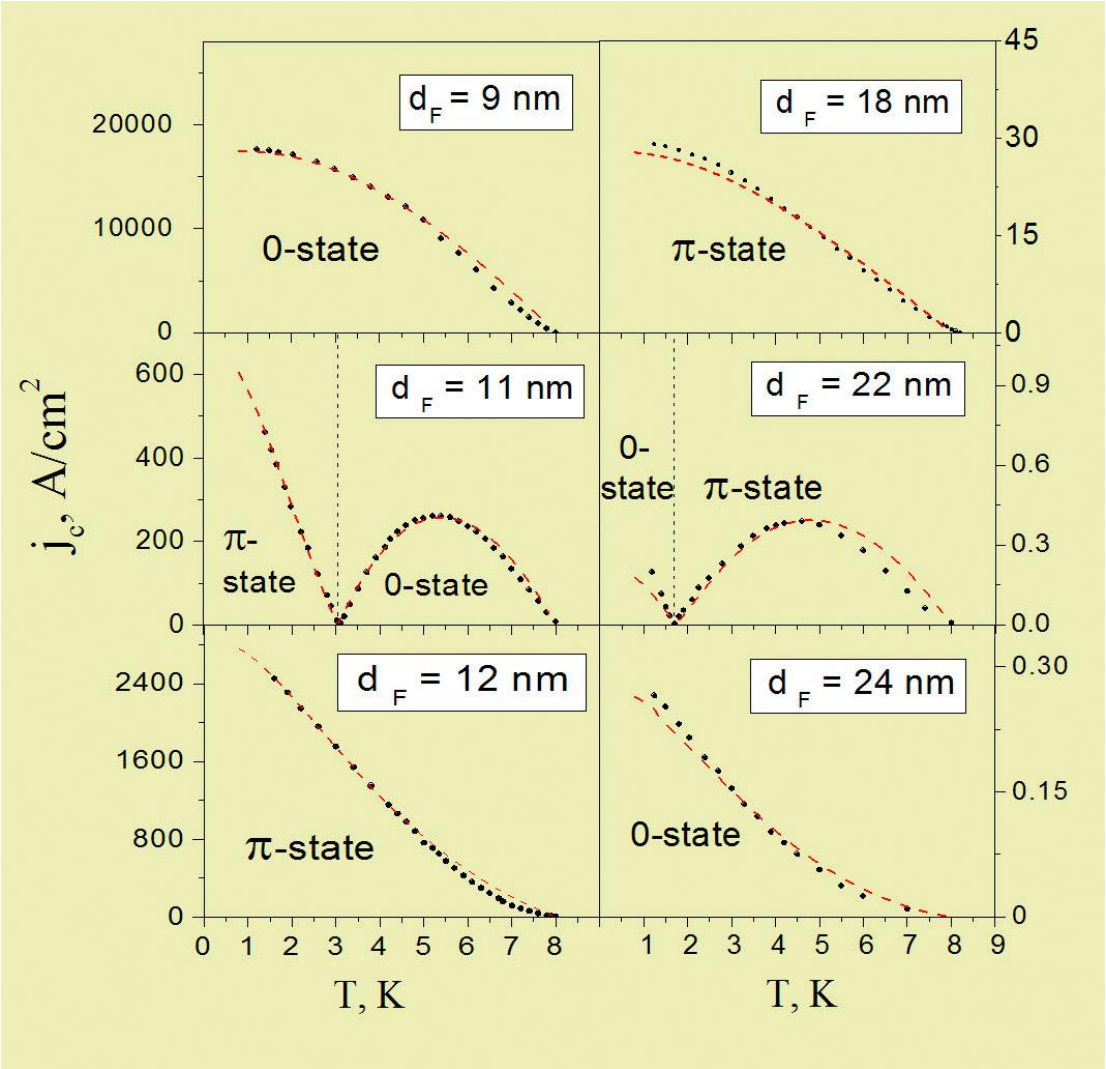
$$\left(\omega + iE_{ex} + \frac{\hbar\cos\Theta}{\tau_s}\right)\sin\Theta - \frac{\hbar D}{2}\frac{\partial^2\Theta}{\partial x^2} = 0.$$

G = cos Θ(T); F = sin Θ(T)

Effective spin-flip rate
$\Gamma(T)$= cos $\Theta(T)/\tau_S$;

Critical current vs. temperature (0-π- and π-0- transitions)

$Nb-Cu_{0.47}Ni_{0.53}-Nb$
d_{F1}=10-11 nm
d_{F2}=22 nm



(V.A.Oboznov et al., PRL, 2006)

Triplet correlations

Bergeret, Volkov Efetov - as a review see Bergeret et al., Rev. Mod. Phys. (2005).

$$D\partial_X^2 \hat{f} - 2|\omega|\hat{f} + i\,\mathrm{sgn}(\omega)\left(\hat{f}\hat{V}^* - \hat{V}\hat{f}\right) = 0$$

$$\hat{V} = J\begin{pmatrix} \cos\alpha & \pm i\sin\alpha \\ \mp i\sin\alpha & -\cos\alpha \end{pmatrix}$$

Structure of the functions f:

$$f = i\hat{\tau}_2\left(f_3(x)\hat{\sigma}_3 + f_0(x)\right) + i\hat{\tau}_1\hat{\sigma}_1 f_1(x)$$

σ, τ -Pauli matrices (spin, Nambu)

$f_3 \propto \langle\psi_\uparrow\psi_\downarrow\rangle - \langle\psi_\downarrow\psi_\uparrow\rangle$ -Singlet condensate

$f_0 \propto \langle\psi_\uparrow\psi_\downarrow\rangle + \langle\psi_\downarrow\psi_\uparrow\rangle$ -Triplet condensate (with projection 0 on z-axis)

$f_1 \propto \langle\psi_\uparrow\psi_\uparrow\rangle \propto \langle\psi_\downarrow\psi_\downarrow\rangle$ -Triplet condensate (with projection +1,-1)

8

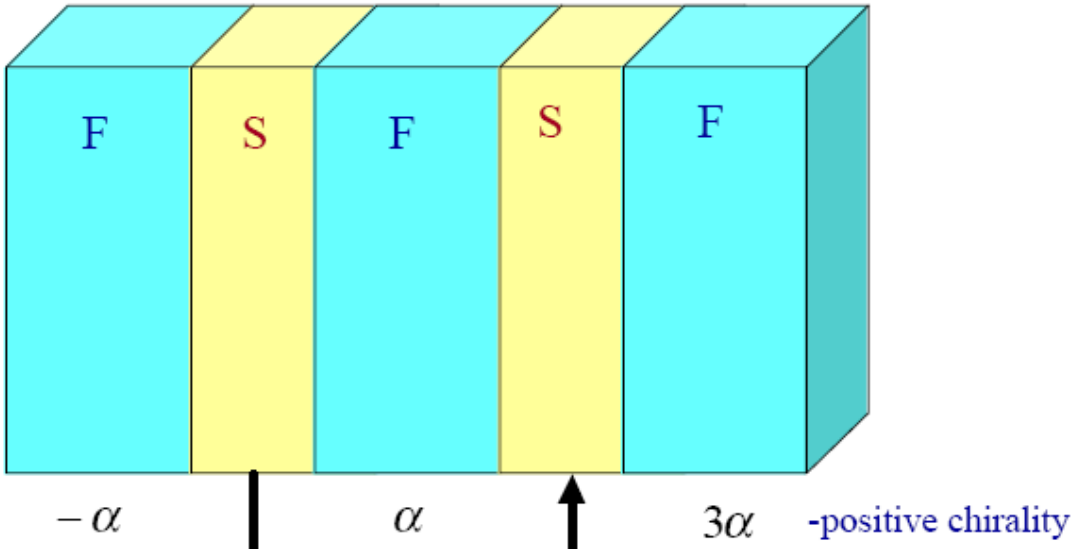


TABLE I. Characteristic length scales of S/F proximity effect.

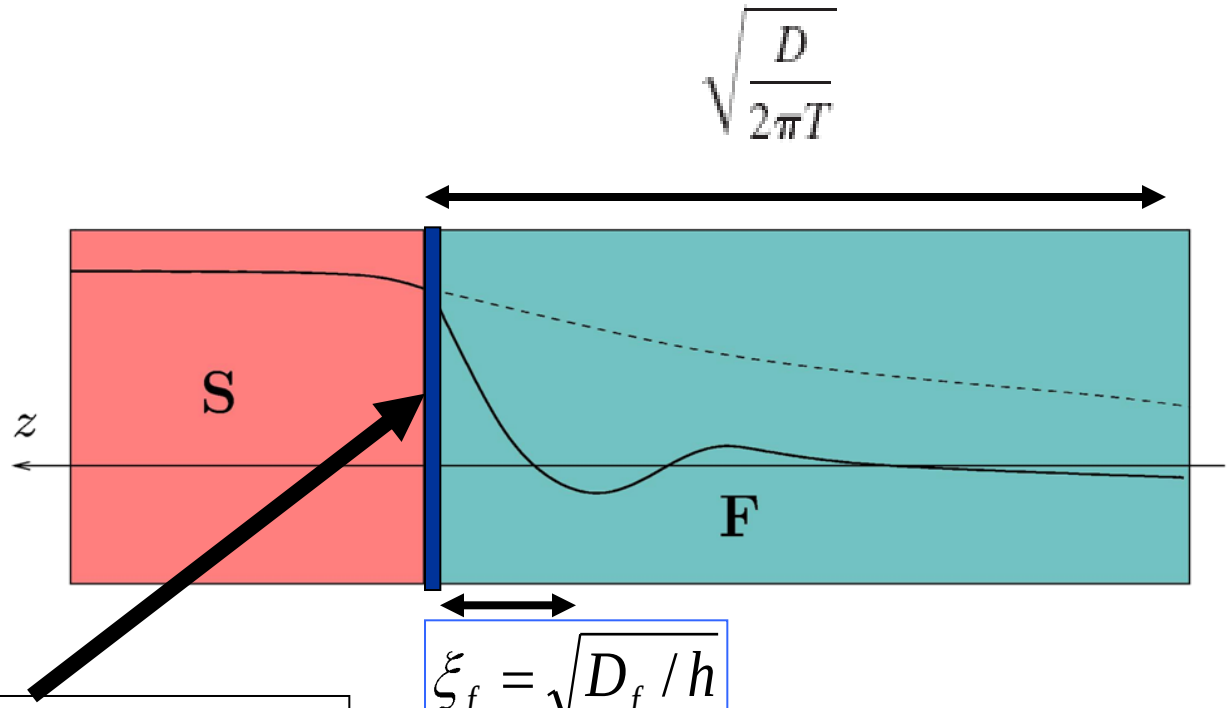
Thermal diffusion length L_T	$\sqrt{\frac{D}{2\pi T}}$
Superconducting coherence length ξ_s	$\frac{v_{Fs}}{2\pi T_c}$ in pure limit
	$\sqrt{\frac{D_s}{2\pi T_c}}$ in dirty limit
Superconducting correlation decay length ξ_{1f} in a ferromagnet	$\frac{v_{Ff}}{2\pi T}$ in pure limit
	$\xi_f = \sqrt{\frac{D_f}{h}}$ in dirty limit
Superconducting correlation oscillating length ξ_{2f} in a ferromagnet	$\frac{v_{Ff}}{2h}$ in pure limit
	$\xi_f = \sqrt{\frac{D_f}{h}}$ in dirty limit

Triplet proximity effect may substantially increase the decaying length in the dirty limit.

The same, but larger amplitude

$$\sqrt{\frac{D}{2\pi T}}$$

No oscillations



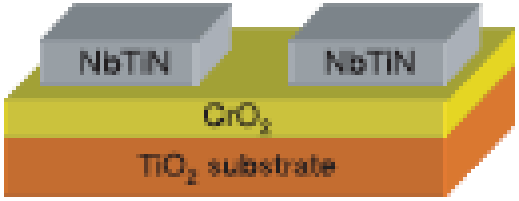
Some source of triplet correlations ?

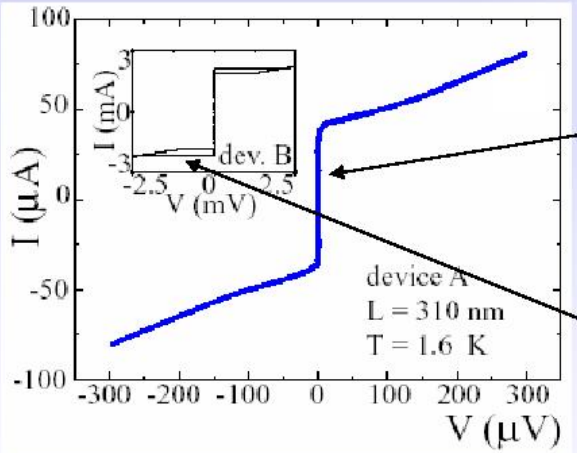
Why difficult to observe ? Magnetic scattering and spin-orbit scattering are harmful for long ranged triplet component.

Magnetic disorder, spin-waves…

Supercurrent measured in NbTiN/CrO2/NbTiN junctions

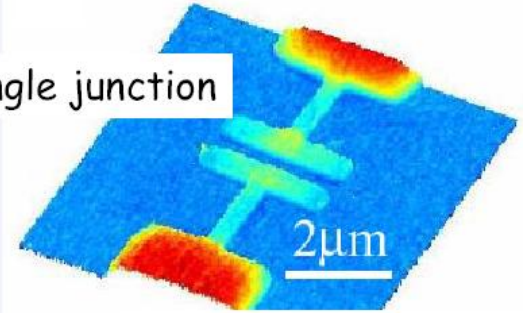
Klapwijk's group in Delft



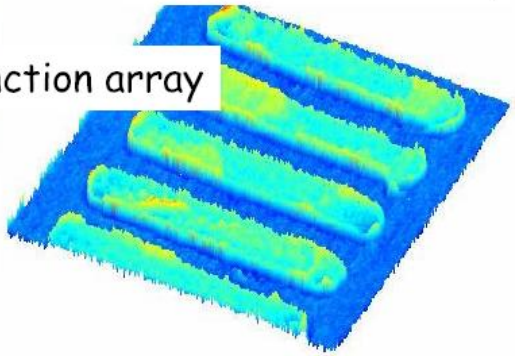


Zero resistance (supercurrent) branch

High I_c devices show hysteresis

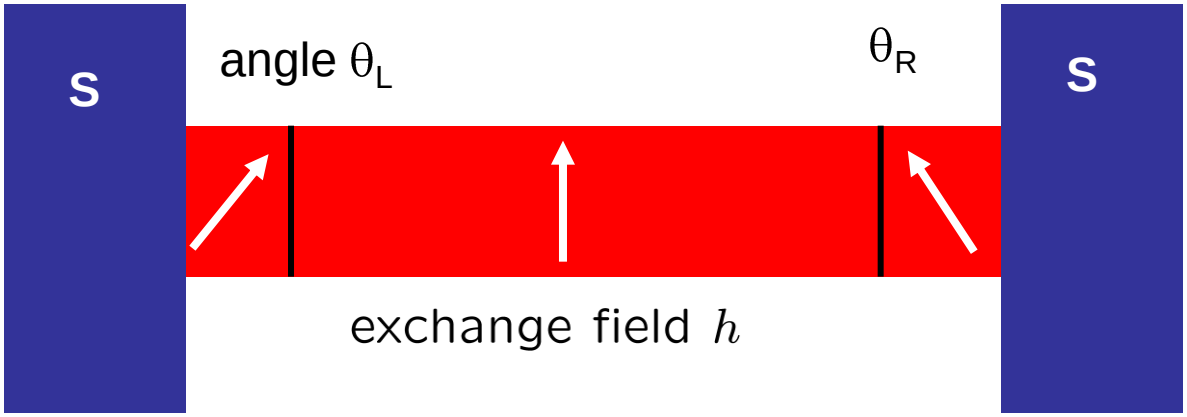


Typical junction length: 300nm - 1 μm



Long junctions with « large » I_c

CrO_2 is half-metallic !



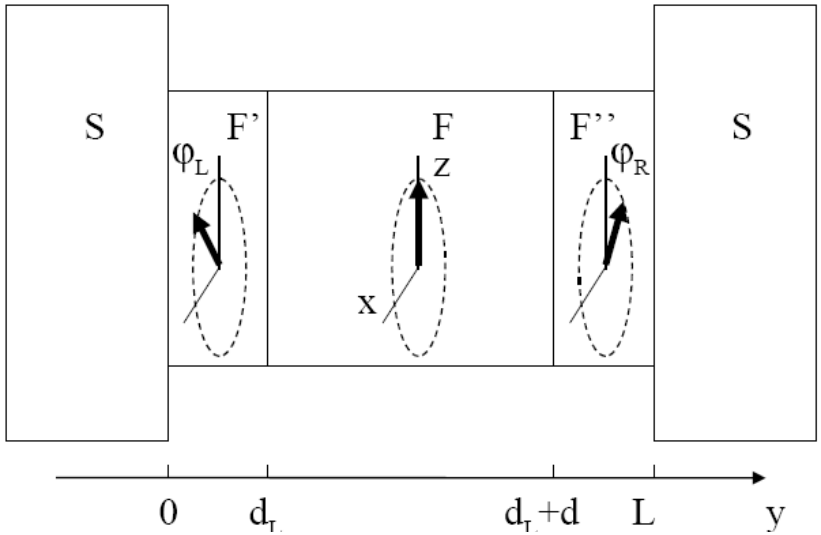


FIG. 1: Geometry of S/F'/F/F''/S junction. The arrows indicate non-colinear orientations of magnetizations in each layer with thickness d_L, d, d_R, respectively ($L = d_L + d + d_R$).

$\xi_f \ll L \ll \xi_0$

$$eR_F I_c = -\frac{2\Delta(T)^2 h_0^2}{\pi^3 T_c^3} \sin\theta_R \sin\theta_L$$

(+ small term)

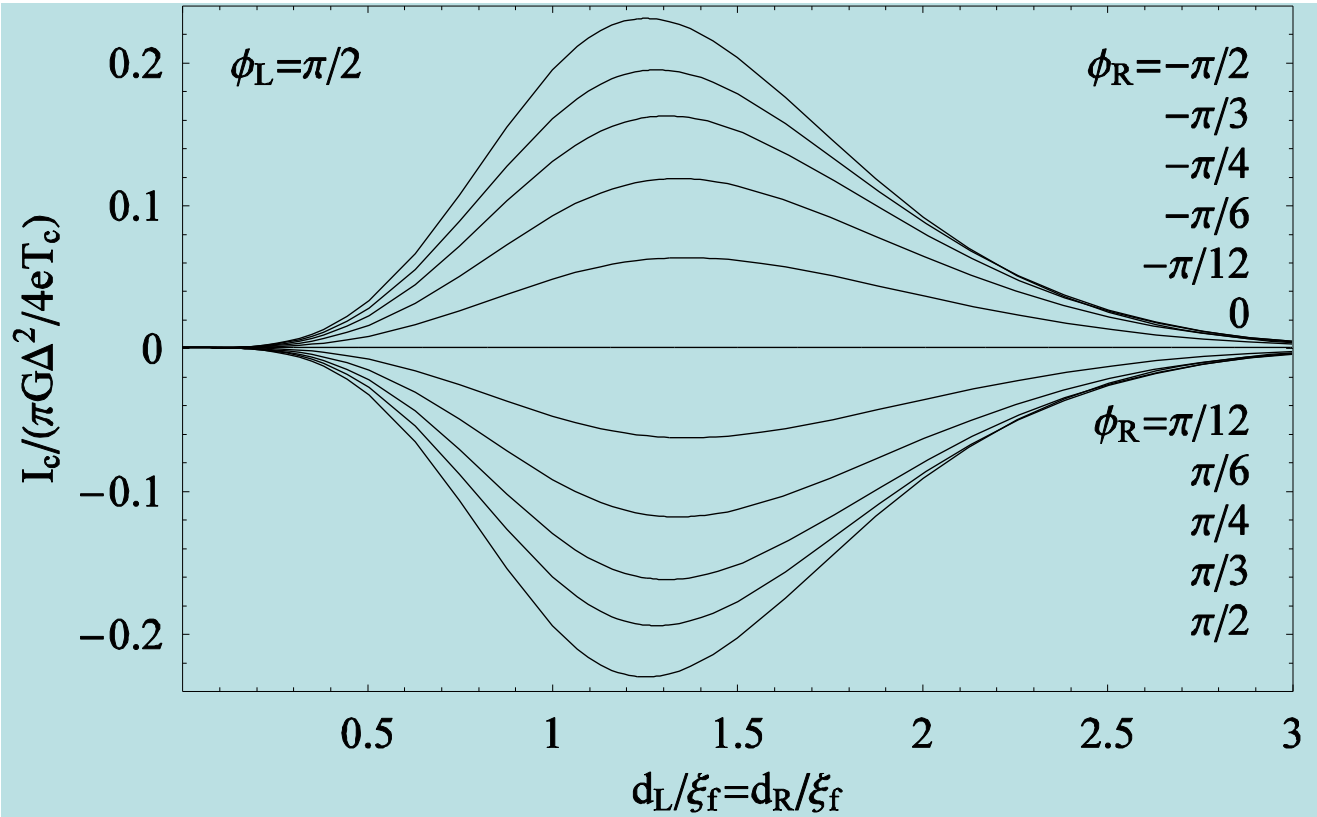


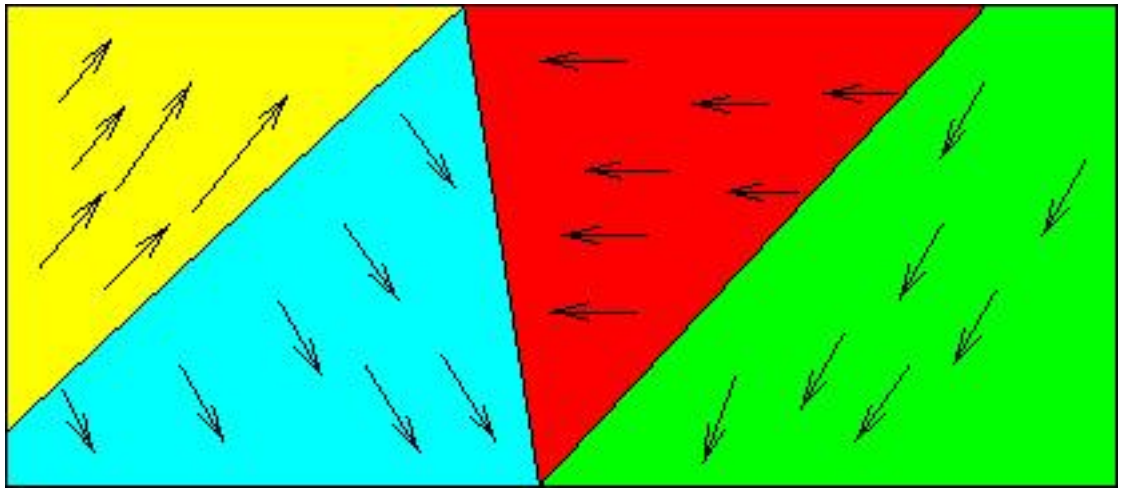
FIG. 2: Critical current induced by long range triplet proximity effect in S/F'/F/F''/S junction, in units of $(\pi G\Delta(T)^2/4eT_c)$, for varying length of F' and F'' layers, at $d_L = d_R \sim \xi_f \ll d \ll \xi_0$, and for different orientations of the magnetization in the layers.

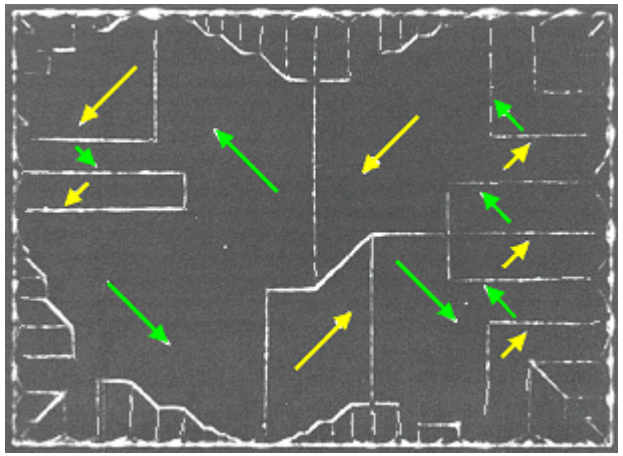
Rather sharp maximum of the critical current at $d_L=d_R=\xi_f$

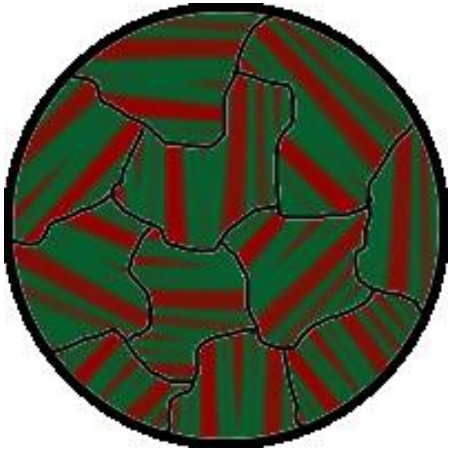
More details - M. Houzet and A. Buzdin « Long range triplet Josephson effect through a ferromagnetic trilayer », PRB, 2007.

SFS junction with of domains

Different types of domains

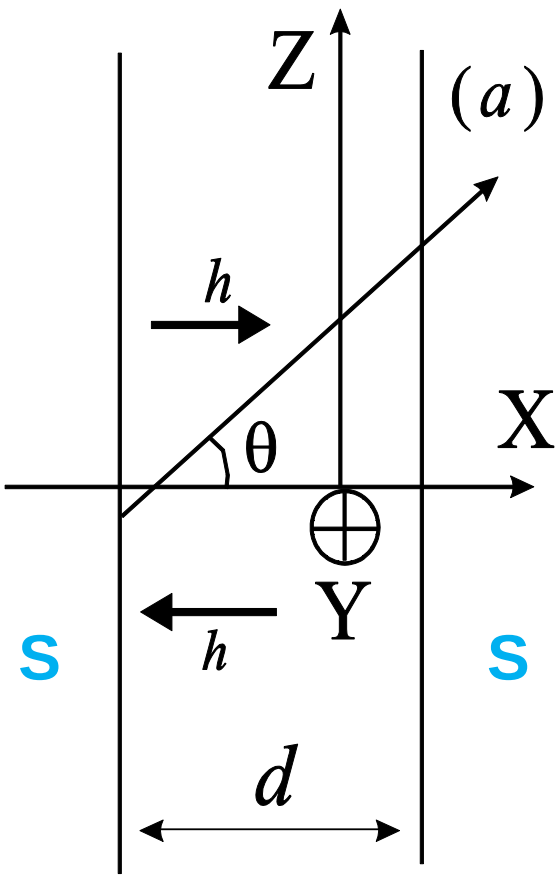


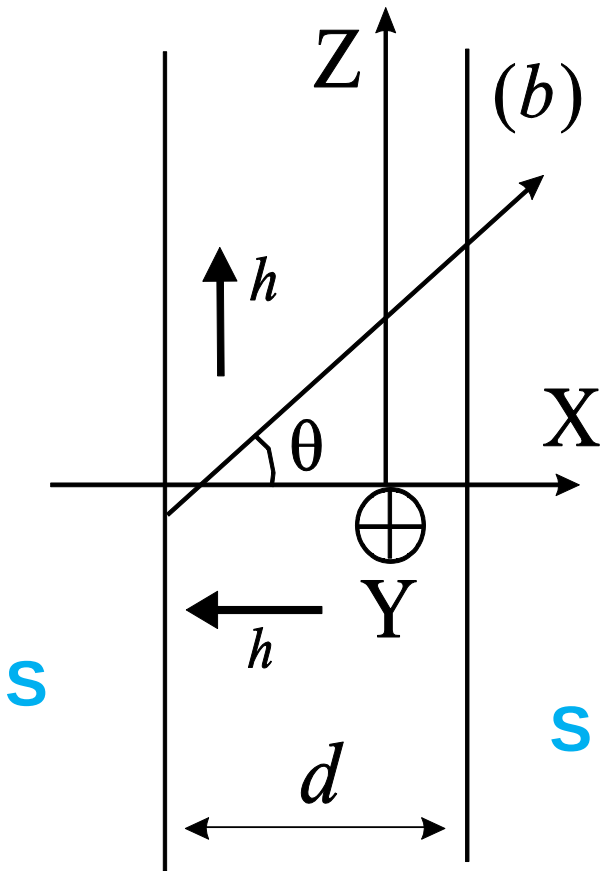




Clean SFS junctions with sharp domain walls

Domain walls with
collinear and non-collinear magnetic moments





Clean SFS junctions, BdG equations

$$-i\hbar v_F \hat{\tau}_z \frac{d}{dt}\hat{g} + \mathbf{h}(\mathbf{R}+t\mathbf{n}_F)\vec{\sigma}\hat{g} + \begin{pmatrix} 0 & \Delta(\mathbf{R}+t\mathbf{n}_F) \\ \Delta^*(\mathbf{R}+t\mathbf{n}_F) & 0 \end{pmatrix}\hat{g} = \epsilon\hat{g}$$

g= (*u,v*), *u and v are* the electronlike and holelike parts of the quasiparticle wave function

$$I_{2D} = \frac{k_F}{2\pi}\int ds \int d\mathbf{n}_F (j(\varphi+\gamma)+j(\varphi-\gamma))(\mathbf{n}_F,\mathbf{n}_j)\ ,$$

$$j(\varphi) = \frac{e\Delta_0}{2\hbar}\sin(\varphi/2)\tanh\left(\frac{\Delta_0\cos(\varphi/2)}{2T}\right)$$

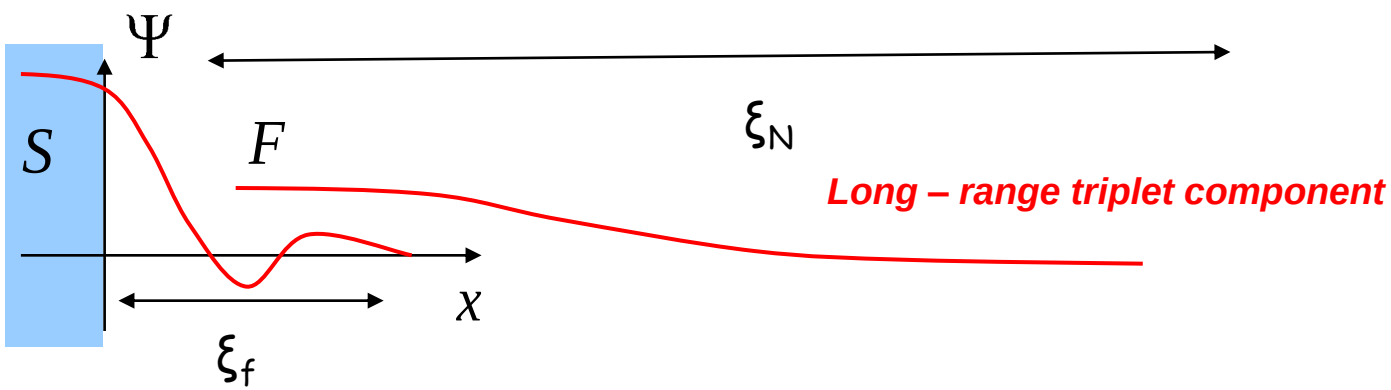
$$I_{3D} = \frac{k_F^2}{4\pi^2}\int dS \int d\mathbf{n}_F (j(\varphi+\gamma)+j(\varphi-\gamma))(\mathbf{n}_F,\mathbf{n}_j)\ ,$$

$2D$: $$\delta I = \frac{e\Delta_0^2 k_F \xi_f}{2\pi\hbar T}\sin\varphi\left(\sin(d/\xi_f) - \frac{d^2}{\xi_f^2}\int_{d/\xi_f}^{\infty}\frac{\sin y}{y^3}dy\right)$$ I~ $\sqrt{\xi_f/d}$

$3D$: $$\delta I \simeq \frac{e\Delta_0^2 k_F^2 \xi_f L_y}{2\pi^2\hbar T}\sqrt{\frac{\pi\xi_f}{2d}}\cos(d/\xi_f+\pi/4)\sin\varphi$$ I~ ξ_f/d

$$\delta I(\alpha) = \sin^2\frac{\alpha}{2}\cdot\delta I(\pi)$$

Long – range triplet proximity effect in dirty SF systems
Bergeret – Volkov – Efetov (2001)



$$f_3 \propto \langle \psi_\uparrow \psi_\downarrow \rangle - \langle \psi_\downarrow \psi_\uparrow \rangle$$ -Singlet condensate

$$f_0 \propto \langle \psi_\uparrow \psi_\downarrow \rangle + \langle \psi_\downarrow \psi_\uparrow \rangle$$ -Triplet condensate (with projection 0 on z-axis)

$$f_1 \propto \langle \psi_\uparrow \psi_\uparrow \rangle \propto \langle \psi_\downarrow \psi_\downarrow \rangle$$ -Triplet condensate (with projection +1,-1)

$$f = i\sigma_y (f_s + i\mathbf{f}_t \vec{\sigma})$$

T. Champel et al. PRL (2008)

$$\left(\frac{D}{2}\nabla^2 - \omega_n\right) f_s = -\mathbf{h}\mathbf{f}_t$$

$$\left(\frac{D}{2}\nabla^2 - \omega_n\right) \mathbf{f}_t = \mathbf{h} f_s$$

Dirty SFS junctions with sharp domain walls. Usadel equations.

$$f = i\sigma_y(f_s + i\mathbf{f}_t\vec{\sigma}) \qquad \left(\frac{D}{2}\nabla^2 - \omega_n\right) f_s = -\mathbf{h}\mathbf{f}_t$$

$$\left(\frac{D}{2}\nabla^2 - \omega_n\right) \mathbf{f}_t = \mathbf{h} f_s$$

$$\left.\frac{\partial f_s}{\partial x}\right|_{\pm d/2} = F_\pm = \pm\frac{\pi\Delta}{\gamma_b\sqrt{\Delta^2+\omega_n^2}} e^{\pm i\varphi/2} ,$$

$$\frac{\partial \mathbf{f}_t}{\partial x} = 0 ,$$

$$\left(\frac{D}{2}\nabla^2 - \omega_n\right) g_1 = ih \operatorname{sign} z \, g_1 + \frac{D}{2} F\delta(x) ,$$

$$\left(\frac{D}{2}\nabla^2 - \omega_n\right) g_2 = -ih \operatorname{sign} z \, g_2 + \frac{D}{2} F\delta(x) .$$

$$I = \frac{2eDN_F T}{\pi} \int_S d^2 s \sum_n Im\left(f_s^* \frac{\partial}{\partial x} f_s - \mathbf{f}_t^* \frac{\partial}{\partial x}\mathbf{f}_t\right)$$

Dirty SFS junctions with sharp domain walls. Collinear magnetic moments

No change in the length of decay of superconducting correlations

No long – range triplet component

In accordance with the result of Crouzy et al. PRB **76** (2007)

Dirty SFS junctions with sharp domain walls. Non-collinear magnetic moments

$$d < \xi_N$$

$$\delta I = \frac{eN_F D L_y}{2} \frac{\xi_h^4}{\gamma_b^2 d^2} \Delta \tanh \frac{\Delta}{2T} \sin \varphi$$

Clean SFS junctions with domain walls

Maximum contribution to the critical current:
Domain walls with antiparallel magnetic moments

Dirty SFS junctions with domain walls

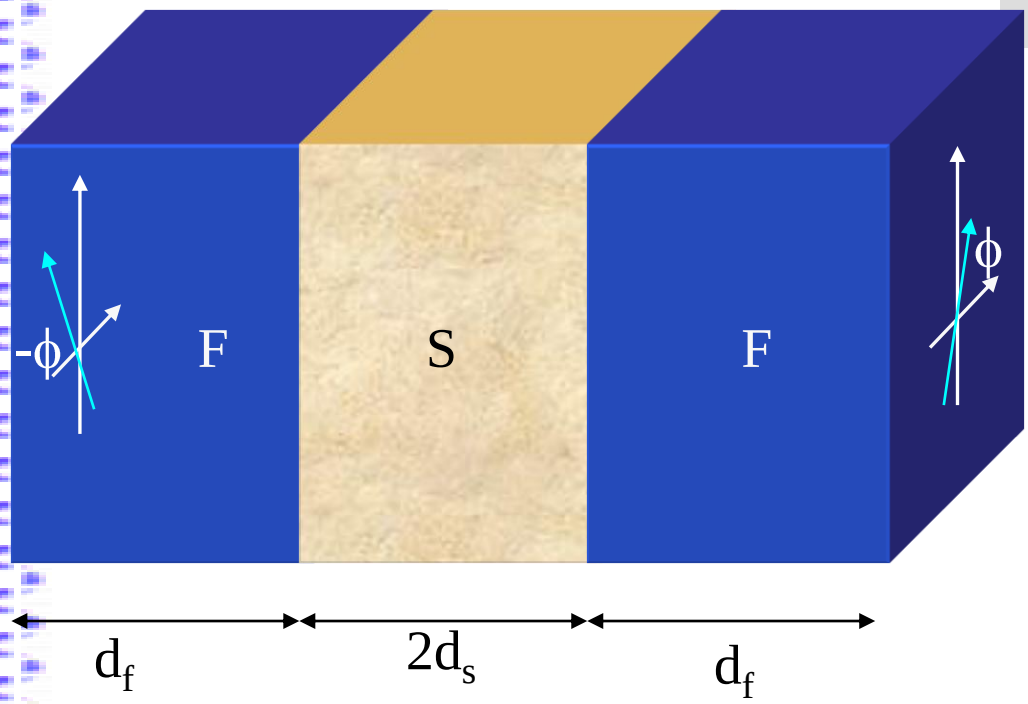
Maximum contribution to the critical current:
Domain walls with non-collinear magnetic moments

Noncollinear domains in SFS junctions are sources of the long range triplet correlation.

The domain walls connecting superconducting leads can be considered as channels for triplet current.

F/S/F trilayers, spin-valve effect

If d_s is of the order of magnitude of ξ_s, the critical temperature is controlled by the proximity effect.



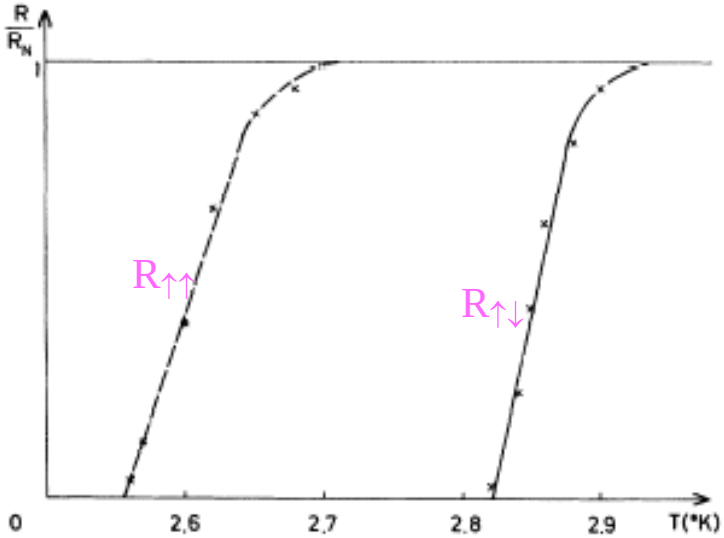


FIG. 1. Resistive measurements of the critical temperatures (R_N = resistance in the normal state) in zero field after the following: dashed line, application of 10 000 G ($T_{c\uparrow\uparrow}$) (all fields are applied parallel to the plane of the films); solid line, application of −10 000 G and subsequently +300 G to return the magnetization of the FeNi layer ($H_1 < 300$ G $< H_2$) ($T_{c\uparrow\downarrow}$).

Firstly the FI/S/FI trilayers has been studied experimentally in 1968 by Deutscher et Meunier.

In this special case, we see that the critical temperature of the superconducting layers is reduced when the ferromagnets are polarized in the same direction

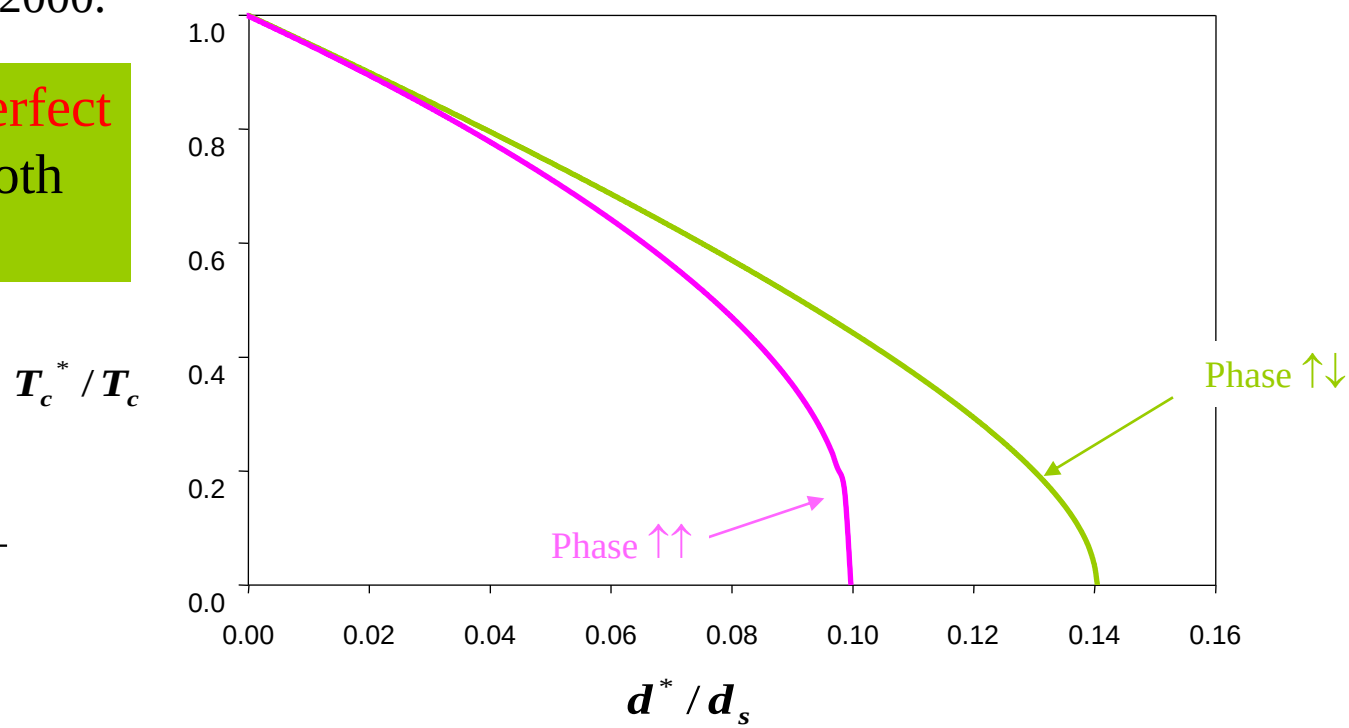
In the dirty limit, we used the quasiclassical Usadel equations to find the new critical temperature $T^*_{c.}$
We solved it self-consistently supposing that the order parameter can be taken as :

$$\Delta = \Delta_0 \left(1 - \frac{x^2}{L^2}\right)$$

with $L >> d_S$

Buzdin, Vedyaev, Ryazhanova, Europhys Lett. 2000,
Tagirov, Phys. Rev. Let. 2000.

In the case of a perfect transparency of both interfaces

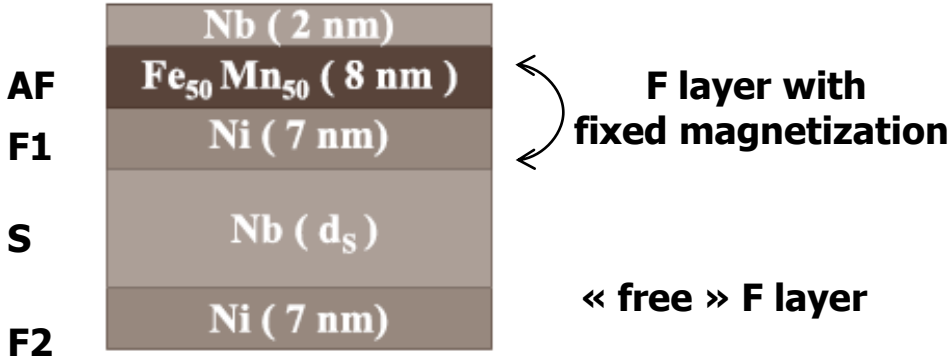


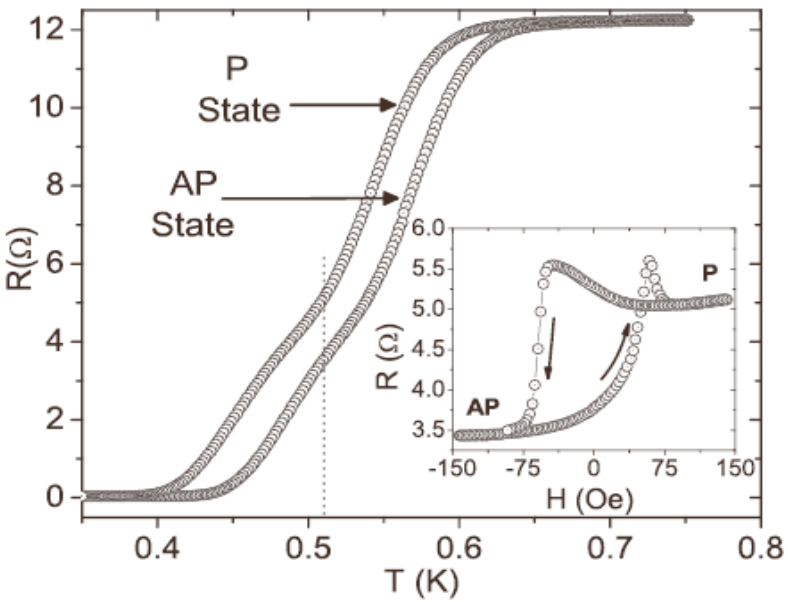
$$d^* = \gamma \sqrt{\frac{h}{D_n}} \frac{D_s}{4\pi T_c}$$

$$\ln\left(\frac{T_{c\uparrow\uparrow}^*}{T_c}\right) = \Psi\left(\frac{1}{2}\right) - \mathrm{Re}\,\Psi\left(\frac{1}{2} + \frac{d^* T_c}{d_s T_{c\uparrow\uparrow}^*}(+i)\right)$$

$$\ln\left(\frac{T_{c\uparrow\downarrow}^*}{T_c}\right) = \Psi\left(\frac{1}{2}\right) - \Psi\left(\frac{1}{2} + \frac{d^* T_c}{d_s T_{c\uparrow\downarrow}^*}\right)$$

Recent experimental verifications

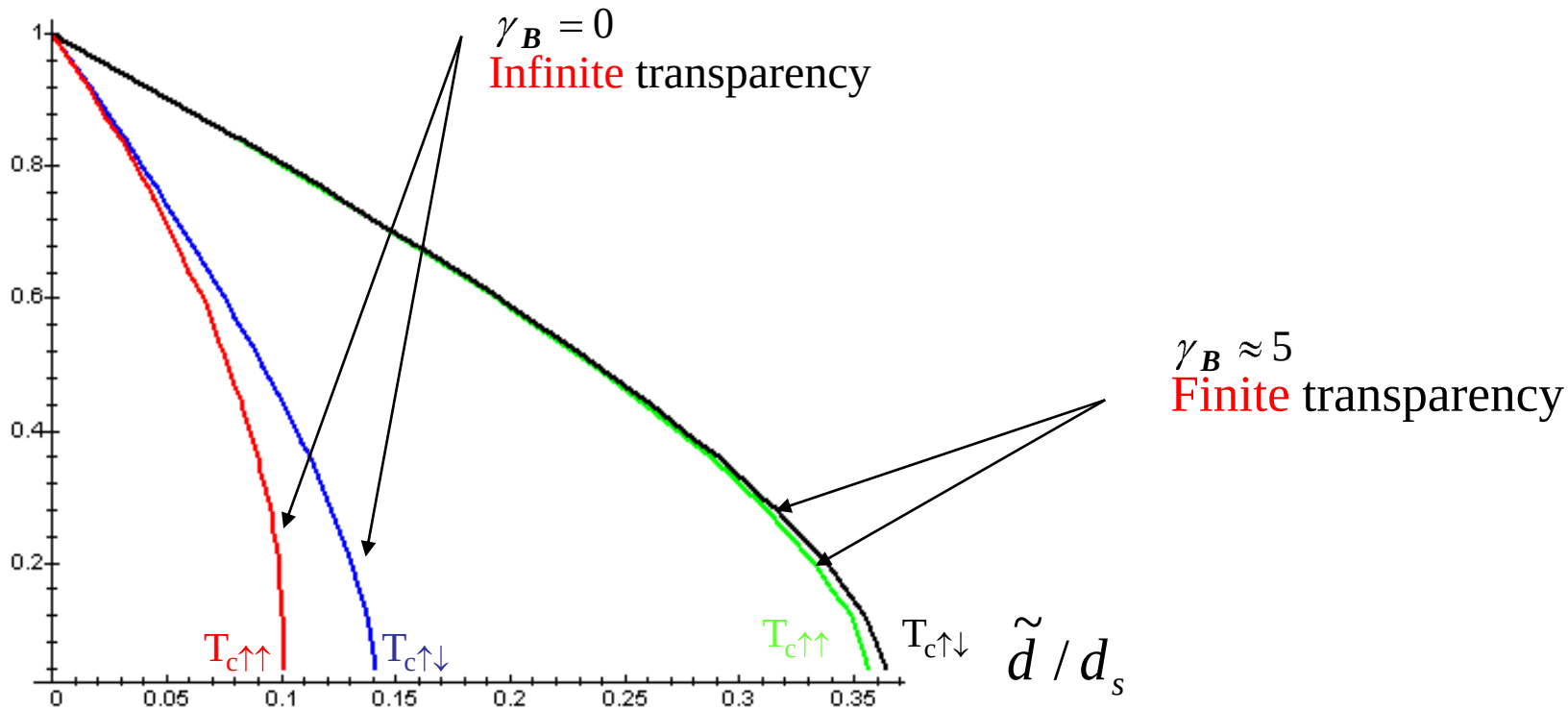




CuNi/Nb/CuNi
Gu, You, Jiang, Pearson, Bazaliy, Bader, 2002

Ni/Nb/Ni
Moraru, Pratt Jr, Birge, 2006

Evolution of the difference between the critical temperatures as a function of interfaces' transparency



$$\ln\left(\frac{T_{c\uparrow\uparrow}^{*}}{T_c}\right) = \Psi\left(\frac{1}{2}\right) - \mathrm{Re}\,\Psi\left(\frac{1}{2} + \frac{\tilde{d}T_c}{d_s T_{c\uparrow\uparrow}^{*}}(1+i)\right)$$

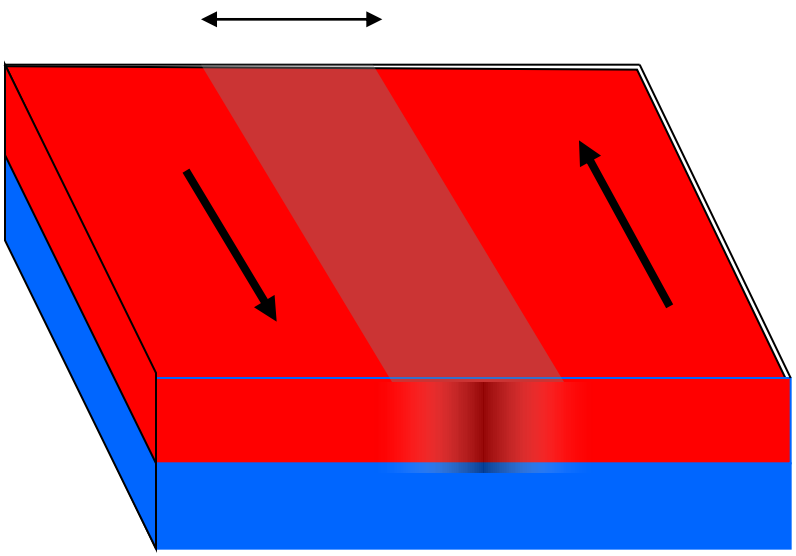
$$\ln\left(\frac{T_{c\uparrow\downarrow}^{*}}{T_c}\right) = \Psi\left(\frac{1}{2}\right) - \Psi\left(\frac{1}{2} + \left(\frac{\tilde{d}T_c}{d_s T_{c\uparrow\downarrow}^{*}}\right)\right)$$

$$\tilde{d} = \frac{D_s}{4\pi T_c}\,\frac{\gamma\sqrt{\dfrac{h}{D_n}}}{1 + (1+i)\,\gamma_B\gamma\sqrt{\dfrac{h}{D_n}}}$$

Similar physics in F/S bilayers

In practice, magnetic domains appear quite easily in ferromagnets

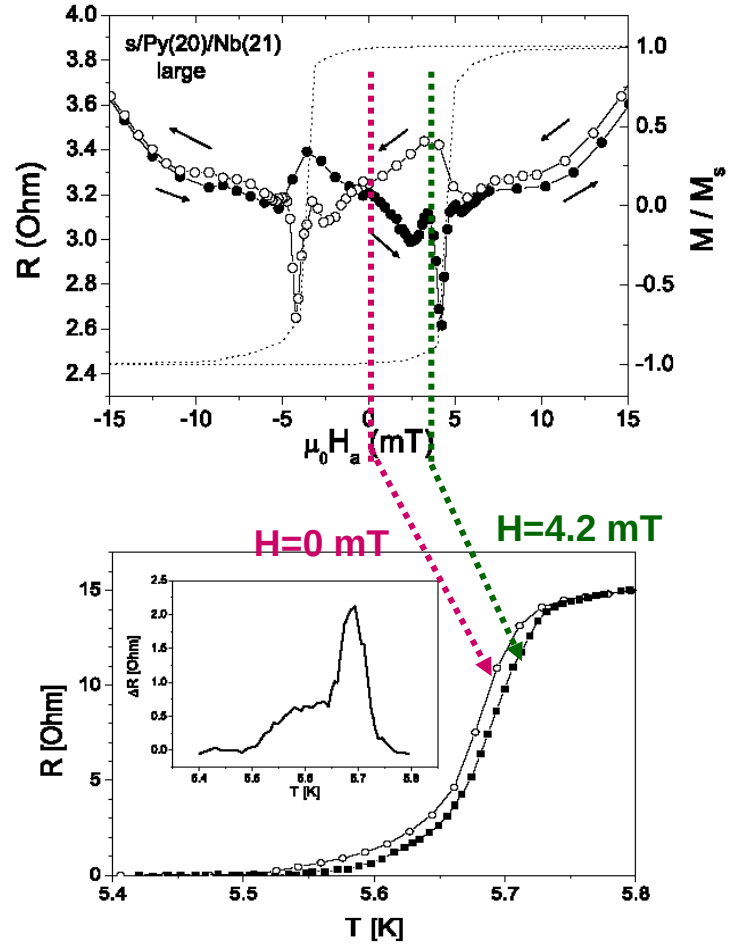
w: width of the domain wall



$Ni_{0.80}Fe_{0.20}$/Nb (20nm)

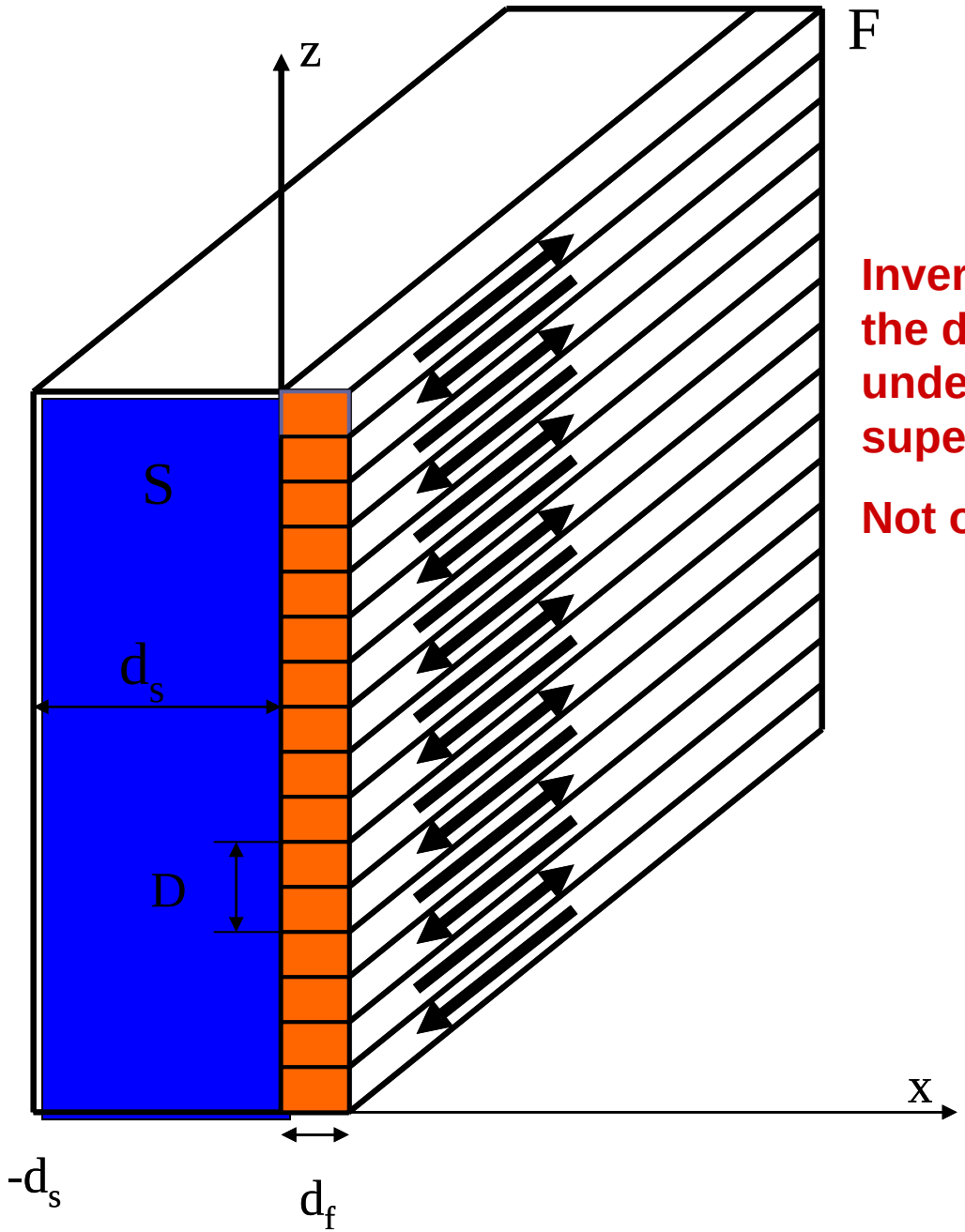
Thin films : Néel domains

Rusanov et al., PRL, 2004



Localized (domain wall) superconducting phase.

Theory - Houzet and Buzdin, PRB (2006).



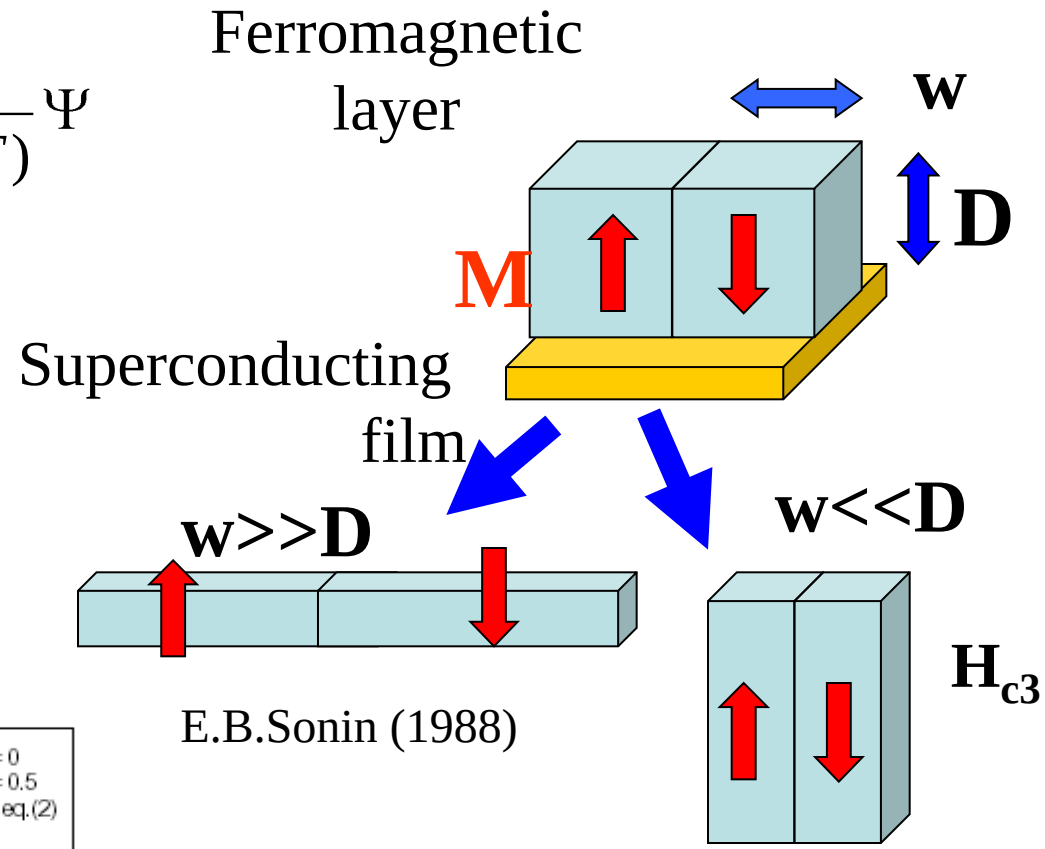
Inverse effect: appearence of the dense domain structure under the influence of superconductivity.

Not observed yet.

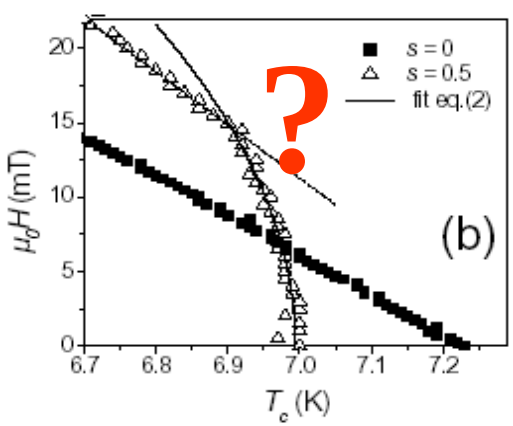
$D \ll \xi$

Domain wall superconductivity in purely electromagnetic model

$$-\left(\nabla + \frac{2\pi i}{\Phi_0}\vec{A}(\vec{r})\right)^2 \Psi = \frac{1}{\xi^2(T)}\Psi$$



E.B.Sonin (1988)

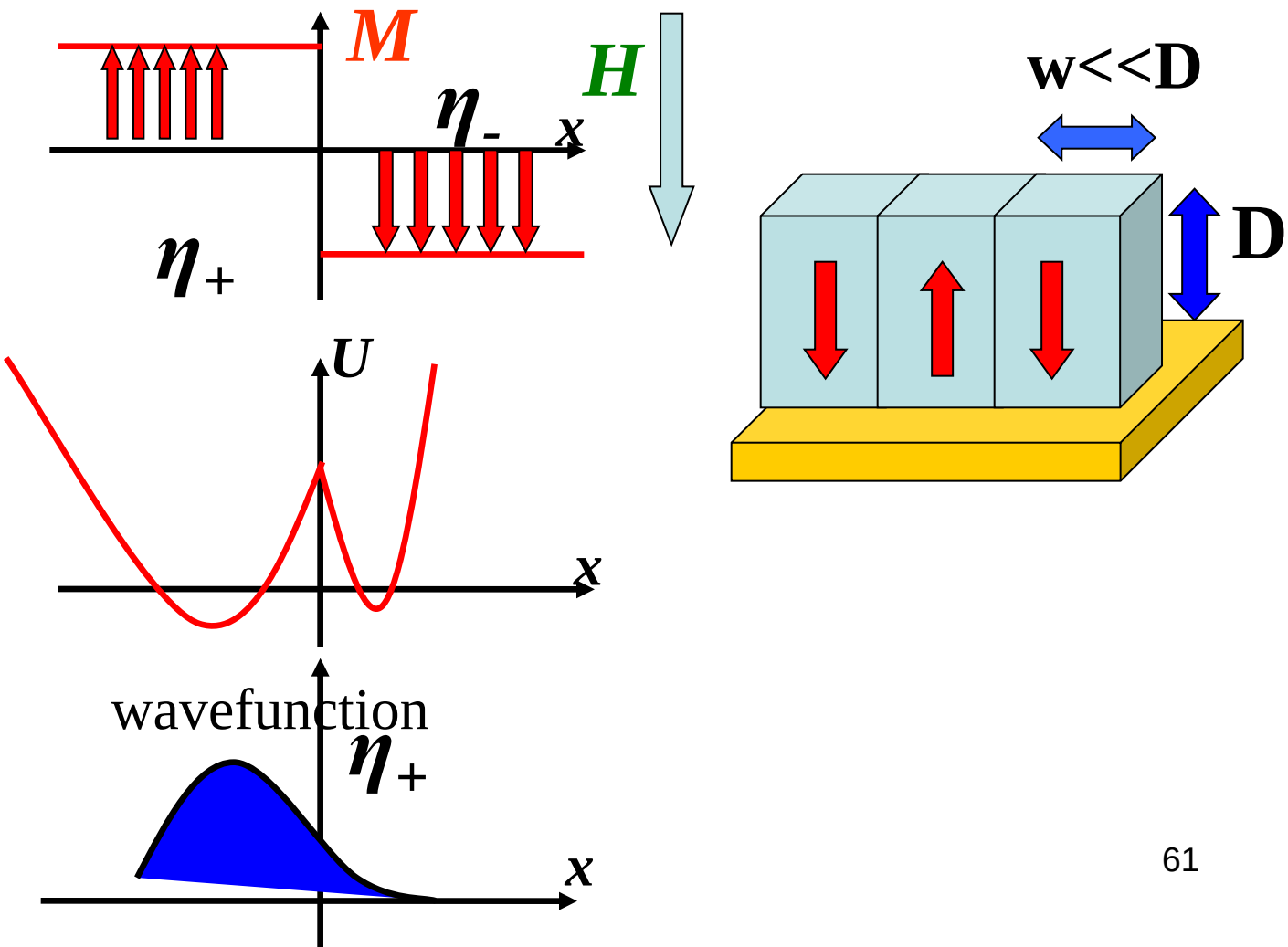


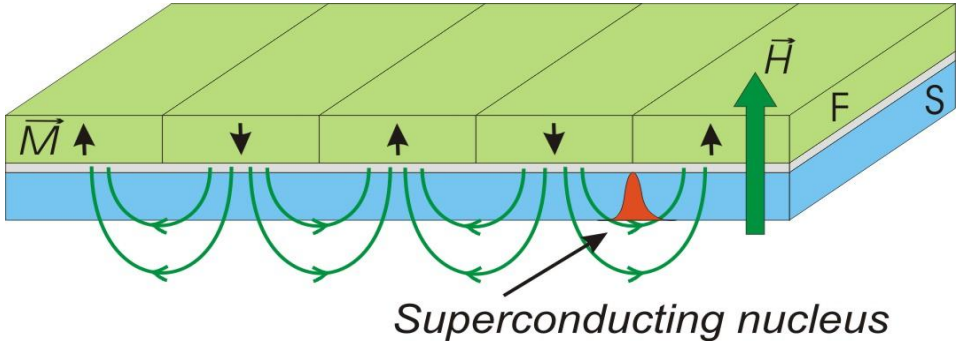
Pb-Co/Pt

Superconductivity nucleation at a single domain wall

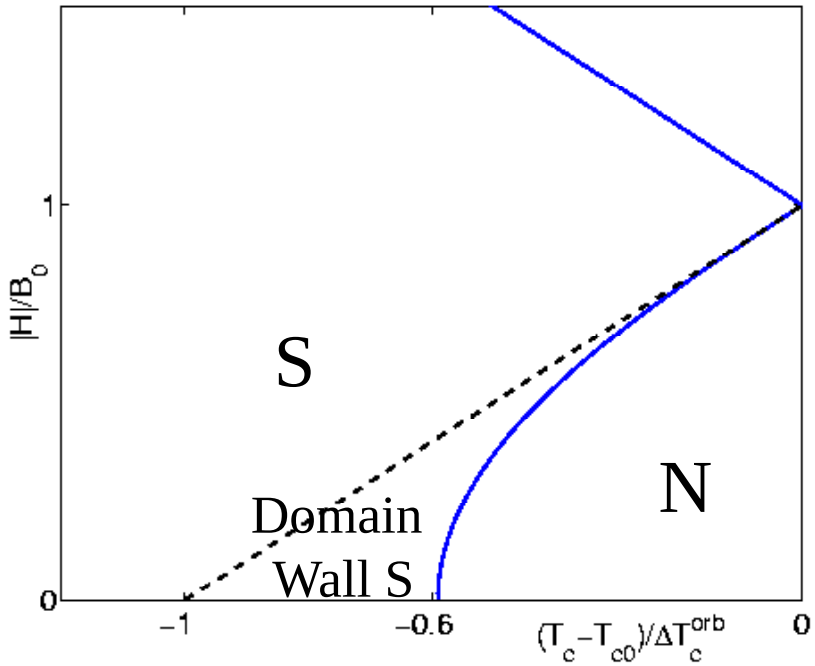
Thin domains

Step-like magnetic field profile





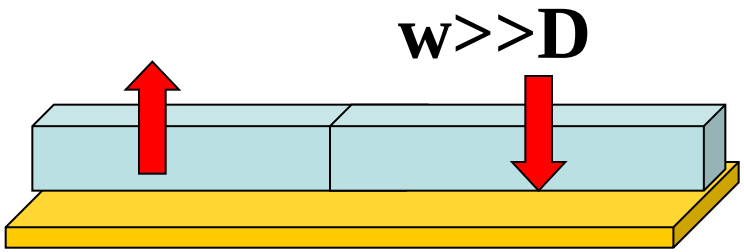
S/F bilayer with domain structure



Nb/F

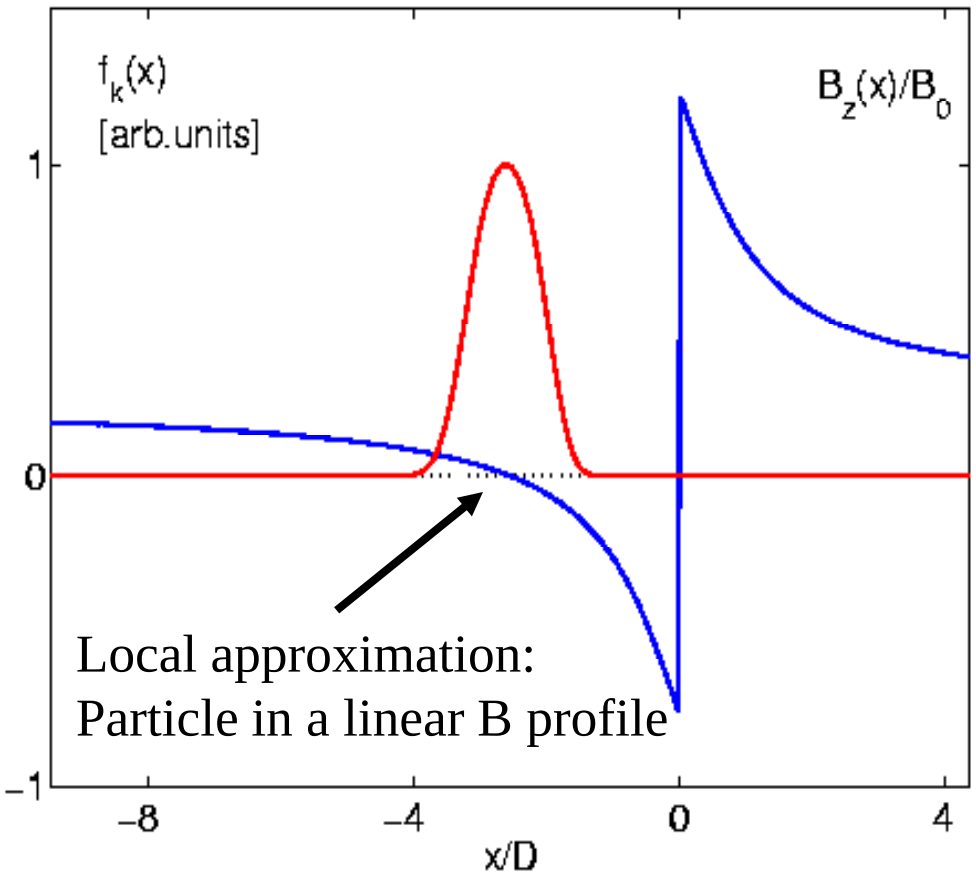
$$B_0 = 4\pi M \sim 1-10kOe \qquad \frac{dH_{c2}}{dT} \sim 0.5kOe/K$$

$$T_c \sim 9K \qquad \delta T_c \sim 1-3K$$



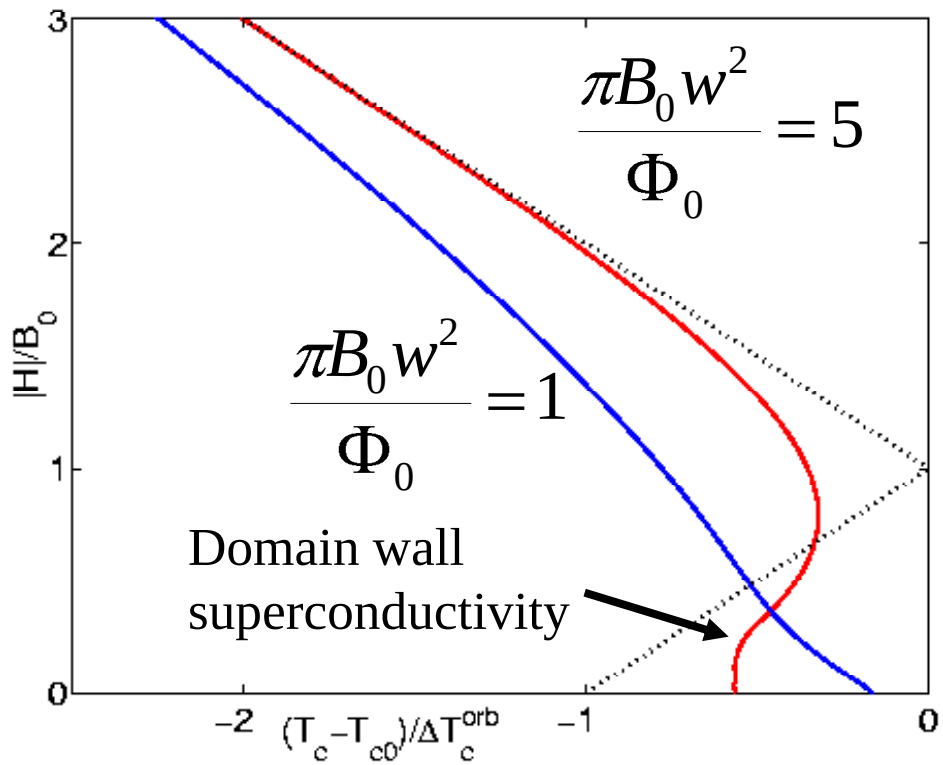
Thick domains

B_0-maximum field induced by the domain wall

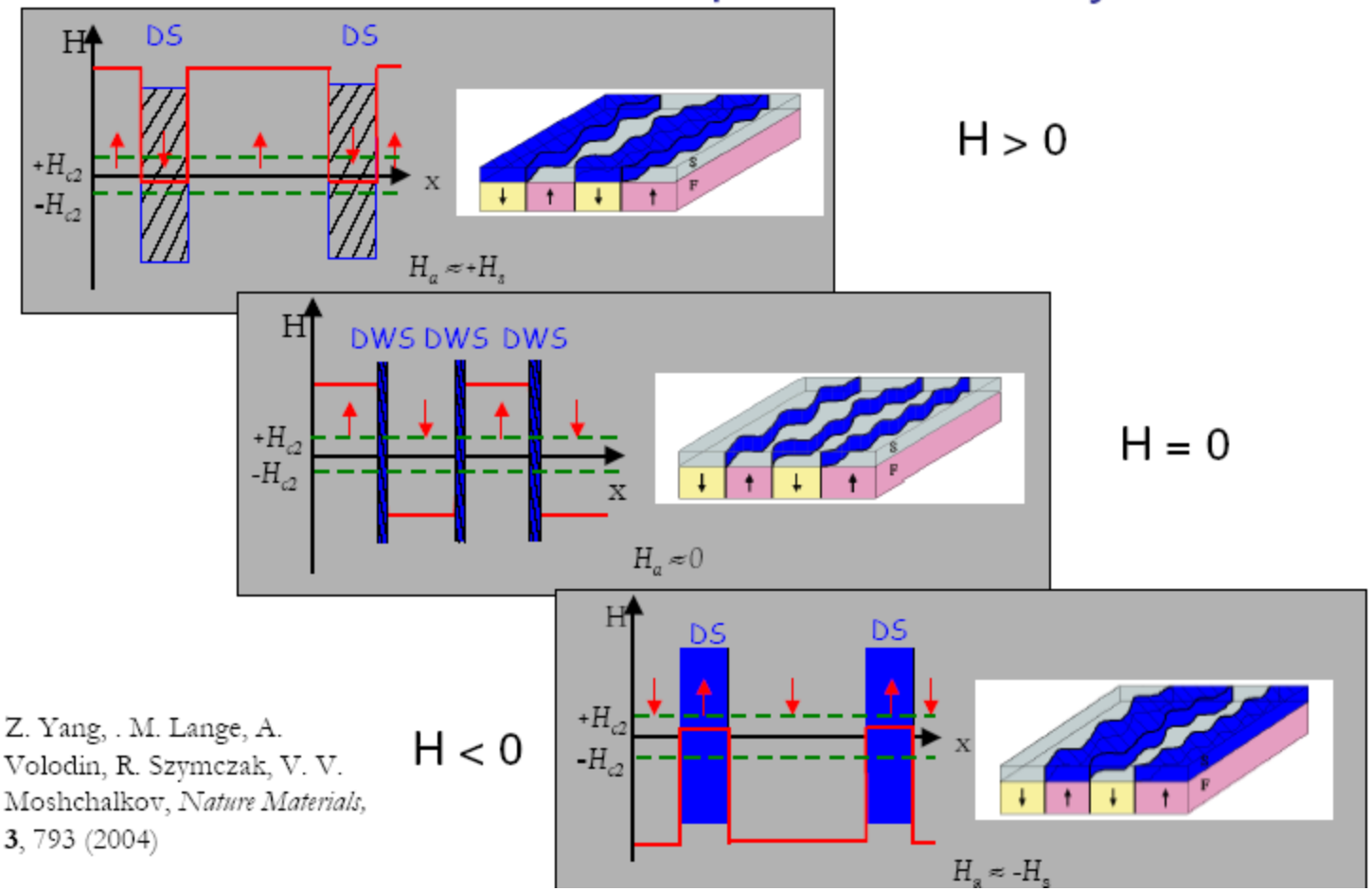


Superconducting nucleus in a periodic domain structure in an external field

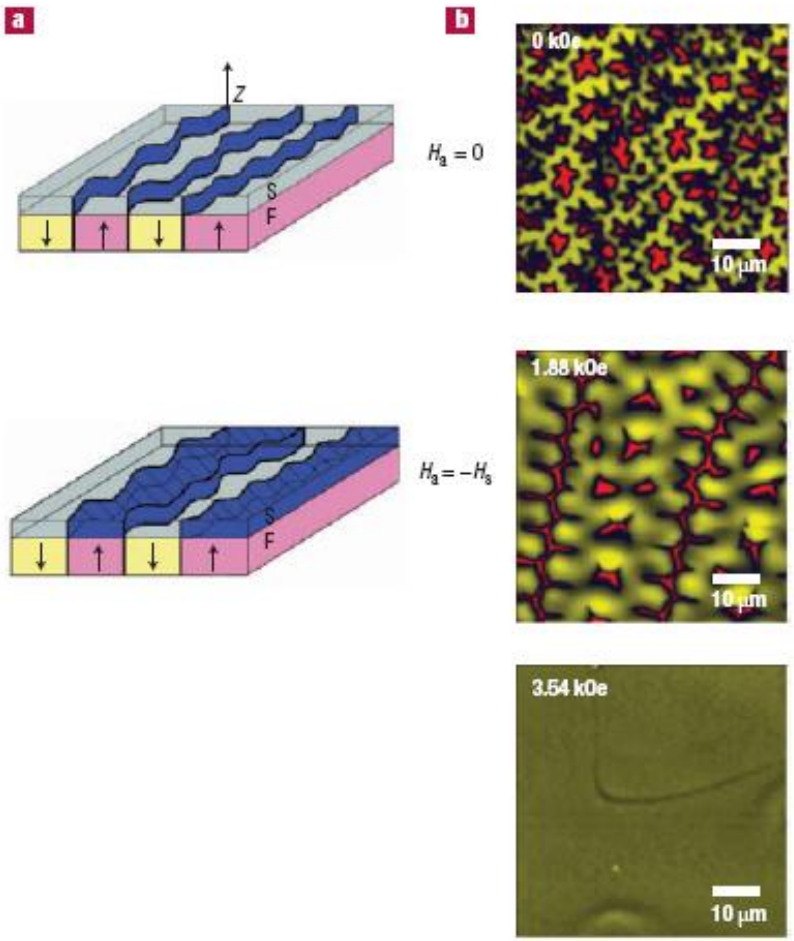
$H \neq 0$



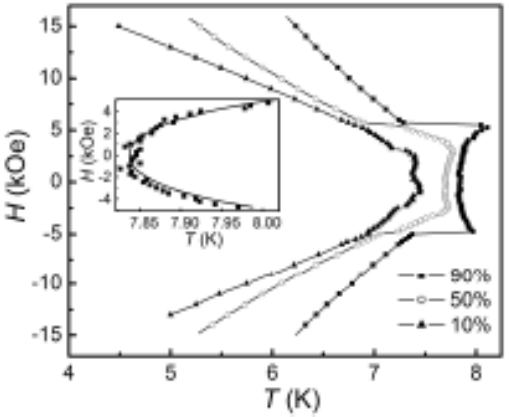
Domain wall superconductivity



Z. Yang, . M. Lange, A. Volodin, R. Szymczak, V. V. Moshchalkov, *Nature Materials*, **3**, 793 (2004)



Nb/BaFe$_{12}$O$_{19}$

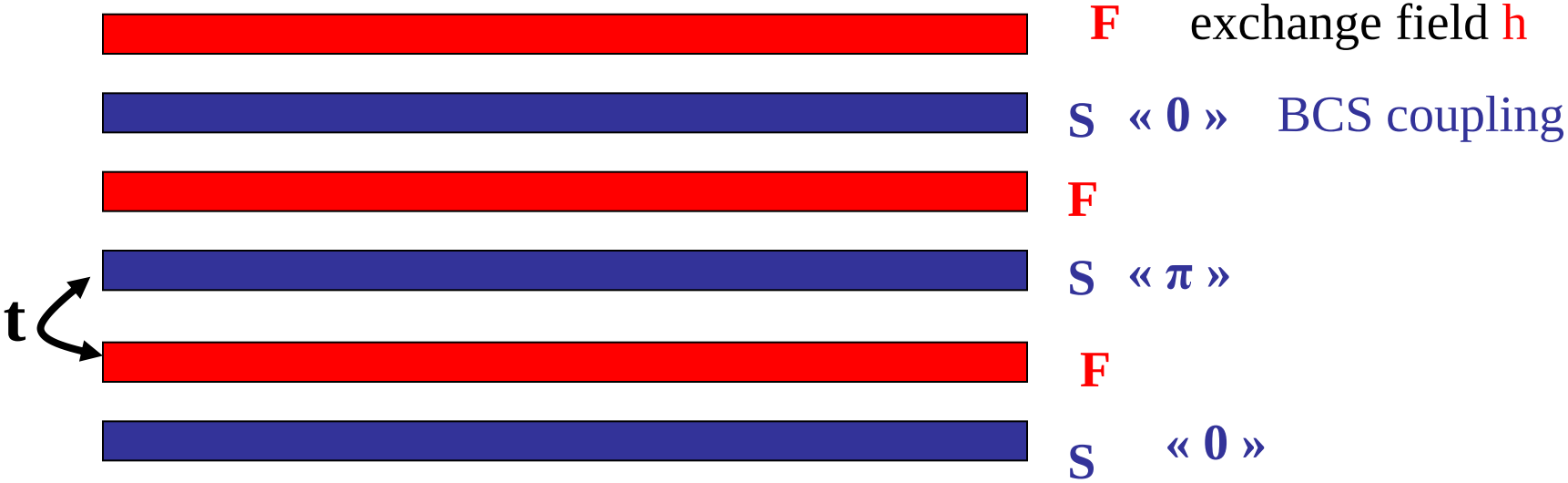


Z. YANG et al, Nature Materials, 2004

Atomic layered S-F systems

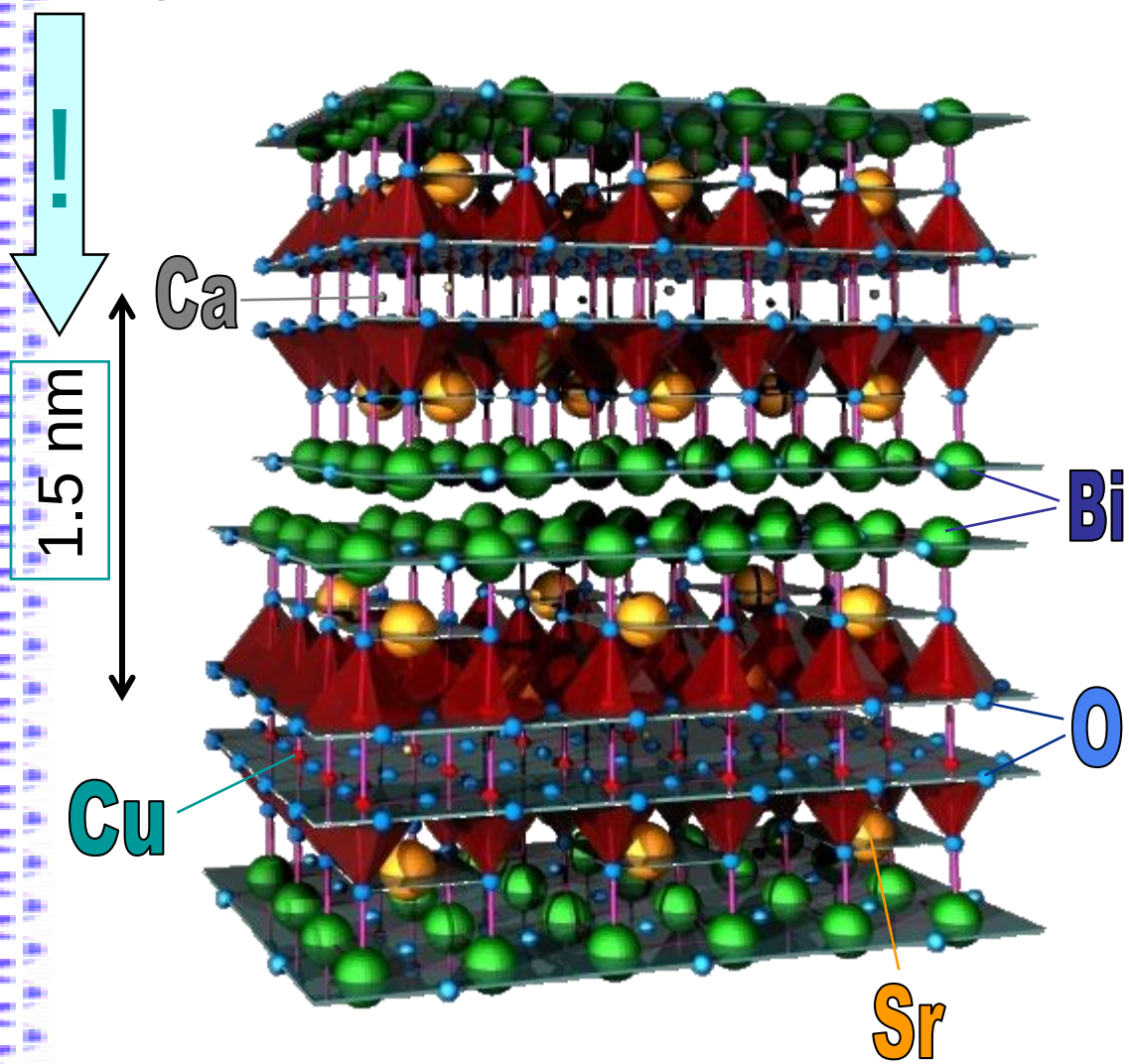
(Andreev et al, PRB 1991, Houzet et al, PRB 2001, Europhys. Lett. 2002)

Magnetic layered superconductors like $RuSr_2GdCu_2O_8$



Also even for the quite small exchange field ($h>T_c$) the **π-phase** must appear.

Crystal structure of layered HTS



- $Bi_2Sr_2CaCu2O_{8+x}$ (Bi2212)
- $Bi_2Sr_2CuO_{6+x}$ (Bi2201)

Also Tl2212, Tl2201 etc.

Hamiltonian of the system

$$H = \sum_{\mathbf{p},n,i,\sigma} \xi(\mathbf{p}) a^{\dagger}_{ni\sigma}(\mathbf{p}) a_{ni\sigma}(\mathbf{p})$$
$$+ t[a^{\dagger}_{ni\sigma}(\mathbf{p}) a_{n,-i,\sigma}(\mathbf{p}) + a^{\dagger}_{n+1,-i,\sigma}(\mathbf{p}) a_{ni\sigma}(\mathbf{p}) + \mathrm{H.c.}]$$
$$\boxed{+ H_{\mathrm{int}1} + H_{\mathrm{int}2}\ ,} \qquad (1a)$$

$$H_{\mathrm{int}1} = \frac{\Lambda}{2} \sum_{\mathbf{p}_1 \mathbf{p}_2 n \sigma} a^{\dagger}_{n1\sigma}(\mathbf{p}_1) a^{\dagger}_{n1-\sigma}(-\mathbf{p}_1)$$

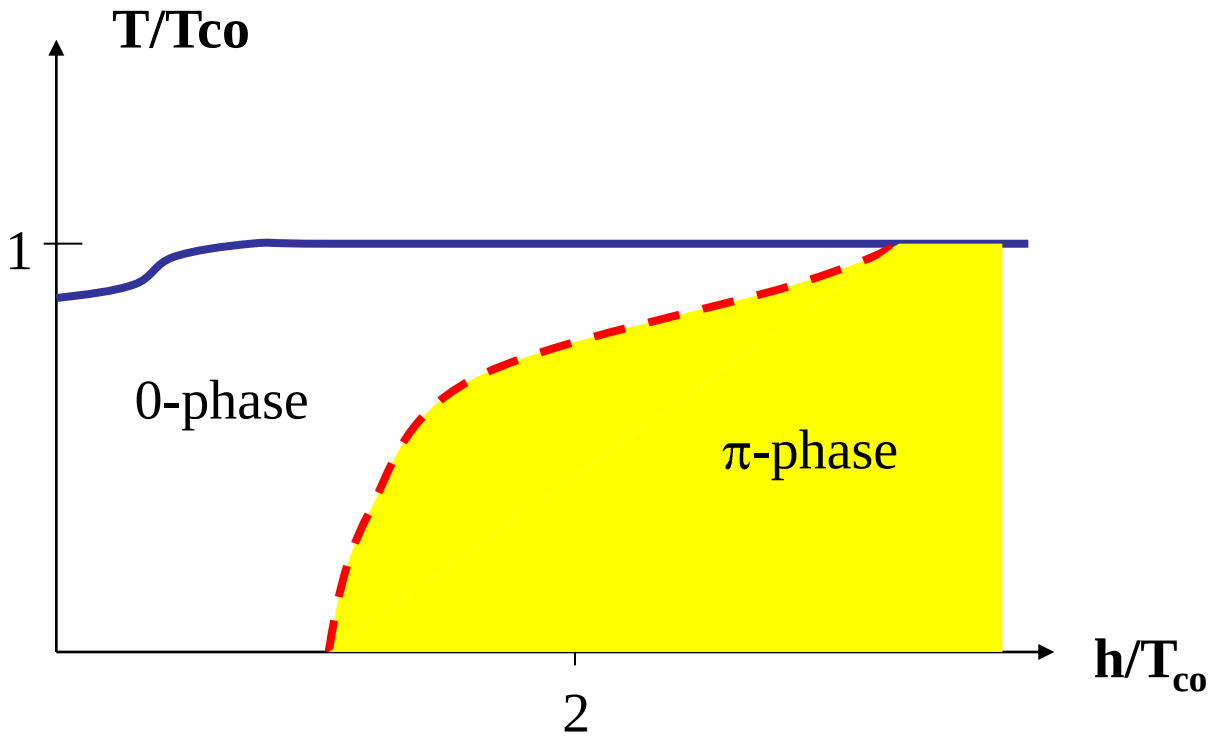
BCS coupling

$$\times a_{n1-\sigma}(-\mathbf{p}_2) a_{n1\sigma}(\mathbf{p}_2)\ , \qquad (1b)$$

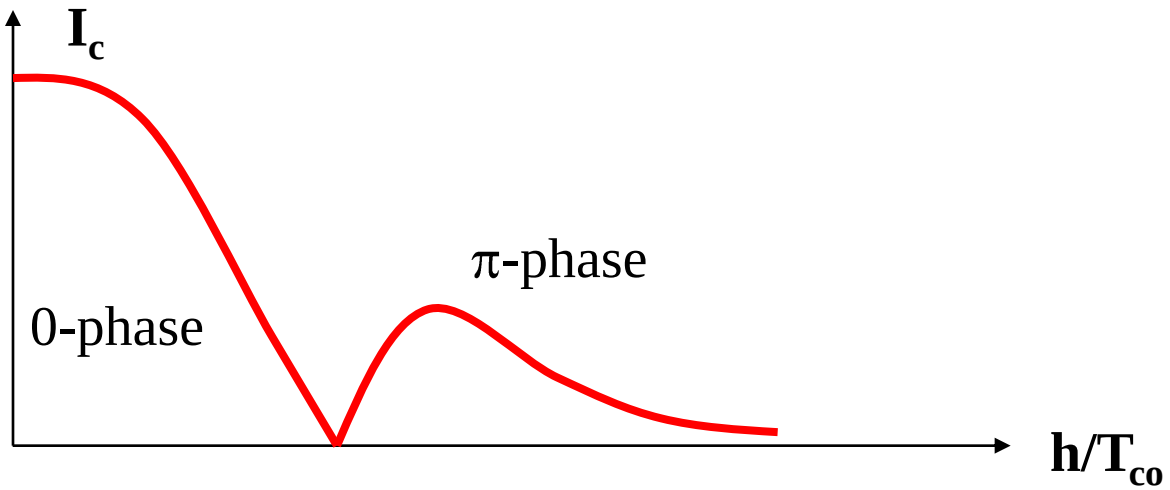
$$H_{\mathrm{int}2} = -\sum_{\mathbf{p},n,\sigma} h\sigma a^{\dagger}_{n,-1,\sigma}(\mathbf{p}) a_{n,-1,\sigma}(\mathbf{p})\ ,$$

Exchange field

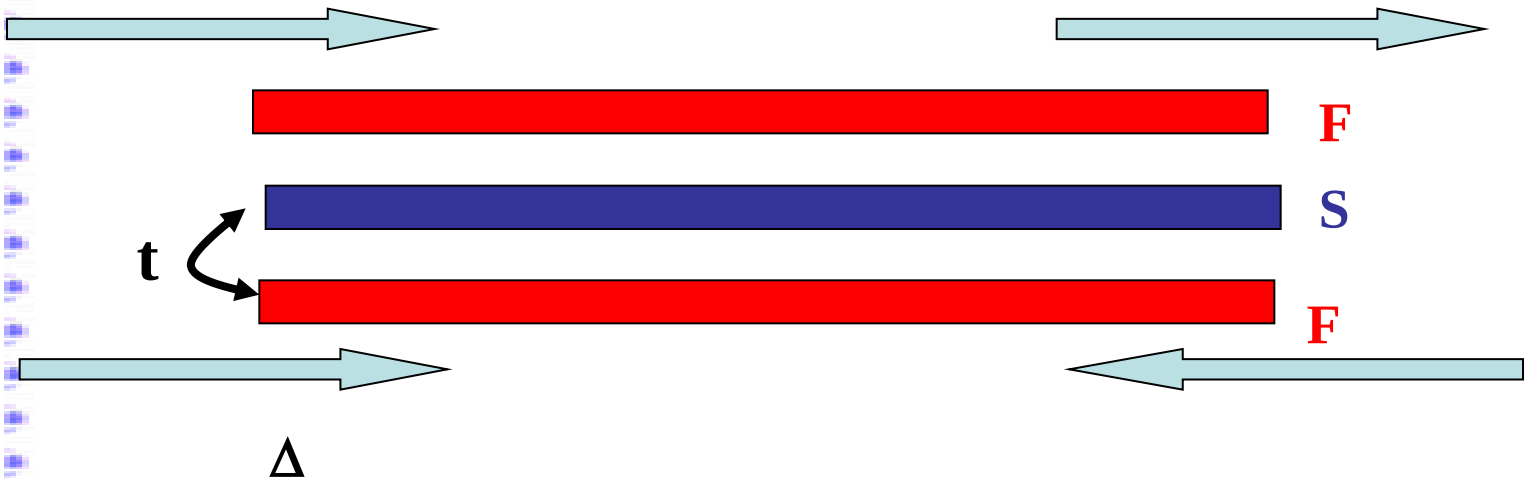
It is possible to obtain the exact solution of this model and to find all Green functions.



The limit **t** <<$\mathbf{T_{co}}$



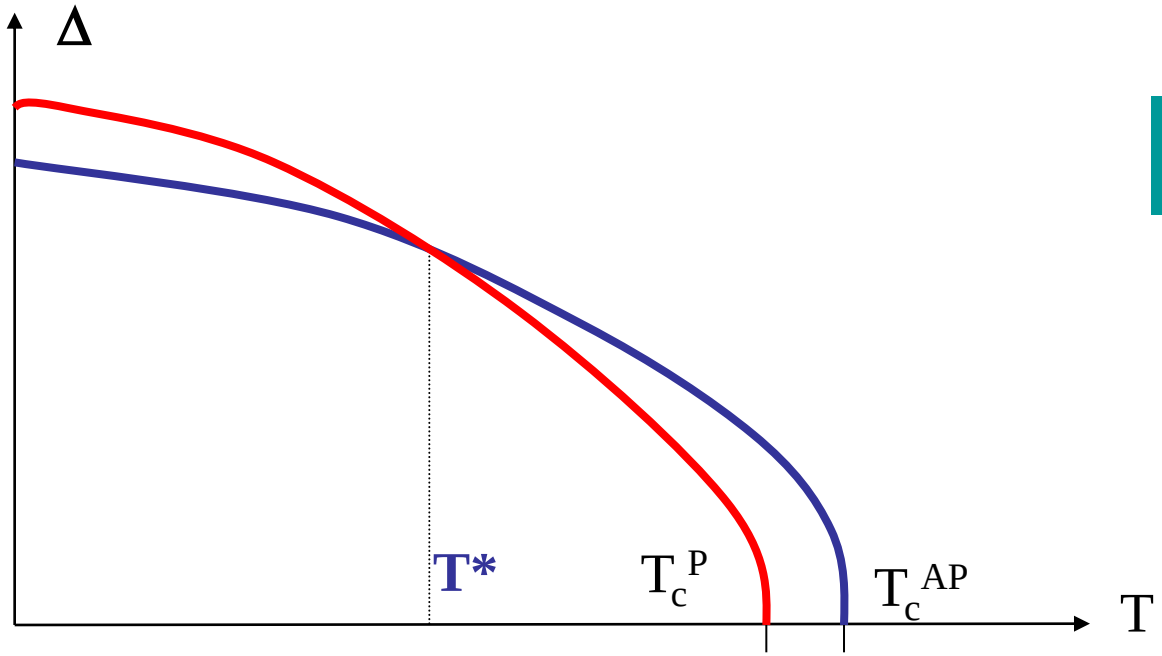
Atomic thickness F/S/F heterostructure. Inversion of the proximity effect at low température.

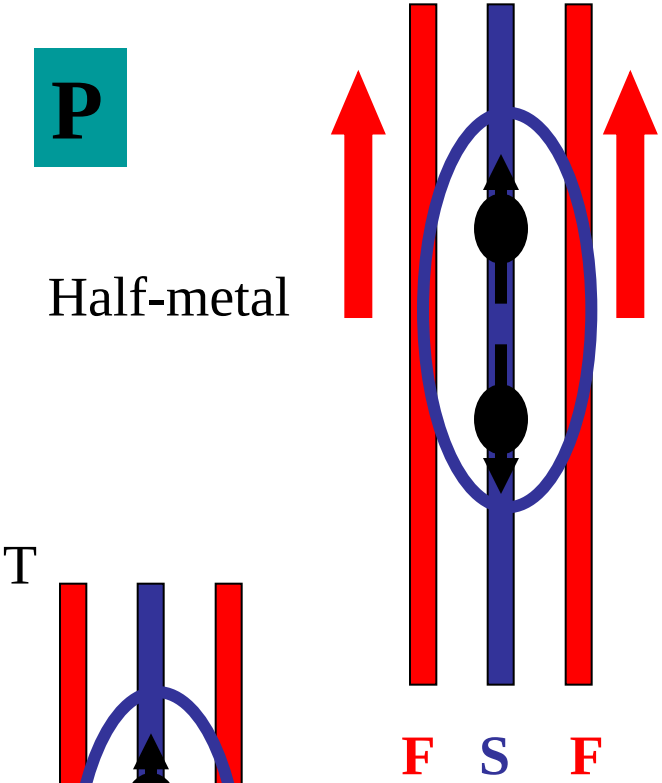


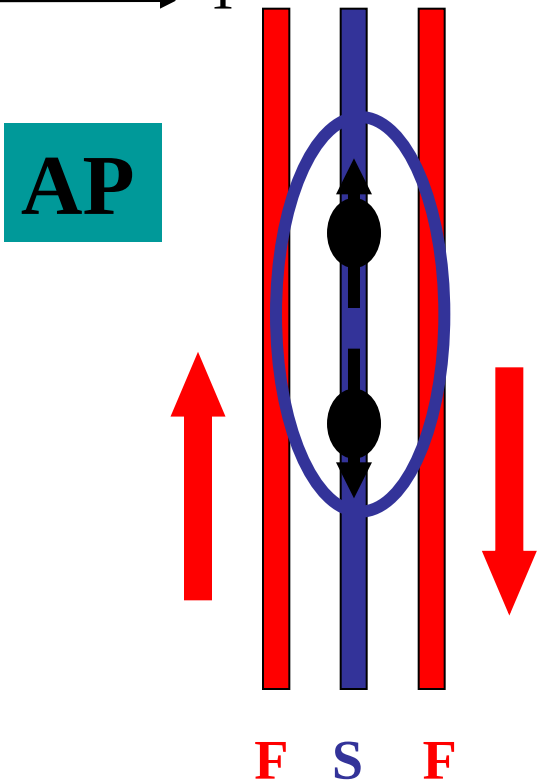
Also the exact solution is possible. (Tollis et al., PRB, 2005)

$T_{cP} < T_{cAP}$

$$\delta T_{\mathrm{P}} - \delta T_{\mathrm{AP}} \simeq -\frac{1397\zeta(7)}{512\pi^6}\frac{t^4h^2}{T_{c0}^6} \simeq -0.0029\frac{t^4h^2}{T_{c0}^6};$$







The inversion of the proximity effect occurs at the temperature

$$T^* = 0.47 T_{c0}$$

for the weak field h

Superconducting multilayered systems

(Buzdin, Cayssol and Tollis, PRL 2005,)

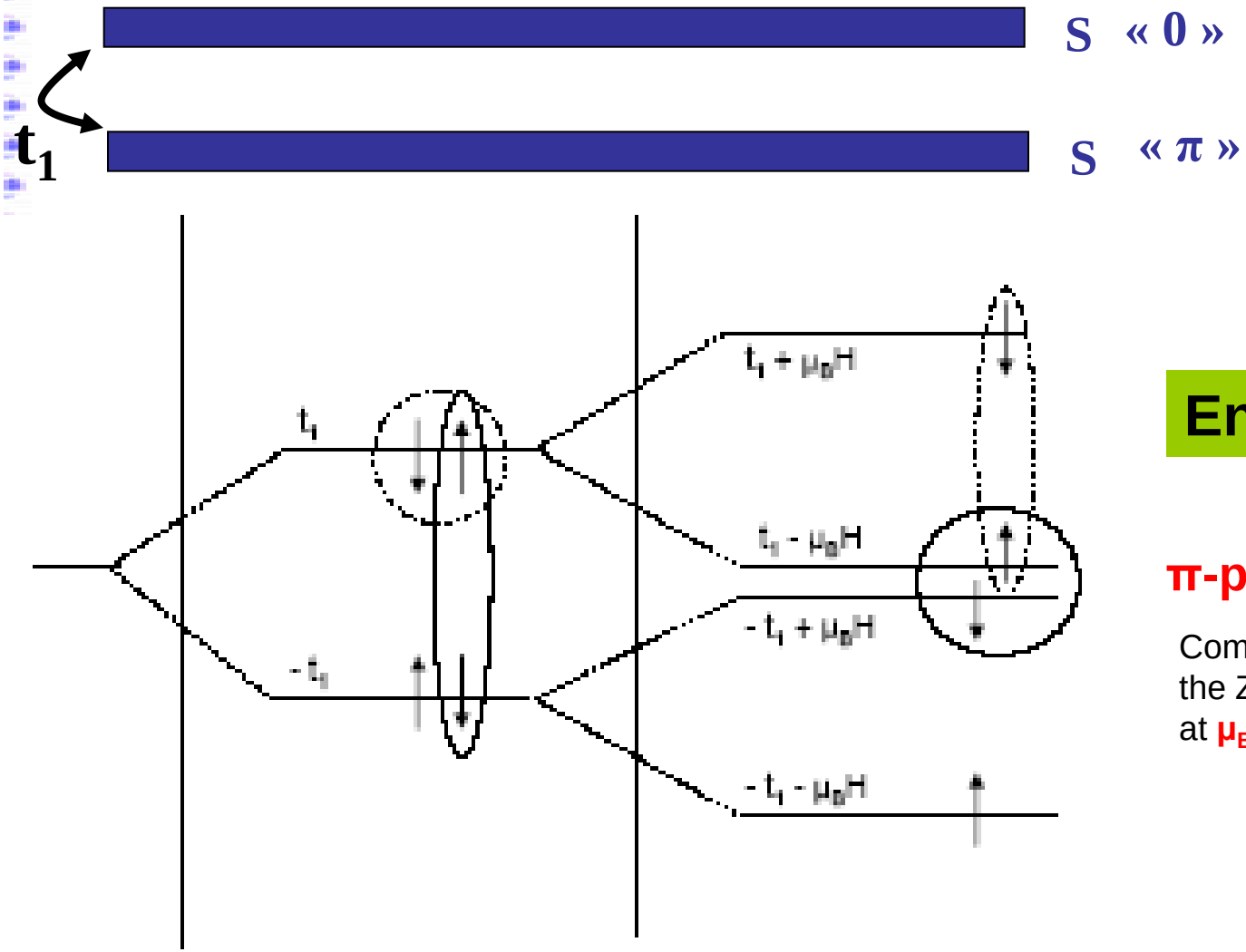
layered superconductors with a structure like high-T_c

Zeeman effect, i.e. the exchange field $\mu_B H$, or ferromagnetic superconductor

BCS coupling and exchange field $h=\mu_B H$

At low temperature the paramagnetic limit may be strongly exceed $\mu_B H \sim t_1$. **π-phase with FFLO modulation in plane.**

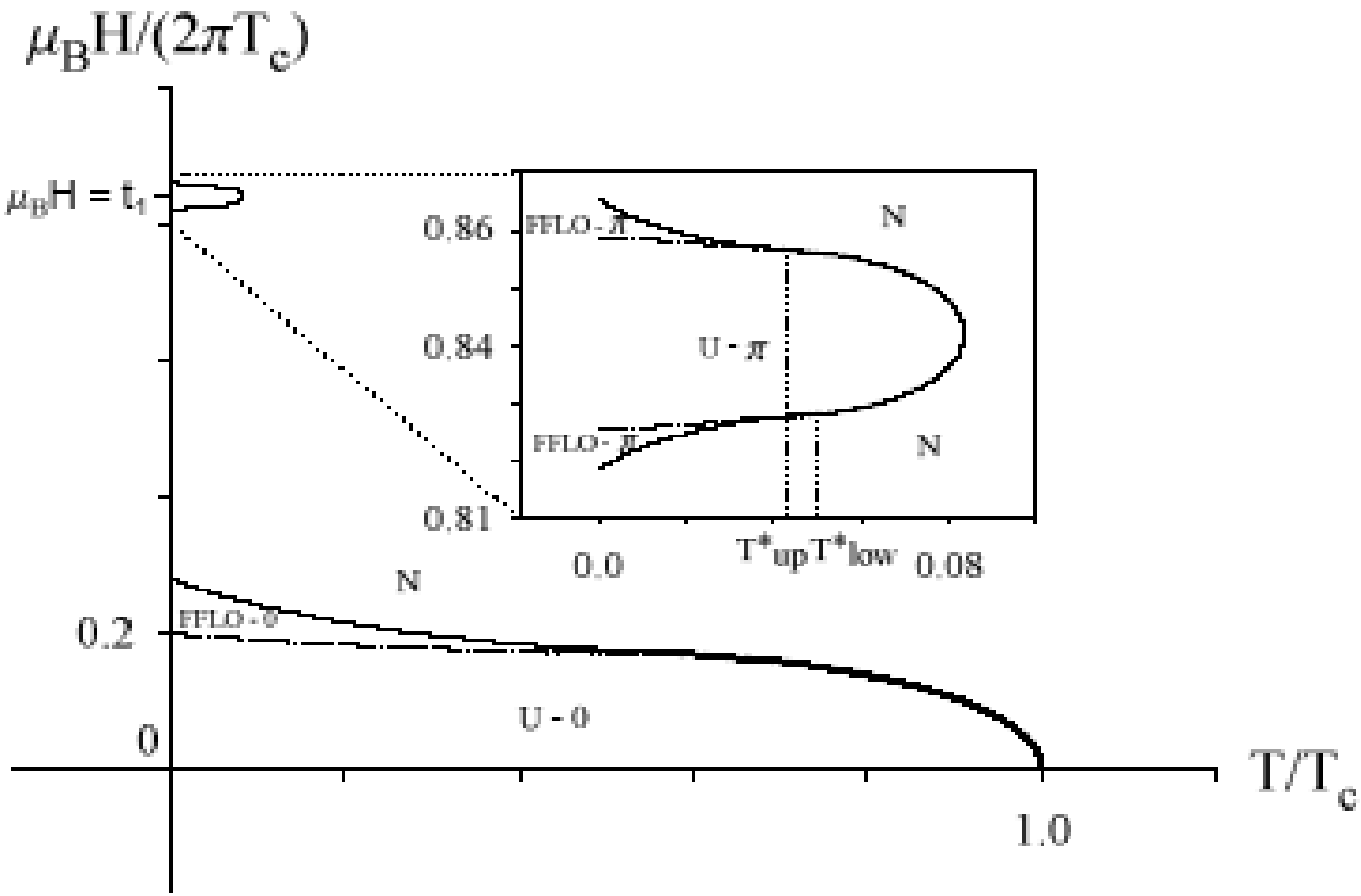
Superconducting bilayer



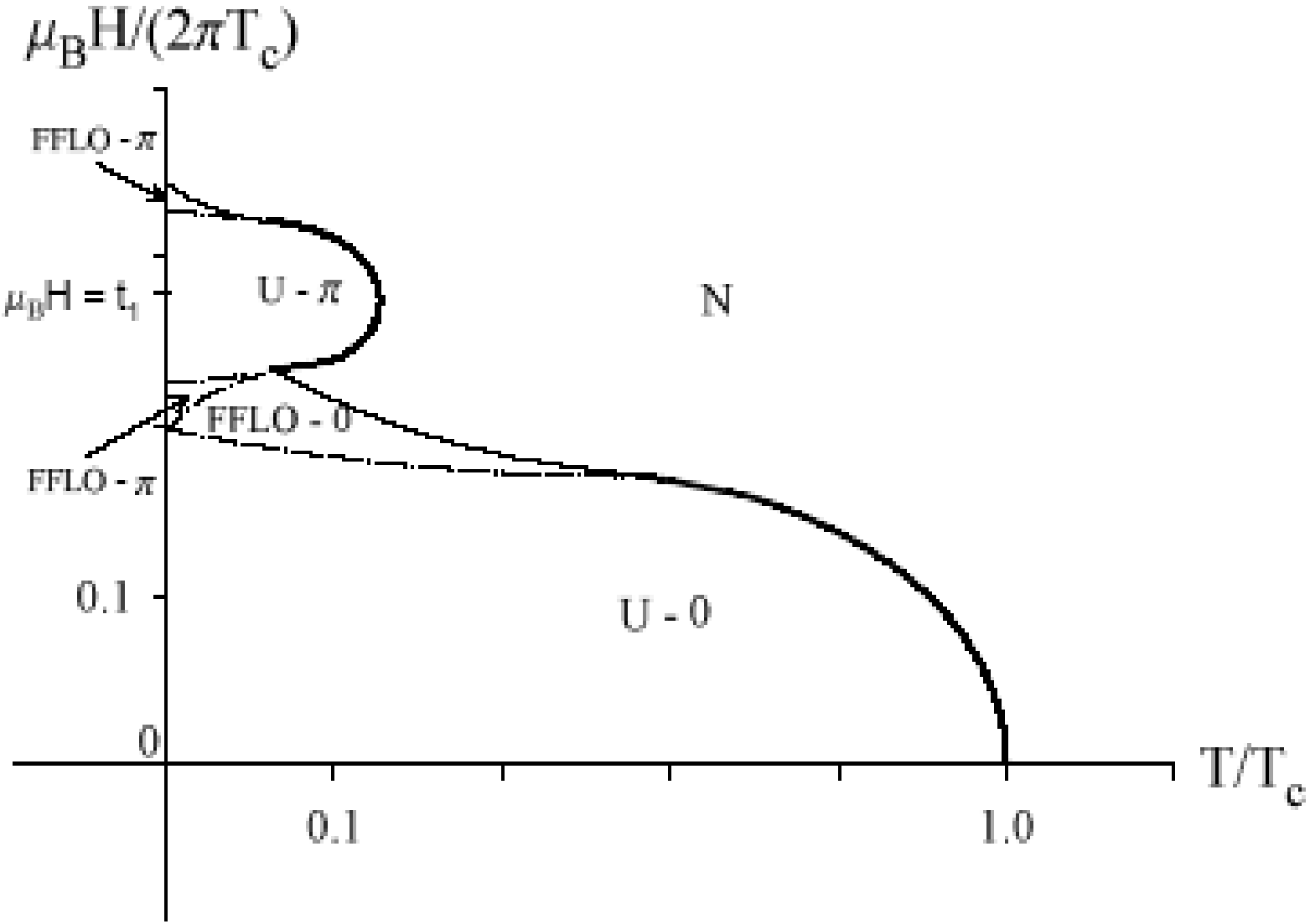
Energy spectrum

π-phase

Compensation of
the Zeeman splitting
at $\mu_B H = h \sim t$



At low temperature the paramagnetic limit may be strongly exceed $\mu_B H \sim t_1$. **π-phase with FFLO modulation in plane. Field-induced superconductivity!**



More complicated phase diagramm

Mechanism for the φ_0 - Josephson junction realization.

Recently the broken inversion symmetry (BIS) superconductors (like $CePt_3Si$) have attracted a lot of interest.

Very special situation is possible when the weak link in Josephson junction is a non-centrosymmetric magnetic metal with broken inversion symmetry !
Suitable candidates : MnSi, FeGe.

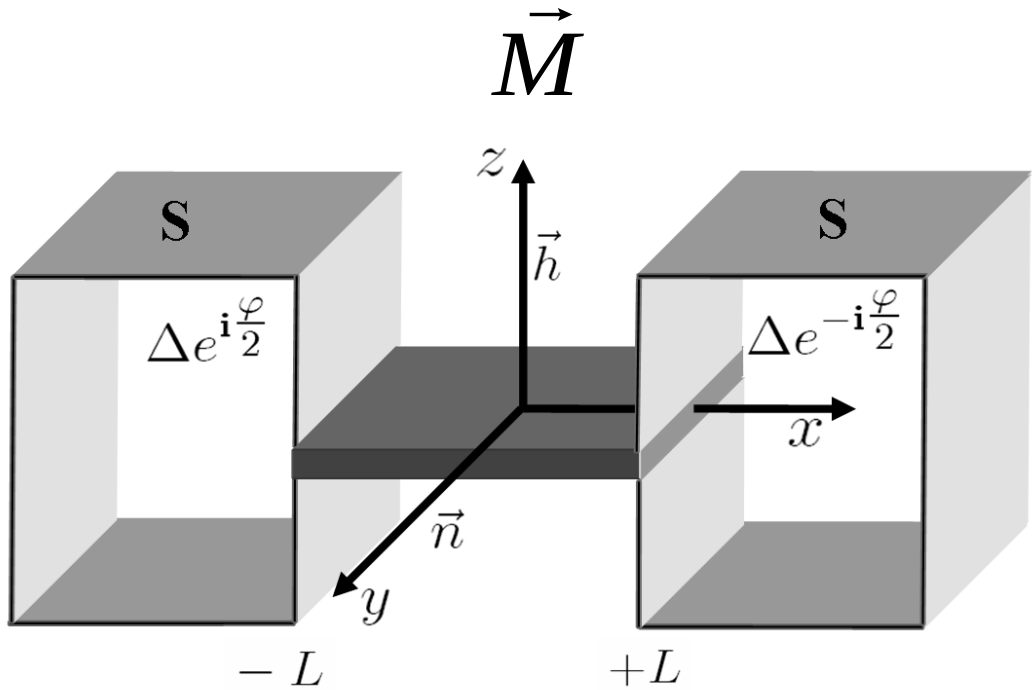
Josephson junctions with time reversal symmetry: $j(-\varphi) = -j(\varphi)$;
i.e. higher harmonics can be observed $\sim j_n \sin(n\varphi)$ –the case of the π junctions.

Without this restriction a more general dependence is possible
$j(\varphi) = j_0 \sin(\varphi + \varphi_0)$.

Rashba-type spin-orbit coupling $\alpha \langle \vec{\sigma} \times \vec{p} \rangle \cdot \vec{n}$

$\vec{n}$ is the unit vector along the asymmetric potential gradient.

Geometry of the junction with BIS magnetic metal



$$F = a|\Psi|^2 + \gamma|\vec{D}\Psi|^2 + \frac{b}{2}|\Psi|^4 - \varepsilon\vec{n}\left[\times\left(\Psi\left(\vec{D}\Psi\right)^* + \Psi^*\left(\vec{D}\Psi\right)\right)\right]$$

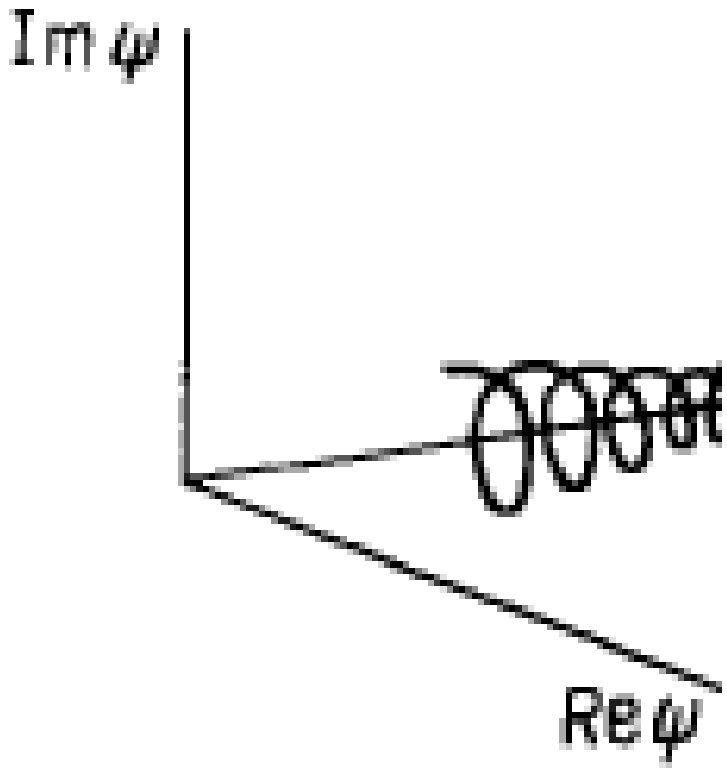
$$D_i = -i\partial_i - 2eA_i$$

$$a\Psi - \gamma\frac{\partial^2\Psi}{\partial x^2} + 2i\varepsilon\hbar\frac{\partial\Psi}{\partial x} = 0,$$

$$\Psi \propto \exp(i\tilde{\varepsilon}x)\exp(-x\sqrt{\frac{a-a_c}{\gamma}}), \qquad where\ \tilde{\varepsilon} = \frac{\varepsilon\hbar}{\gamma}$$

φ_0 - Josephson junction (A. Buzdin, PRL, 2008).

$$\Psi \propto \exp(i\tilde{\varepsilon}x)\exp(-x\sqrt{\frac{a-a_c}{\gamma}}), \qquad where\ \tilde{\varepsilon} = \frac{\varepsilon h}{\gamma}$$



In contrast with a π junction it is not possible to choose a real ψ function !

φ_0 Josephson junction

$$j(\varphi) = j_c \sin(\varphi + \varphi_o)$$

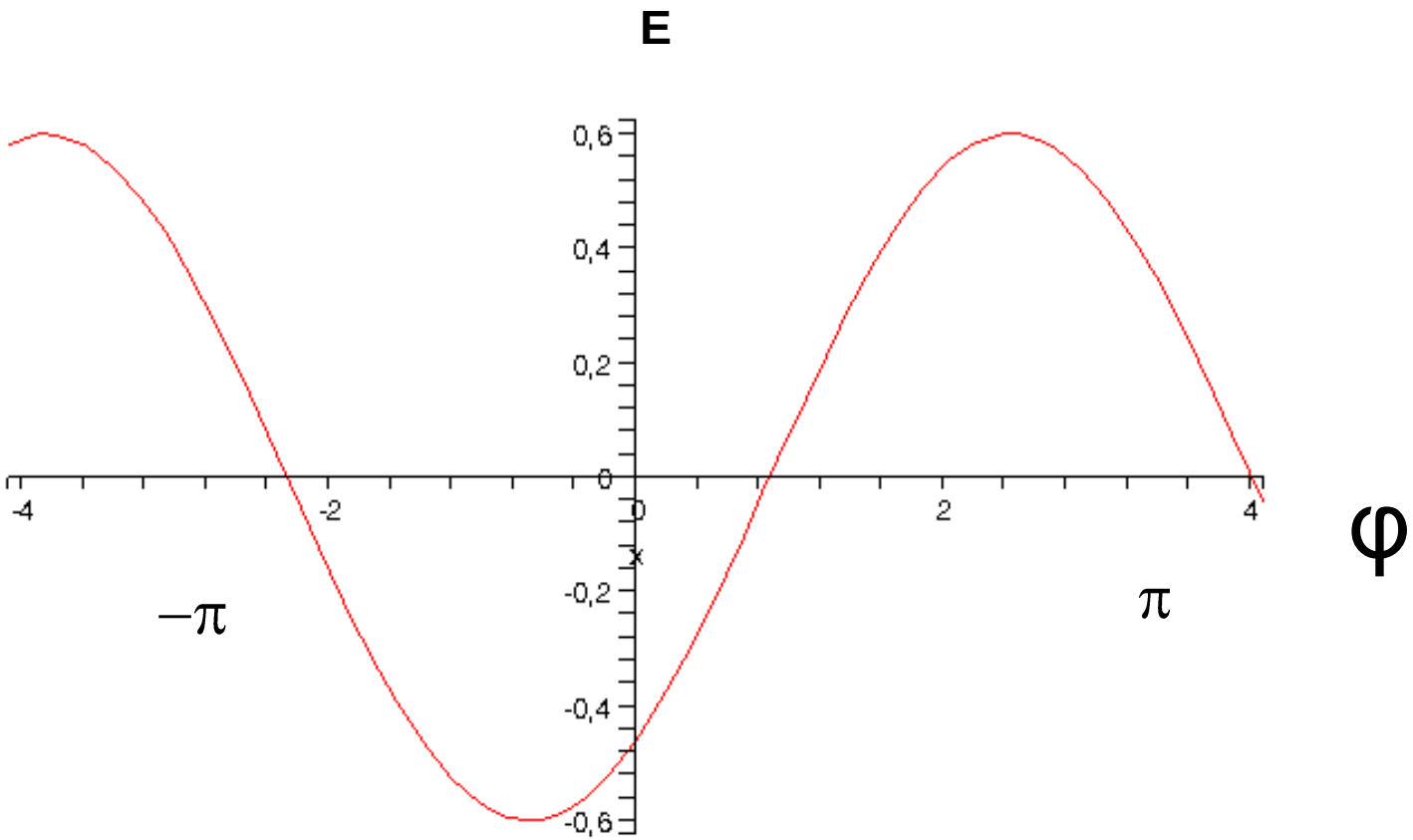
where $$\varphi_o = \frac{2\varepsilon\hbar L}{\gamma}$$

The phase shift φ_0 is proportional to the length and the strength of the BIS magnetic interaction.

The φ_0 Junction is a natural phase shifter.

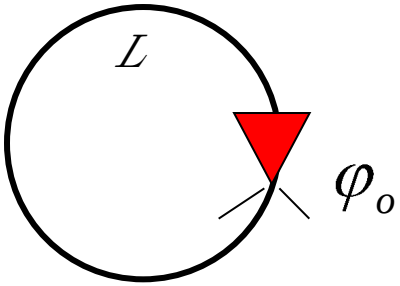
Energy $E_J(\varphi) \sim -j_c \cos(\varphi + \varphi_0)$

$$E_J(\varphi) \sim -j_c \cos(\varphi + \varphi_0)$$



$$E(\varphi = \varphi_0) \neq E(\varphi = -\varphi_0)$$

Spontaneous flux (current) in the superconducting ring with φ_0 - junction.



$$E(\varphi) = \frac{j_c}{2e}\left(-\cos(\varphi + \varphi_0) + \frac{k\varphi^2}{2} \right)$$

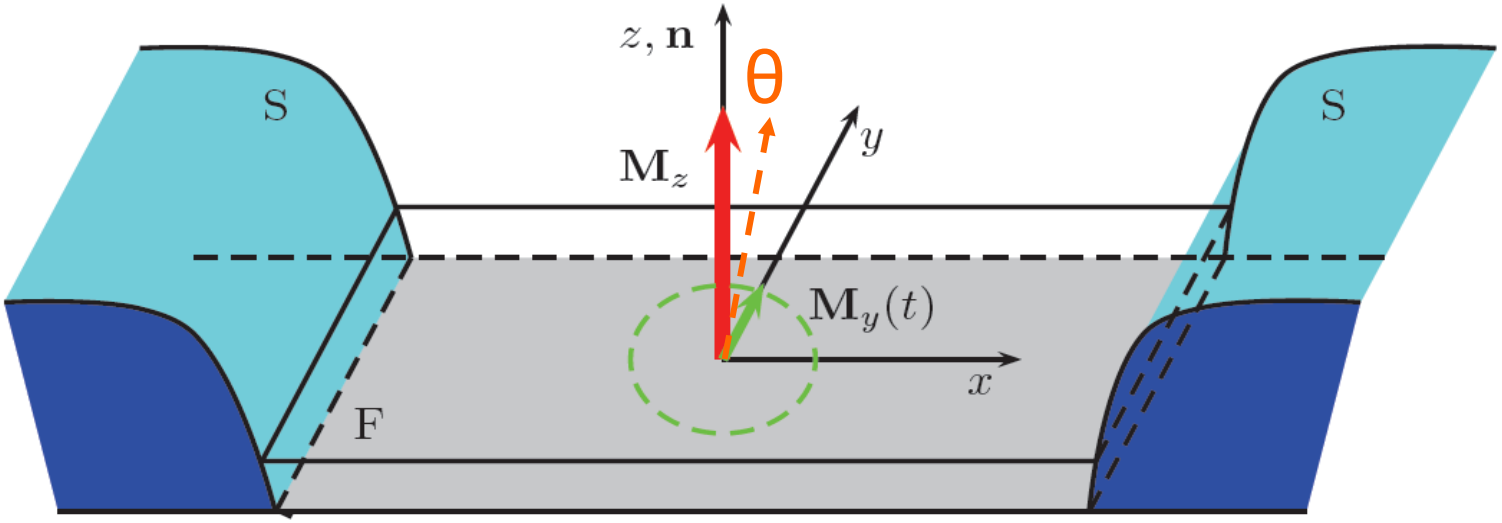
$$k = \frac{c\Phi_0}{2\pi L j_c}$$

In the k<<1 limit the junction generates the flux $\Phi=\Phi_0(\varphi_0/2\pi)$

$$\varphi_o = \frac{2\varepsilon\hbar L}{\gamma}$$

Very important : The φ_0 junction provides a mechanism of a **direct coupling** between supercurrent (superconducting phase) and magnetic moment (z component).

Let us consider the following geometry :



$$\varphi_0 = x \frac{v_{\rm so}}{v_F} \frac{M_y}{M_0}$$

$$\sin\theta = \frac{I}{I_c}\Gamma \quad \text{with } \Gamma = \frac{E_J}{K\mathcal{V}} x \frac{v_{\rm so}}{v_F}$$

voltage-biased Josephson junction

$$\varphi(t) = \omega_J t$$

Magnetic anisotropy (easy z-axis) energy :

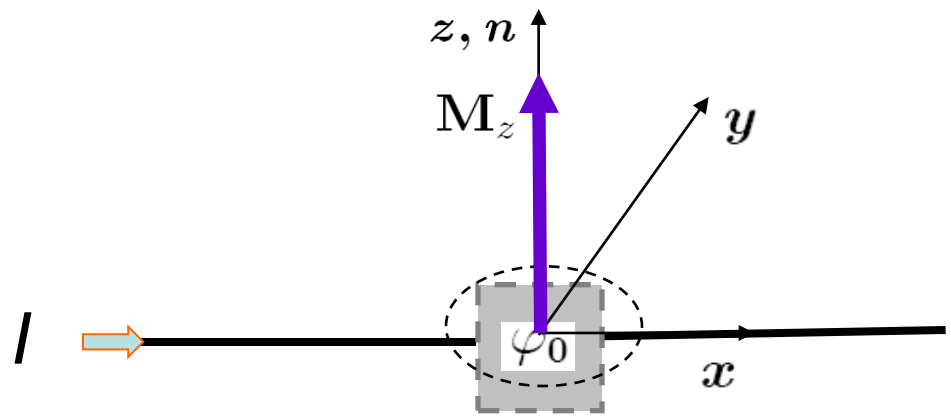
$$E_M = -\frac{K\mathcal{V}}{2}\left(\frac{M_z}{M_0}\right)^2$$

Coupling parameter :

$$\Gamma = \frac{E_J}{K\mathcal{V}} x \frac{v_{\rm so}}{v_F}$$

Weak coupling regime : Γ<1.
Srong coupling regime : Γ>1.

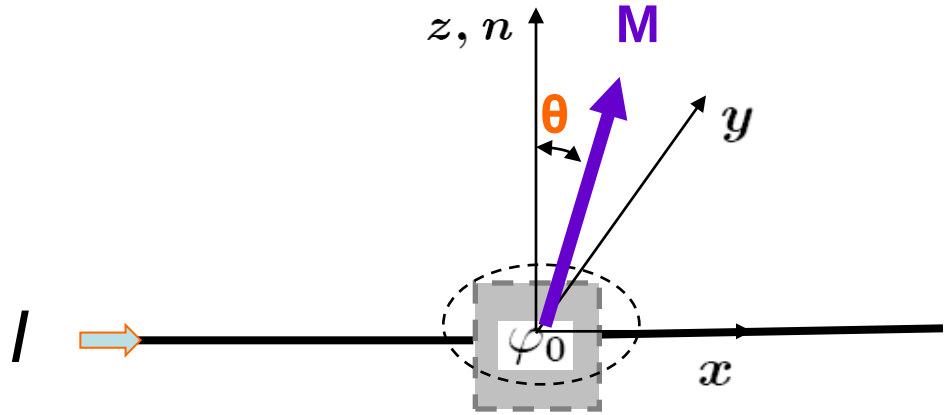
Let us consider first the φ_0 - junction when a constant current $I<I_c$ is applied :



$$\varphi_0 = x\frac{v_{\rm so}}{v_F}\frac{M_y}{M_0}$$

$$E_{tot} = -\frac{\Phi_0}{2\pi}\varphi I + E_s(\varphi,\varphi_0) + E_M(\varphi_0)$$

Minimum energy condition: $\partial_\varphi E_{\rm tot} = \partial_{\varphi_0} E_{\rm tot.} = 0.$



The current provokes rotation of the magnetic moment : $\sin\theta = \frac{I}{I_c}\Gamma$

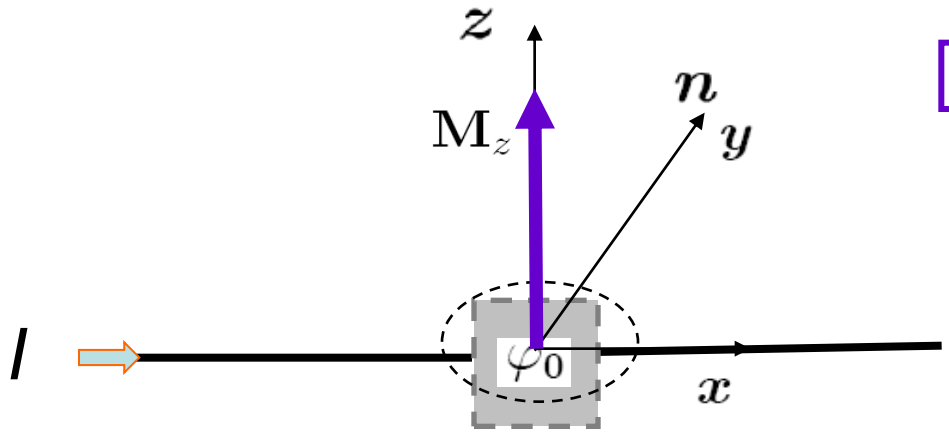
$$M_y = M_0 \sin\theta$$

For the case $\Gamma>1$ when $I>I_c/\Gamma$ the moment will be oriented along the y-axis.

Applying to the φ_0 - junction a current (phase difference) we can generate the magnetic moment rotation.

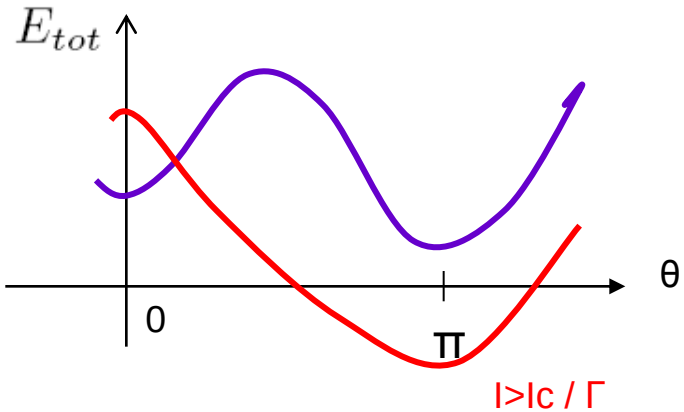
a.c. current -> moment's precession!

What happens if the SO gradient is along **y**-axis?



$$\varphi_0 = x\,(v_{so}/v_F)\cos\theta$$

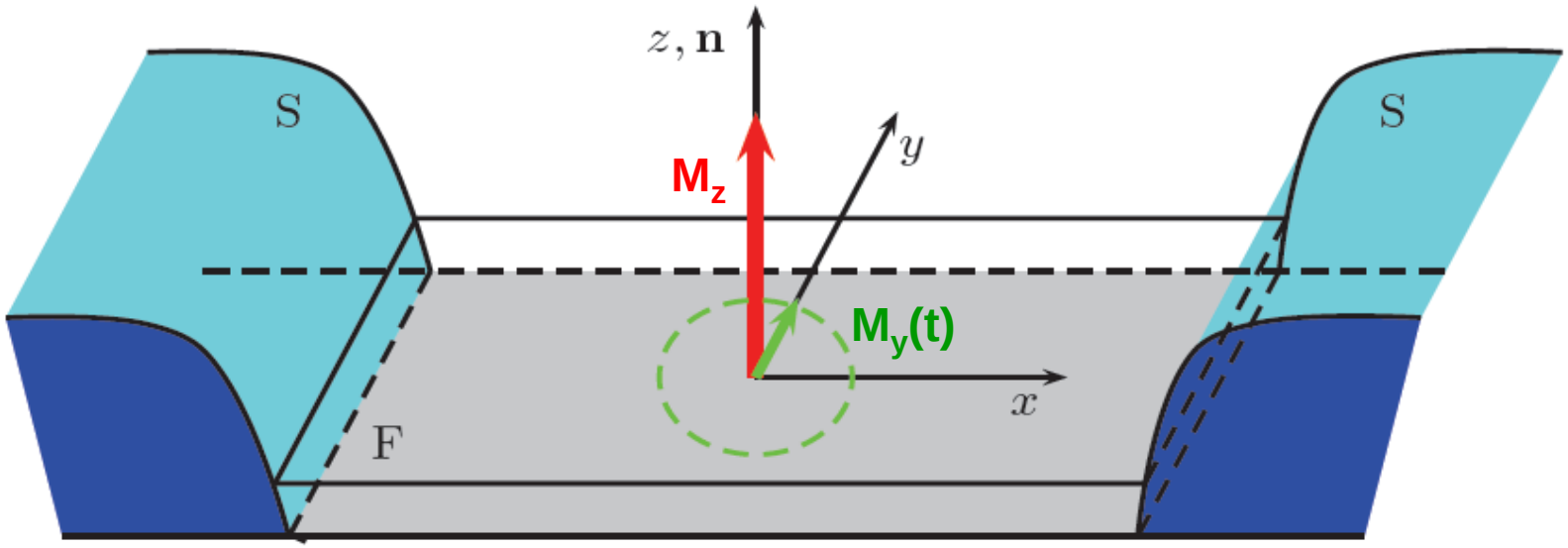
The total energy has two minima θ=(0, π).



The current pulses would provoke the switches of **M** between θ=0 and θ= π orientations. This corresponds to the transition of the junction from $+\varphi_0$ to $-\varphi_0$ state.

Magnetic moment precession – voltage-biased φ_0- junction

$\varphi(t) = \omega_J t$



$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \wedge \mathbf{H}_{\text{eff}} + \frac{\alpha}{M_0}\left(\mathbf{M} \wedge \frac{d\mathbf{M}}{dt}\right),$$

where $\mathbf{H}_{\text{eff}} = -\delta F / \mathcal{V} \delta \mathbf{M}$ is the effective magnetic field

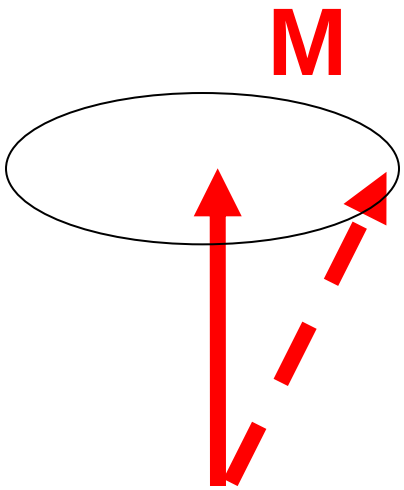
$$\mathbf{H}_{\text{eff}} = \frac{K}{M_0}\left[\Gamma \sin\left(\omega_J t - r\frac{M_y}{M_0}\right)\hat{\boldsymbol{y}} + \frac{M_z}{M_0}\hat{\boldsymbol{z}}\right]$$

$$r = x v_{\text{so}} / v_F$$

Landau-Lifshitz equation :

$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \wedge \mathbf{H}_{\text{eff}} + \frac{\alpha}{M_0}\left(\mathbf{M} \wedge \frac{d\mathbf{M}}{dt}\right)$$

$$\mathbf{H}_{\text{eff}} = \frac{K}{M_0}\left[\Gamma \sin\left(\omega_J t - r\frac{M_y}{M_0}\right)\hat{y} + \frac{M_z}{M_0}\hat{z}\right]$$



Magnetic moment precession :

$$\frac{I}{I_c} = \sin\omega_J t + \frac{\Gamma r}{2}\frac{1}{\omega^2 - 1}\sin 2\omega_J t + \ldots,$$

Weak coupling regime : Γ<<1.

$$\omega = \omega_J/\omega_F$$

Without damping

$$m_x(t) = \frac{\Gamma\omega\cos\omega_J t}{1-\omega^2} \text{ and } m_y(t) = -\frac{\Gamma\sin\omega_J t}{1-\omega^2}.$$

$$\frac{I}{I_c} = \sin\omega_J t + \frac{\Gamma r}{2}\frac{1}{\omega^2-1}\sin 2\omega_J t + \ldots,$$

With damping

$$m_y(t) = \frac{\omega_+ - \omega_-}{r}\sin\omega_J t + \frac{\alpha_- - \alpha_+}{r}\cos\omega_J t,$$

$$\omega_\pm = \frac{\Gamma r}{2}\frac{\omega\pm 1}{\Omega_\pm} \text{ and } \alpha_\pm = \frac{\Gamma r}{2}\frac{\alpha}{\Omega_\pm} \quad \text{with } \Omega_\pm = (\omega\pm 1)^2 + \alpha^2.$$

$$I(t) \approx I_c\sin\omega_J t + I_c\frac{\omega_+ - \omega_-}{2}\sin 2\omega_J t + \\ + I_c\frac{\alpha_- - \alpha_+}{2}\cos 2\omega_J t + I_0(\alpha)$$

The current acquires a d..c. component !

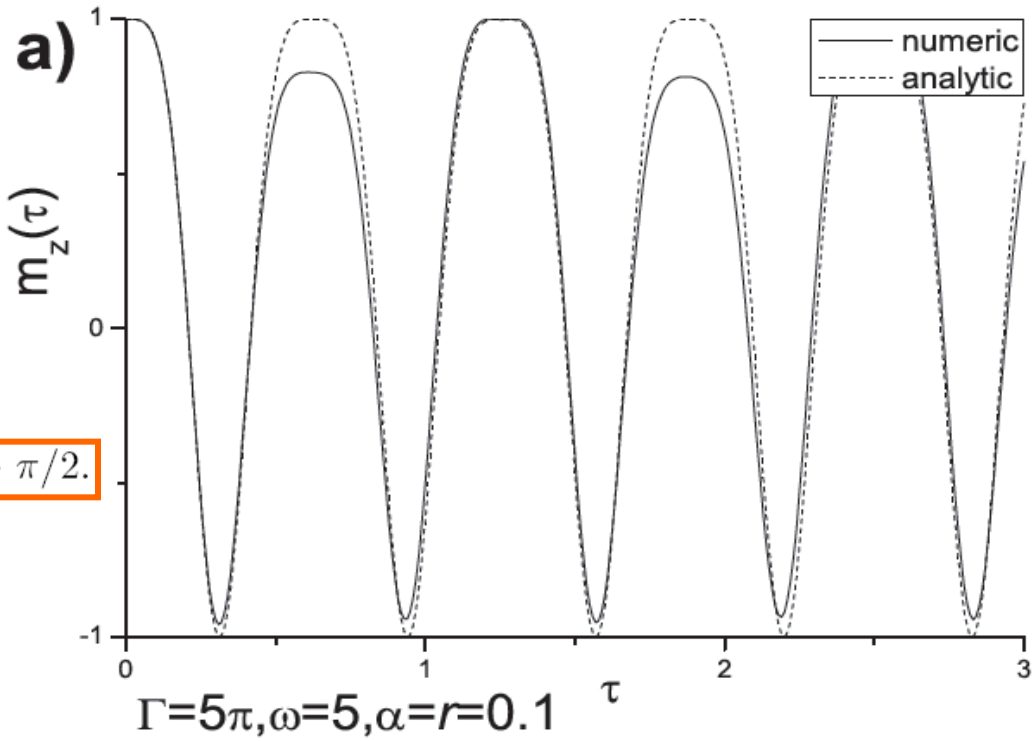
$$I_0(\alpha) = \frac{\alpha\Gamma r}{4}\left(\frac{1}{\Omega_-} - \frac{1}{\Omega_+}\right)$$

Srong coupling regime : Γ>>1.

if r <<1 $\boxed{r = x v_{so}/v_F}$ then $m_y \approx 0$ and without damping we have :

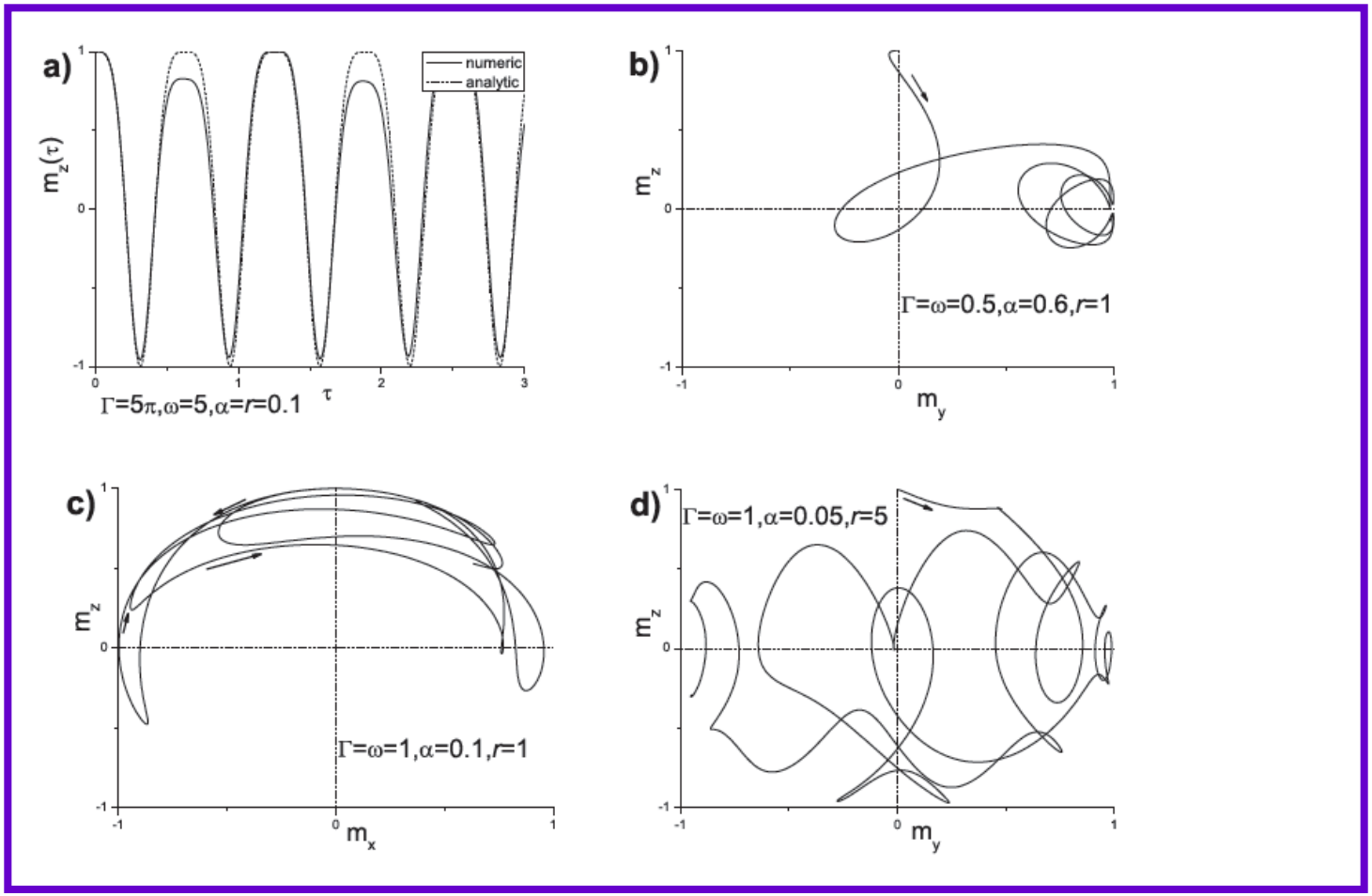
$$\begin{cases} m_x(t) = \sin\left[\frac{\Gamma}{\omega}(1-\cos\omega_J t)\right] \\ m_z(t) = \cos\left[\frac{\Gamma}{\omega}(1-\cos\omega_J t)\right] \end{cases}$$

complete reversal being induced by $\Gamma/\omega > \pi/2$.



Comparison between analytic results (dashed line) and numerical computation.

Complicated regime of the magnetic dynamics :



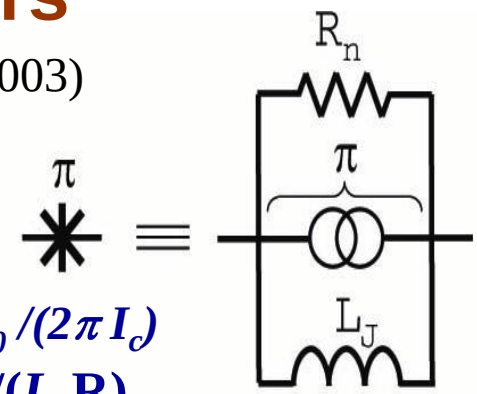
For more details – see (F. Konschelle and A. Buzdin, PRL, 2009).

Complementary Josephson logic

RSFQ-logic using π-shifters

A.V.Ustinov, V.K.Kaplunenko. Journ. Appl. Phys. **94**, 5405 (2003)

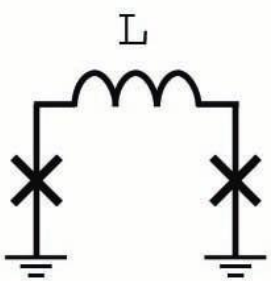
RSFQ- logic: Rapid Single Quantum logic



$L_J = \Phi_0/(2\pi I_c)$

$\tau \sim 1/(I_c R)$

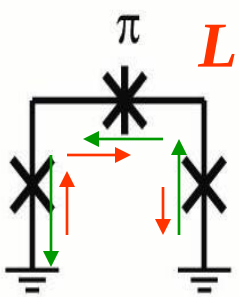
Conventional RSFQ-cell



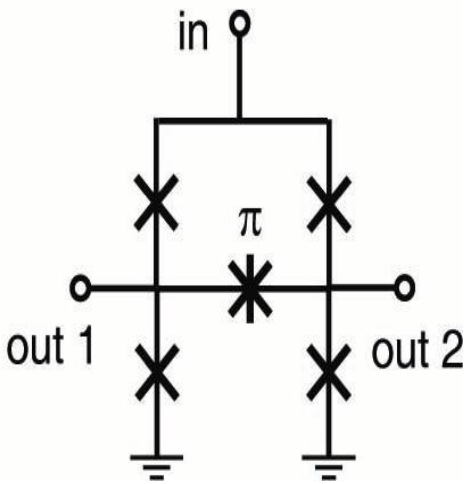
$LI_c > \Phi_0$

Fluxon memorizing cell

RSFQ -π– cell



$L \rightarrow 0$



π-RSFQ –***Toggle Flip-Flop***

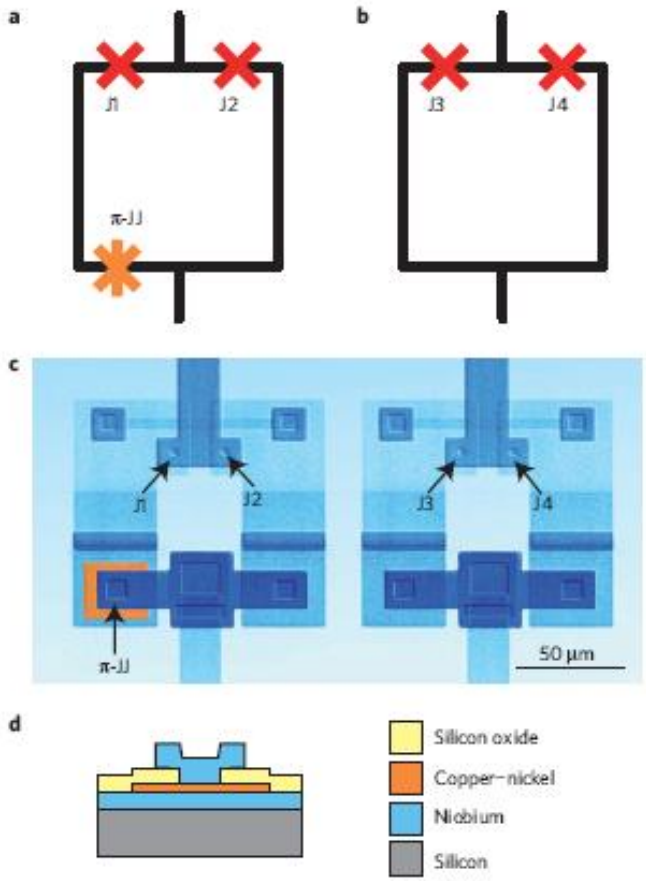
To operate at 20 GHz clock rate
$I_c R \sim 50\ \mu V$ has to be
We have $I_c R > 0.1\ \mu V$ for the present

nature physics

July 2010

Implementation of superconductor/ferromagnet/superconductor π-shifters in superconducting digital and quantum circuits

A. K. Feofanov[1], V. A. Oboznov[2], V. V. Bol'ginov[2], J. Lisenfeld[1], S. Poletto[1], V. V. Ryazanov[2], A. N. Rossolenko[2], M. Khabipov[3], D. Balashov[3], A. B. Zorin[3], P. N. Dmitriev[4], V. P. Koshelets[4] and A. V. Ustinov[1]*



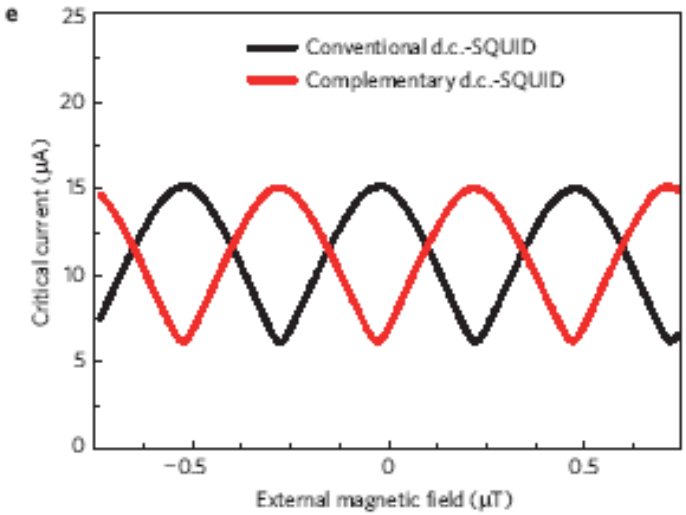
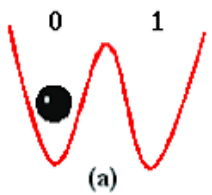
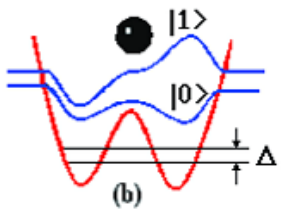


Figure 1 | Complementary d.c.-SQUIDs. a, Schematic diagram of a complementary d.c.-SQUID employing two conventional Josephson

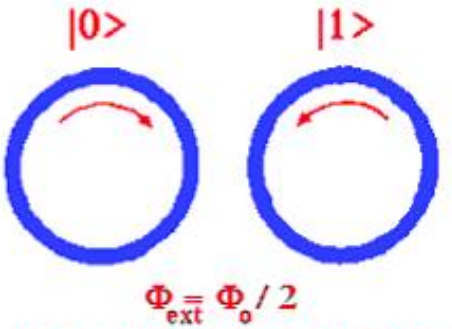
Superconducting phase qubit

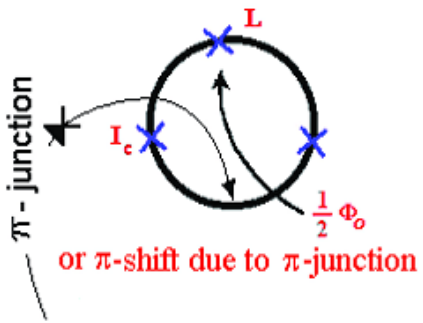


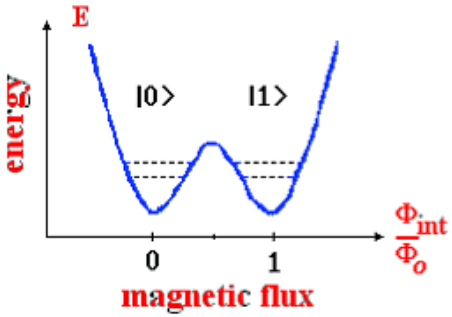
Digital bit

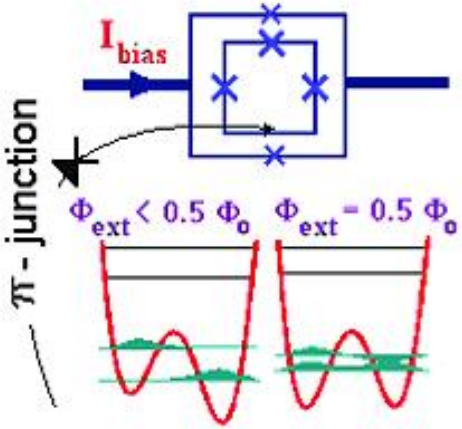


Quantum bit









qubit operation

nature physics

July 2010

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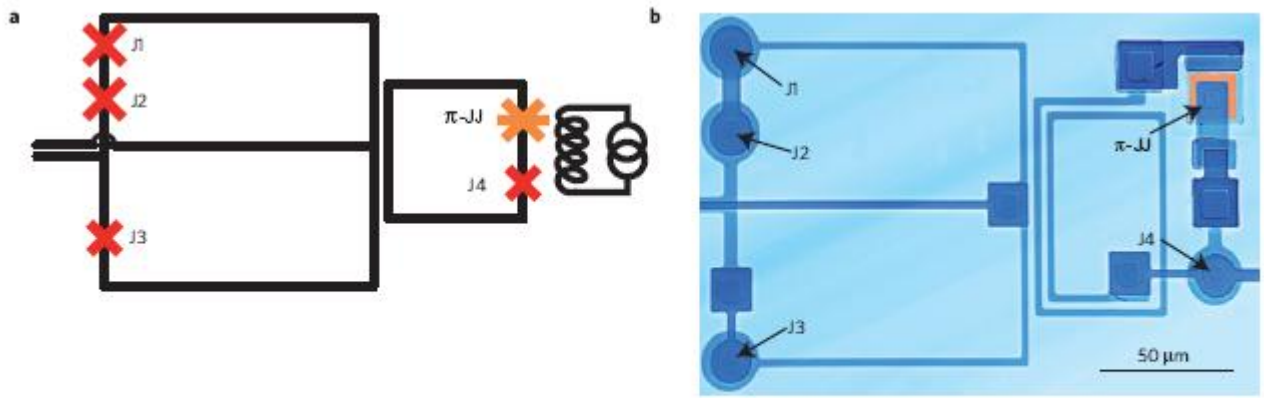


Figure 3 | Self-biased phase qubit. a, Schematic diagram of a phase qubit circuit used to test the decoherence properties of the π-junction. The qubit is realized by the central loop with embedded conventional and π-Josephson junctions. The larger loop to its left is a d.c.-SQUID for qubit readout. To the right of the qubit is a coupled weakly flux bias coil. **b**, Scanning electron microscope picture of the realized phase qubit employing a π-junction in the qubit loop. The flux bias coil is not shown.

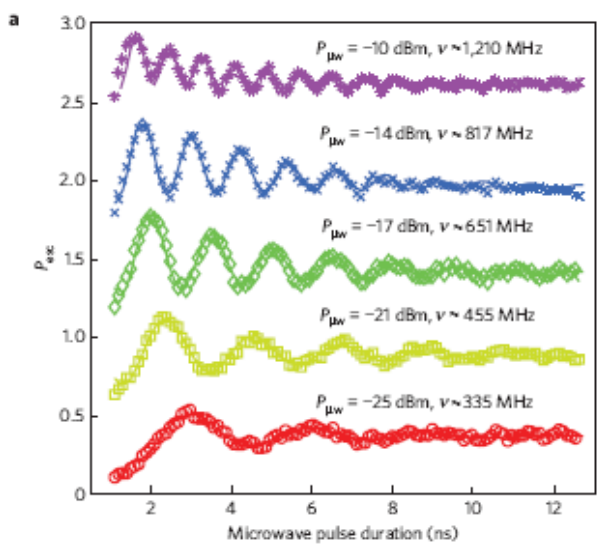


Figure 4 | Rabi oscillations between the ground and the excited qubit states resulted from resonant microwave driving. a,b, Rabi oscillations observed in the phase qubit with an embedded π-junction (**a**) and a conventional phase qubit made on the same wafer as a reference (**b**). Each

I. conclusions

- **Unusual non-uniform magnetic and superconducting phases.**
- **Ferromagnetic superconductors –unconventional types of superconductivity. Fundamental properties of this superconductivity?**
- **Superconductor-ferromagnet heterostructures permit to study superconductivity under huge exchange field ($h \gg T_c$).**
- **The π-junction realization in S/F/S structures is quite a general phenomenon.**
- **The BIS magnets provide a mechanism of the realization of the novel φ_0 - junctions . In these φ_0 - junctions a direct (linear) coupling between superconductivity and magnetism is realized. Seems to be ideal for superconducting spintronics.**

Some general Refs.: **Magnetic superconductors-** M. Kulic and A. Buzdin in **Superconductivity,** Springer, 2008 (eds. Benneman and Ketterson).
S/F proximity effect - A. Buzdin, Rev. Mod. Phys. (2005). S. Bergeret et al. Rev. Mod. Phys. (2005).
Coexistence of Superconductivity and Magnetism –Bulaevskii et al., ADV. IN PHYSICS, 1985, VOL. 34, 175.

Non-uniform (FFLO) states and quantum oscillations in superconductors and superfluid ultracold Fermi gases

1. Larkin-Ovchinnikov-Fulde-Ferrell state.
LOFF or FFLO states.
The origin of the non-uniform modulated superconducting state.

2. Experimental evidences of FFLO state.

3. Exactly solvable models of FFLO state.

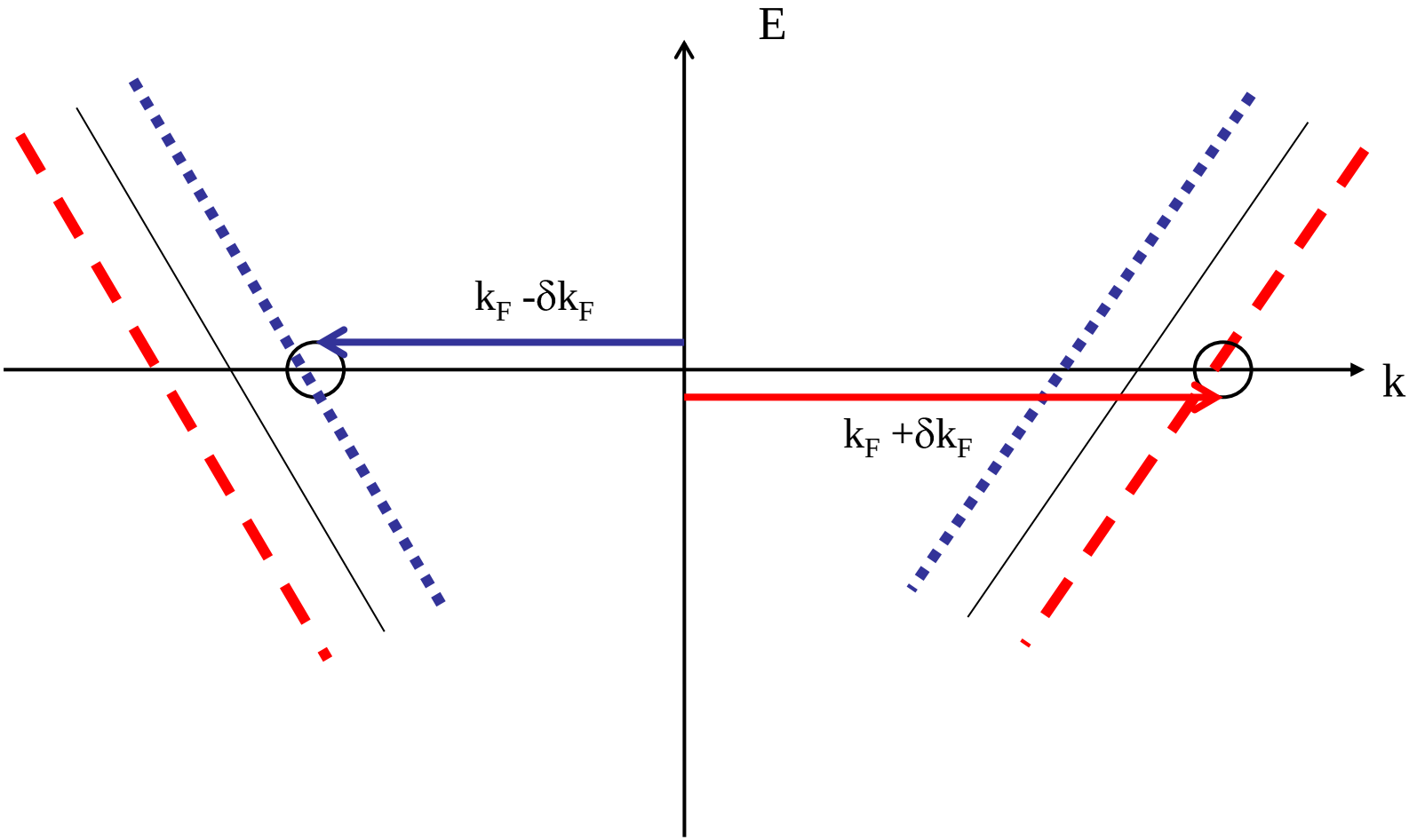
4. Vortices in FFLO state. Role of the crystal structure.

5. Supefluid ultracold Fermi gases with imbalanced state populations: one more candidate for FFLO state?

FFLO inventors

Fulde and Ferrell

Larkin and Ovchinnikov



The total momentum of the Cooper pair is
$-(k_F - \delta k_F) + (k_F - \delta k_F) = 2\,\delta k_F$

Pairing of electrons with opposite spins and momenta unfavourable :

$$[\epsilon(\mathbf{k}) - \mu_B H^{\text{eff}}] \neq [\epsilon(-\mathbf{k}) + \mu_B H^{\text{eff}}]$$

But :

$$\boxed{[\epsilon(\mathbf{k}+\mathbf{q}) - \mu_B H^{\text{eff}}] \approx [\epsilon(-\mathbf{k}+\mathbf{q}) + \mu_B H^{\text{eff}}] \quad \text{if} \quad q \approx \frac{\mu_B H^{\text{eff}}}{v_F}}$$

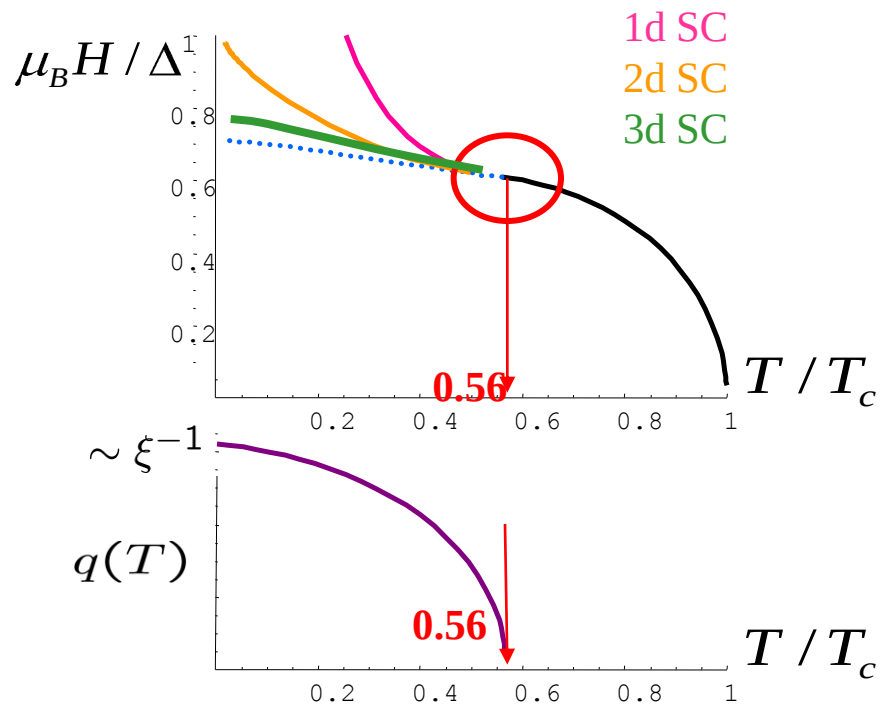
$$\rightarrow \Delta(\mathbf{r}) = \Delta \exp(i\mathbf{q}\cdot\mathbf{r})$$

At $T = 0$, Zeeman energy compensation is exact in 1d, partial in 2d and 3d.

- the upper critical field is increased
- Sensivity to the disorder and to the orbital effect:

(clean limit)

$$\boxed{q(T) \gg \frac{1}{\ell_{\text{imp}}}, \frac{1}{L_H}}$$



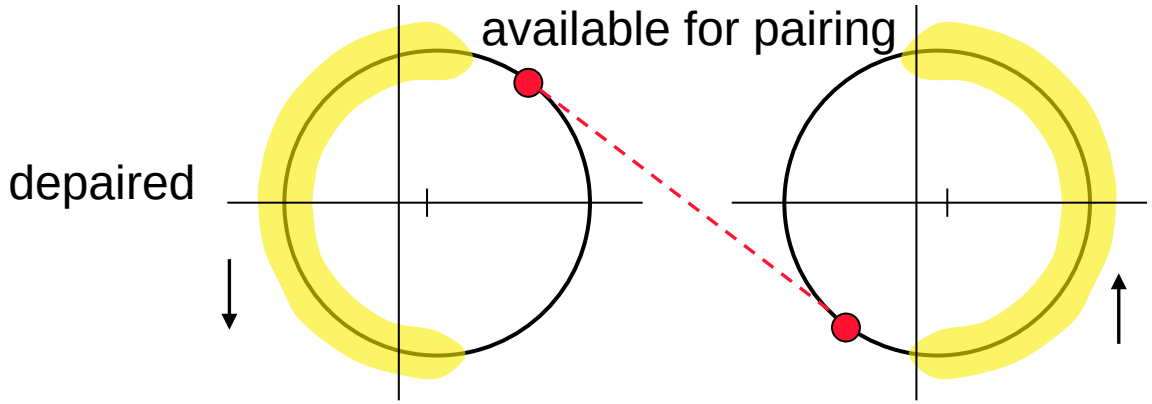
FF state

$(\mathbf{k}\uparrow, -\mathbf{k}\downarrow)$

$(\mathbf{k}\uparrow, -\mathbf{k}+\mathbf{q}\downarrow)$

uniform

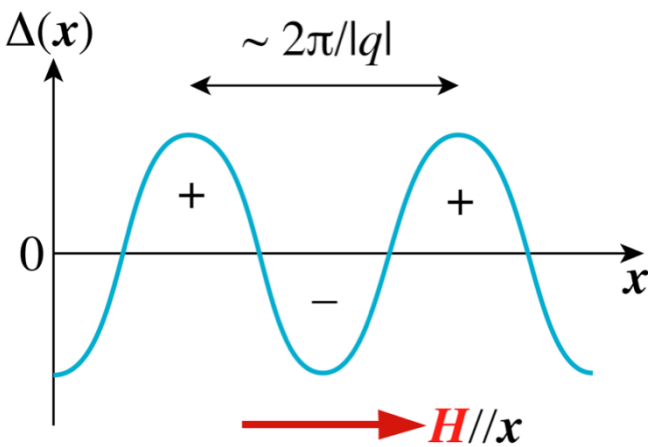
$$\Delta(\mathbf{r}) = \Delta(\mathbf{q})\exp(i\mathbf{q}\cdot\mathbf{r})$$

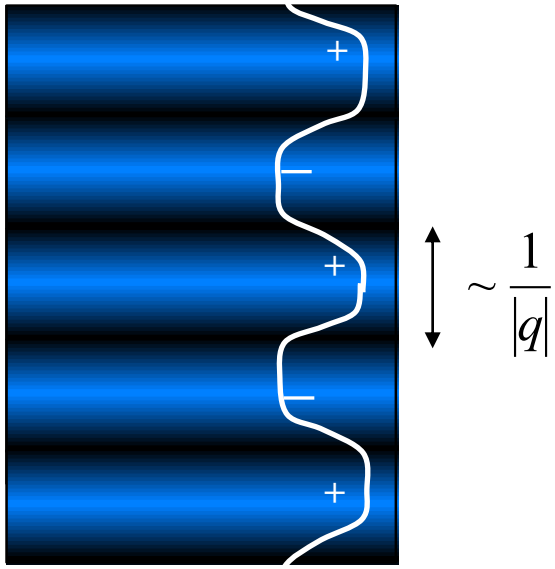


LO state

spatially nonuniform

$$\Delta(\mathbf{r}) = \Delta_0 \cos(\mathbf{q}\cdot\mathbf{r})$$





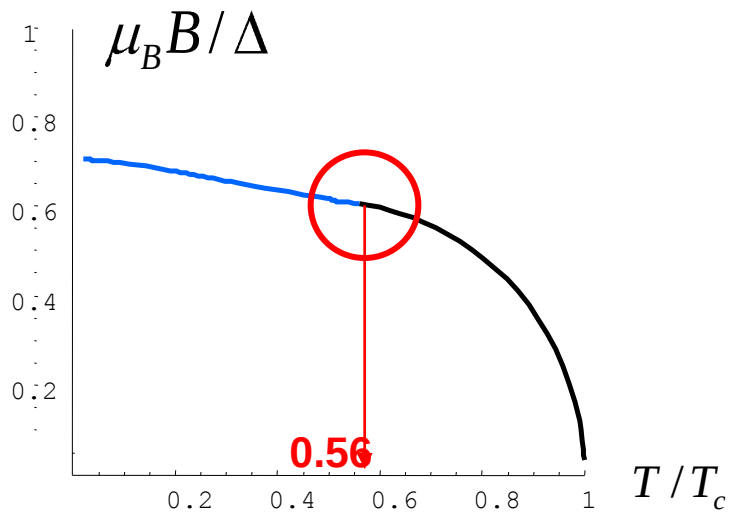
The SC order parameter performs one-dimensional spatial modulations along H, forming planar nodes

Modified Ginzburg-Landau functional :

$$F = a|\Psi|^2 - \gamma|\nabla\Psi|^2 + \eta|\nabla^2\Psi|^2 - \gamma'|\Psi|^4 + \beta|\nabla\Psi|^2|\Psi|^2$$
$$+ \beta'\left[\Psi^{*2}(\nabla\Psi)^2 + \Psi^2(\nabla\Psi^*)^2\right] + \delta|\Psi|^6 + \ldots$$

$$\tilde{\nabla} = \nabla - \frac{2ie}{\hbar c}\mathbf{A}$$

May be 1st order transition at $T < T^\star \approx 0.56 T_c$



2. Experimental evidences of FFLO state.

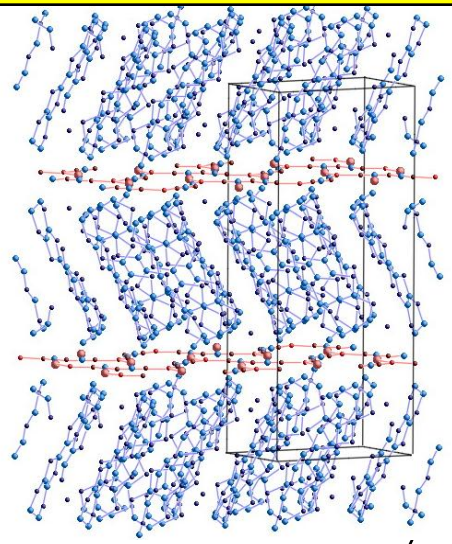
- Unusual form of $H_{c2}(T)$ dependence
- Change of the form of the NMR spectrum
- Anomalies in altrasound absorbtion
- Unusual behaviour of magnetization
- Change of anisotropy ….

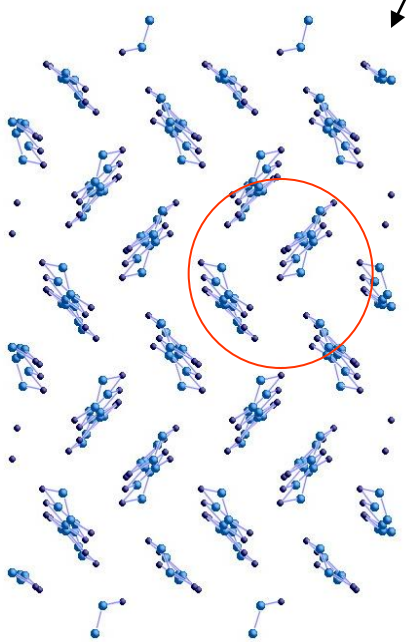
Organic superconductor

κ-(BEDT-TTF)$_2$Cu(NCS)$_2$ (T_c=10.4K)

Layered structure ⇒ Suppression of orbital effects in ***H*** parallel to the planes

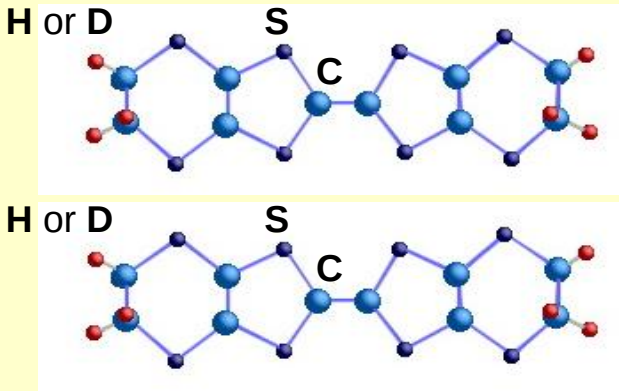
~ 15 Å



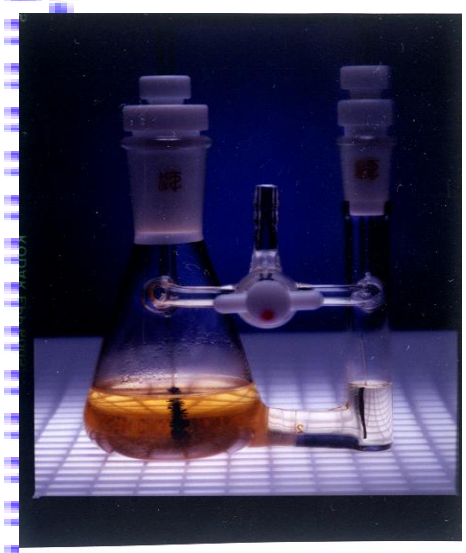


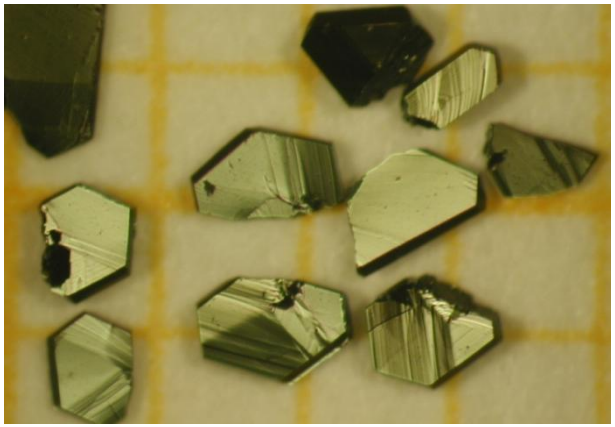
Cu[N(CN)$_2$]Br layer

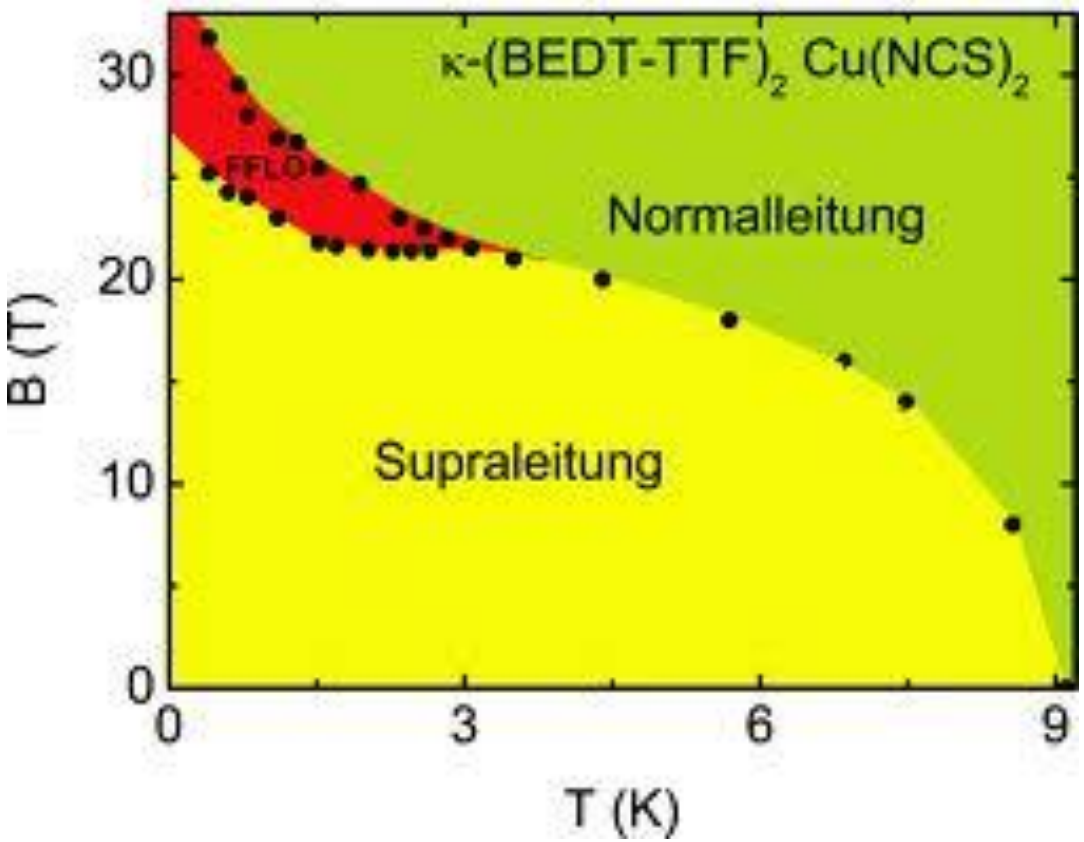
BEDT-TTF layer



BEDT-TTF (donor molecule)



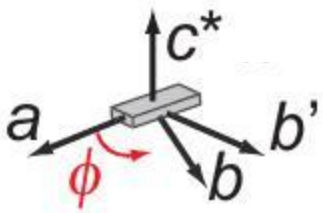


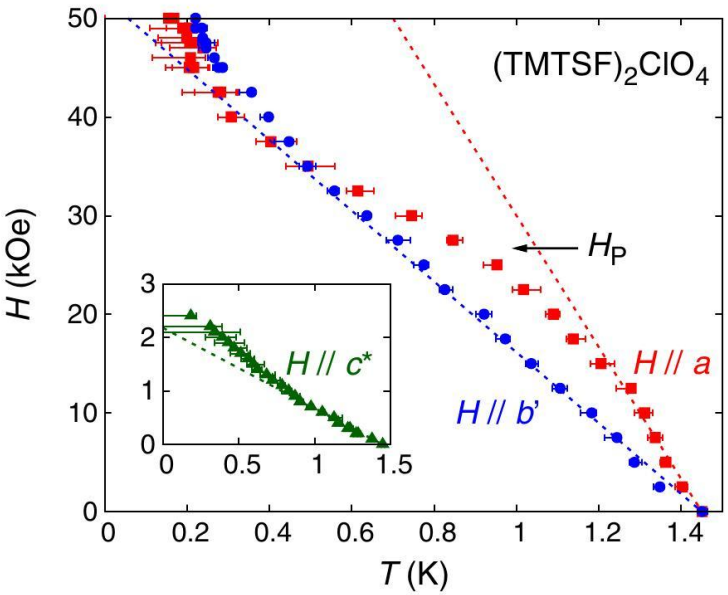


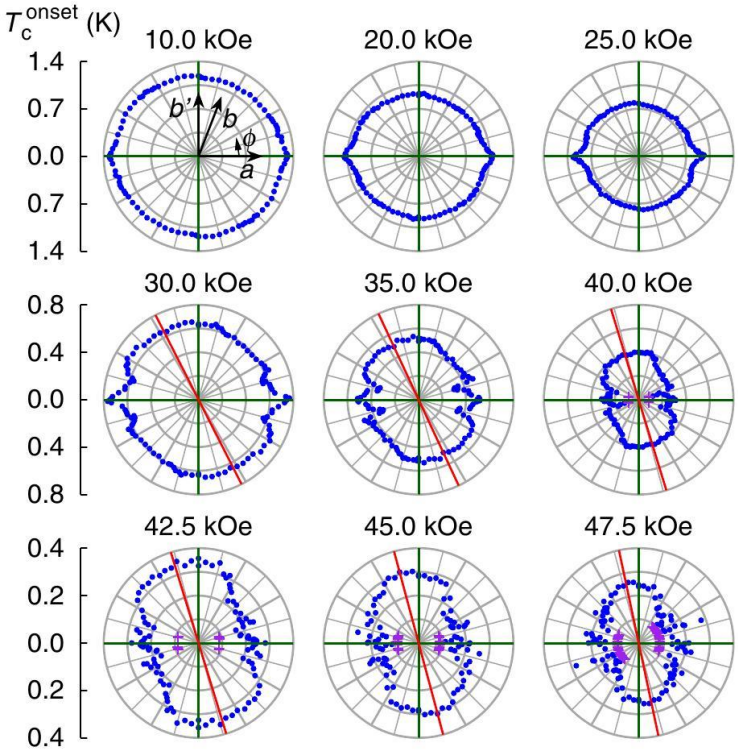
The Fulde-Ferrell-Larkin-Ovchinnikov State in the Organic Superconductor κ-(BEDT-TTF)$_2$Cu(NCS)$_2$ as Observed in Magnetic Torque Experiments

B. Bergk[a], A. Demuer[b], I. Sheikin[b], Y. Wang[c], J. Wosnitza[a], Y. Nakazawa[d], and R. Lortz[c]

Anomalous in-plane anisotropy of the onset of SC in $(TMTSF)_2ClO_4$

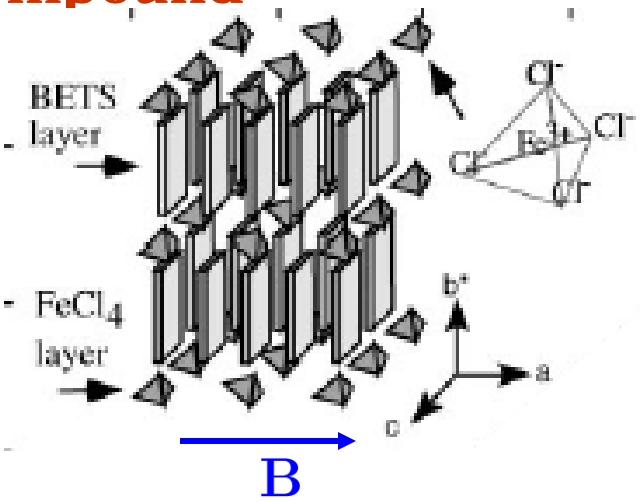


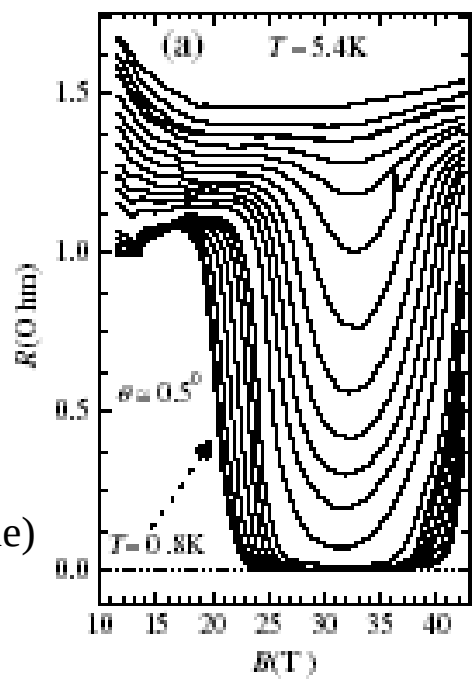




S.Yonezawa, S.Kusaba, Y.Maeno, P.Auban-Senzier, C.Pasquier, K.Bechgaard, and D. Jerome, Phys. Rev. Lett. **100**, 117002 (2008)

Field induced superconductivity (FISC) in an organic compound



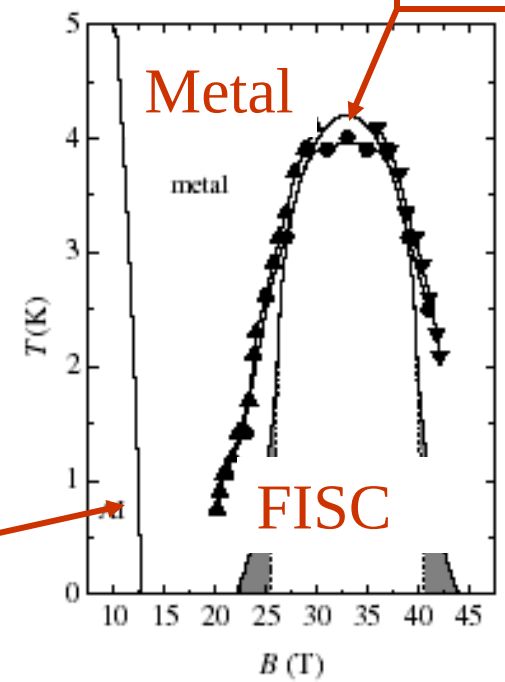


c-axis (in-plane) resistivity

λ-$(BETS)_2FeCl_4$

$T_c^{max} \approx 4.2K$

$B_0 \approx 33Tesla$

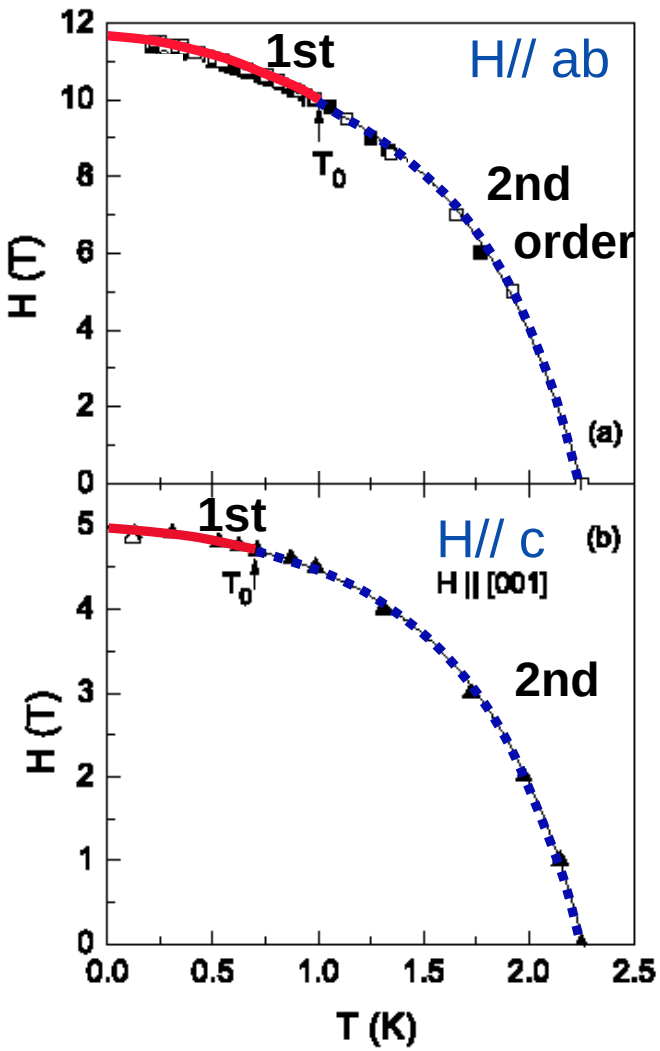


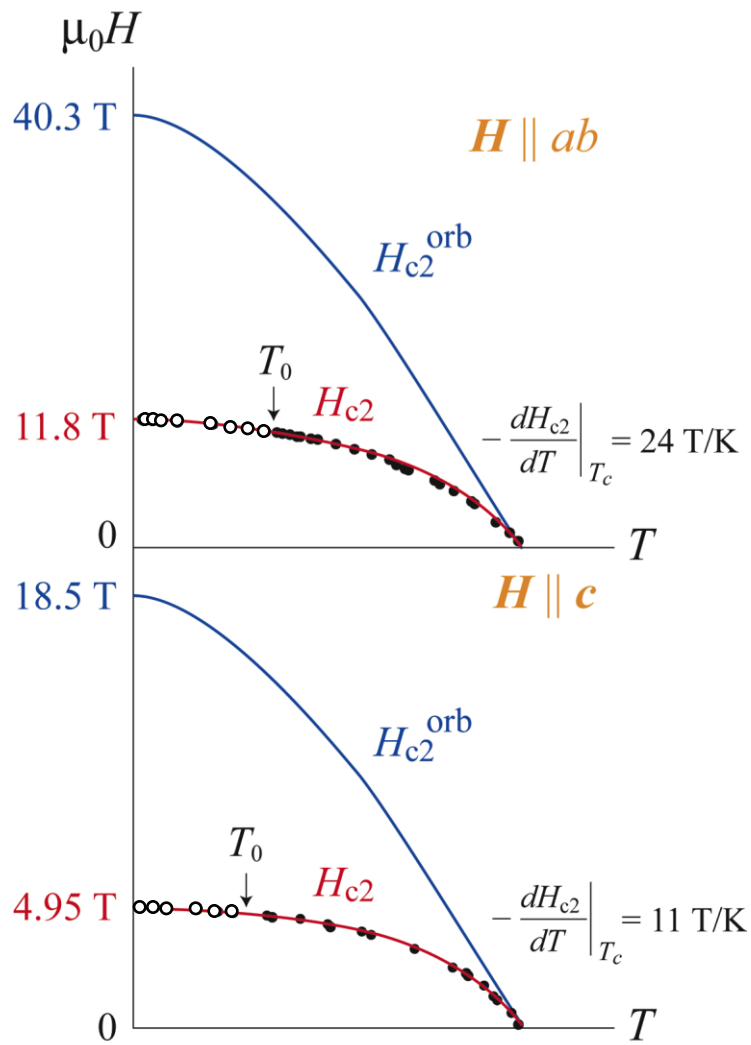
FISC for $18T < B < 45T$

S. Uji *et al.*, Nature **410** 908 (2001)

L. Balicas *et al.*, PRL **87** 067002 (2001)

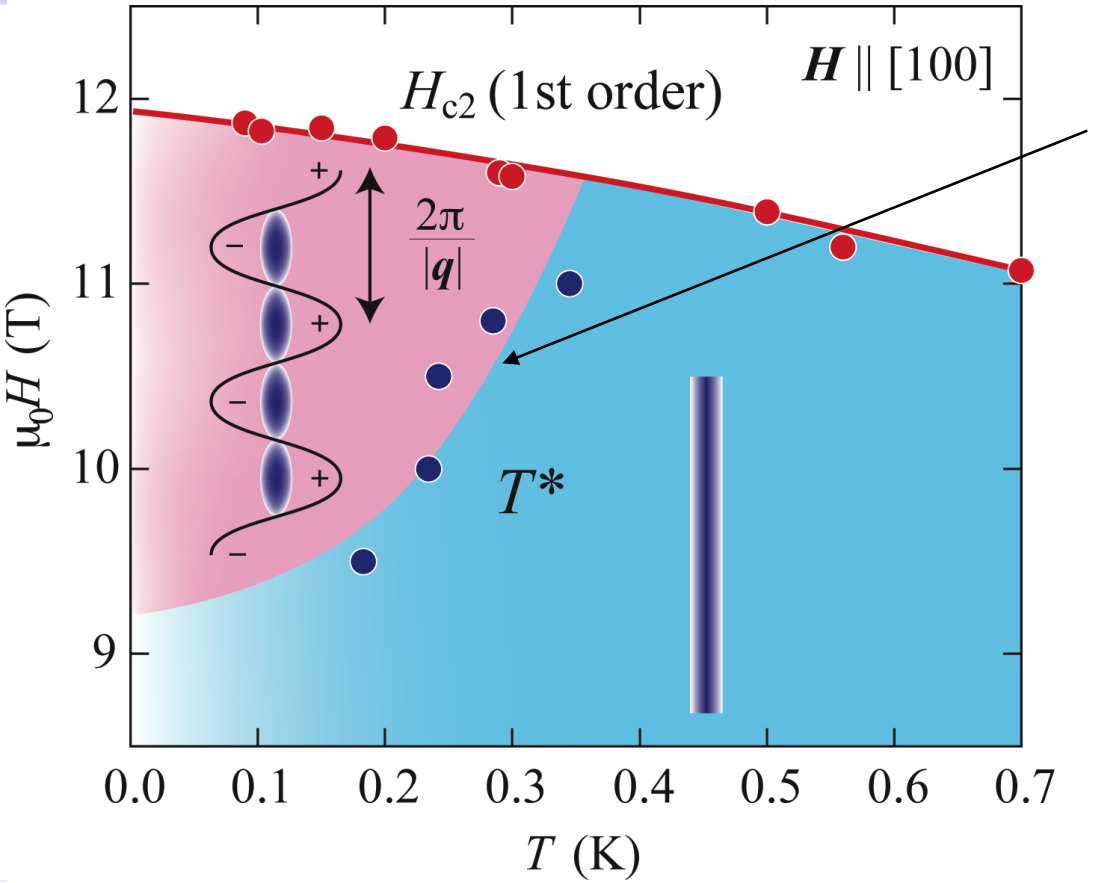
H-T phase diagram of $CeCoIn_5$





Pauli paramagnetically limited superconducting state

New high field phase of the flux line lattice in $CeCoIn_5$



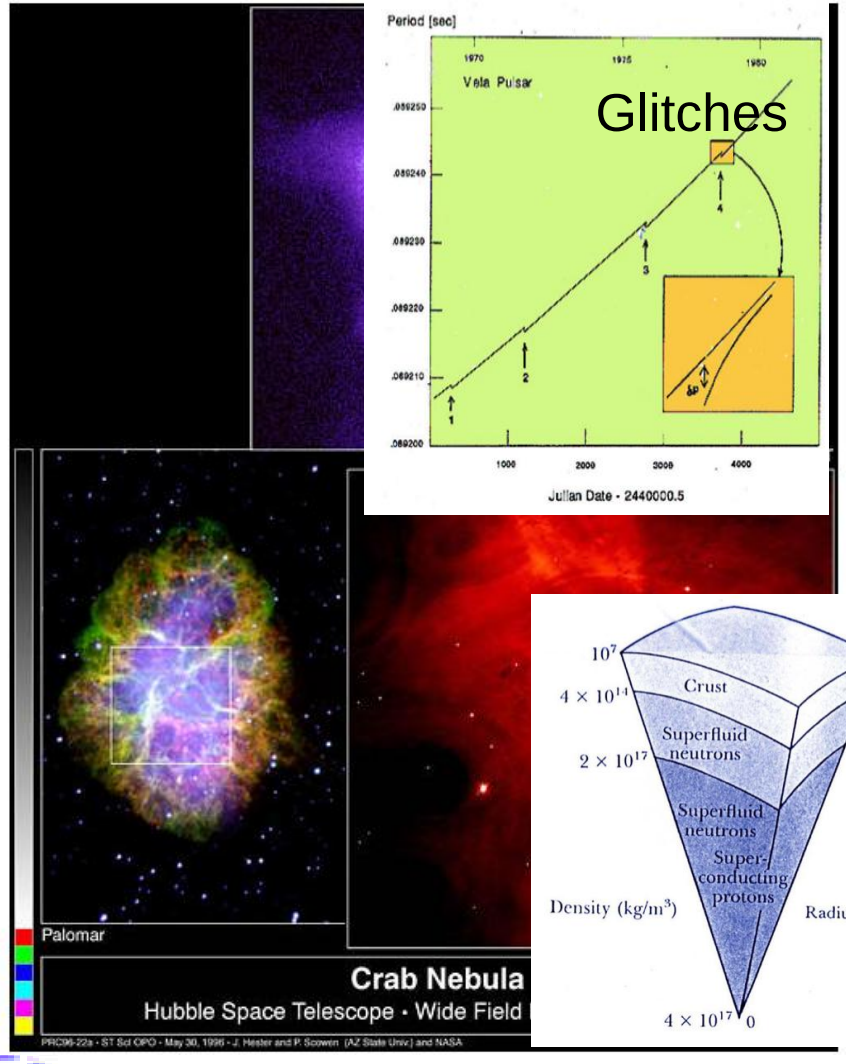
This 2nd order phase transition is characterized by a structural transition of the flux line lattice

Ultrasound and NMR results are consistent with the FFLO state which predicts a segmentation of the flux line lattice

FFLO State in Neutron Star

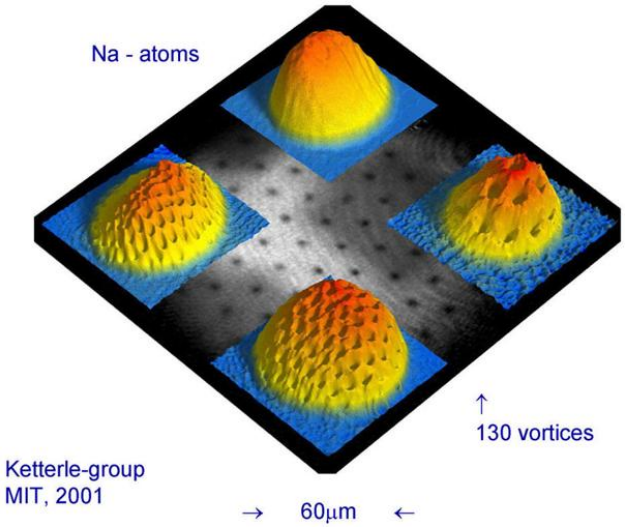
Color superconductivity

R.Casalbuoni and G.Nardulli
Rev. Mod. Phys. (2004)



Bose-Einstein-Condensate

Vortices



Supefluid ultracold Fermi gases with imbalanced state populations: one more candidate for FFLO state?

Massachusetts Institute of Technology:
M.W. Zwierlein, A. Schirotzek, C. H. Schunck, W.Ketterle (2006)

Rice University, Houston:
Guthrie B. Partridge, Wenhui Li, Ramsey I. Kamar, Yean-an Liao, Randall G. Hulet (2006)

3. Exactly solvable models of FFLO state.

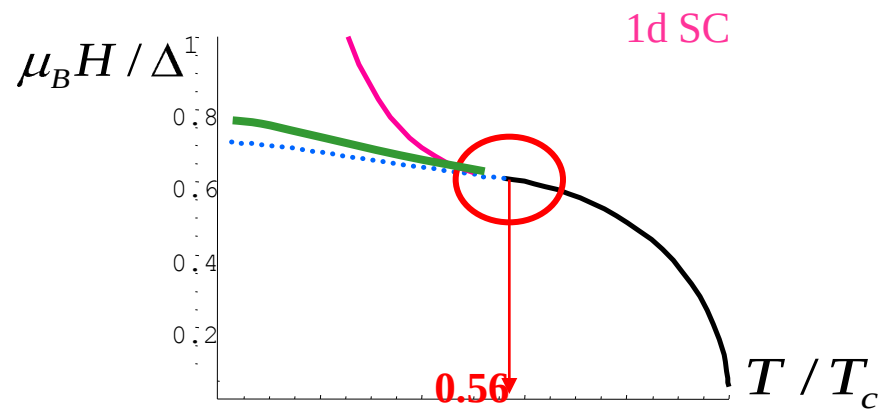
FFLO phase in the case of pure paramagnetic interaction and BCS limit

Exact solution for the 1D and quasi-1D superconductors ! (Buzdin , Tugushev 1983)

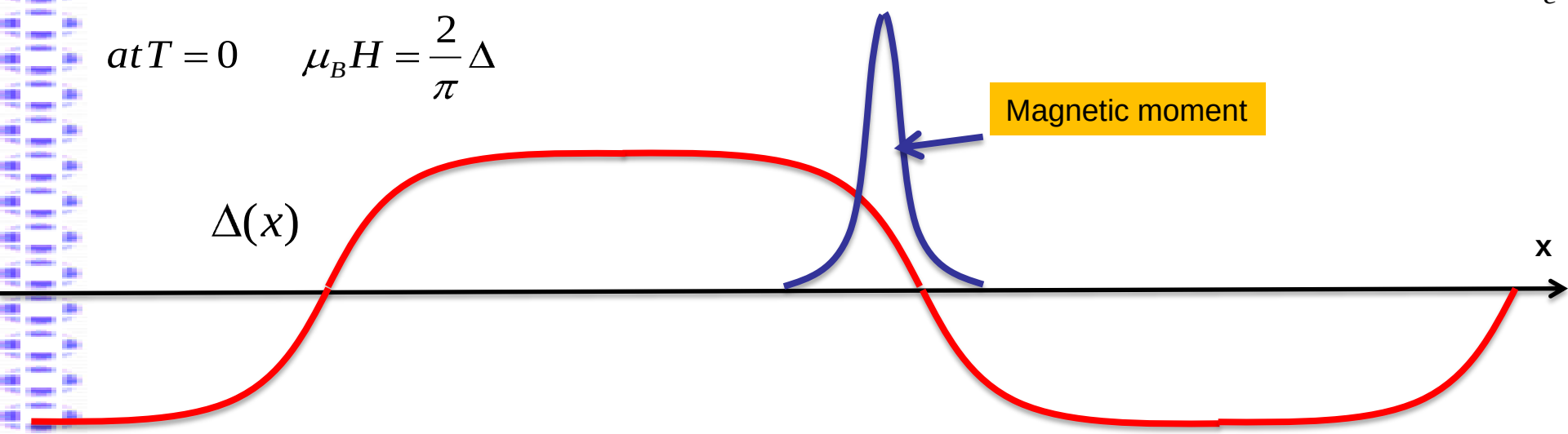
- The FFLO phase is the soliton lattice,

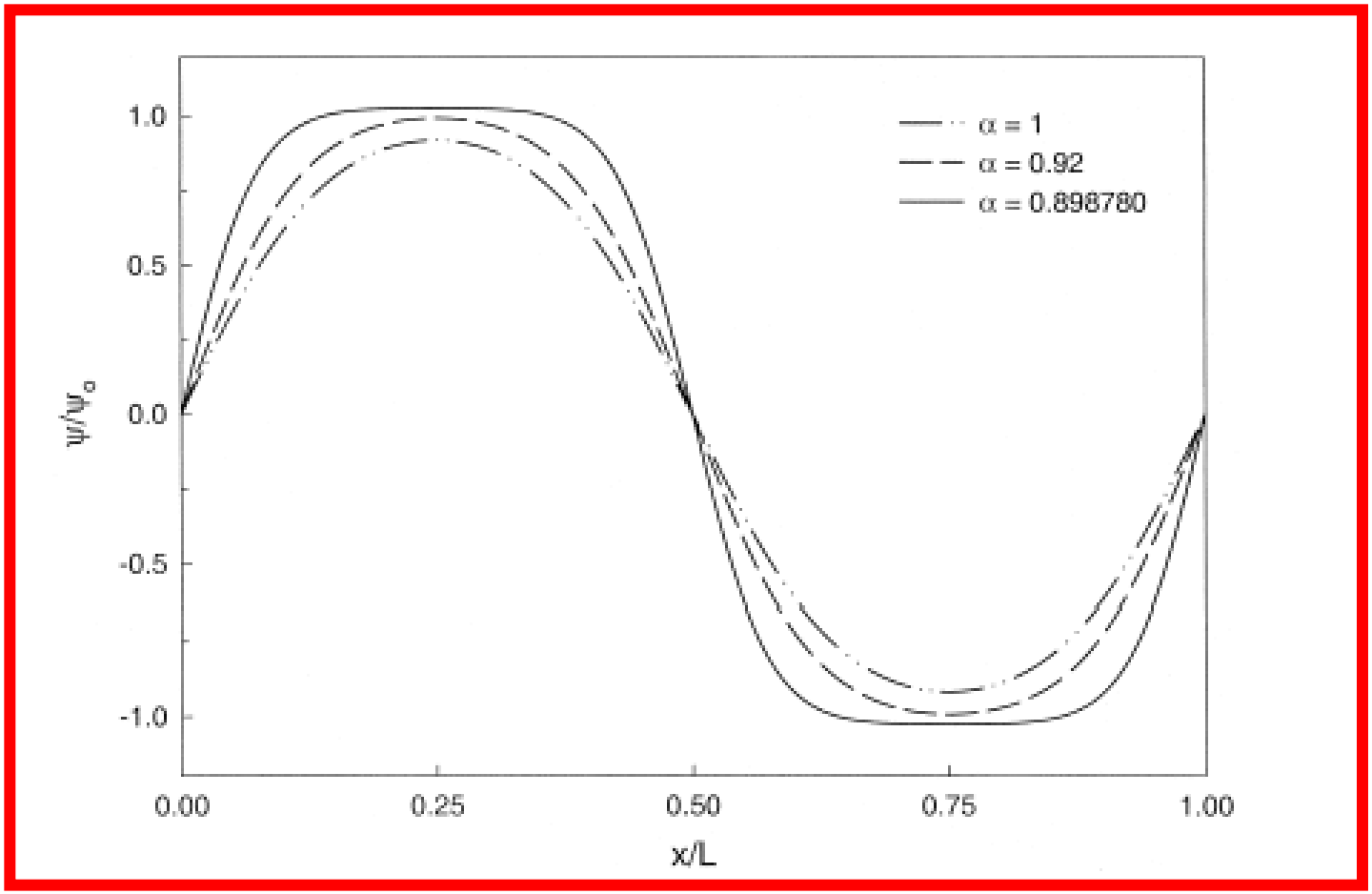
first proposed by **Brazovskii, Gordyunin and Kirova in 1980** for polyacetylene.

$$\Delta(x) = \Delta_0 sn(x/\xi, k)$$

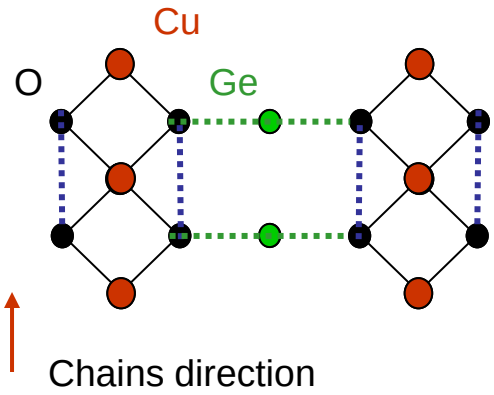


$$at\, T = 0 \qquad \mu_B H = \frac{2}{\pi}\Delta$$

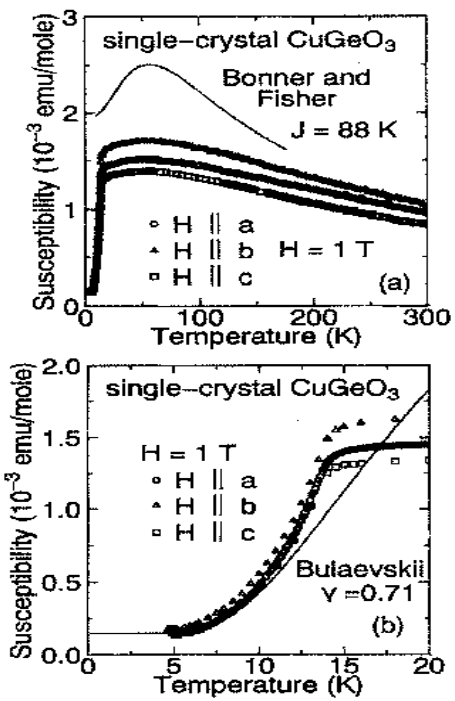


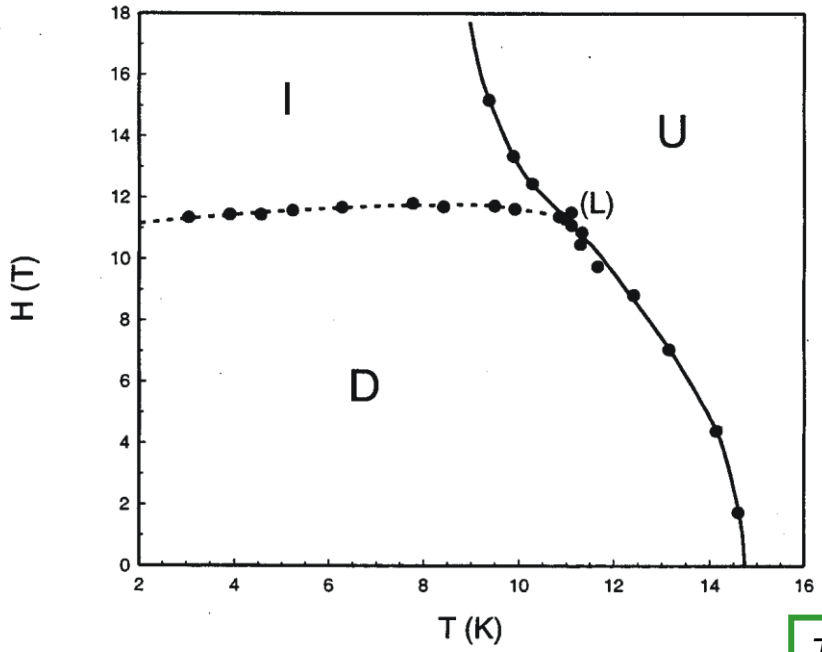

α = 1
α = 0.92
α = 0.898780
1.0
0.5
0.0
-0.5
-1.0
ψ/ψ₀
0.00
0.25
0.50
0.75
1.00
x/L

Spin-Peierls transitions - e.g. $CuGeO_3$



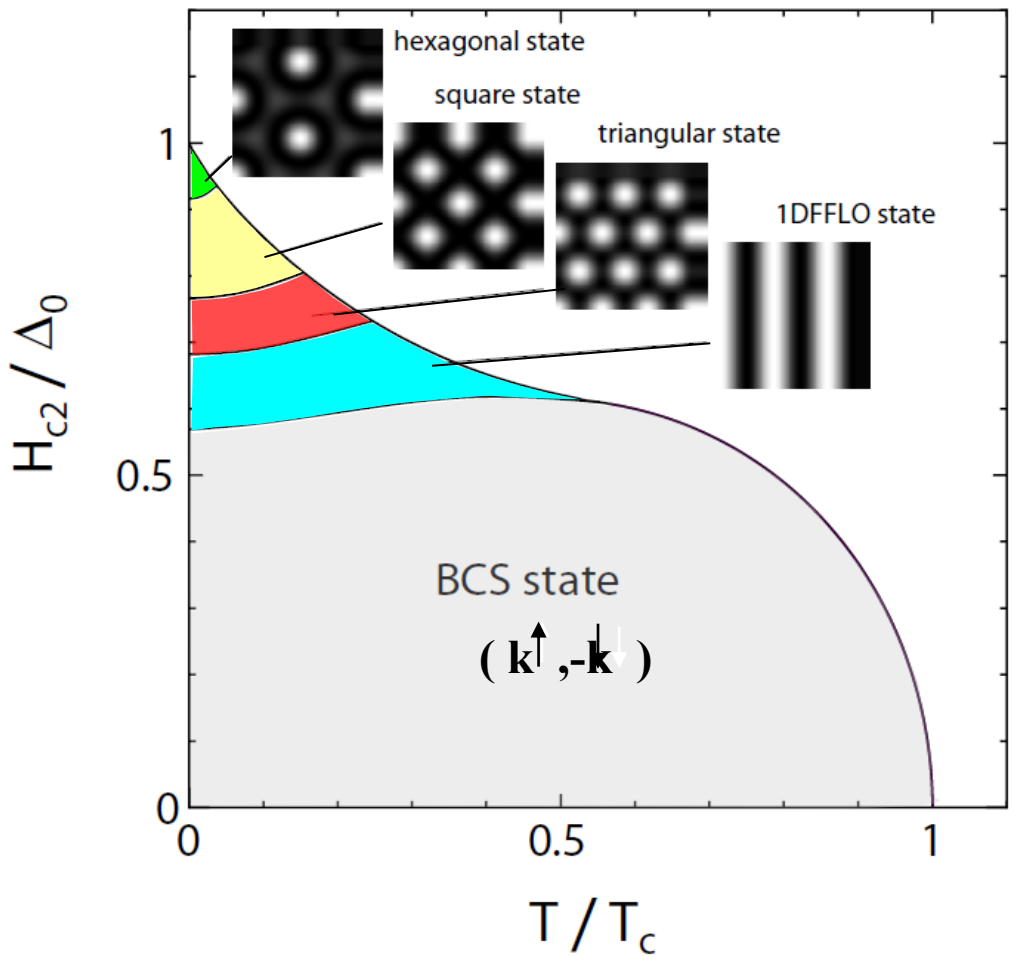
$$u_n = (-1)^n \Delta(x = na)$$





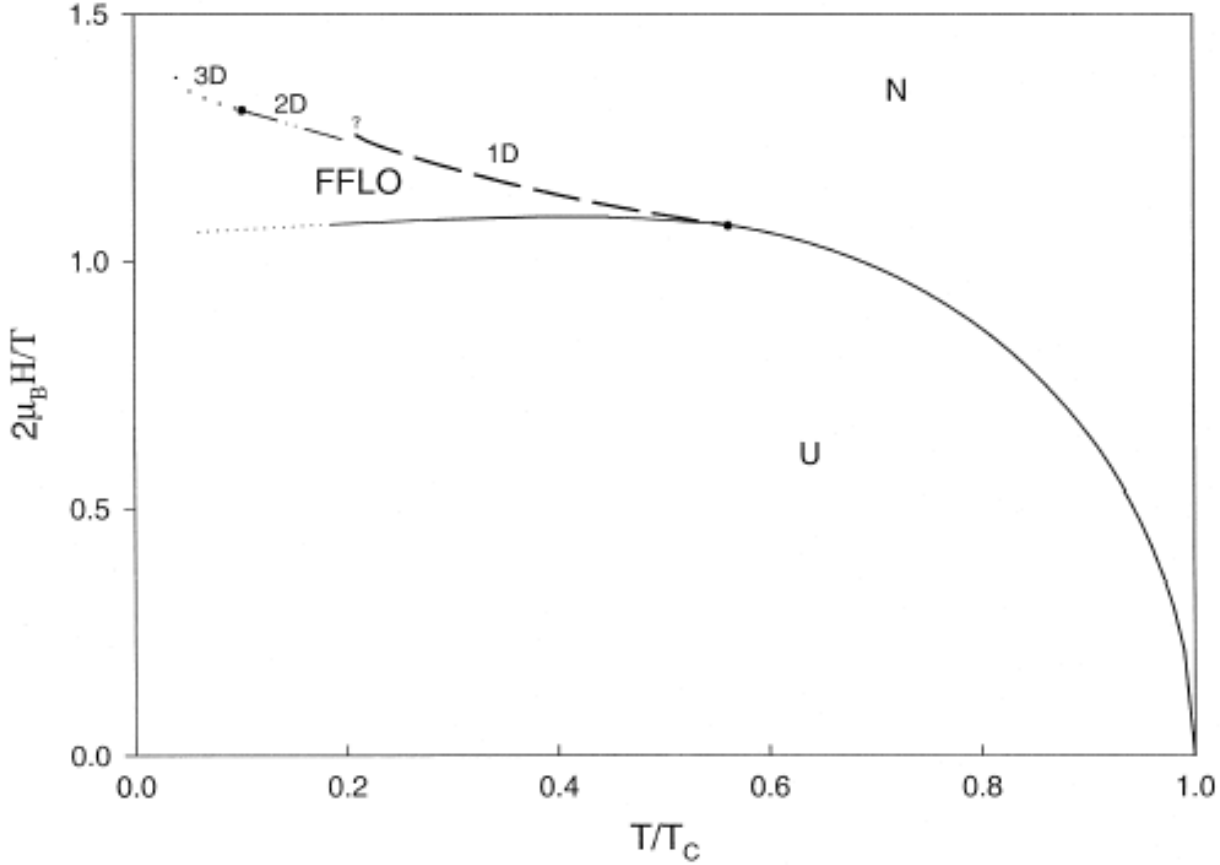
T_{SP}=14.2 K

In 2D superconductors



Y.Matsuda and H.Shimahara
J.Phys. Soc. Jpn (2007)

In 3D superconductors



The transition to the FFLO state is 1st order. The sequence of phases is similar to 2D case. **Houzet et al. 1999**; **Mora et al. 2002**

4. Vortices in FFLO state. Role of the crystal structure.

FFLO phase in the case of paramagnetic and orbital effect (3D BCS limit) – upper critical field

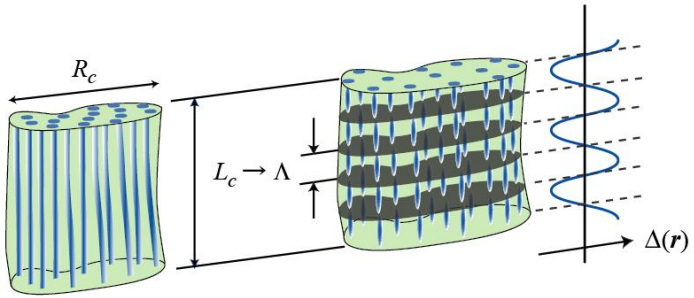
Note : The system with elliptic Fermi surface can be tranformed by scaling transformation to ihe isotropic one. Sure the direction of the magnetic field will be changed.

$$\Delta(\boldsymbol{r}) \sim \exp(\mathrm{i}Qz)\,\exp(-\rho^2 eH/2\hbar c)$$

Lowest m=0 Landau level solution, **Gruenberg and Gunter, 1966**

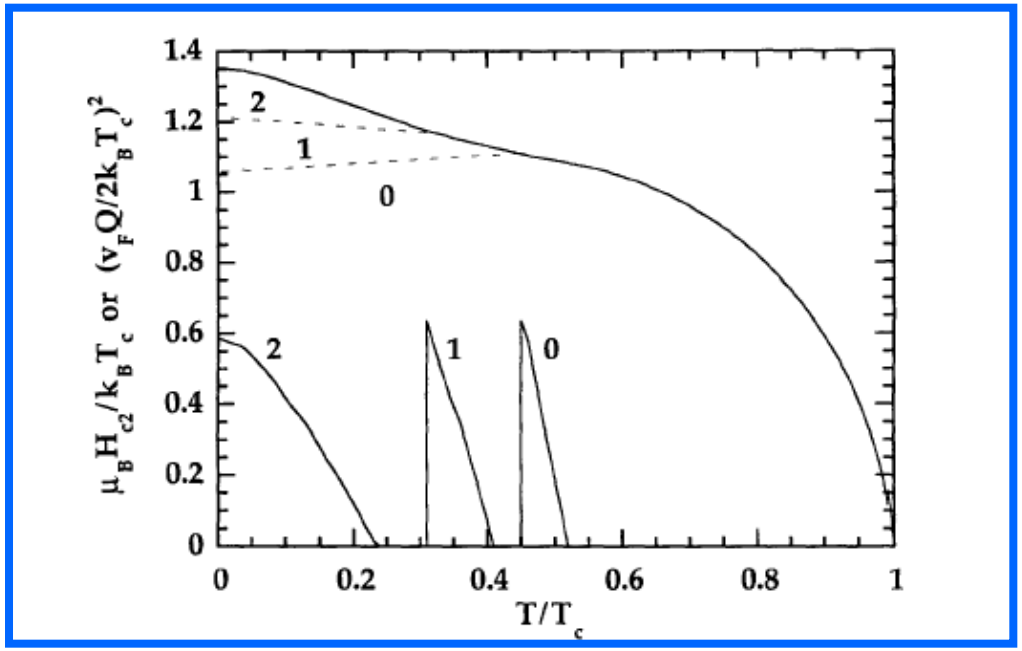
$$\alpha \equiv \sqrt{2}\,\frac{H_{c2}^{orb}}{H_{c2}^{P}}$$

FFLO exists for Maki parameter α>1.8.



For Maki parameter α>9 the highest Landau level solutions are realized – **Buzdin and Brison, 1996.**

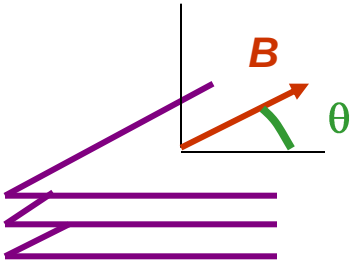
$$\Delta(\boldsymbol{r}) \sim \mathrm{e}^{-\mathrm{i}m\varphi}\rho^m\,\exp(\mathrm{i}Qz)\,\exp(-\rho^2 eH/2\hbar c),$$

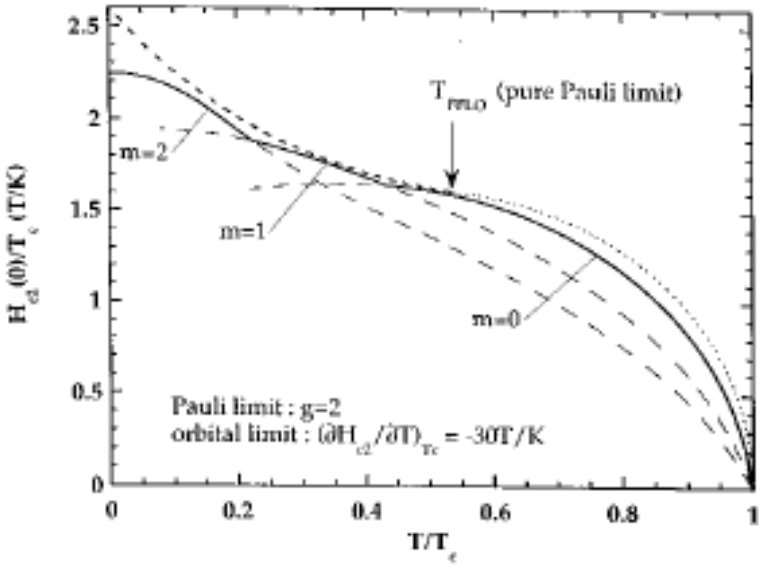


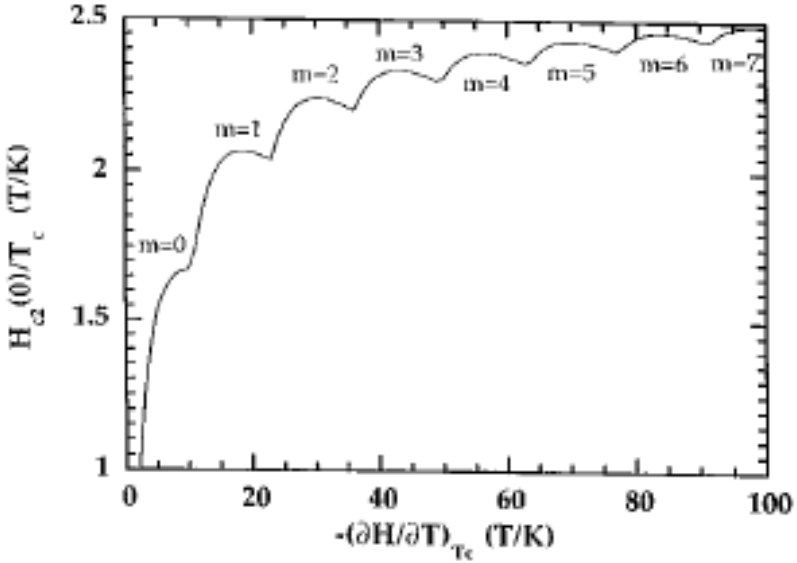
FFLO phase in 2D superconductors in the tilted magnetic field - upper critical field

Highest Landau level solutions are realized –
Bulaevskii, 1974; **Buzdin and Brison, 1996; Houzet and Buzdin, 2000.**

$$\Delta(r) \sim e^{-im\varphi}\rho^m \exp(iQz)\exp(-\rho^2 eH/2\hbar c),$$







Exotic vortex lattice structures in tilted magnetic field

Generalized Ginzburg-Landau functional

Near the tricritical point, the characteristic length is $q(T)^{-1} \underset{T \to T^\star}{\longrightarrow} \infty$

Microscopic derivation of the Ginzburg-Landau functional :

$$\frac{\mathcal{F}}{N(0)T_c^2} = 0.86\,\frac{B - H^{\text{eff}}(T)}{H^{\text{eff},\star}}|\Delta|^2 + 3.0\,\frac{T - T^*}{T^*}\xi_0^2|\nabla\Delta|^2 + 0.15\,\frac{T - T^*}{T^*}|\Delta|^4$$

Instability toward FFLO state

Instability toward 1st order transition

Next orders are important :

$$\frac{\mathcal{F}}{N(0)T_c^2} = 0.86\,\frac{B - H^{\text{eff}}(T)}{H^{\text{eff},\star}}|\Delta|^2 + 3.0\,\frac{T - T^*}{T^*}\xi_0^2|\tilde{\nabla}\Delta|^2 + 3.1\,\xi_0^4|\tilde{\nabla}^2\Delta|^2 + 0.15\,\frac{T - T^*}{T^*}|\Delta|^4$$
$$+0.85\,\xi_0^2\left\{|\Delta|^2|\tilde{\nabla}\Delta|^2 + \frac{1}{8}\left[(\Delta^*\tilde{\nabla}\Delta)^2 + (\Delta\tilde{\nabla}\Delta^*)^2\right]\right\} + 0.011\,|\Delta|^6$$

Validity:

- large scale for spatial variation of Δ : vicinity of T^*

  small orbital effect, introduced with $\tilde{\nabla} = \nabla - \frac{2ie}{\hbar c}\mathbf{A}$

- we neglect diamagnetic screening currents (high-κ limit)

• 2nd order phase transition at

$$0.86\,\frac{B-H_{eff}(T)}{H^*_{eff}}\Delta - 3.0\,\frac{T-T^*}{T^*}\xi_0^2\tilde{\nabla}^2\Delta + 3.1\,\xi_0^4\tilde{\nabla}^4\Delta = 0$$

→ higher Landau levels $\tilde{\nabla}^2\Delta_N = -\frac{4eH_\perp}{\hbar c}(N+\frac{1}{2})\Delta_N$

• Near the transition: minimization of the free energy with solutions in the form

$$\boxed{\Psi_{\zeta=\rho+i\sigma}(x,y) = \frac{(2\sigma)^{\frac{1}{4}}}{(2^N N!)^{\frac{1}{2}}} e^{-\frac{\pi y^2 B_\perp}{\phi_0}} \sum_p \mathcal{H}_N\left(y\sqrt{\frac{2\pi B_\perp}{\phi_0}} + p\sqrt{2\pi\sigma}\right) e^{2i\pi p(x+iy)\sqrt{\frac{\sigma B_\perp}{\phi_0}} + i\pi p\zeta^2}}$$

gauge $\mathbf{A} = (0, -yB_\perp, 0)$

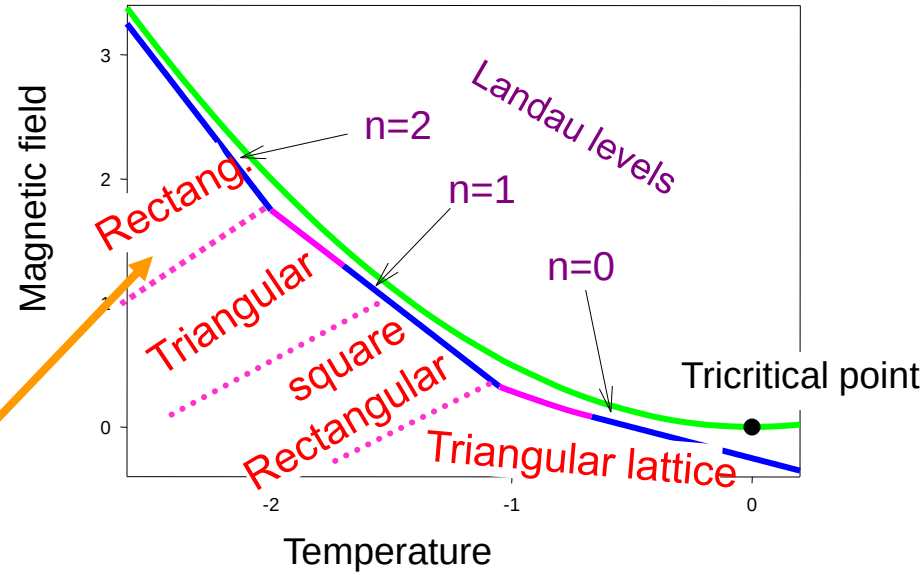
ζ Parametrizes all vortex lattice structures at a given Landau level N

$$(\mathbf{r}_1, \mathbf{r}_2) = \left(\sqrt{\frac{\phi_0}{\sigma B_\perp}}, \zeta\sqrt{\frac{\phi_0}{\sigma B_\perp}}\right)$$ is the unit cell

All of them are decribed in the subset : $\left[|\zeta| > 1; 0 < \rho < \frac{1}{2}\right]$

Analysis of phase diagram :

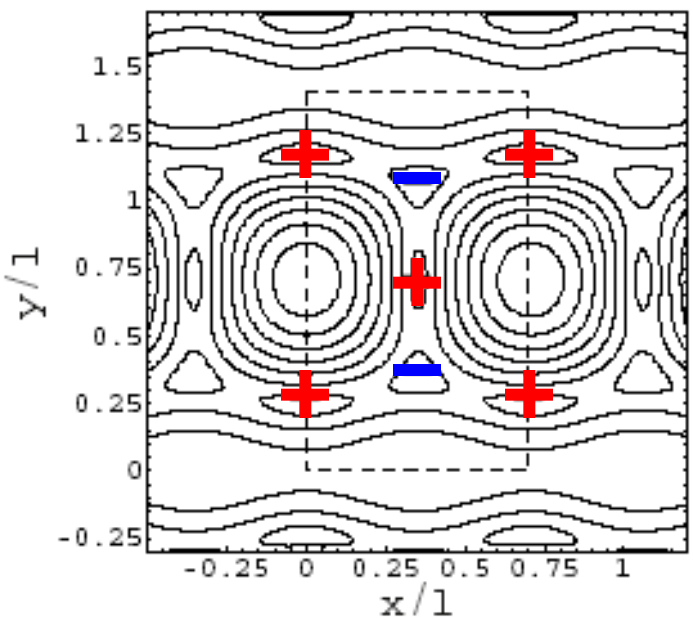
- cascade of 2nd and 1st order transitions between S and N phases
- 1st order transitions within the S phase
- exotic vortex lattice structures



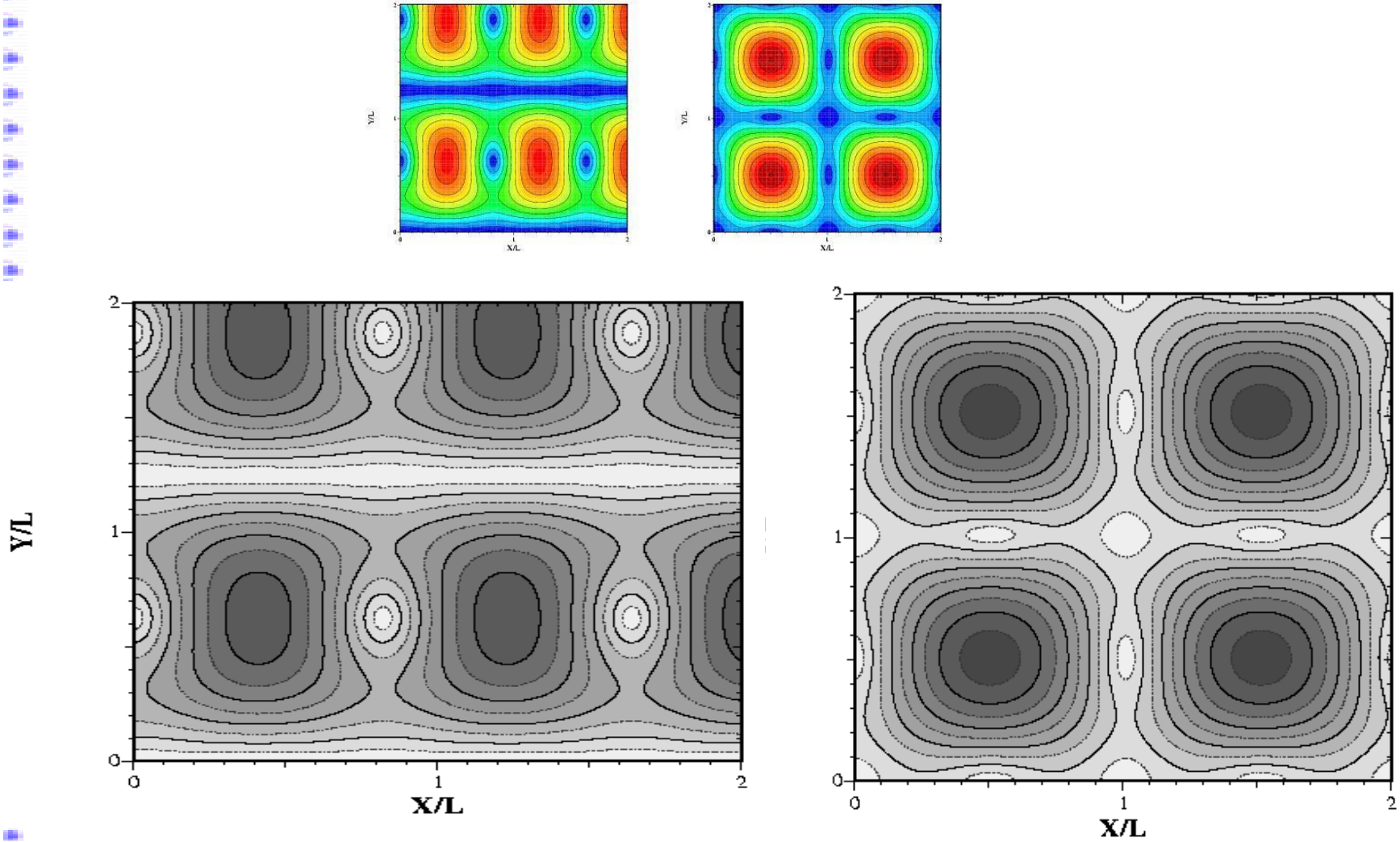
1st order transition

2nd order transition

2nd order transition in the paramagnetic limit



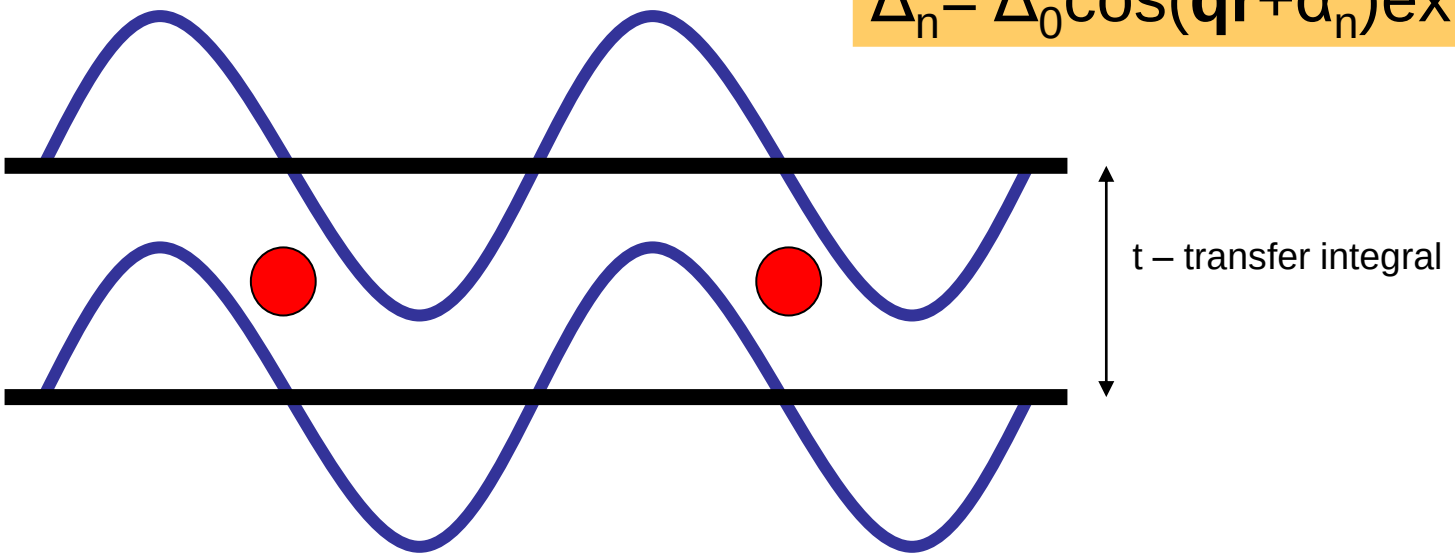
At Landau levels n > 0, we find structures with several points of vanishment of the order parameter in the unit cell and with different winding numbers w = ± 1, ± 2 …



Order parameter distribution for the asymmetric and square lattices with Landau level n=1.
The dark zones correspond to the maximum of the order parameter and the white zones to its minimum.

Intrinsic vortex pinning in LOFF phase for parallel field orientation

$$\Delta_n = \Delta_0 \cos(\mathbf{qr} + \alpha_n) \exp(i\varphi_n(r))$$



Josephson coupling between layers is modulated

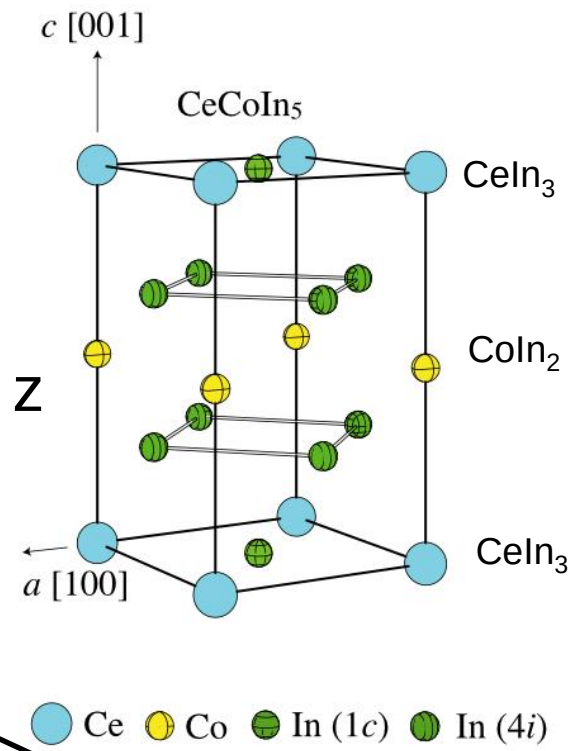
$$F_{n,n+1} = [-I_0 \cos(\alpha_n - \alpha_{n+1}) + I_2 \cos(\mathbf{qr}) \cos(\alpha_n + \alpha_{n+1})] \cos(\varphi_n - \varphi_{n+1})$$

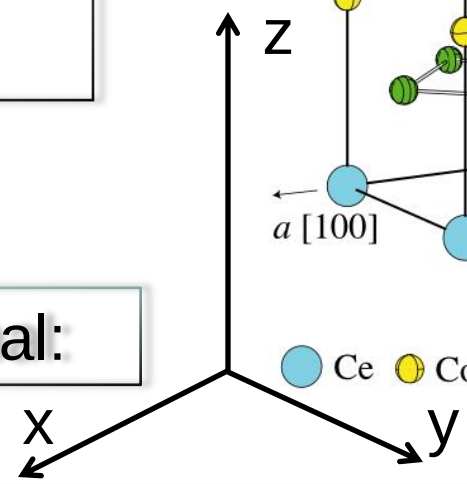
$\varphi_n - \varphi_{n+1} = 2\pi x H s / \Phi_0 + \pi n$

s-interlayer distance, x-coordinate along **q**

$CeCoIn_5$

- Quasi-2D heavy fermion (T_c=2.3K)
- Strong antiferromagnetic fluctuation
- *d*-wave symmetry
- Tetragonal symmetry



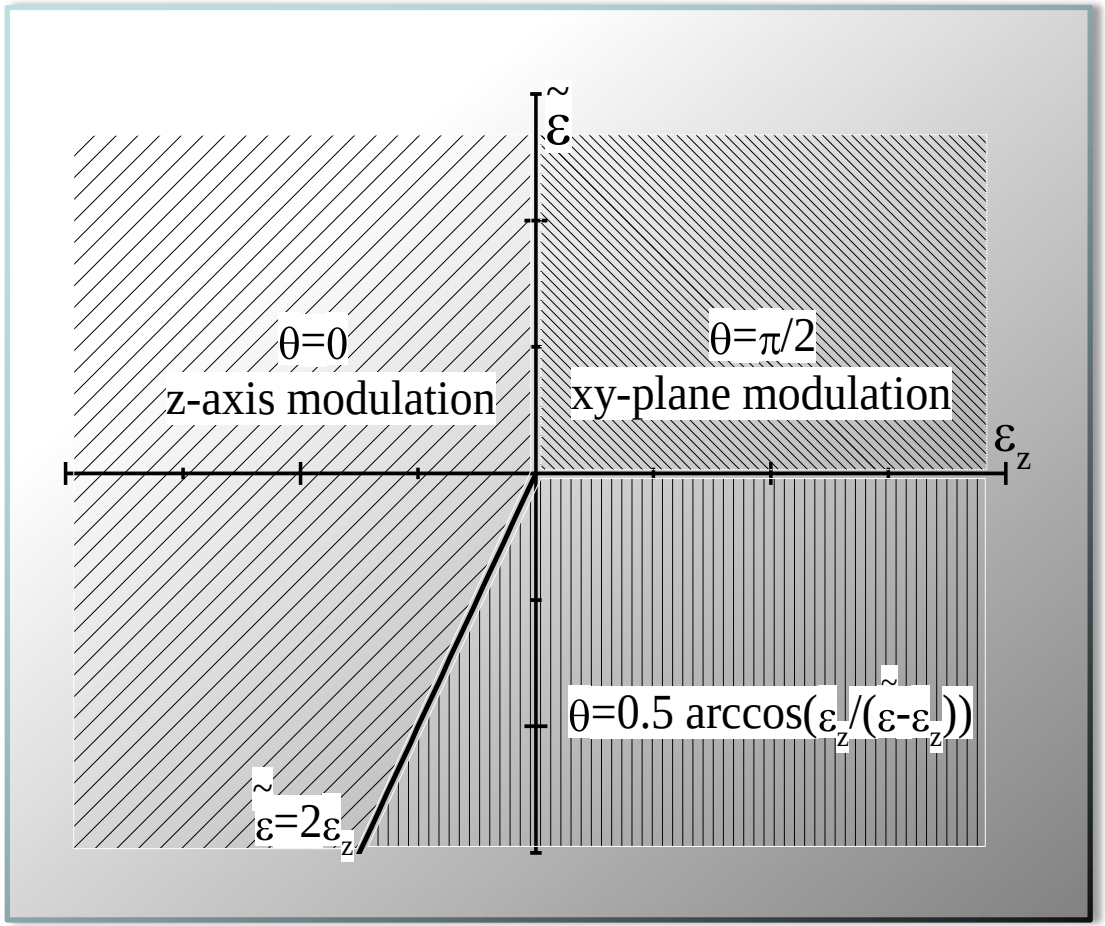


Modified Ginzburg-Landau functional:

$$\mathcal{F} = \underbrace{\alpha\,|\Psi|^2 - g\sum_{i=1}^{3}|\Pi_i\Psi|^2 + \gamma\left|\sum_{i=1}^{3}\Pi_i^2\Psi\right|^2}_{\text{isotropic part}} + \varepsilon_z\left|\Pi_z^2\Psi\right|^2 + \frac{\varepsilon_x}{2}\left(|\Pi_x\Pi_y\Psi|^2 + |\Pi_y\Pi_x\Psi|^2\right)$$
$$+\frac{\widetilde{\varepsilon}}{2}\left(|\Pi_z\Pi_x\Psi|^2 + |\Pi_x\Pi_z\Psi|^2 + |\Pi_z\Pi_y\Psi|^2 + |\Pi_y\Pi_z\Psi|^2\right).$$

$$\Pi_j = \nabla_j - \frac{2e}{\hbar c} iA_j$$

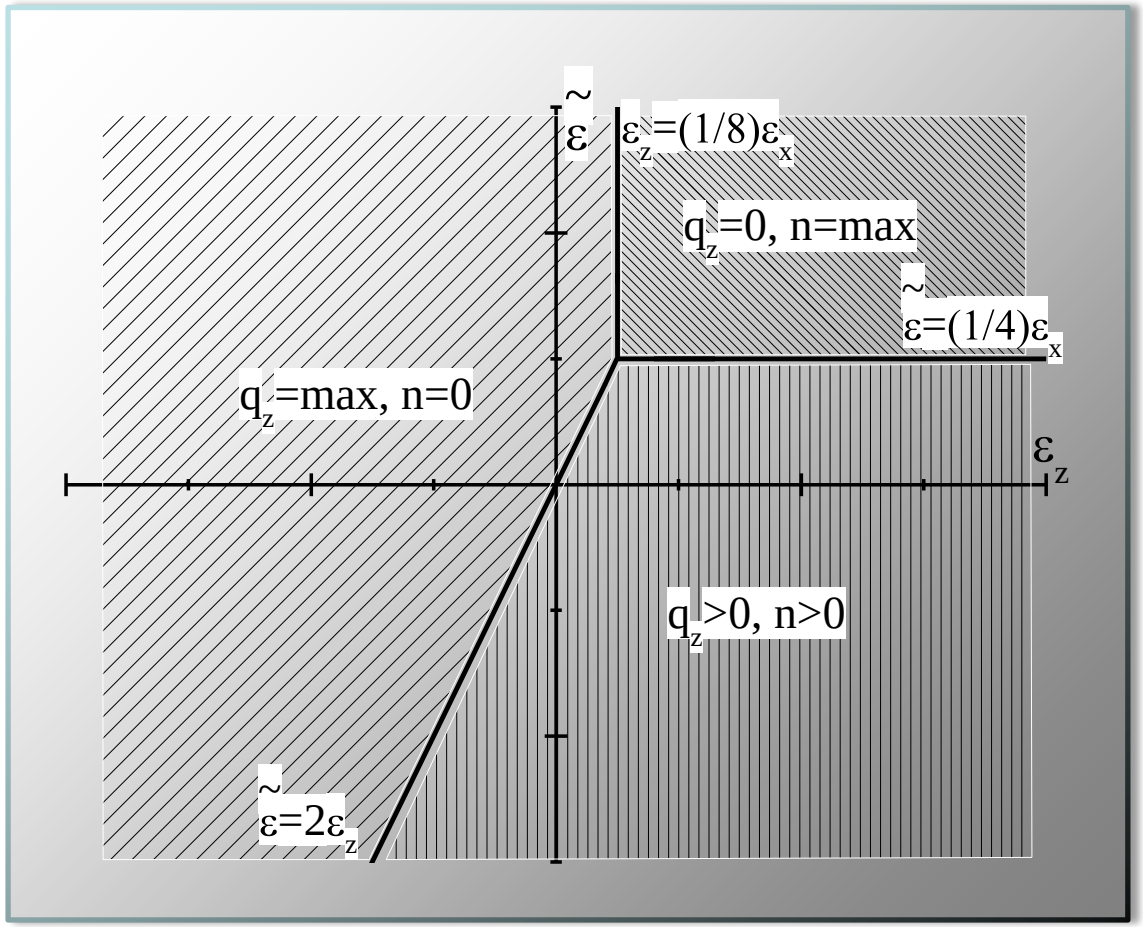
No orbital effect

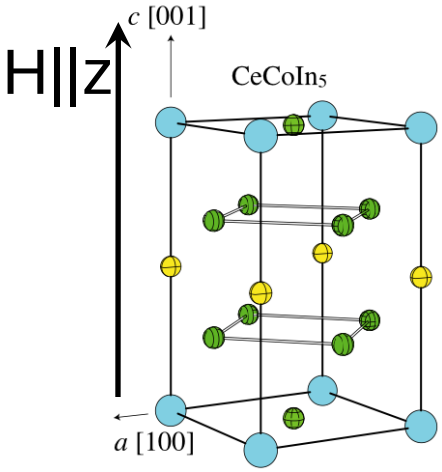


Modulation $(\tilde{\varepsilon}, \varepsilon_z)$ diagram in the case of the absence of the orbital effect (pure paramagnetic limit).

Areas with different patterns correspond to different orientation of the wave-vector modulation. The phase diagram does not depend on the ε_x value.

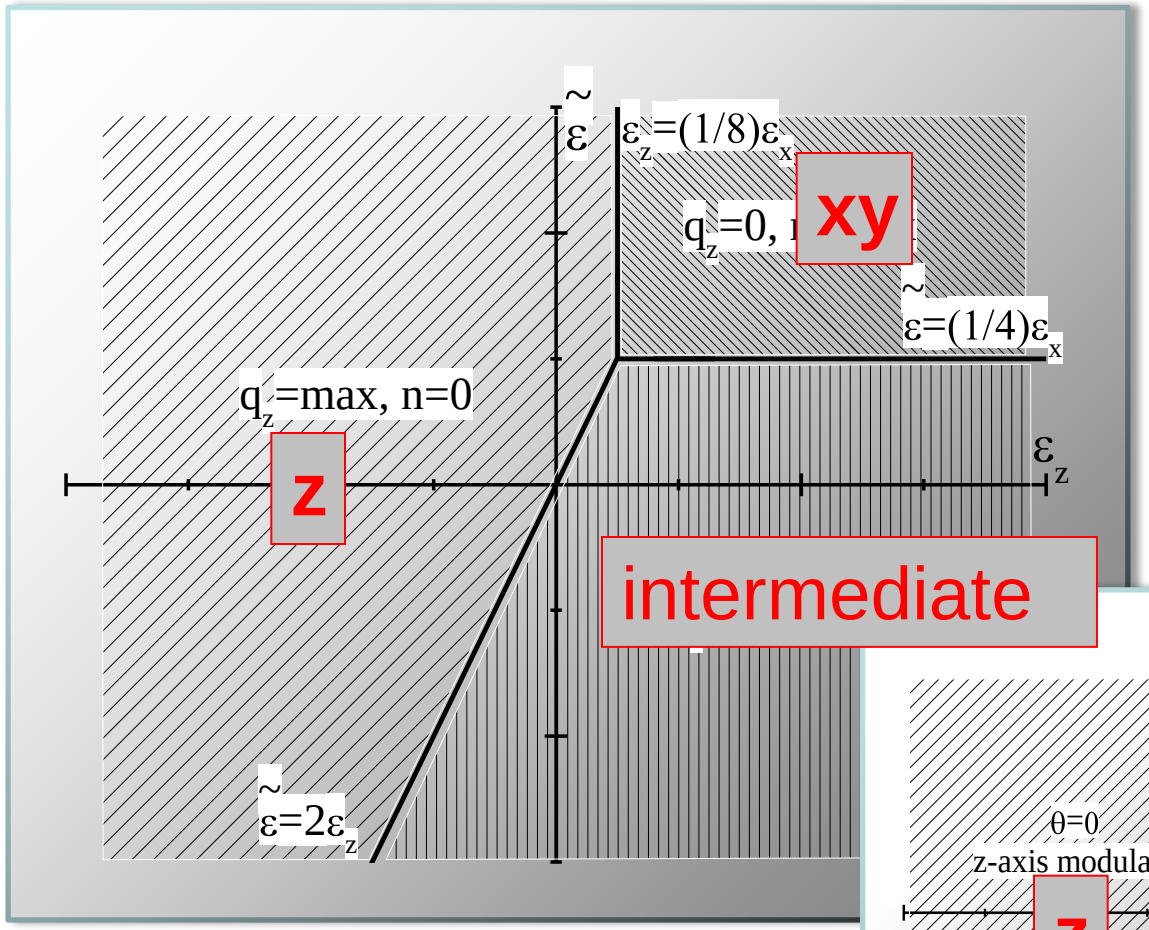
Magnetic field along z axis

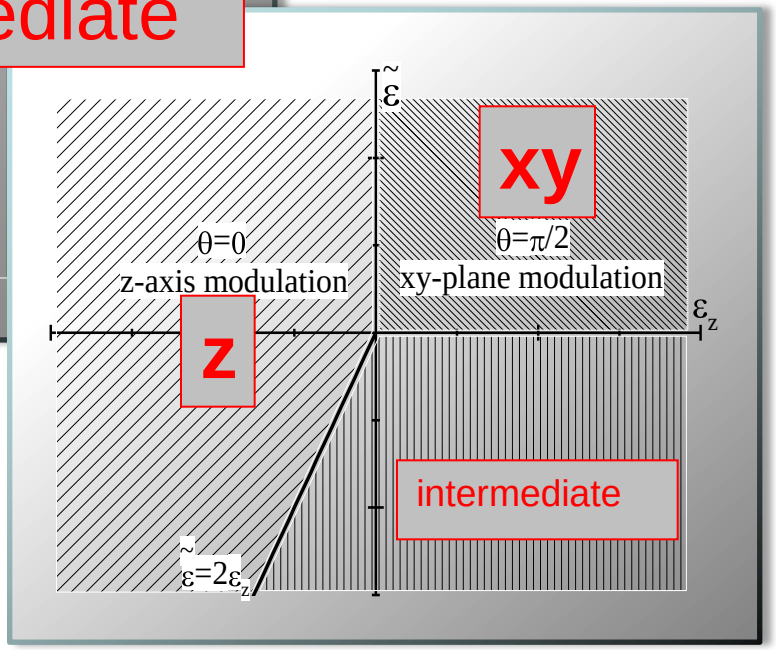




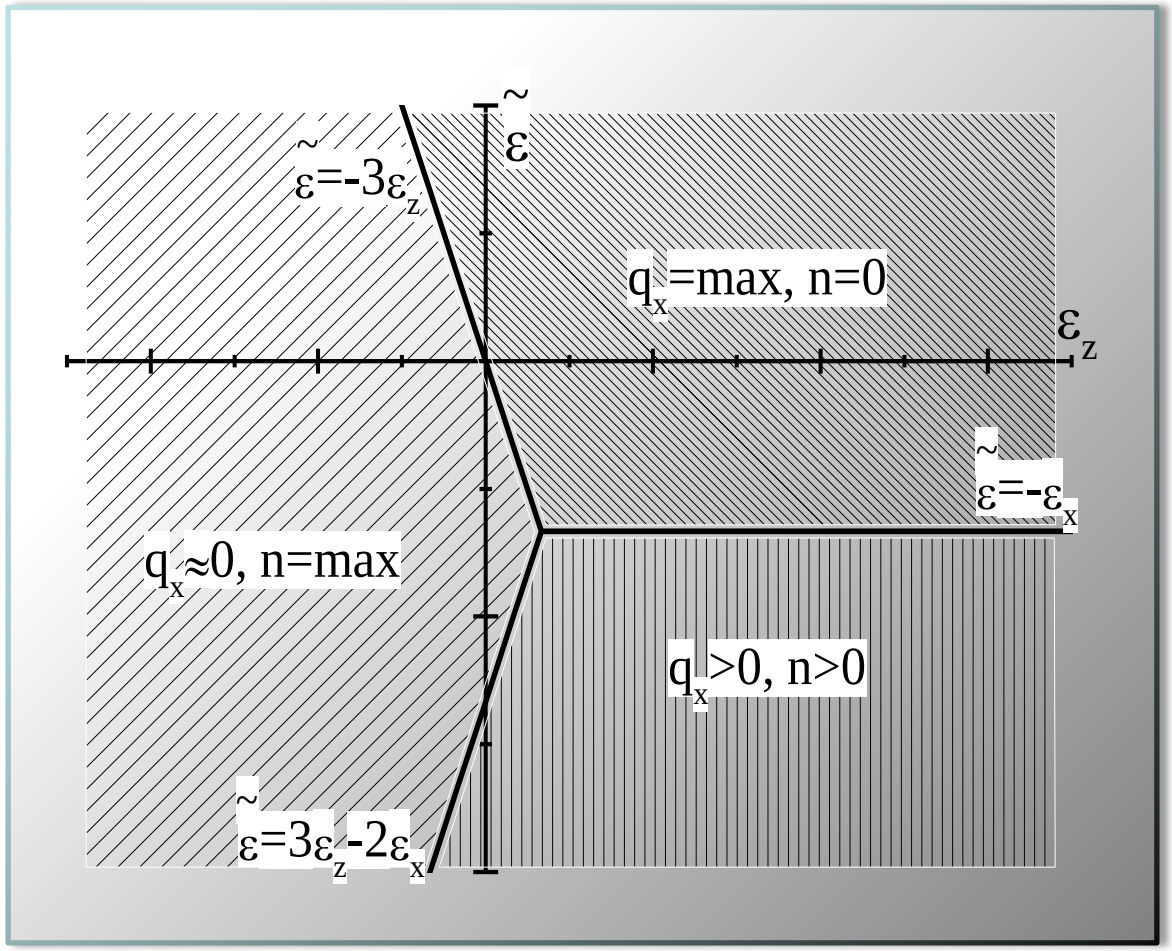
Modulation diagram in the case when the magnetic field applied along z axis. There are 3 areas on the diagram corresponding to 3 types of the solution for modulation vector q_z and Landau level n. Modulation direction is always parallel to the applied field and ε_x here is treated as a constant.

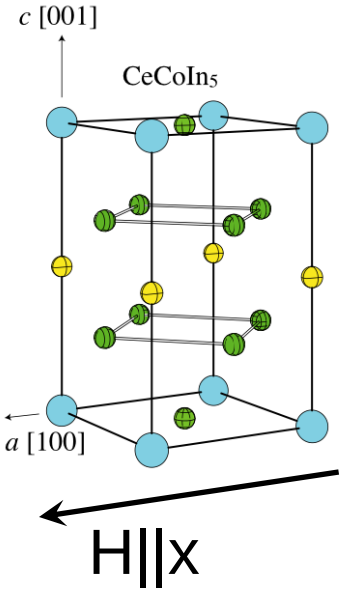
Magnetic field along z axis





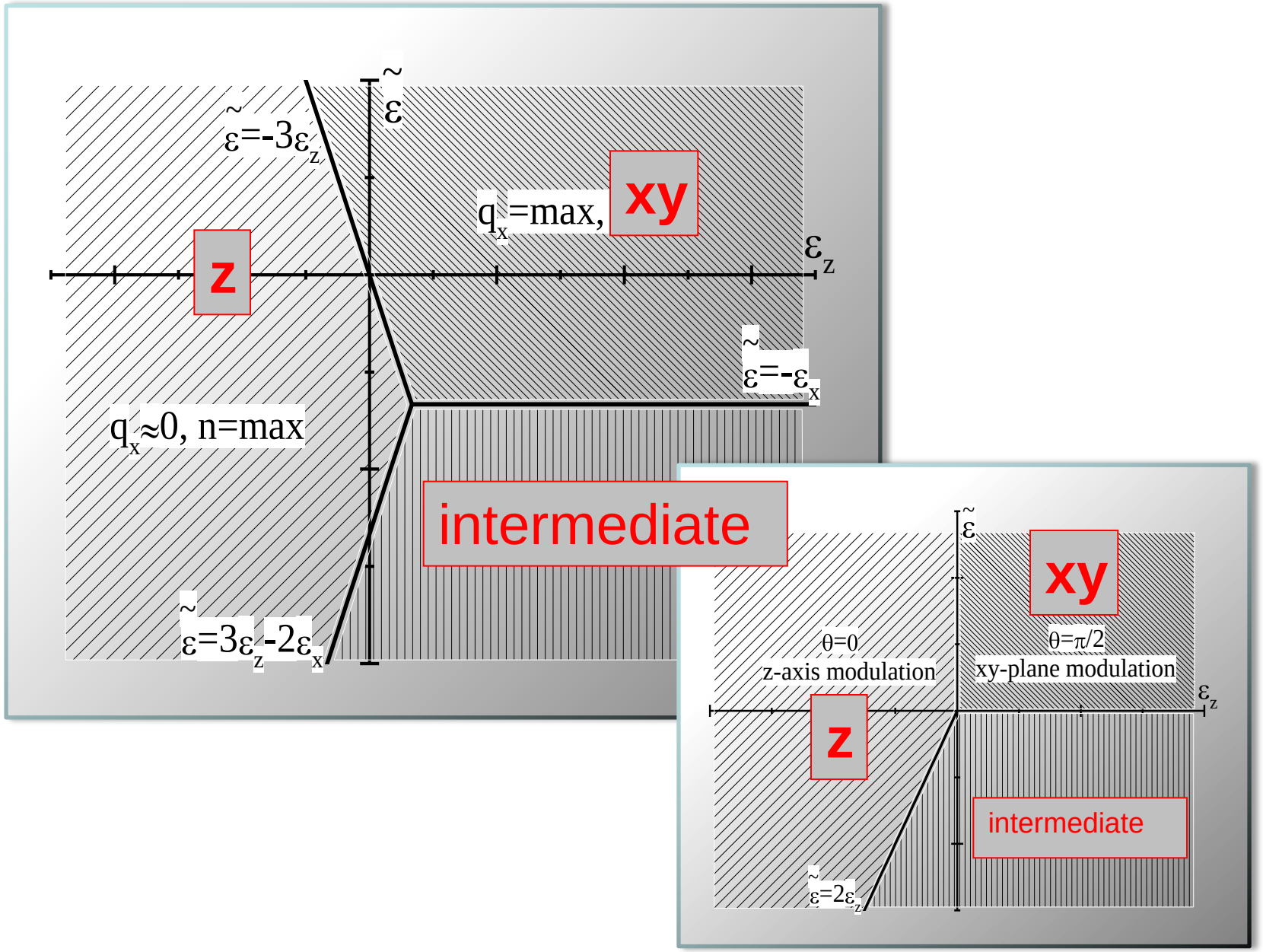
Magnetic field along x axis





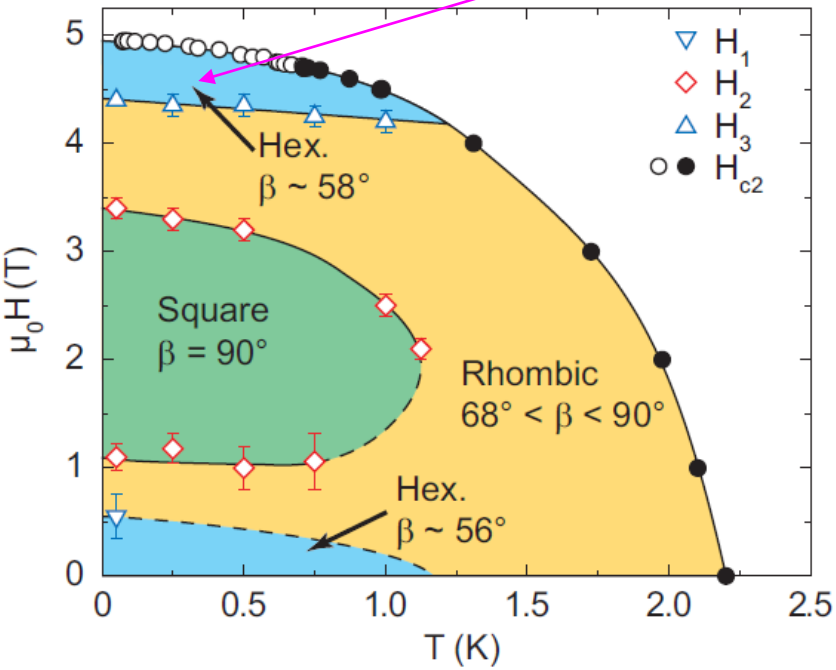
Modulation diagram ($\tilde{\varepsilon}$, ε_z) in the case when the magnetic field is applied along x axis. There are three areas on the diagram corresponding to different types of the solution for modulation vector q_x and Landau level n. Modulation direction is always parallel to the applied field. The choice of the intersection point is determined by the coefficient ε_x.

Magnetic field along x axis

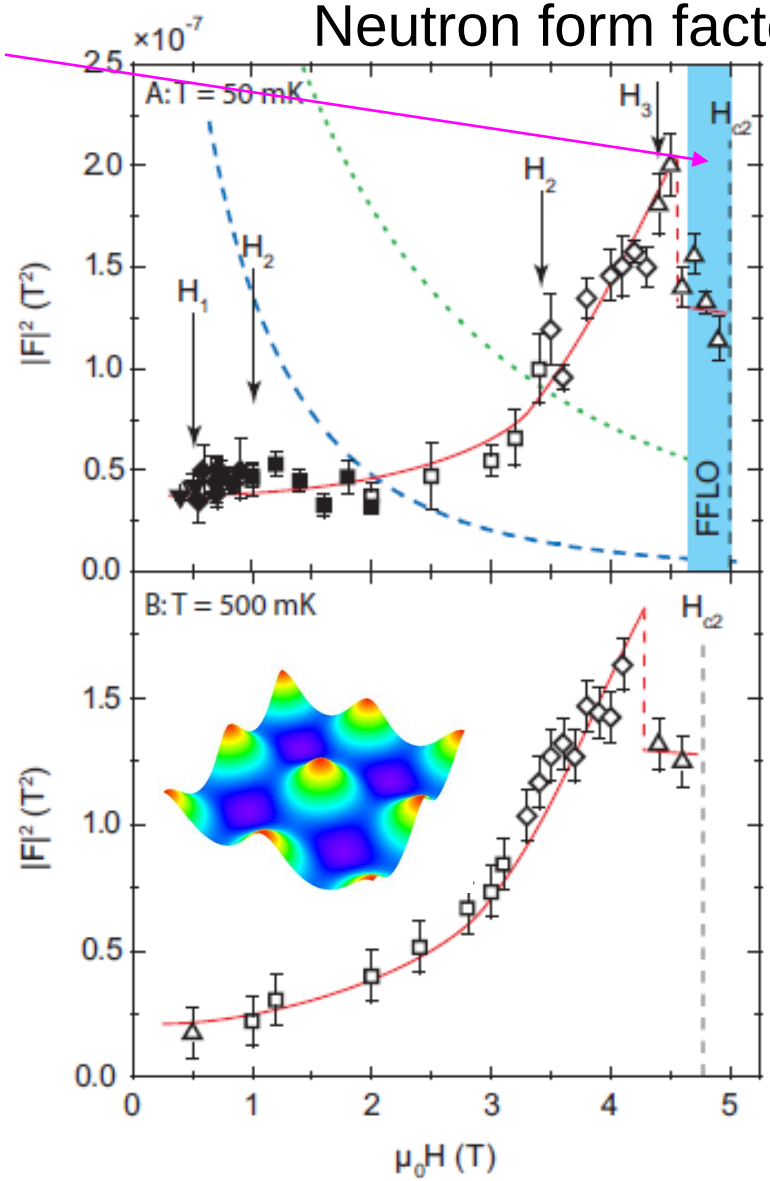


Small angle neutron scattering from the vortex lattice for *H //c*

FFLO?



Neutron form factor



Neutron form factor seems to be consisitent with FFLO state

A.D.Bianch *et al*.Science (2007)

- The crystal structure effects influence on the FFLO state is very important.
- The FFLO states with higher Landau level solutions could naturally exist in real 3D compounds (without any restrictions to the value of Maki parameter).
- Wave vector of FFLO modulation along the magnetic field could be zero.
- In the presence of the orbital effect the system tries in some way to reproduce optimal directions of the FFLO modulation by varying the Landau level index n and wave-vector of the modulation along the field.

5. Supefluid ultracold Fermi gases with imbalanced state populations: one more candidate for FFLO state?

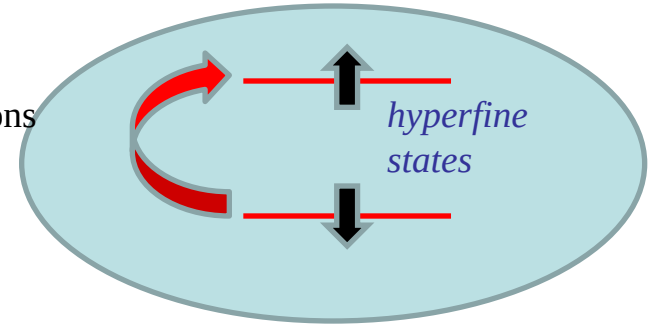
Massachusetts Institute of Technology:
M.W. Zwierlein, A. Schirotzek, C. H. Schunck, W.Ketterle (2006)

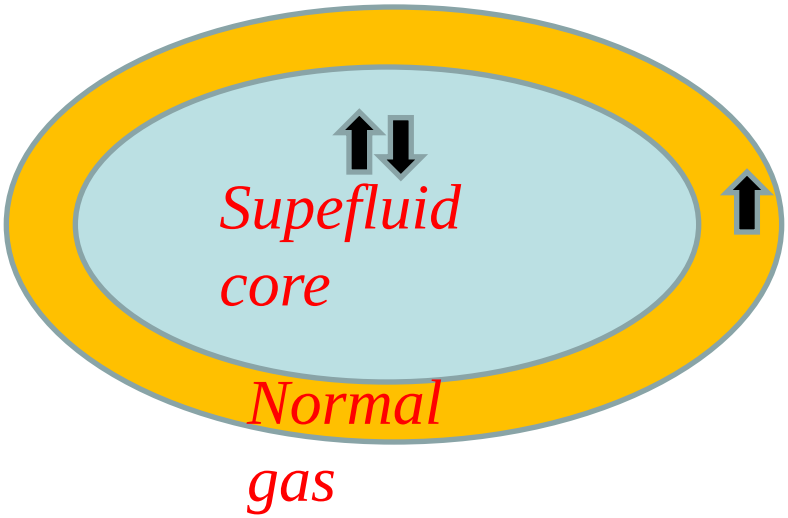
Rice University, Houston:
Guthrie B. Partridge, Wenhui Li, Ramsey I. Kamar, Yean-an Liao, Randall G. Hulet (2006)

Experimental system: Fermionic ^{6}Li atoms cooled in magnetic and optical traps (mixture of the two lowest hyperfine states with different populations)

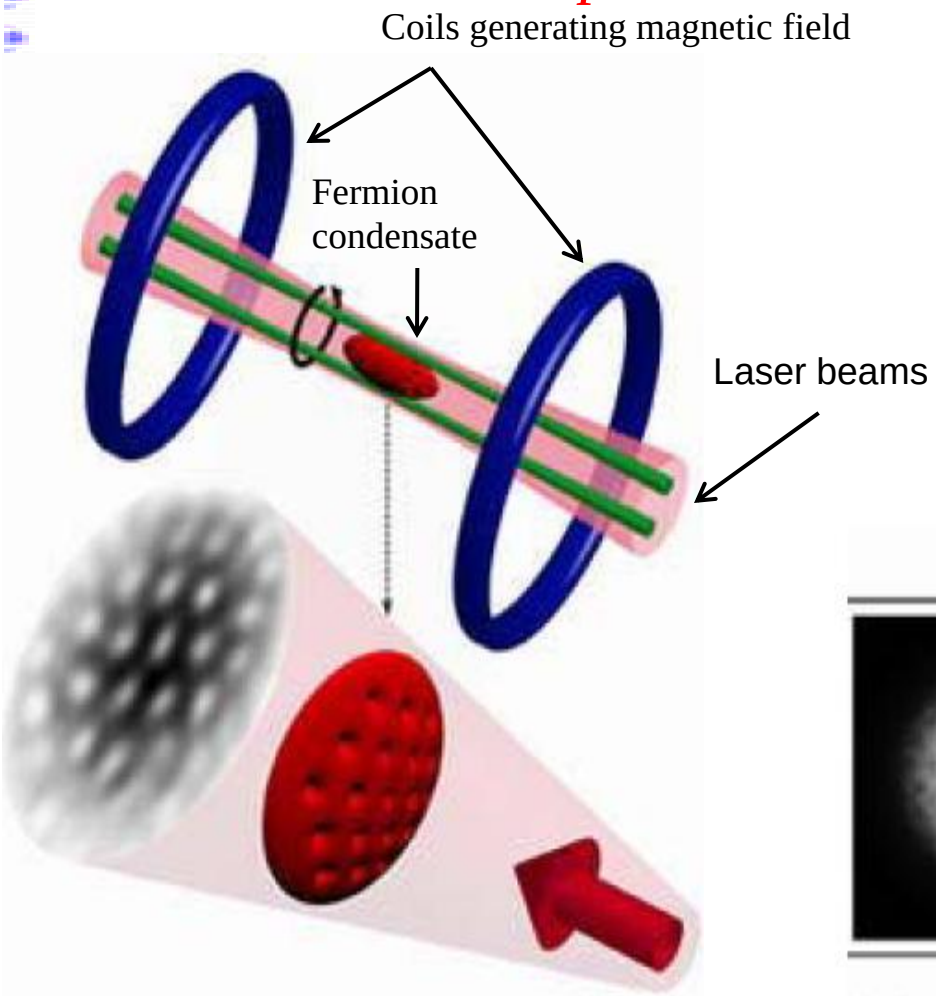
Experimental result: phase separation

rf transitions
76 MHz



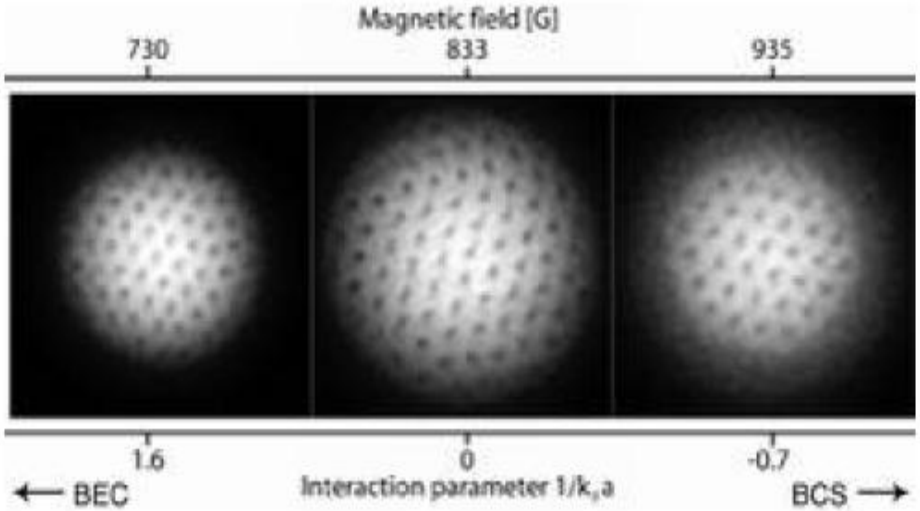


Rotating supefluid ultracold Fermi gases in a trap



Vortex as a test for superfluidity

Images of vortex lattices



MIT: Ketterle et al (2005)

Questions:

1. *What is the effect of confinement (finite system size) on FFLO states?*

2. *Effect of rotation on FFLO states in a trap (effect of magnetic field on FFLO state in a small superconducting sample).*

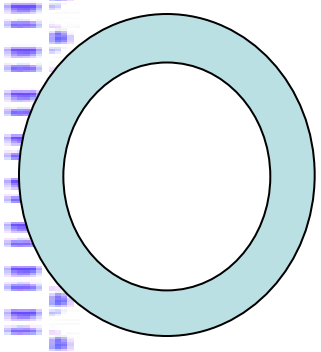
3. *Possible quantum oscillation effects.*

Examples of quantum oscillation effects.

Little-Parks effect. Switching between the vortex states.

Multiply-connected systems

Superconducting thin-wall cylinder

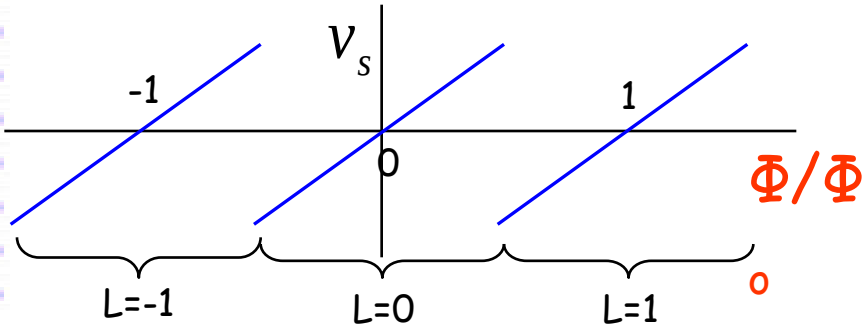


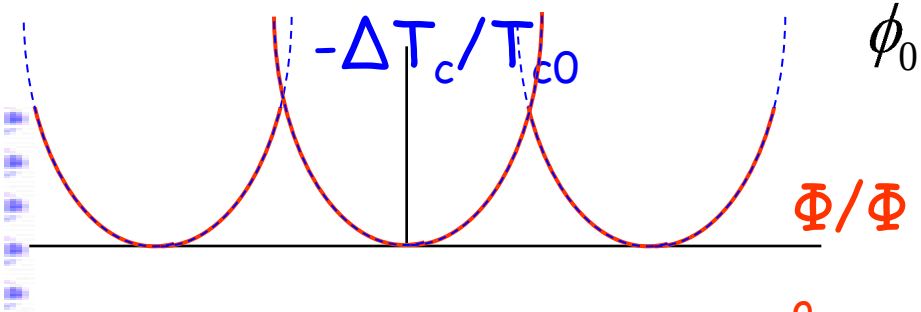
$$v_s = \frac{\hbar}{mR}\left(L - \frac{\Phi}{\Phi_0}\right)$$

$$\frac{\Delta T_c(H)}{T_{c0}} \sim \left(\frac{m v_s}{\hbar}\right)^2$$

T_c (H) oscillations

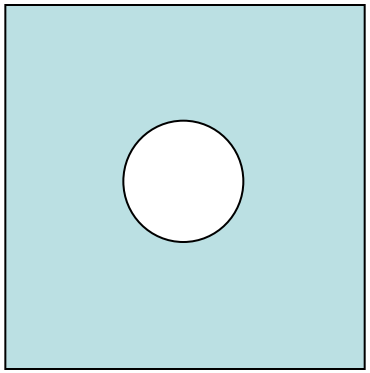
Multiquantum vortices





$$\phi_0 = \frac{\pi\hbar c}{e}$$

Superconductor with a columnar defect or hole



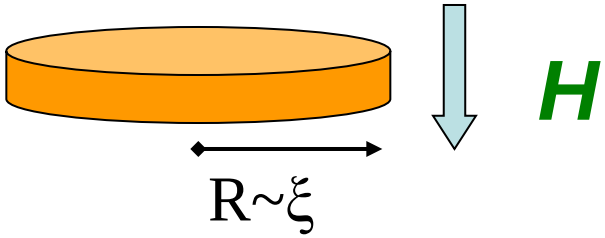
A.Bezryadin, A.I. Buzdin, B. Pannetier (1994)

Examples of quantum oscillation effects.

Little-Parks effect.

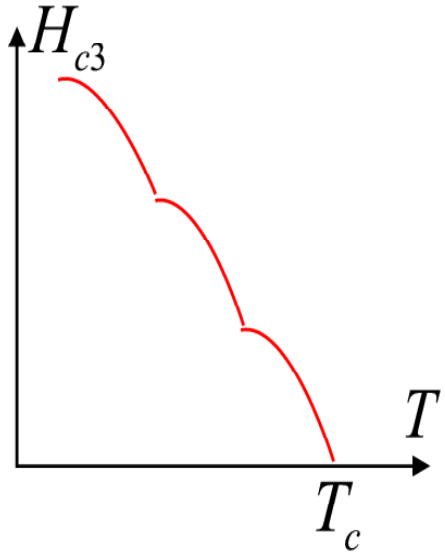
Simply-connected systems

Mesoscopic samples
dimensions ~ several coherence lengths



O.Buisson et al (1990)
R.Benoist, W.Zwerger (1997)
V.A.Schweigert, F.M.Peeters (1998)
H.T.Jadallah, J.Rubinstein, Sternberg (1999)

T_c (H) oscillations



Origin of T_c oscillations: Transitions between the states with different vorticity L

$$\Delta = |\Delta(r)| e^{iL\theta}$$

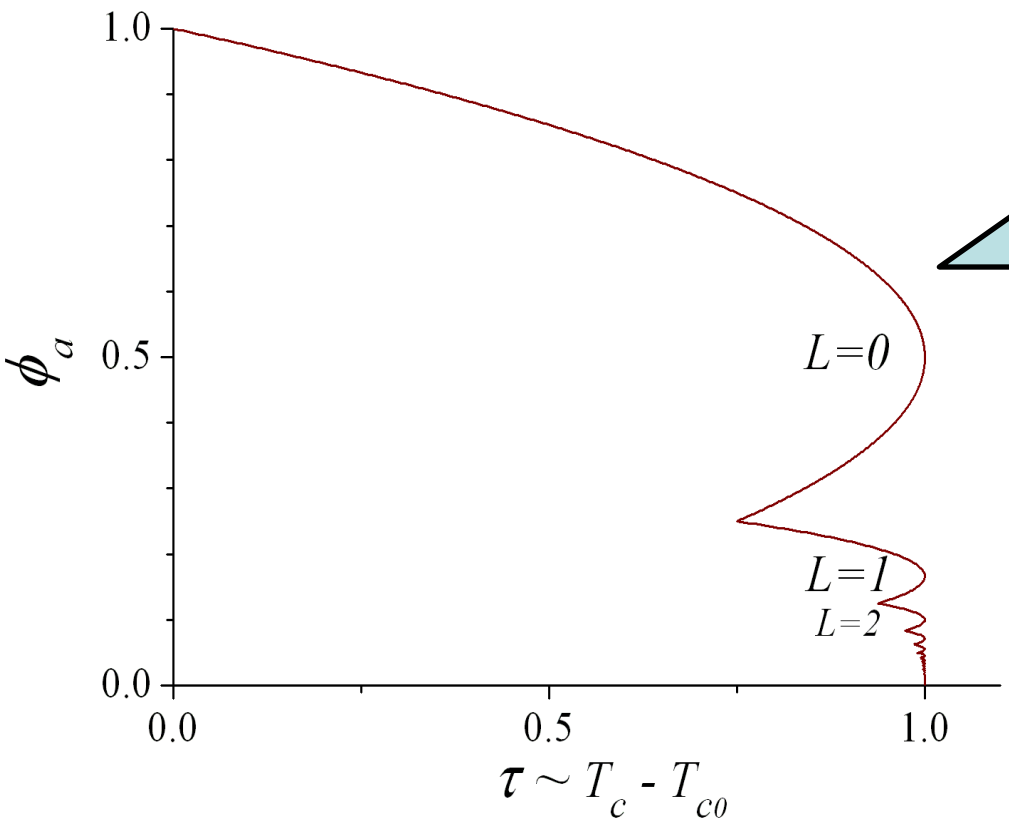
L - Vorticity
(orbital momentum)

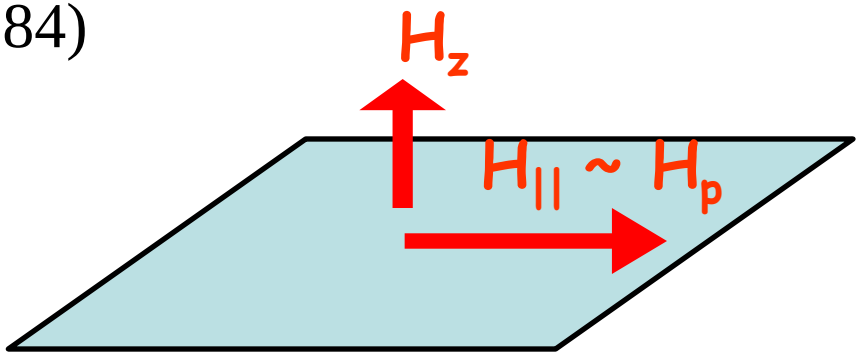
Multiquantum vortices

Examples of quantum oscillation effects.

FFLO states and Tc(H) oscillations in infinite 2D superconductors

A.I. Buzdin, M.L. Kulic (1984)





$$\phi_a = \frac{\pi H_z}{\phi_0 k_0^2}$$

Model: Modified Ginzburg-Landau functional (2D)

$$F = \int ((a + V(\vec{r}))|\Psi|^2 - \beta|\vec{D}\Psi|^2 + \gamma|\vec{D}^2\Psi|^2)dxdy$$

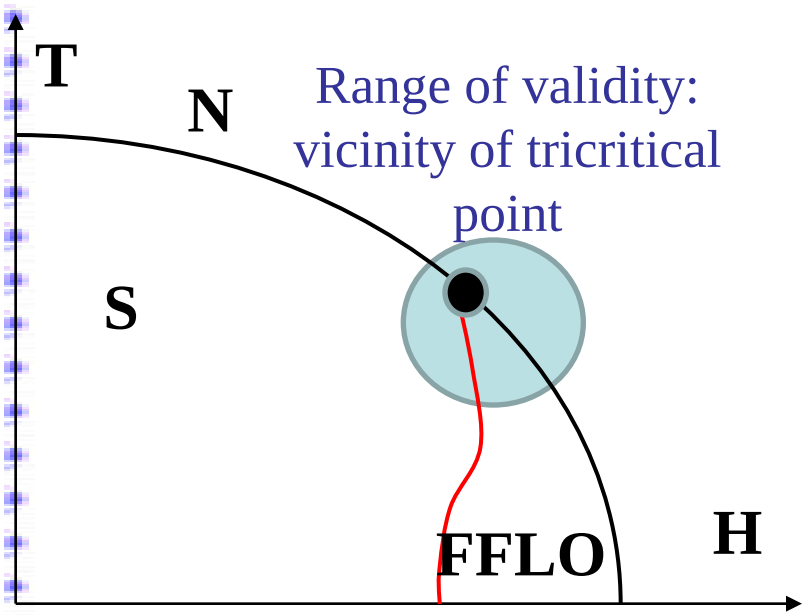
$$a = \alpha(T - T_{c0})$$

Trapping potential

FFLO instability

$$\vec{D} = \nabla - 2iM[\vec{\Omega}, \vec{r}]/\hbar$$

$$\vec{D} = \nabla - 2ie\vec{A}/\hbar c$$



Range of validity: vicinity of tricritical point

Confinement mechanisms:

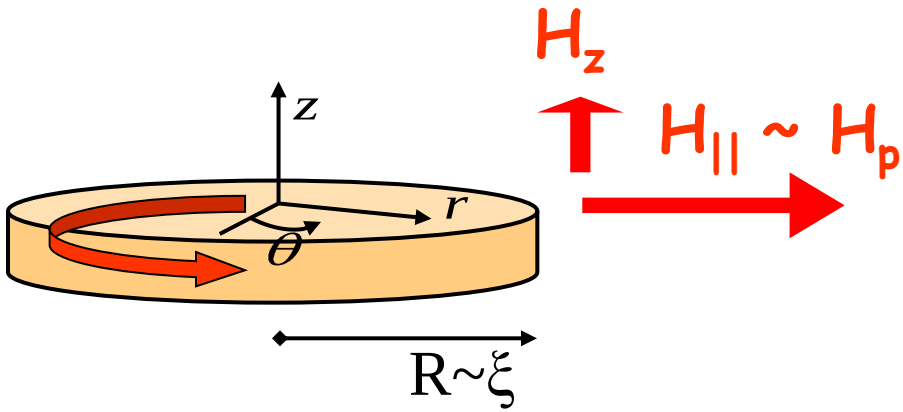
1. Zero trapping potential. Boundary condition at the system edge

$$\vec{n}\vec{D}\Psi = 0$$

2. nonzero trapping potential

$$V(\vec{r}) = M\omega^2 r^2/2$$

FFLO states in a 2D mesoscopic superconducting disk



Interplay between the system size, magnetic length, and FFLO length scale

Perpendicular magnetic field component $H_z = 0$

$$G = a|\Psi|^2 - \beta|\nabla\Psi|^2 + \gamma|\Delta\Psi|^2 \ ,$$

Eigenvalue problem:

$$-\Delta\Psi = k_0^2 q^2 \Psi \ , \qquad \frac{\partial\Psi}{\partial r}|_{r=R} = 0 \ .$$

$k_0 = \beta/2\gamma$ Wave number of FFLO instability

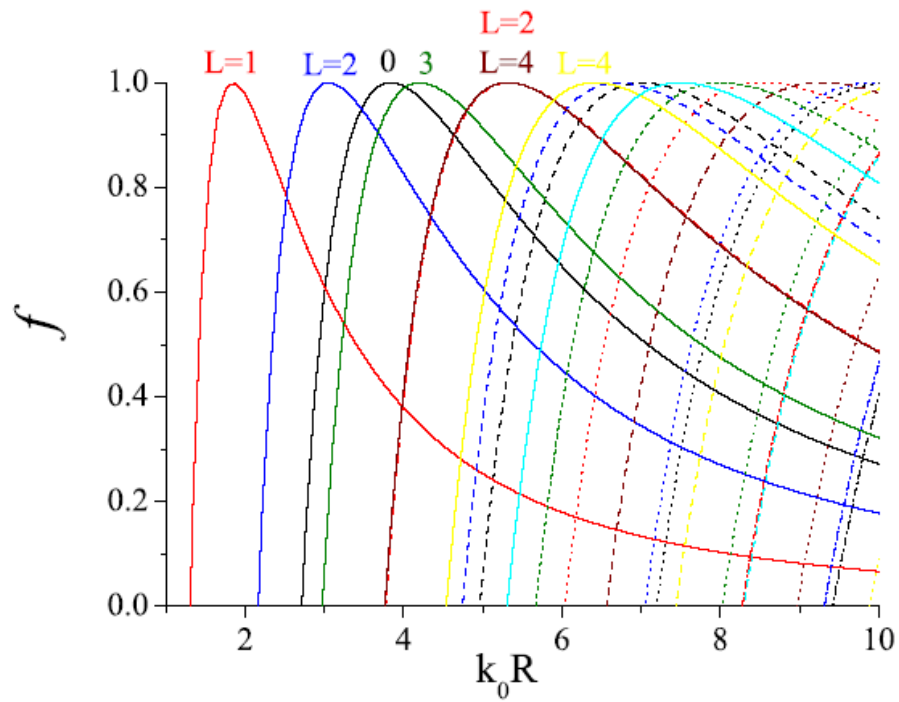
$$\Psi = e^{iL\theta} J_L(q\rho) \qquad \frac{dJ_L(x)}{dx}|_{qk_0R} = 0$$

x_{Ln} a set of zeros of the derivative of the Bessel function of the L-th order

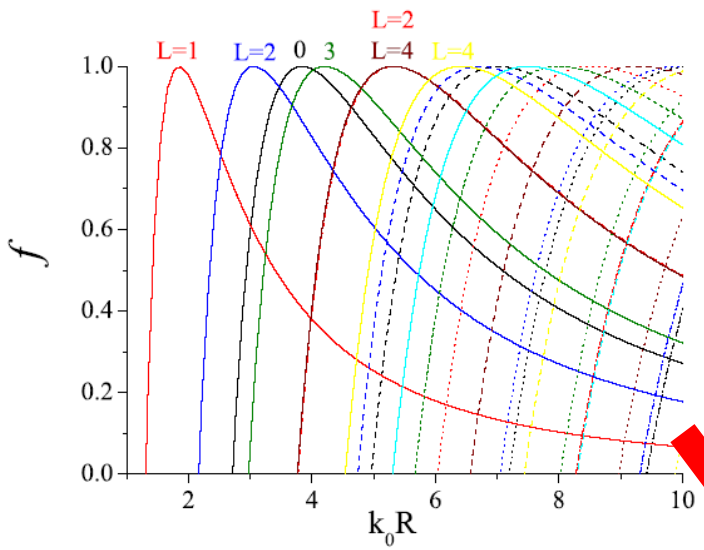
The critical temperature:

$$T_c = T_{c0} + \frac{\beta^2}{4\gamma\alpha} f(q) \qquad f(q) = 2q^2 - q^4$$

$$f\left(\frac{x_{Ln}}{k_0 R}\right) = \frac{2x_{Ln}^2}{(k_0R)^2} - \frac{x_{Ln}^4}{(k_0R)^4} \ .$$



H - T Phase diagram: $H_z = 0$

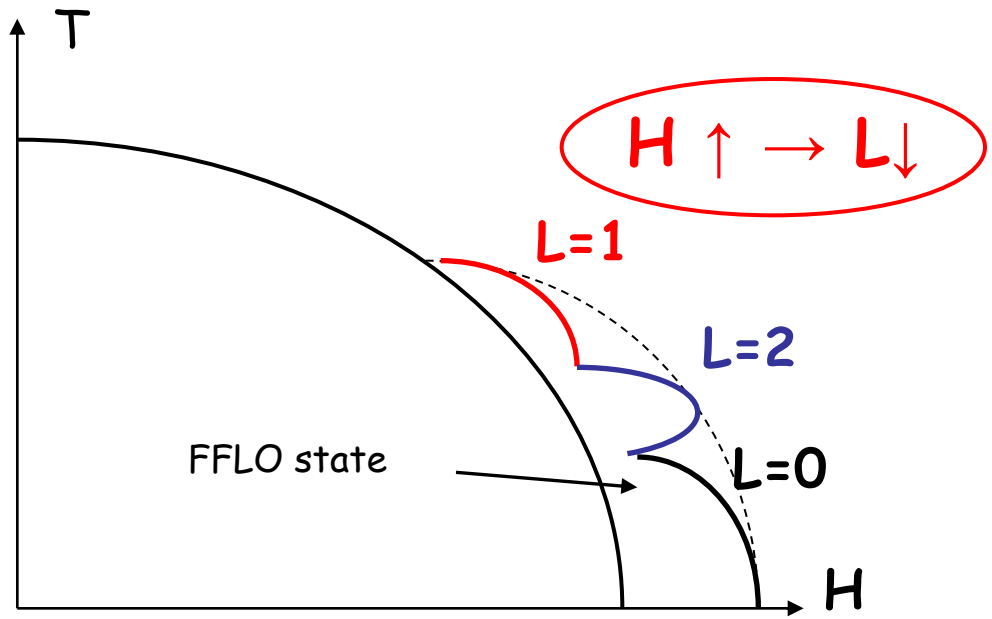


$$a = \alpha(T - T_{c0}),\ \beta = \beta(H/T),\ T_{c0} = T_{c0}(H)$$

$$\beta/2\gamma = k_0(H,T)$$

$$T_c = T_{c0} + \frac{\beta^2}{4\gamma\alpha} f(q)\ ,$$

$$\boxed{T_c = \max_L \{T_L\}\ .}$$



Tilted magnetic field: $H_z \neq 0$

$$\mathbf{H} = \mathbf{H}_{\|} + H_z \mathbf{z}_0$$

$$G = a|\Psi|^2 - \beta|\mathbf{D}\Psi|^2 + \gamma|\mathbf{D}^2\Psi|^2 ,$$

$$\mathbf{D} = \nabla + \frac{2\pi i}{\Phi_0}\mathbf{A}_{\|} \qquad A_\theta = H_z r/2.$$

$$\beta = \beta(H_{\|}, T)$$

$$T_c = T_{c0} + \frac{\beta^2}{4\gamma\alpha_0} f(q) , \qquad f(q) = 2q^2 - q^4 .$$

Eigenvalue problem:

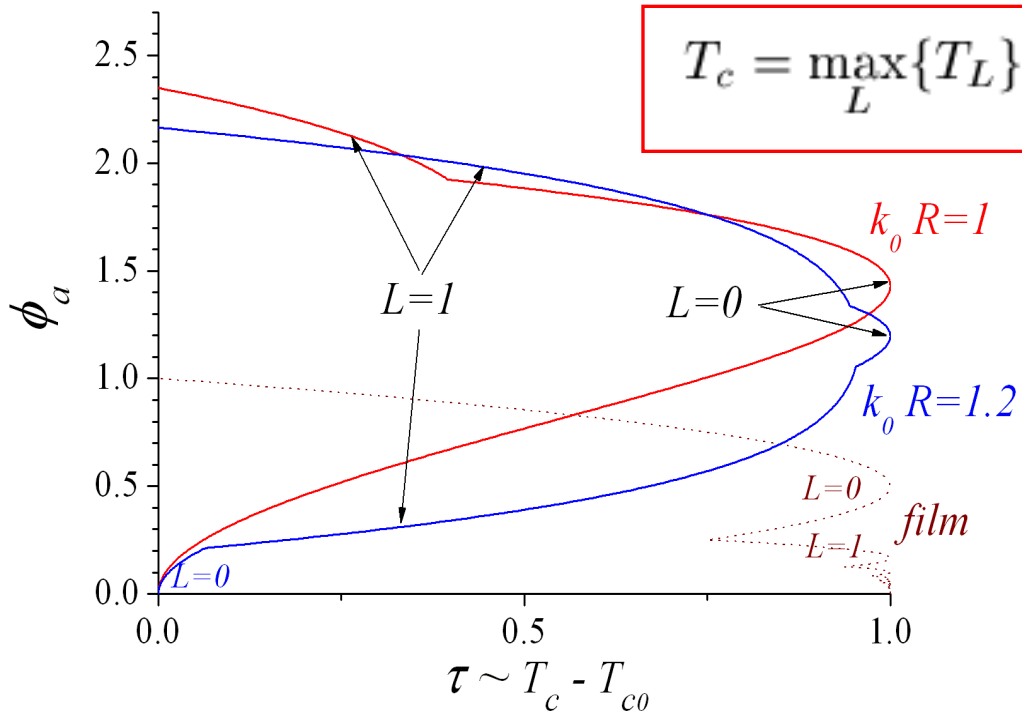
$$-\mathbf{D}^2\Psi = k_0^2 q^2 \Psi \qquad \left.\frac{\partial\Psi}{\partial r}\right|_{r=R} = 0 .$$

$$\psi(\rho) = \mathrm{e}^{-\phi/2}\phi^{|L|/2} F(a_L, b_L, \phi) ,$$

$$\phi_a = \frac{\pi R^2 H_z}{\Phi_0}$$

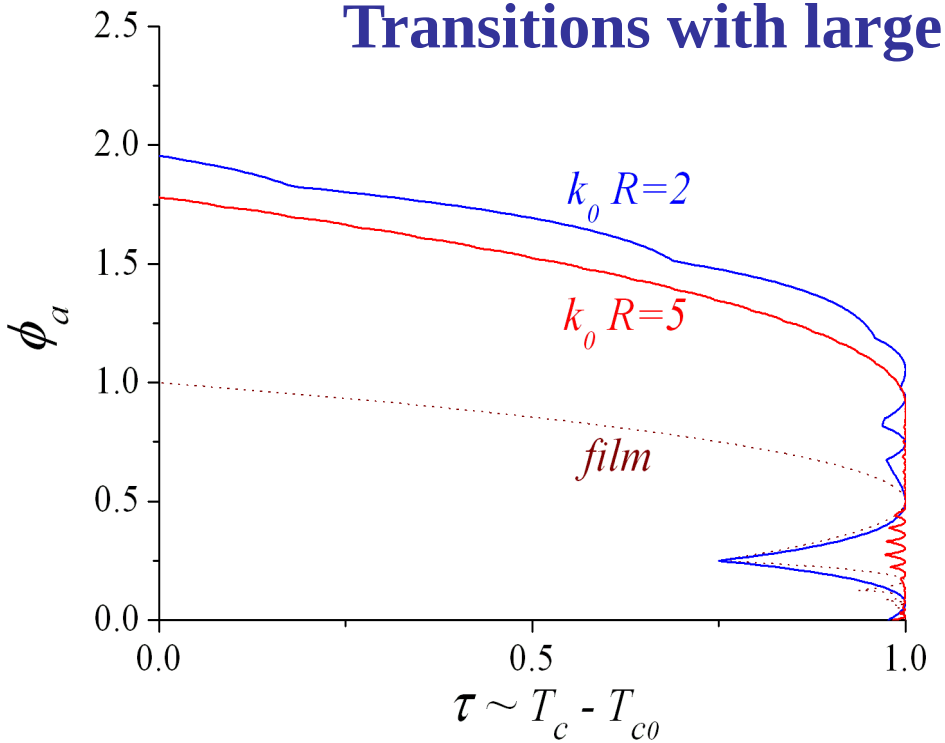
Field induced superconductivity

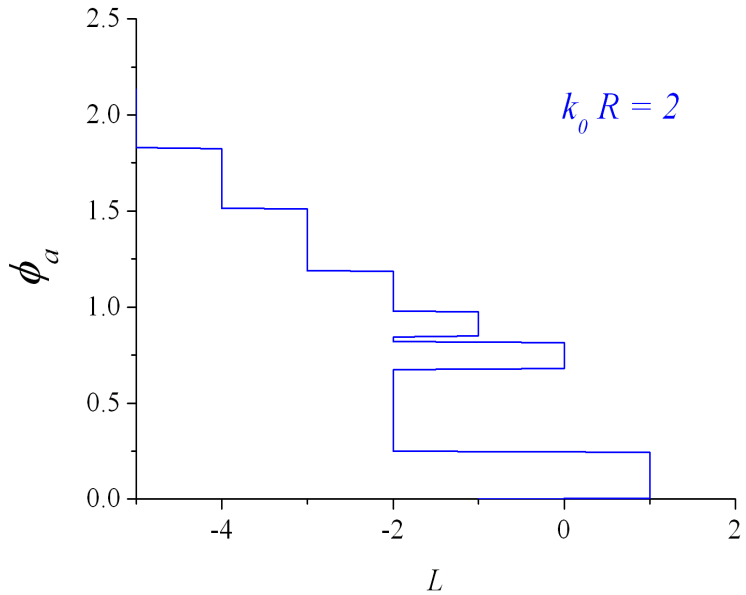
$$T_c = \max_L \{T_L\} .$$



Tilted magnetic field: $H_z \neq 0$

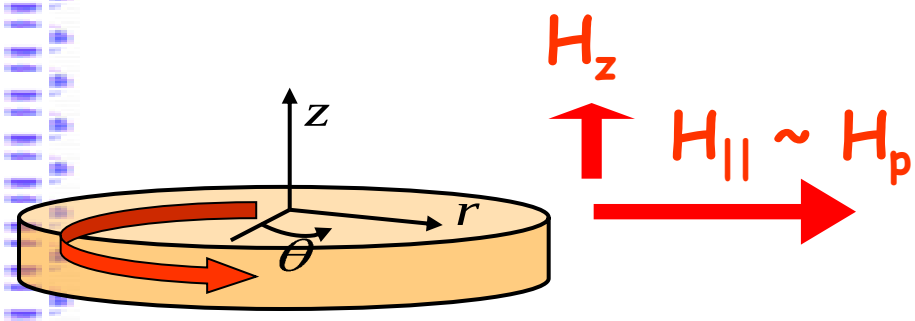
Transitions with large jumps in vorticity





$$T_c = \max_L \{T_L\} .$$

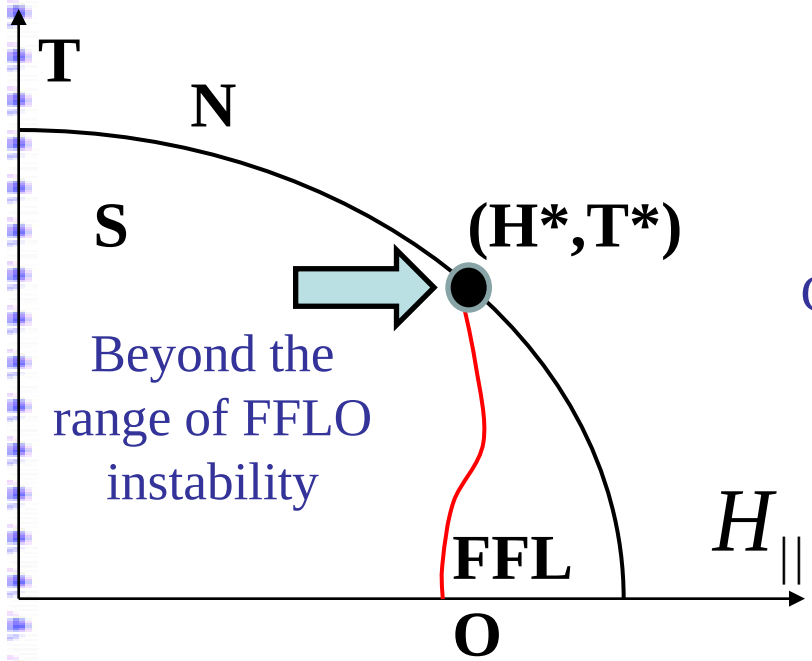
Vortex solutions beyond the range of FFLO instability. Critical field of the vortex entry.



$$F \propto \int (\xi_1^2 |\vec{D}\Psi|^2 + \xi_2^4 |\vec{D}^2\Psi|^2) dxdy$$

We focus on the limit $\xi_1 << \xi_2$

$$\Psi \approx e^{iL\theta}$$



Condition of the first vortex entry:

$$F(L=1) - F(L=0) = 0$$

Condition of the first vortex entry:

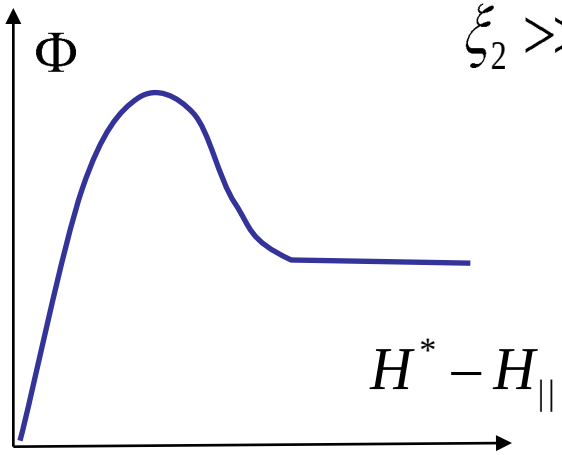
$$\ln\frac{R}{\xi_m} - \Phi + \frac{\alpha}{2} + \frac{\alpha\xi_m^2}{R^2}\left(3\Phi^2 - 4\Phi\ln\frac{R}{\xi_m} - \Phi^3\right) = 0$$

$$\xi_m = \max(\xi_1, \xi_2)$$

$$\Phi = \frac{\pi H_z R^2}{\phi_0}$$

$$\alpha = \frac{\xi_2^4}{\xi_m^2 \xi_1^2}$$

$$\frac{R}{\xi_m} >> 1$$



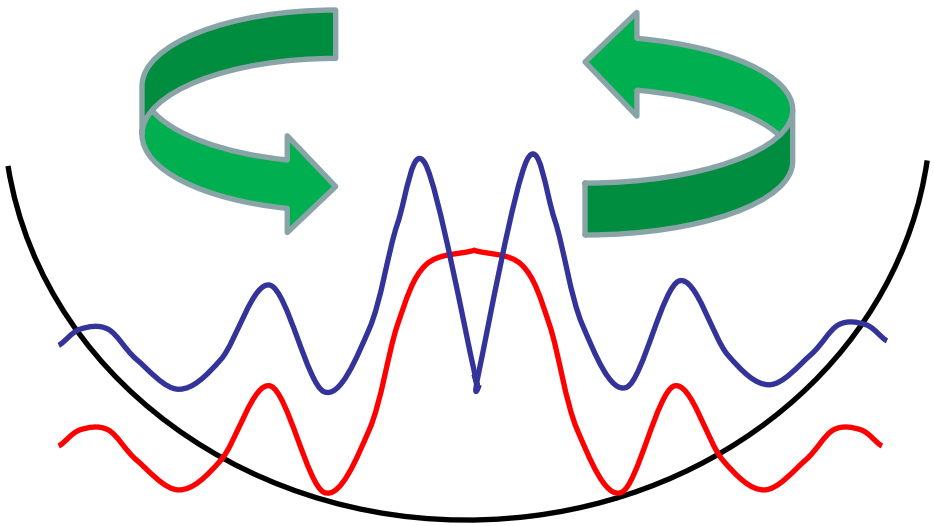
Limiting cases:

$$\xi_2 << \xi_1 \qquad \Phi = \ln\frac{R}{\xi_1} \qquad H_z \propto \frac{1}{R^2}\ln\frac{R}{\xi_1}$$

$$\xi_2 >> \xi_1 \qquad \Phi = \left(\frac{R}{\xi_2}\right)^{2/3} \qquad H_z \propto \left(\frac{1}{R}\right)^{4/3}$$

Change in the scaling law

FFLO states in a trapping potential



Interplay between the rotation effect, confinement, and FFLO instability

FFLO states in a 2D system in a parabolic trapping potential (no rotation)

$$\Delta^4\Psi + 2\Delta^2\Psi + (\tau + v_0\rho^2)\Psi = 0$$

$k_0 = \beta/2\gamma$ FFLO length scale

$\tau = a/\gamma k_0^4$ Temperature shift

$v_0 = M\omega^2/2\gamma k_0^6$ (Trapping frequency)^2

$\vec{\rho} = k_0\vec{r}$ Dimensionless coordinate

Fourier transform:

$$\Psi = \int e^{i\vec{q}\vec{\rho}}\psi_{\vec{q}}\, d^2\vec{q}$$

$$-v_0\frac{\partial^2}{\partial\vec{q}^2}\Psi + \left(q^4 - 2q^2\right)\Psi = -\tau\Psi$$

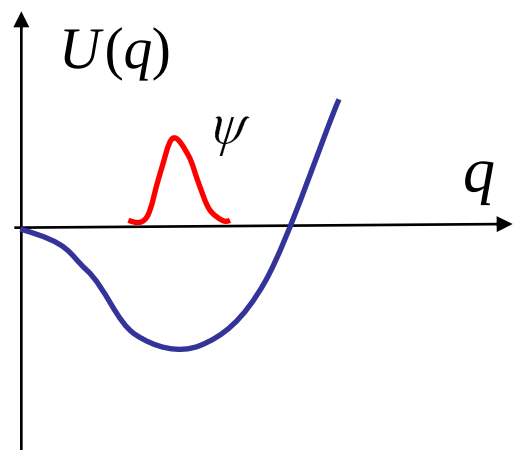
$$L = 0$$

$$q = 1 + x$$

$$U(q) = q^4 - 2q^2 \approx -1 + 4x^2$$

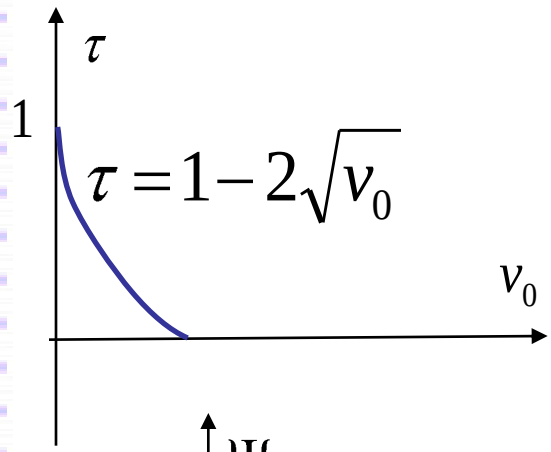
$$\psi_{\vec{q}} = e^{-\lambda x^2} \qquad \lambda = \frac{1}{\sqrt{v_0}}$$

$v_0 \to 0$



FFLO states in a 2D system in a parabolic trapping potential (no rotation)

Phase diagram

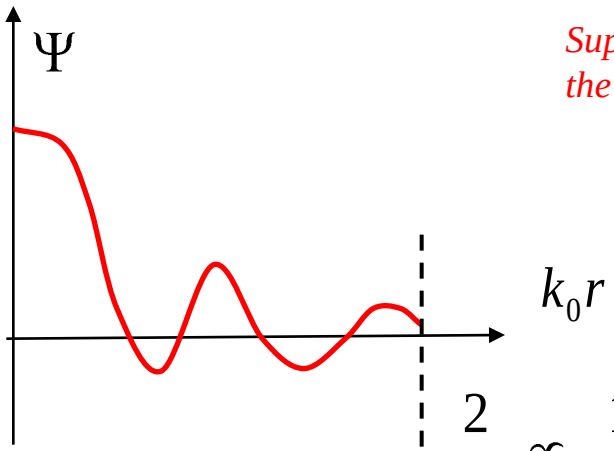


Condensate wave function

$$k_0 r >> 1$$

$$\Psi \propto \sqrt{\frac{\pi}{k_0 r}} \cos(k_0 r - \pi/4) e^{-k_0^2 r^2 \sqrt{v_0}/4}$$

Suppression of wave function oscillations by the increase in the trapping frequency



$$\frac{2}{v_0^{1/4}} \propto \frac{1}{\sqrt{\omega}}$$ *=Number of observable oscillations*

FFLO states in a rotating 2D gas in a parabolic trapping potential.

$$\Psi(\vec{r}) = f_L(r)e^{iL\theta} \qquad D^4 f_L + 2D^2 f_L + (\tau + v_0\rho^2) f_L = 0$$

$$D^2 = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right) - \left(\frac{L}{\rho} + \phi_a\rho\right)^2$$

Expansion in eigenfunctions of the problem without trap:

$$f_L(\rho) = \sum_{n=0}^{\infty} c_n u_{nL}(\rho)$$

$$-D^2 u_{nL} = q_{nL}^2 u_{nL} = 2\phi_a(2n + |L| + L + 1)u_{nL}$$

$$(2q_{nL}^2 - q_{nL}^4)c_n - \sum_m v_{nm}c_m = \tau c_n$$

$$v_{nm} = v_0 \int_0^{\infty} u_{mL}u_{nL}\rho^3 d\rho$$

$$k_0 = \beta/2\gamma$$ FFLO length scale

$$\tau = a/\gamma k_0^4$$ Temperature shift

$$v_0 = M\omega^2/2\gamma k_0^6$$ (Trapping frequency)^2

$$\phi_a = 2M\Omega/\hbar k_0^2$$ rotation frequency

$$\vec{\rho} = k_0\vec{r}$$ Dimensionless coordinate

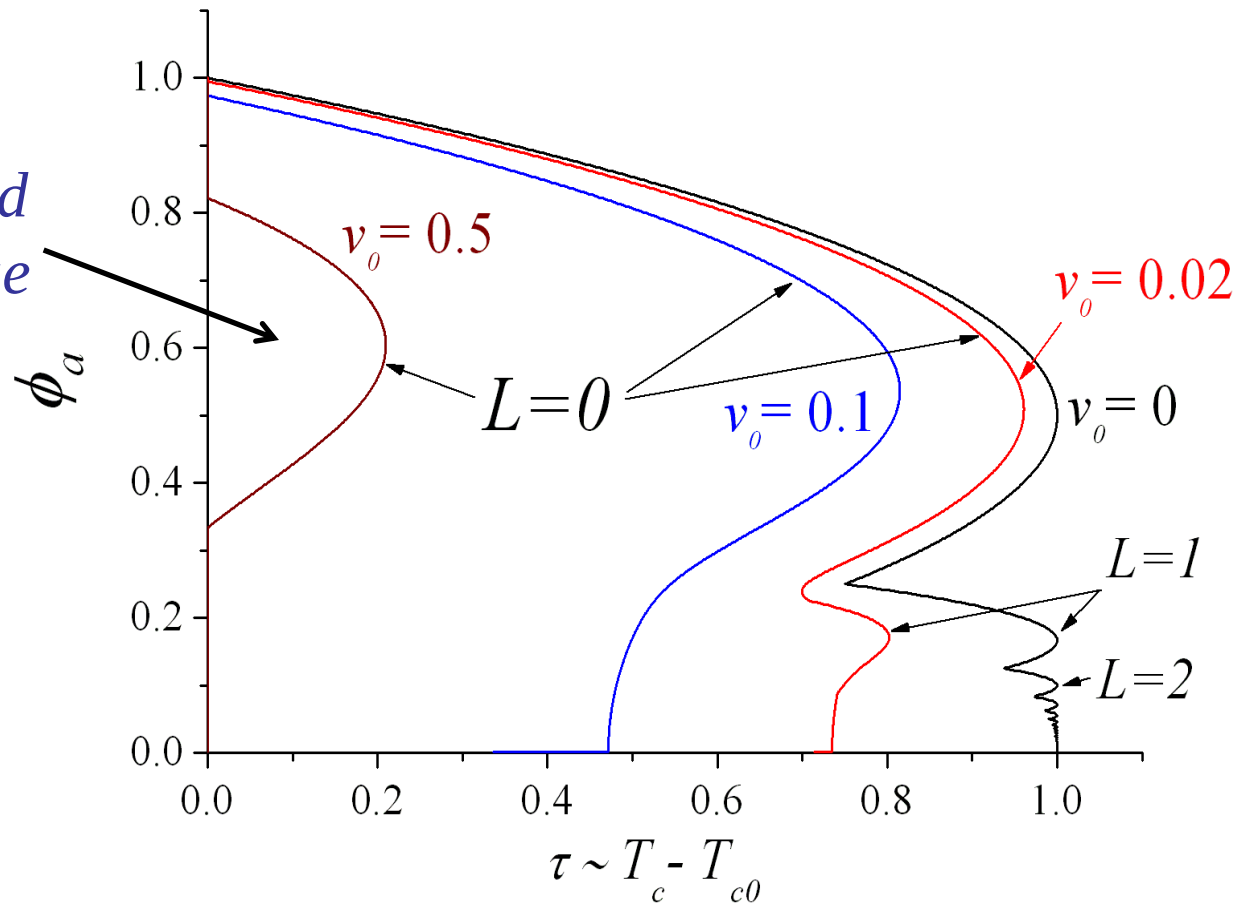
FFLO states in a rotating 2D gas in a parabolic trapping potential.

Suppression of quantum oscillations by the increase in the trapping frequency.

First-order perturbation theory:

$$\tau = \max_{L\geq 0}\left[(4\phi_a - v_0/\phi_a)(2L+1) + v_0 L/\phi_a - 4\phi_a^2(2L+1)^2\right]$$

rotation induced superfluid phase



II. Conclusions

- **There are strong experimental evidences of the existence of the the FFLO state in organic layered superconductors and in heavy fermion superconductor** $CeCoIn_5$
- **The interplay between FFLO modulation and orbital effect results in new type of the vortex structures, non-monotonic critical field behavior in layered superconductor in tilted field.**
- **Special behavior of fluctuations near the FFLO transition – vanishing stiffness.**
- **FFLO phase in ultracold Fermi gases with imbalanced state populations?**