ГЕНЕРАЦИЯ ЭЛЕКТРОМАГНИТНОГО ИЗЛУЧЕНИЯ ВНУТРЕННИМИ ДЖОЗЕФСОНОВСКИМИ КОНТАКТАМИ В ВЫСОКОТЕМПЕРАТУРНЫХ СВЕРХПРОВОДНИКАХ

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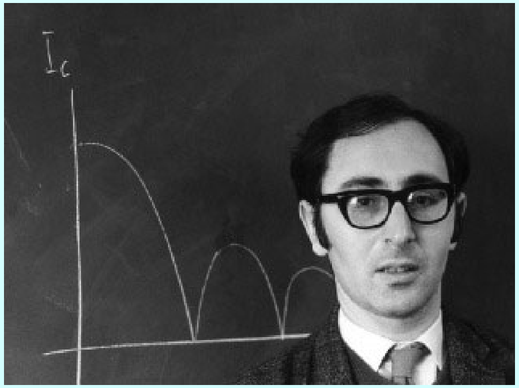
23 февраля — 1 марта 2014 года

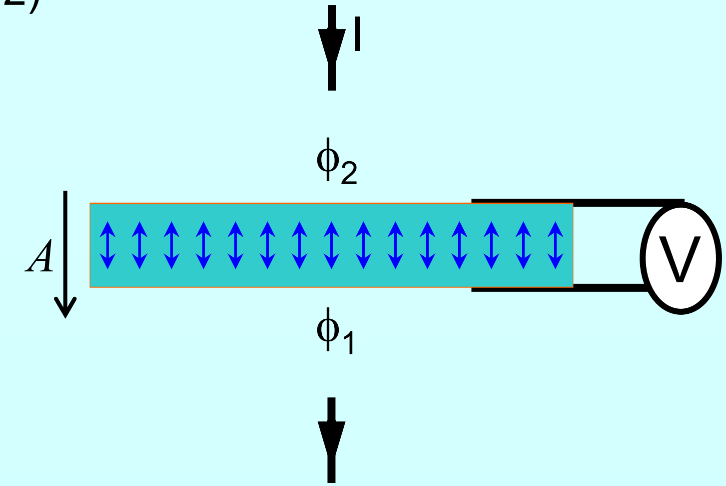
Outline

- History
  - Josephson effects and generation of *em* waves
  - Josephson junctions arrays, synchronization
  - Intrinsic JJ stacks in layered superconductors ($Bi_2Sr_2CaCu_2O_{8+\delta}$)
- Terahertz radiation from high-temperature superconductors
  - Review of experiments
- Synchronization by internal resonance mode
  - Mechanisms of coupling & structure of coherent state
  - Radiated power, mode damping, features in IV
  - Synchronization in inhomogeneous junction stacks
  - Mechanism of line width
- Mesa arrays

Josephson effects in superconducting tunneling junctions

B. D. Josephson, Phys. Lett. 1, 251 (1962)





- phase difference $\theta = \phi_2 - \phi_1 - \frac{2e}{\hbar c}\int_1^2 A\,dl$
- dc Josephson effect $I \propto \sin\,\theta$
- ac Josephson effect $V \propto d\theta/dt$
- alternating tunnel current $I \propto \sin(2\pi f\, t)$

$f = 2eV/h$ $\quad f/V = 0.483$ THz/mV

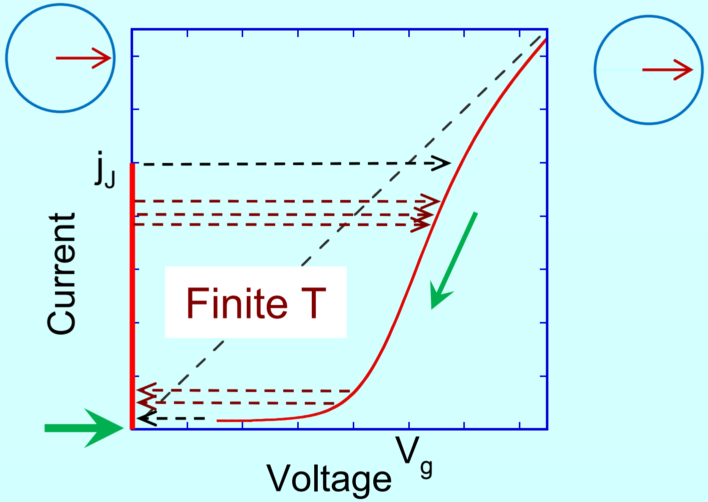
Nobel prize 1973 (with Giaever and Esaki)

Phase dynamics in a single junction

Resistively Shunted Junction Model

Sine-Gordon equation

$$\boxed{\frac{1}{\omega_p^2}\frac{\partial^2\theta}{\partial t^2}+\frac{\nu}{\omega_p}\frac{\partial\theta}{\partial t}-\lambda_J^2\frac{\partial^2\theta}{\partial x^2}+\sin\theta=\frac{j_{ext}}{j_J}}$$



Parameters:

λ_J Josephson length

ω_p plasma frequency

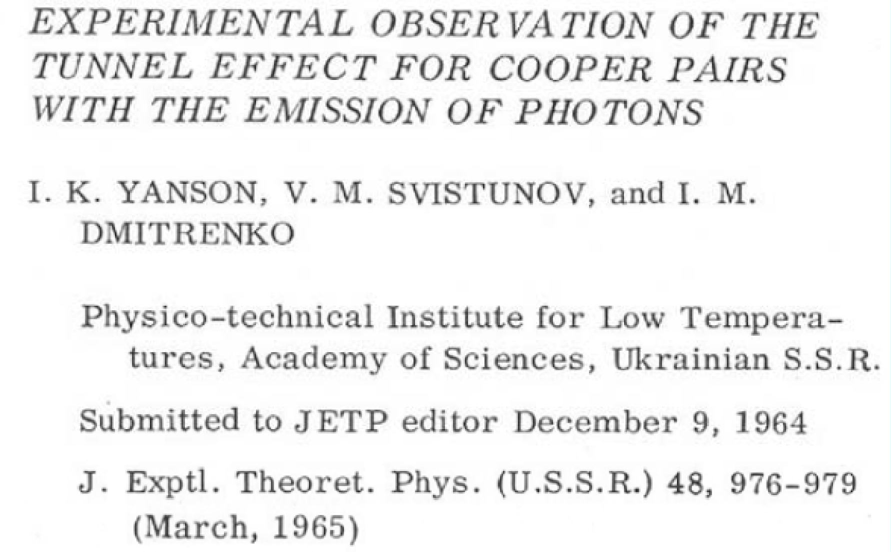
$c_s = \lambda_J\omega_p$ Swihart velocity

ν damping parameter

$\beta = 1/\nu^2$ McCumber parameter

underdamped (ν □ 1) vs overdamped (ν □ 1)

First observations of radiation out of Josephson junctions



EXPERIMENTAL OBSERVATION OF THE TUNNEL EFFECT FOR COOPER PAIRS WITH THE EMISSION OF PHOTONS

I. K. YANSON, V. M. SVISTUNOV, and I. M. DMITRENKO

Physico-technical Institute for Low Temperatures, Academy of Sciences, Ukrainian S.S.R.

Submitted to JETP editor December 9, 1964

J. Exptl. Theoret. Phys. (U.S.S.R.) 48, 976-979 (March, 1965)

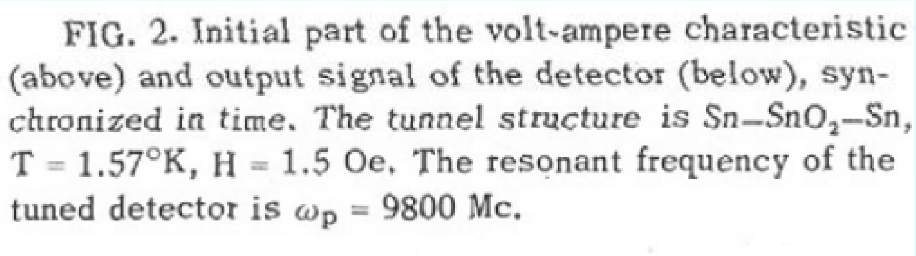
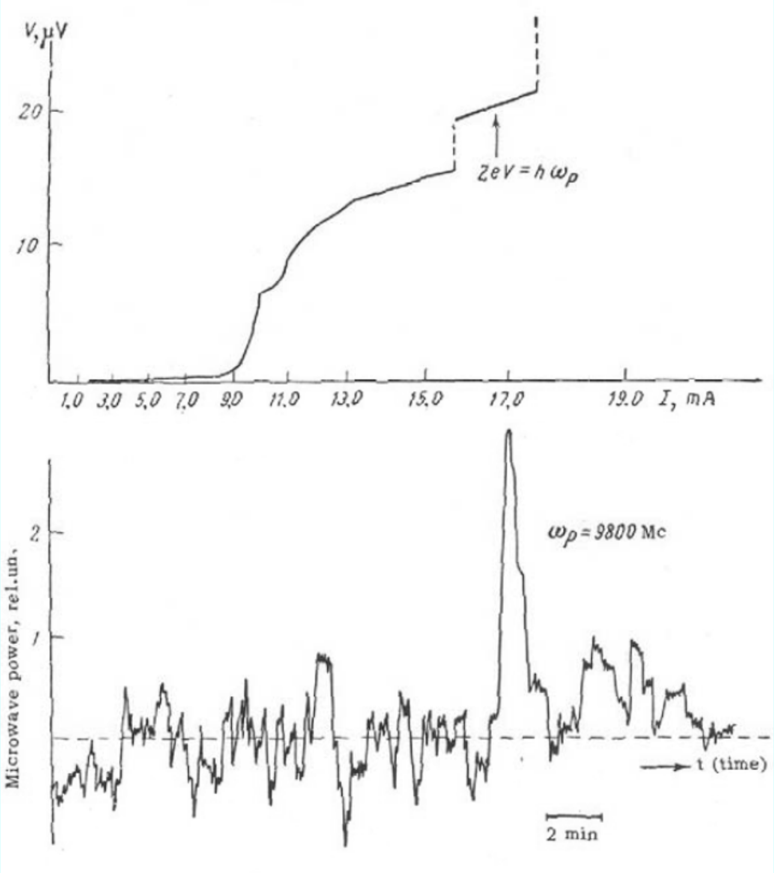


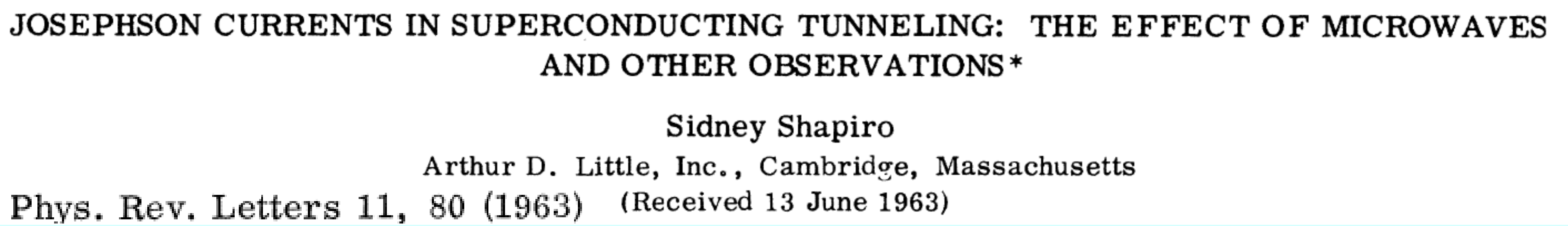
FIG. 2. Initial part of the volt-ampere characteristic (above) and output signal of the detector (below), synchronized in time. The tunnel structure is Sn–SnO$_2$–Sn, T = 1.57°K, H = 1.5 Oe. The resonant frequency of the tuned detector is ω_p = 9800 Mc.



Also: D. N. Langenberg *et al*., PRL **15**, 294 (1965)

Radiation power is very small: $< 10^{-12}$ W

Dynamic Josephson effects:
Shapiro steps



JOSEPHSON CURRENTS IN SUPERCONDUCTING TUNNELING: THE EFFECT OF MICROWAVES AND OTHER OBSERVATIONS*

Sidney Shapiro

Arthur D. Little, Inc., Cambridge, Massachusetts

Phys. Rev. Letters 11, 80 (1963) (Received 13 June 1963)

Irradiate Jos. junction →
steps at $V_m = mhf_{ext}/2e$

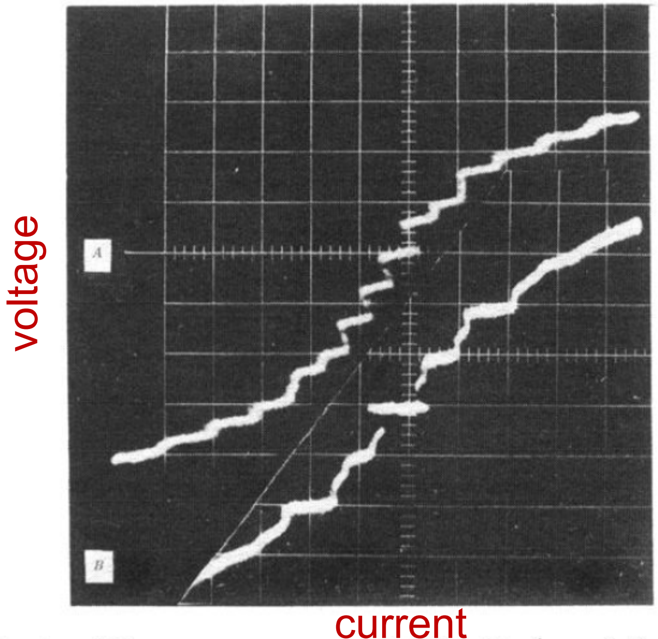


FIG. 3. Microwave power at 9300 Mc/sec (*A*) and 24850 Mc/sec (*B*) produces many zero-slope regions spaced at $h\nu/2e$ or $h\nu/e$. For *A*, $h\nu/e = 38.5\ \mu\text{V}$, and for *B*, 103 μV. For *A*, vertical scale is 58.8 μV/cm, horizontal scale is 67 nA/cm; for *B*, vertical scale is 50 μV/cm, horizontal scale is 50 μA/cm.

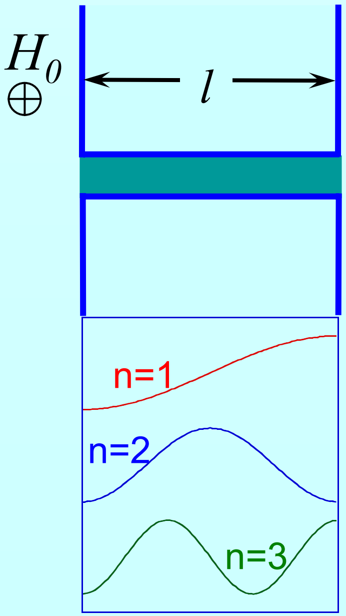
Dynamic Josephson effects:

Fiske resonances

= cavity modes excited by oscillating Josephson current

Coon and Fiske, Phys. Rev., **138**, A744 (1965)
Kulik, Pis'ma ZhETF, **2**, 134 (1965)

$\omega_n = c_s \pi n / l$



Coupling can be tuned by magnetic field

D. N. Langenberg *et al.*,

PRL **15**, 294 (1965)

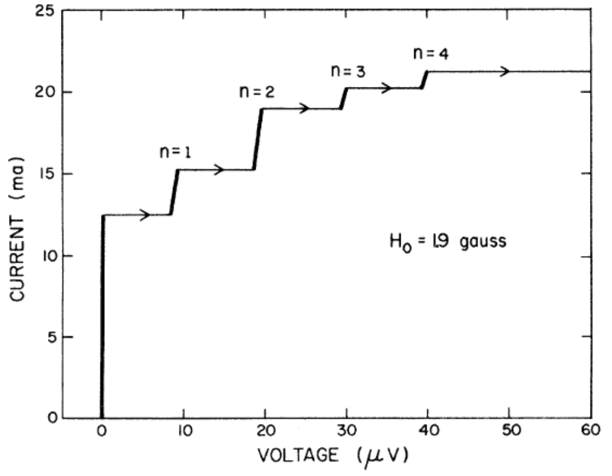
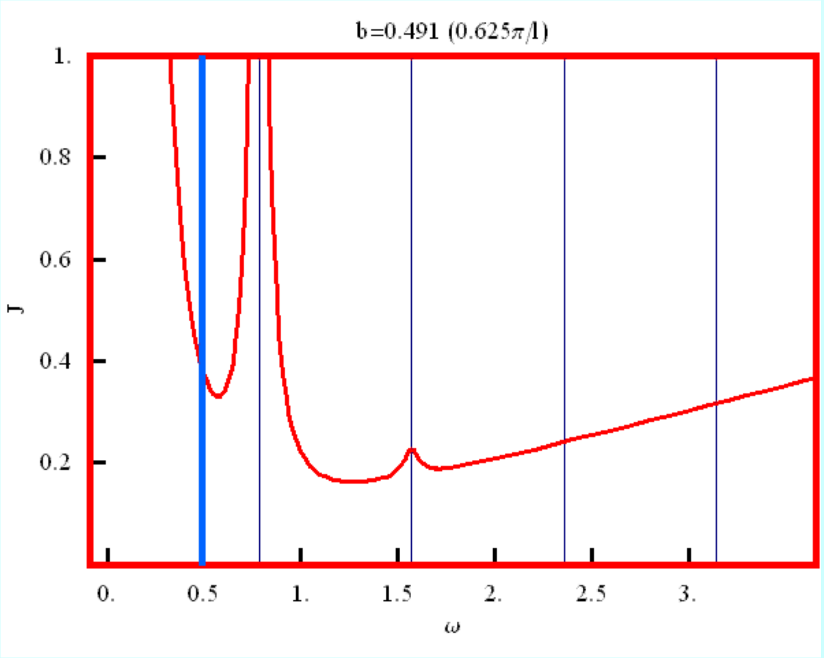


FIG. 2. A typical *I–V* curve for the Sn–Sn-oxide–Sn junctions used in these experiments. The voltage separation of the modes (labeled by *n*) corresponds to a frequency separation of approximately 4.6 Gc/sec. By adjusting the magnitude of H_0, a greater portion of each mode could be observed as well as several other higher modes.

Field evolution of IVs

$l = 4\lambda_J,\ \nu = 0.1$

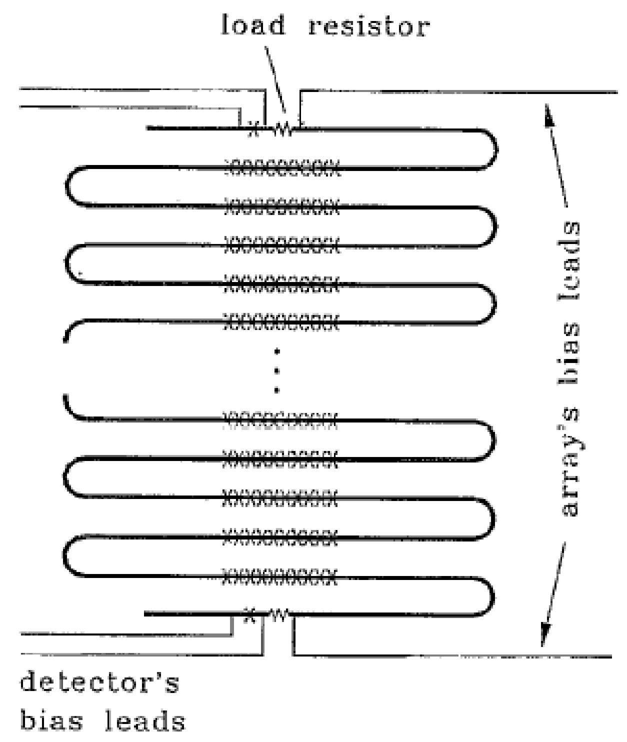


Josephson junction arrays

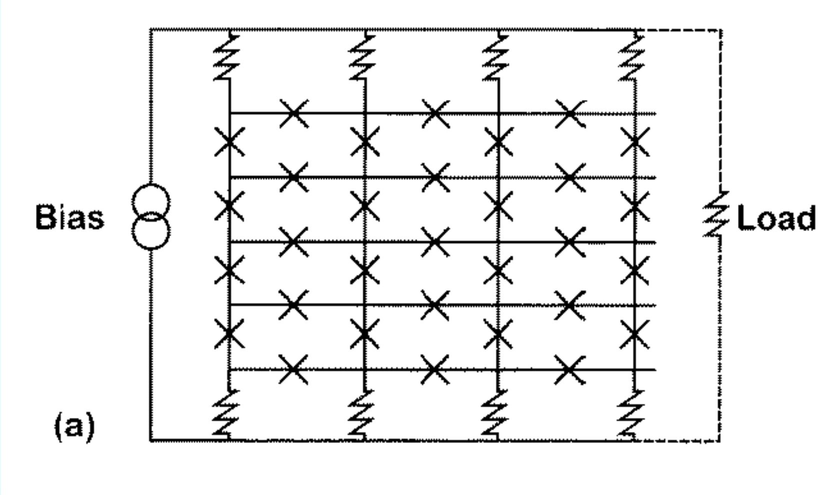
T. D. Clark, Phys. Lett. **A 27**, 585 (1968)
…

Reviews:
A. K. Jain *et al*., Phys. Rep. **109**, 309 (1984)
M. Darula *et al*., Sup. Sci. Tech. **12**, R1 (1999)

One-dimensional array



Two-dimensional array



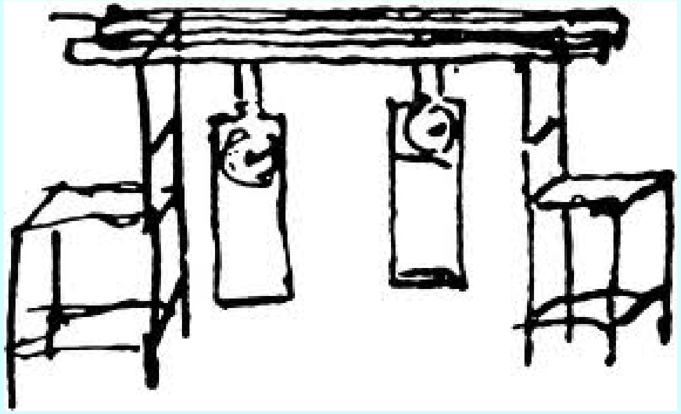
Achievement:
S. Han *et al*., APL **64**, 1425 (1994):
500 Nb/AlO$_x$/Nb junctions, 47 μW at 394 GHz

Synchronized oscillations: P $\propto$ N^2

Synchronization

adjustment of rhythms of oscillating objects due to their weak interaction

Christiaan Huygens, 1665



Large Number of oscillators:
Synchronization transition
Kuramoto, 1975

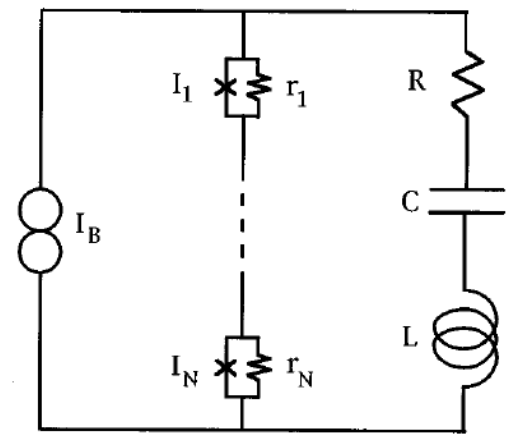
Synchronization problem

Linear array of point junctions

Wiesenfeld, Colet, Strogatz, Phys. Rev. Lett., **76**, 404 (1996)

$$\frac{\hbar}{2er_j}\dot{\phi}_j + I_j \sin\phi_j + \dot{Q} = I_B, \quad j = 1, \dots, N \qquad (1)$$

$$L\ddot{Q} + R\dot{Q} + \frac{1}{C}Q = \frac{\hbar}{2e}\sum_{k=1}^{N} \dot{\phi}_k, \qquad (2)$$



$$\Downarrow$$

Kuramoto model (1975)

$$\dot{\theta}_j = \omega_j - \frac{K}{N}\sum_{k=1}^{N} \sin(\theta_j - \theta_k + \alpha),$$

Synchronization order parameter

$$\sigma e^{i\psi} = \frac{1}{N}\sum_{k=1}^{N} e^{i\theta_k},$$

$$0 \le \sigma \le 1$$

Self-consistent analysis:

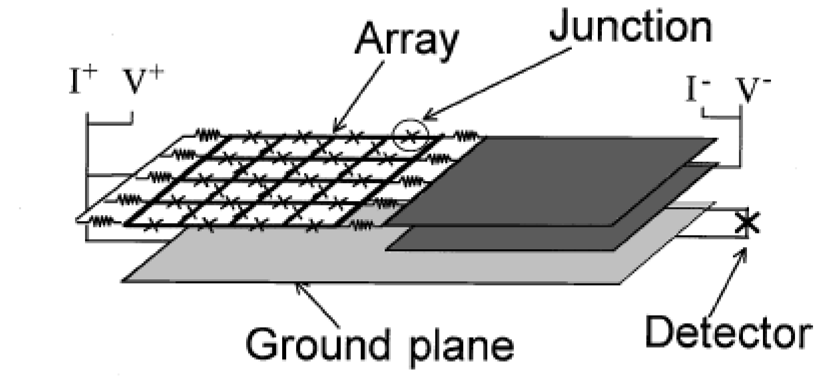
E.g., for $\alpha = 0$ and $g(\omega_j) = \dfrac{\gamma/\pi}{(\omega_j - \bar{\omega})^2 + \gamma^2}$

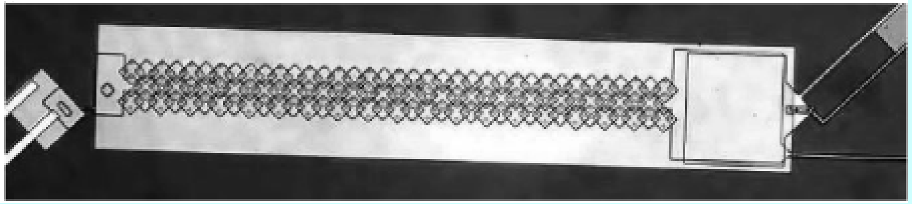
Synchronization transition at $K_c = 2\gamma$

$\sigma = \sqrt{1 - 2\gamma / K}$ for $K > K_c$

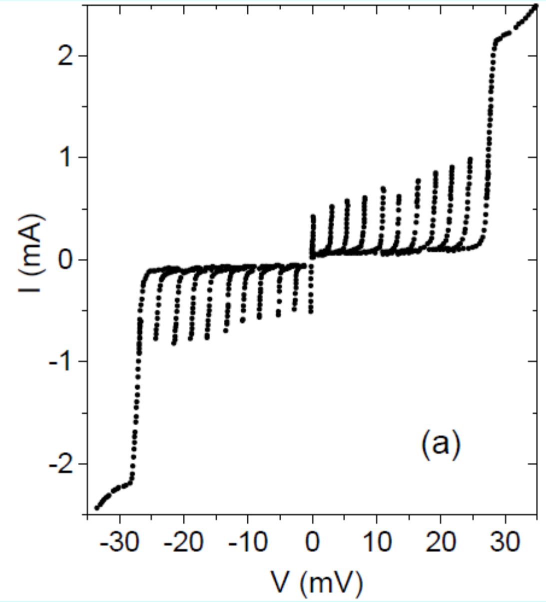
Synchronization via coupling to common resonance

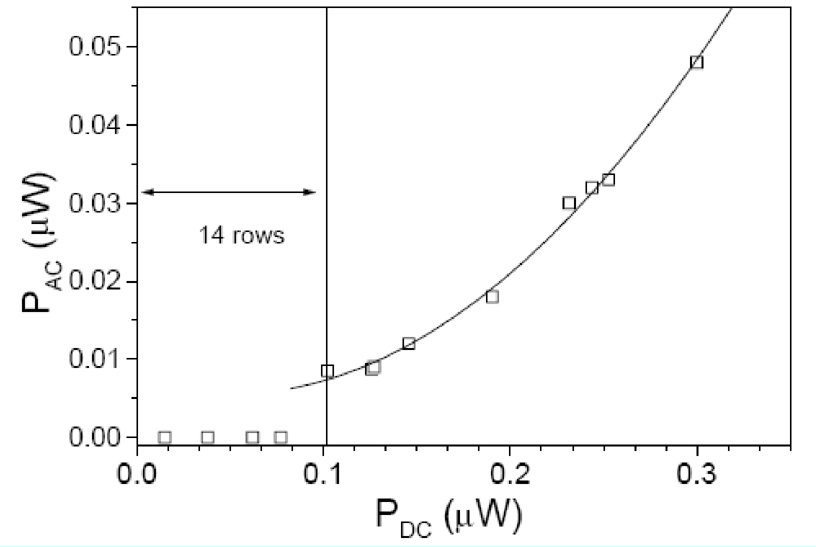
P. Barbara *et al*., PRL **82**, 1963 (1999)
Nb/Al/AlO$_x$/Nb-junctions, 150 GHz

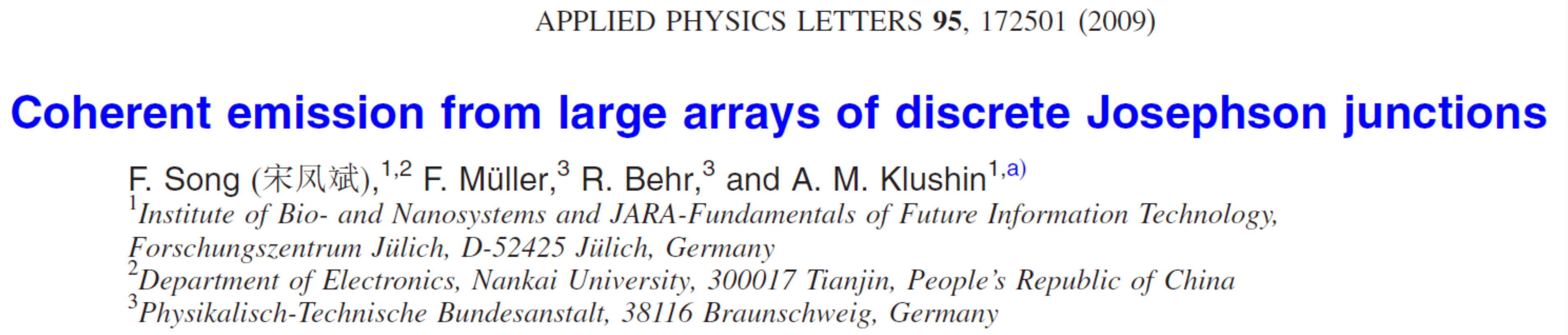




3×36







APPLIED PHYSICS LETTERS **95**, 172501 (2009)

Coherent emission from large arrays of discrete Josephson junctions

F. Song (宋风斌),[1,2] F. Müller,[3] R. Behr,[3] and A. M. Klushin[1,a)]
[1]*Institute of Bio- and Nanosystems and JARA-Fundamentals of Future Information Technology, Forschungszentrum Jülich, D-52425 Jülich, Germany*
[2]*Department of Electronics, Nankai University, 300017 Tianjin, People's Republic of China*
[3]*Physikalisch-Technische Bundesanstalt, 38116 Braunschweig, Germany*

1D array of Nb-Al_2O_3-Al-Al_2O_3-Nb SINIS junctions

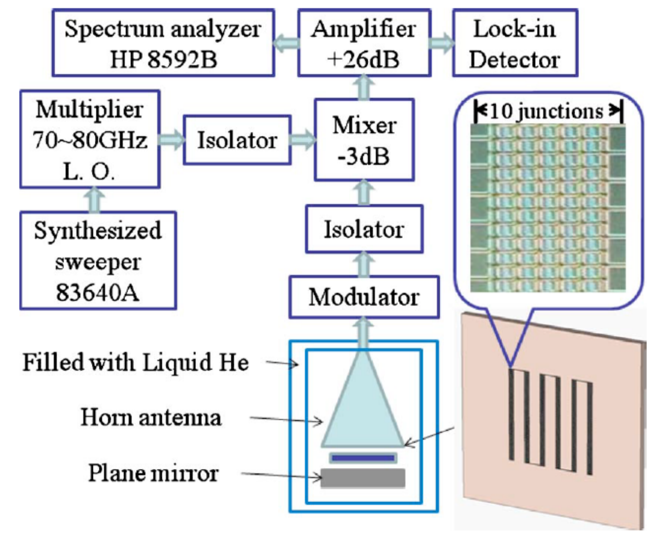
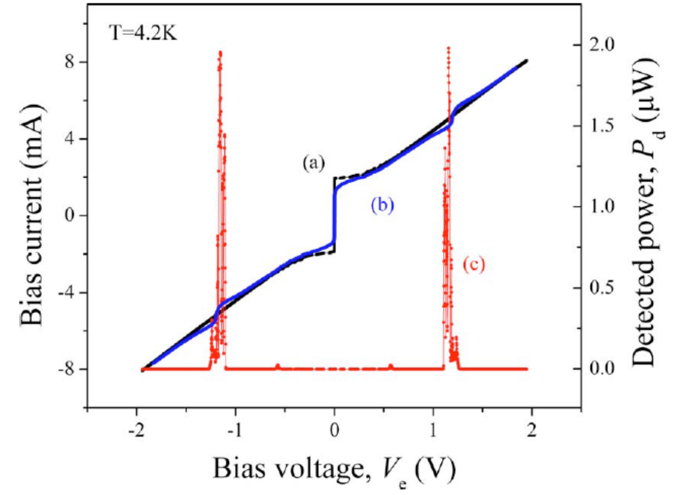
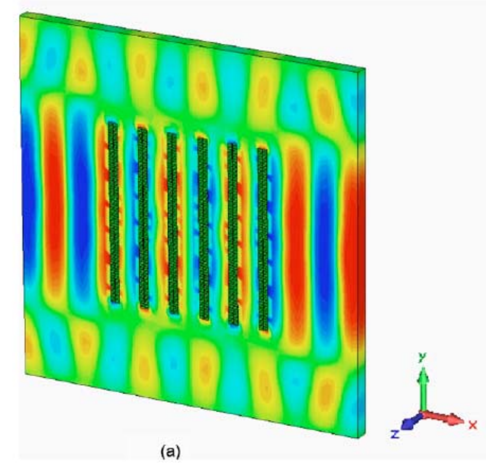


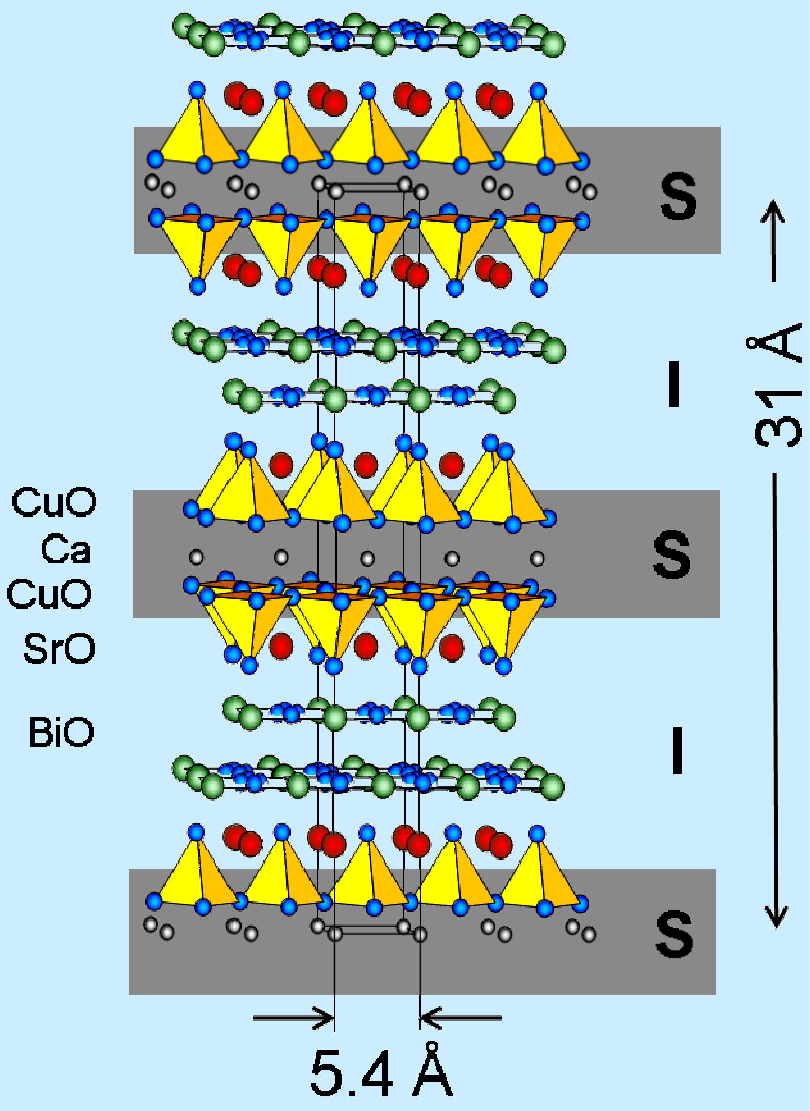
FIG. 1. (Color online) Schematic diagram of the measurement setup. Right side shows schematically a large series array of $N=7500$ SINIS Josephson junctions, equally divided into six subarrays on the substrate. Above it, the enlarged view is a photo of a small part of the meandering structure.





Frequency is limited by the superconducting gap:
~ 0.7 THz for Nb

Layered High-T_c superconductors



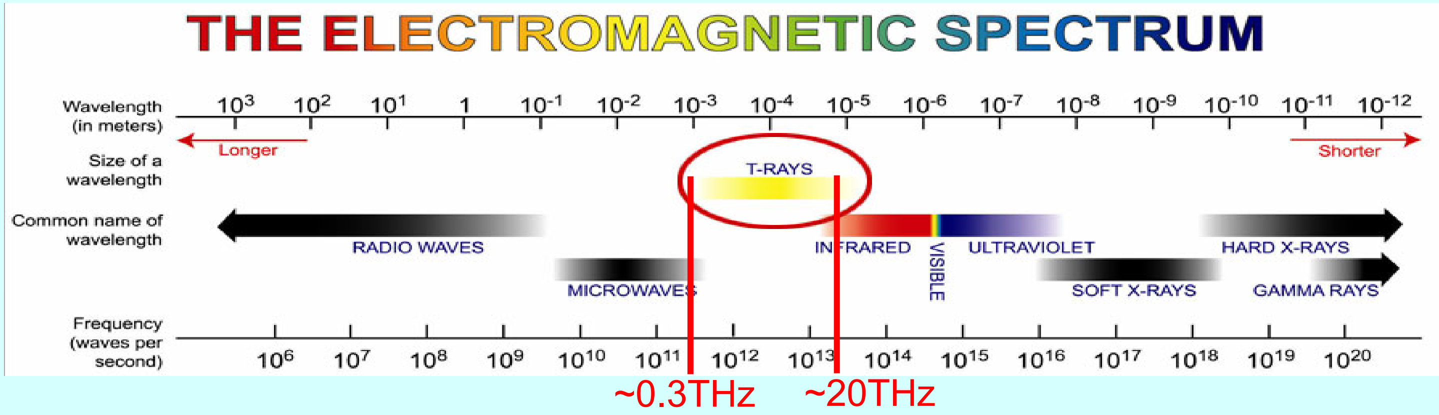
$Bi_2Sr_2CaCu_2O_{8+\delta}$ (BSCCO) T_c up to 90 K
H. Maeda *et al.* 1988

anisotropy 400-1000

s =1.56 nm   640 junctions/μm

Δ = 30-60 meV
f < 15 THz

Terahertz em waves



Potential applications

- new spectroscopy
- medical imaging
- security screening
- quality control

No commercial efficient compact continuous coherent THz sources

QCL: Quantum Cascade Lasers
DFG: Difference-Frequency Generation

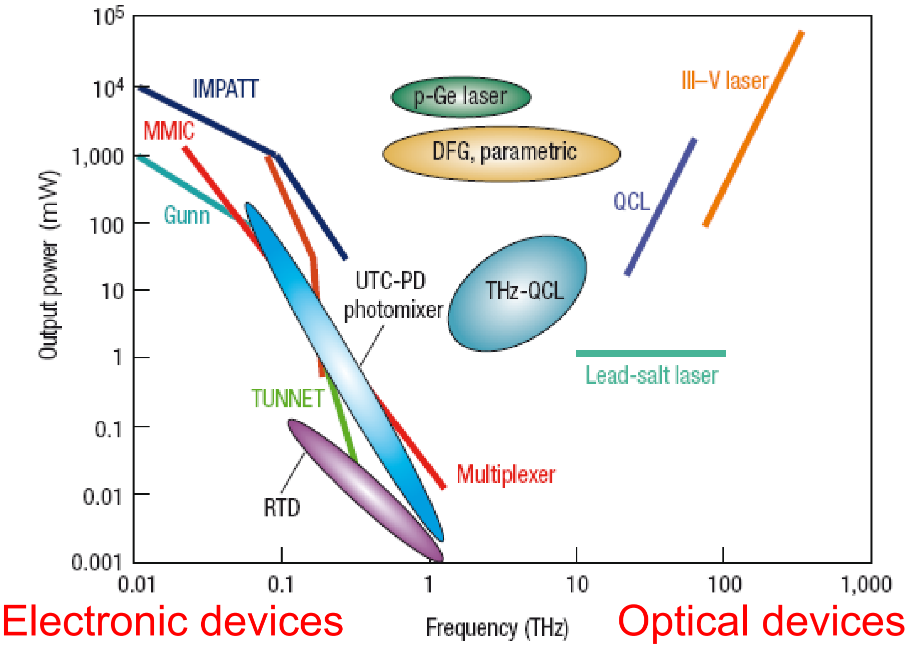
IMPATT: impact ionization avalanche transit-time diode

MMIC: microwave monolithic integrated circuit

TUNNET: tunnel injection transit-time diode

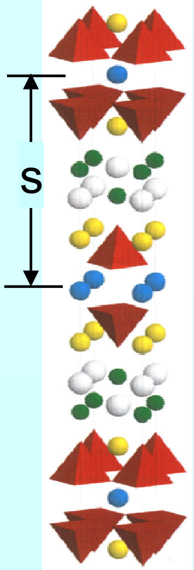
RTD: resonant tunnel diode

Tonouchi, “*Cutting-edge THz technology*”, Nature Photonics **1**, 97 (2007)



Intrinsic Josephson effect

BSCCO



- Multiple branches

  Kleiner *et al.*, 1992 …
- Fiske resonances

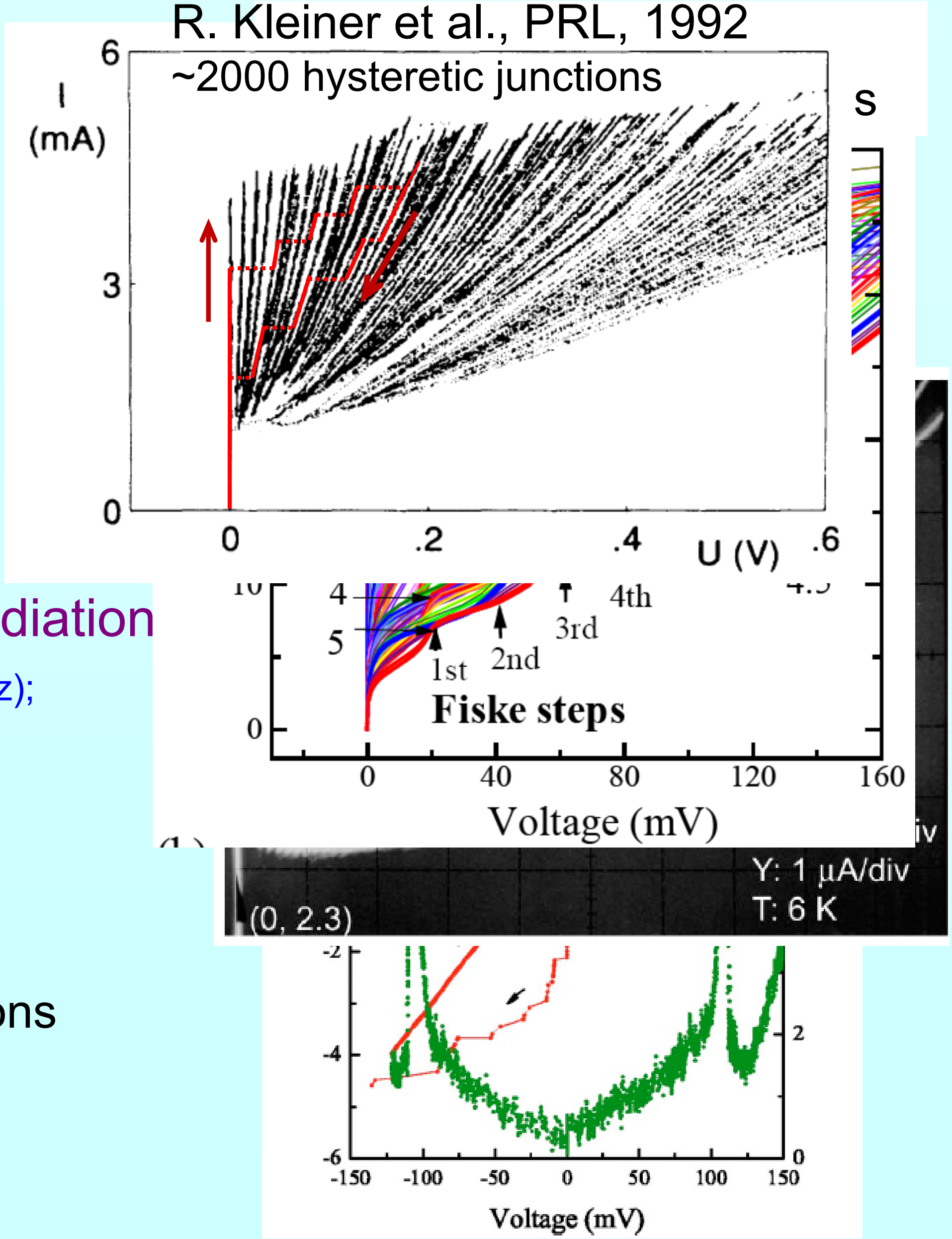
  Irie *et al.*, 1998; … Kim *et al.*, 2005
- Shapiro steps

  Wang *et al.*, 2000, 2001;

  Latyshev *et al.*, 2001 …
- Detection of Josephson radiation

  Hechtfischer *et al.*, 1997 (6-120GHz);

  Batov *et al.*, 2006 (0.5THz)…
- …


R. Kleiner et al., PRL, 1992

~2000 hysteretic junctions

Challenge:

Synchronize oscillations in all junctions

Concept: Using <u>internal</u> resonance as a synchronizator

Phase dynamics in stacks

Maxwell equations
+ material relations
for superconductor

→

Reduced equations for phases φ_n and fields h_n

$$j_z = \sigma_c E_z + j_J \sin\varphi_n$$

$$j_x = \sigma_{ab} E_x + \frac{c\Phi_0}{8\pi^2\lambda_{ab}^2} p_n$$

$$E_z \approx \frac{\Phi_0}{2\pi c s}\frac{\partial\varphi_n}{\partial t}; E_x \approx \frac{\Phi_0}{2\pi c}\frac{\partial p_n}{\partial t}$$

$$\boxed{\begin{gathered}\frac{\partial^2\varphi_n}{\partial t^2} + \nu_c \frac{\partial\varphi_n}{\partial t} + \sin\varphi_n - l^2 \frac{\partial h_n}{\partial x} = 0 \\ \left(l^2\nabla_n^2 - 1\right)h_n + \frac{\partial\varphi_n}{\partial x} + \nu_{ab}\frac{\partial}{\partial t}\left(\frac{\partial\varphi_n}{\partial x} - h_n\right) = 0\end{gathered}}$$

Sakai *et. al.*, 1993; Bulaevskii *et. al.*, 1994

Parameters:

λ_{ab}, λ_c London penetration depths

$\gamma = \lambda_c/\lambda_{ab}$ anisotropy

$\omega_p = c/\sqrt{\varepsilon_c}\lambda_c$ plasma frequency

σ_{ab}, σ_c quasiparticle conductivities

s interlayer spacing

$\lambda_J = \gamma s$ Josephson length

Reduced parameters:

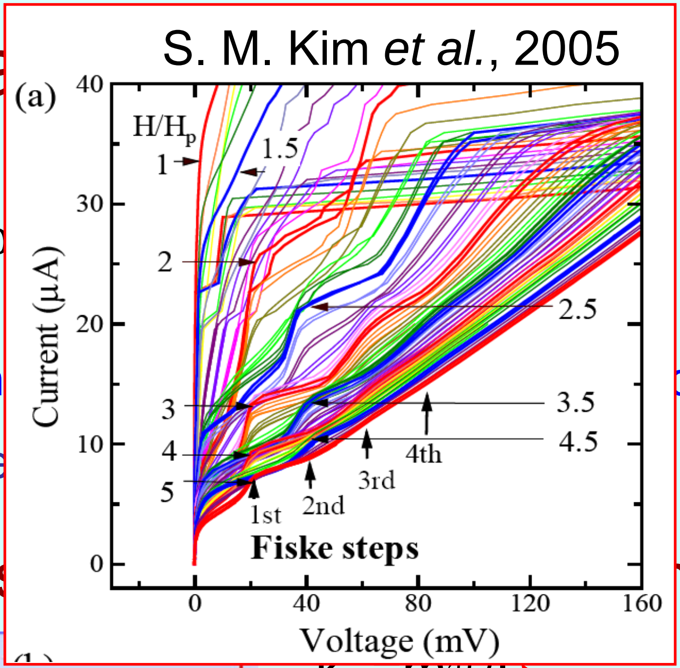
$$h = \frac{2\pi\gamma s^2 B}{\Phi_0} \quad t \to \omega_p t \quad x \to x/\lambda_J$$

$$\nu_c = \frac{4\pi\sigma_c}{\varepsilon_c\omega_p} \ (\sim 0.002)$$

$$\nu_{ab} = \frac{4\pi\sigma_{ab}}{\varepsilon_c\gamma^2\omega_p} \ (\sim 0.1) \quad l = \lambda_{ab}/s$$

Plasma and Fiske modes

S. M. Kim *et al.*, 2005



$\omega_p^2(k,q) = \omega$

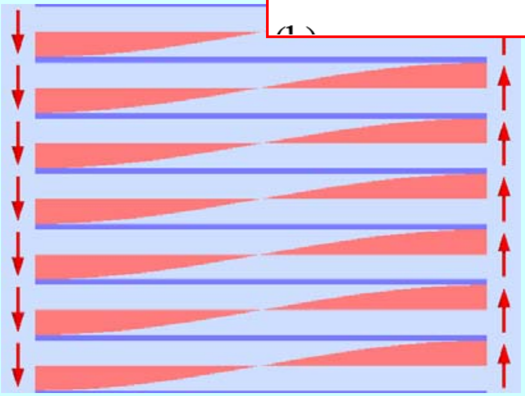
$c_0 = c/\sqrt{\varepsilon_c}$

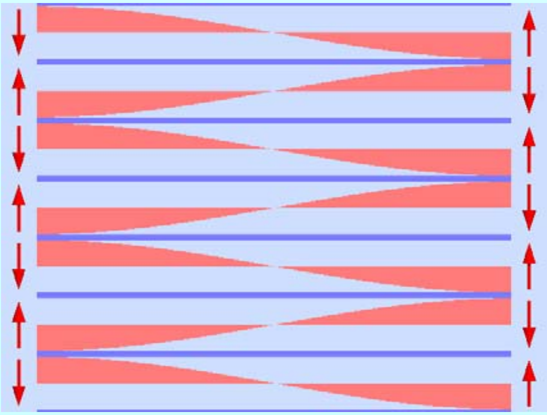
ω_{p0} Joseph don penetration depth

q = 0: in-phase phase mode, $c_\pi = \underbrace{(s/2\lambda_{ab})}_{\sim 1/300} c_0$

Finite-size s nodes

$k_m = m\pi/L$





In-phase mode, $\omega_{m,0} = c_0 k_m$

$\omega_{1,0}/2\pi$ = 1THz at L=43 μm

- Can synchronize oscillations in many junctions
- Generates outgoing radiation

Anti-phase, $\omega_{m,\pi} = c_\pi k_m$

- Excited by Josephson vortex lattice

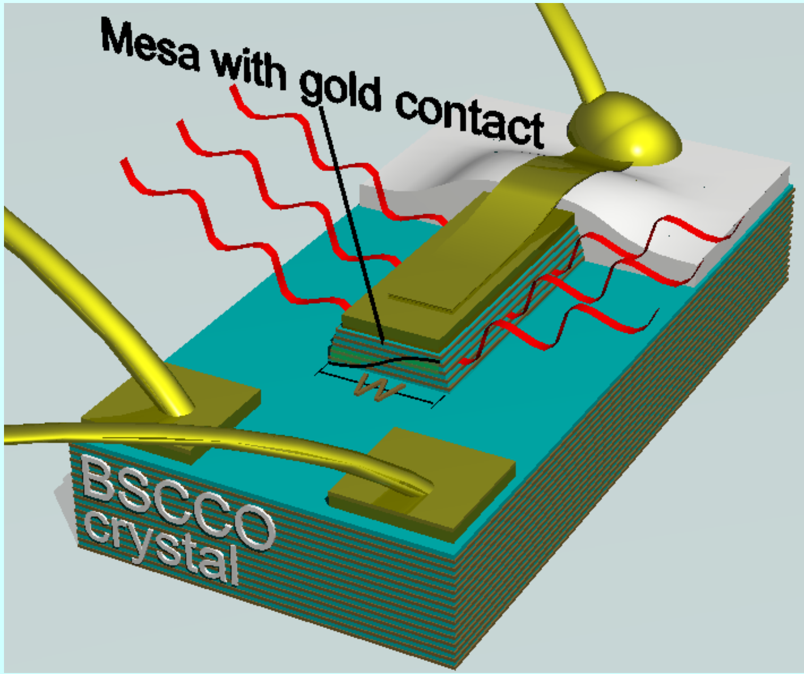
(Fiske steps in magnetic field)

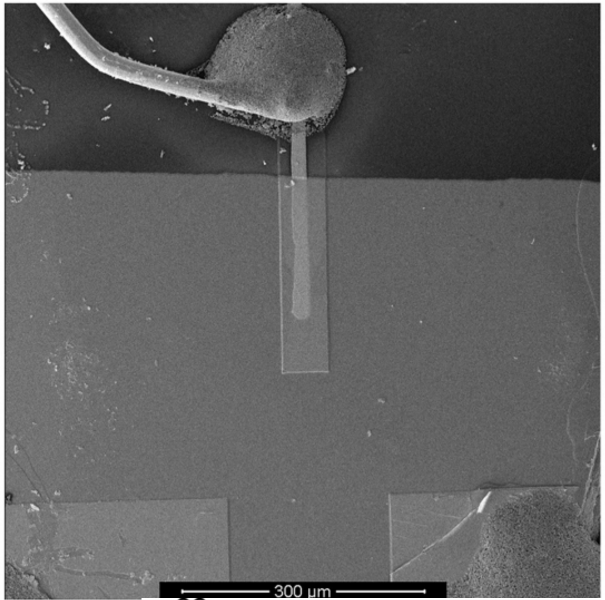
Homogeneous oscillations at H=0 → no direct coupling

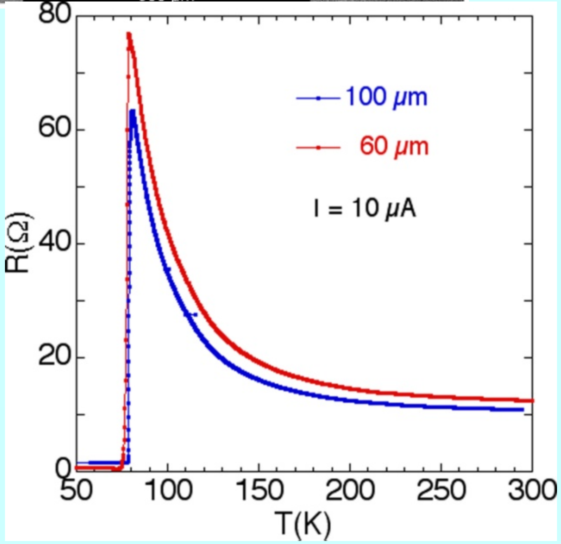
Coherent THz radiation from *$Bi_2Sr_2CaCu_2O_8$* mesas

Ozyuzer, *et al*., Science **318**, 1291 (2007)

Recent Review: Welp, Kadowaki and Kleiner, NATURE PHOTONICS **7**, 702(2013)





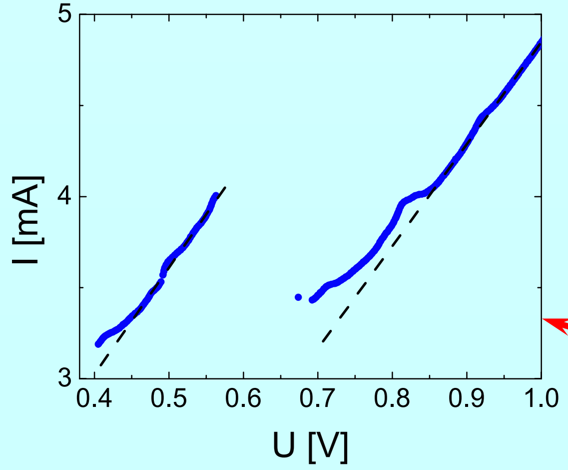


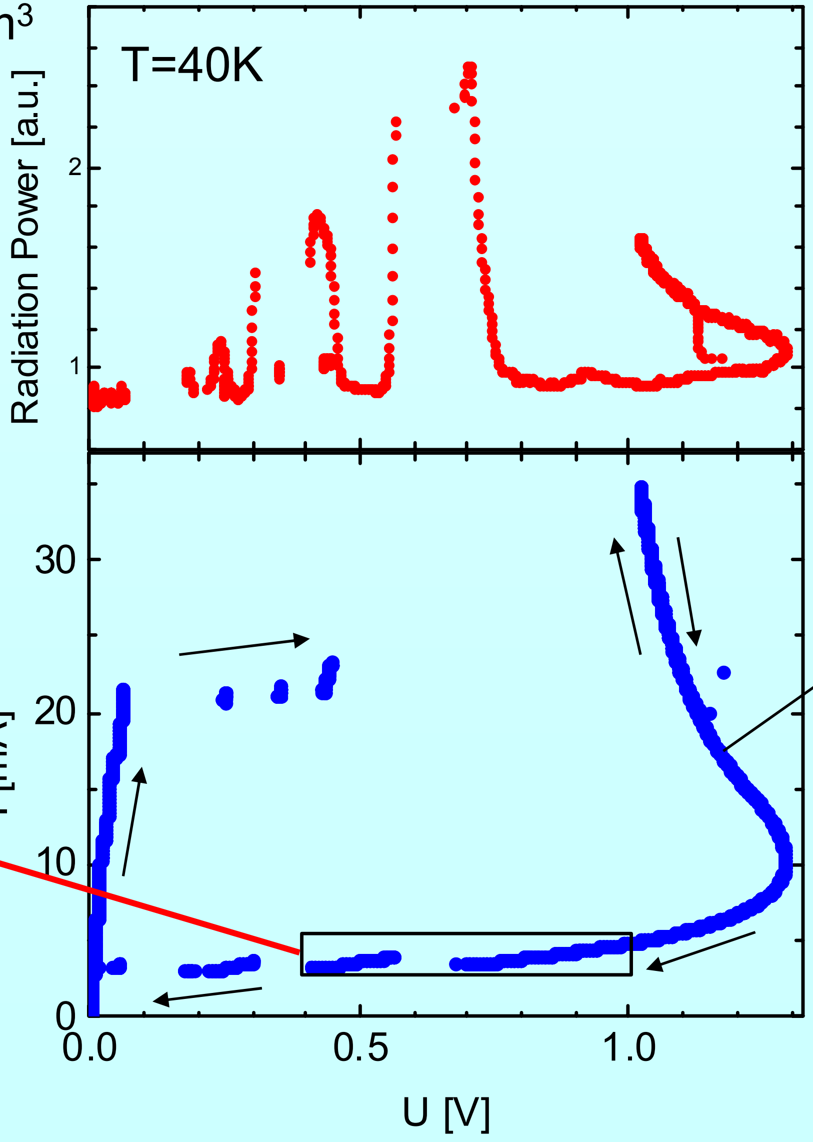
Ar-ion milling, photolithography;

w **= 40-100 µm, ~1µm high, 300 µm long**

Transport and radiation (bolometer)

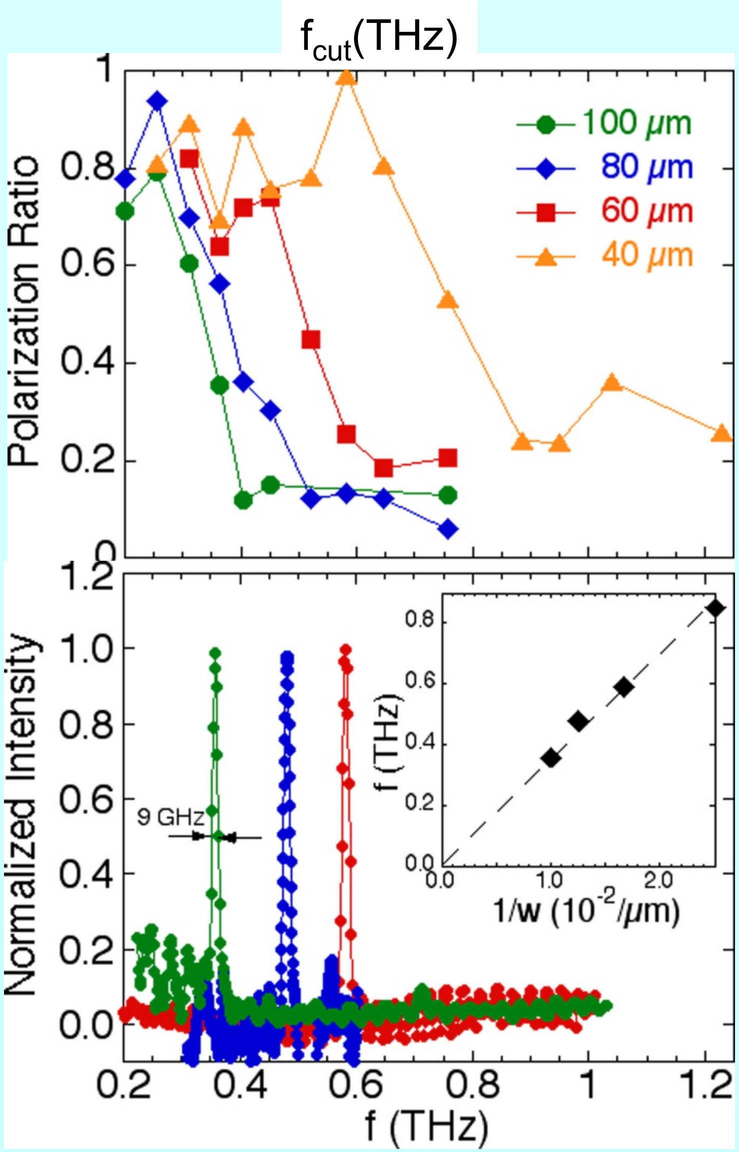
L. Ozyuzer, Mesa KK03: 100×300×1 μm^3





Heating:
larger T
↓
smaller R
↓
smaller V

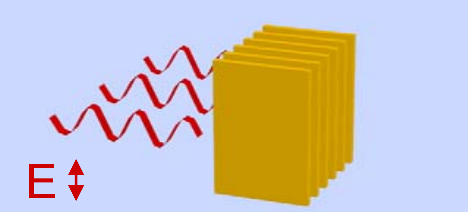
Radiation frequency

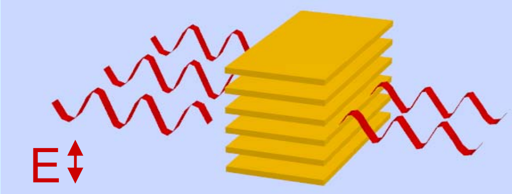


Parallel-plate filters:

TE: cut-off for waves with $f < f_{cut} = c/2d$

TM: no cut-off



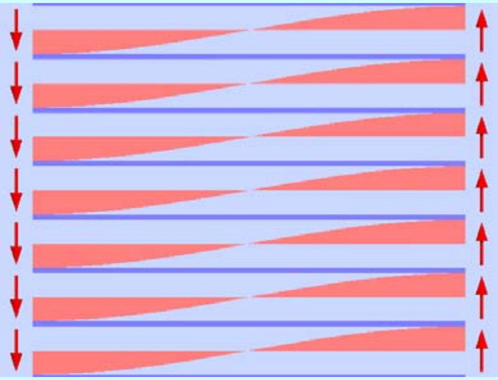


Frequency:

1. Satisfies Josephson relation
2. Increases with decreasing width, roughly ~*1/w*:

cavity resonance $f = c_0/2nw$;

f = 0.52 THz for w = 80 μm, n ≈ 3.5



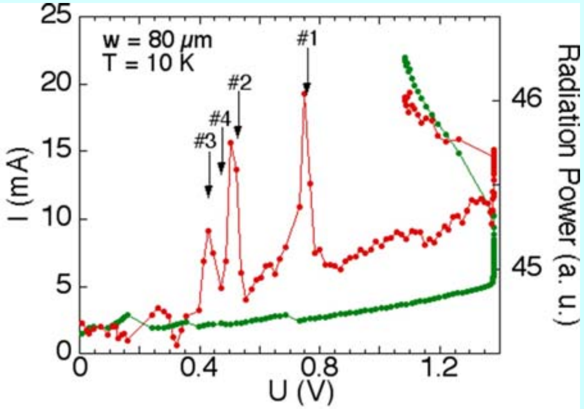
Emitted power:
Initial: ~0.5 μW
Now: up to 50 μW

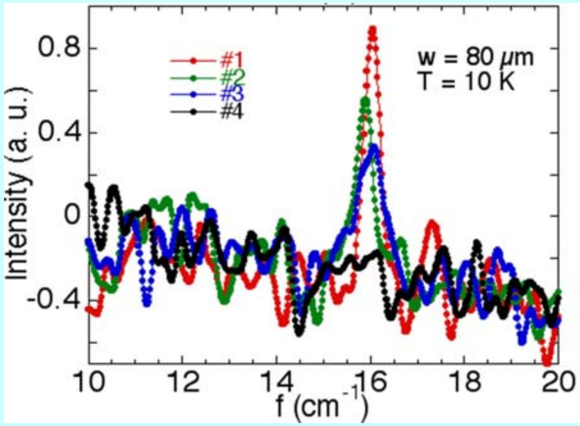
linewidth ~ 9 GHz, instrument resolution is 7.5 GHz

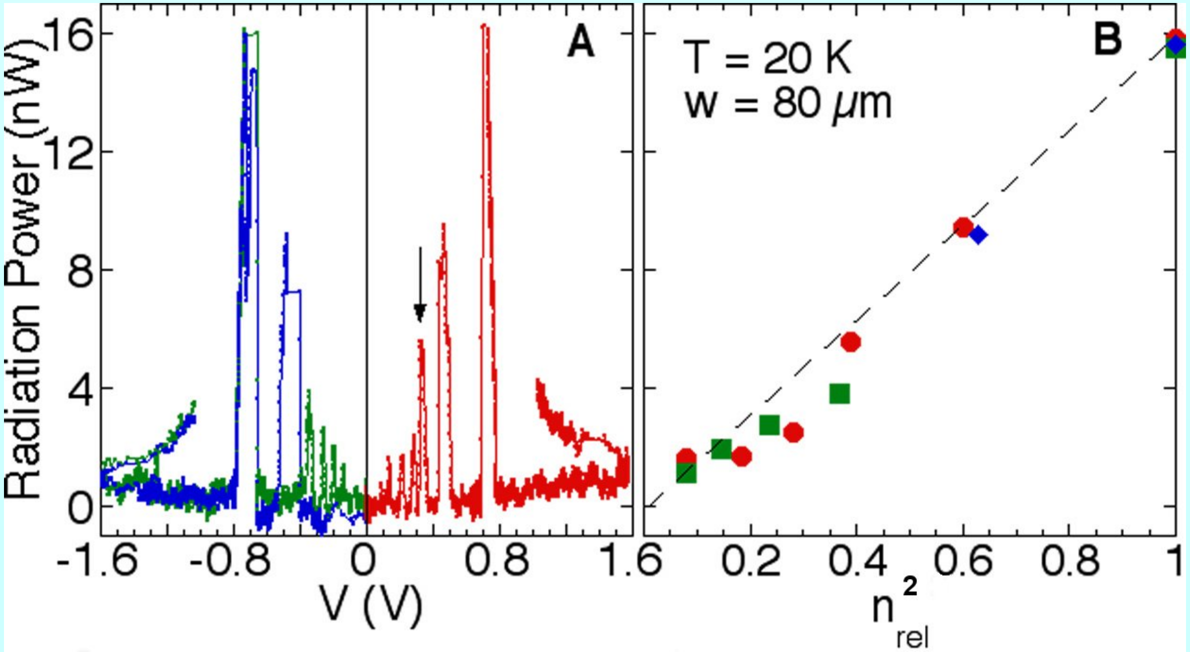
Coherent emission

Variable number of emitters *n* (= resistive junctions)

Radiation frequency does not depend on *n*

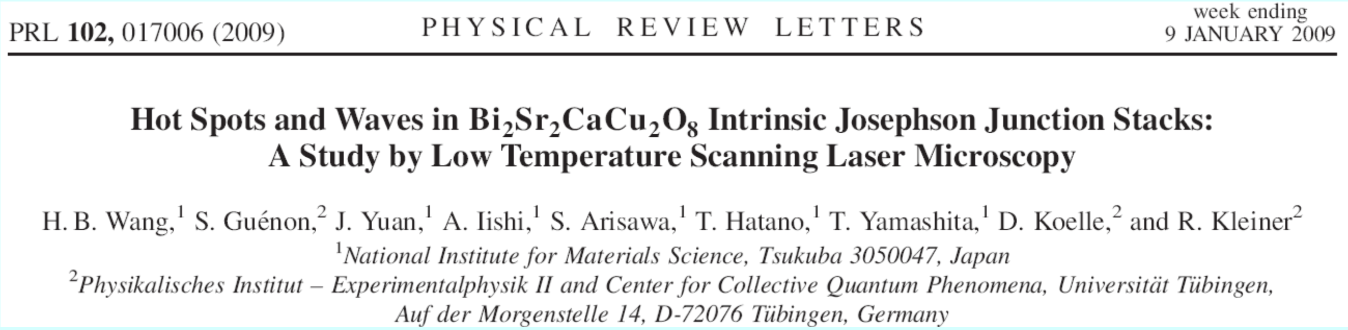






Coherent: *$P \sim n^2$*

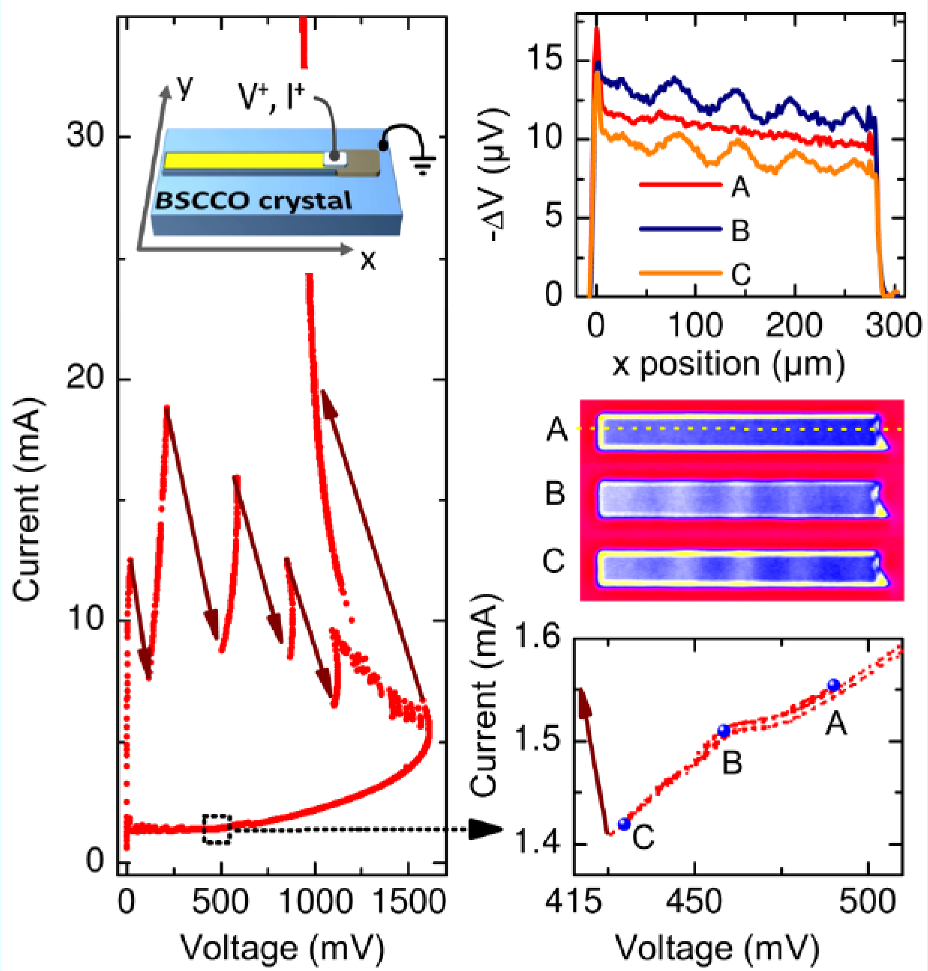
Direct observation of standing waves in mesas



PRL **102,** 017006 (2009) PHYSICAL REVIEW LETTERS week ending 9 JANUARY 2009

Hot Spots and Waves in $Bi_2Sr_2CaCu_2O_8$ Intrinsic Josephson Junction Stacks: A Study by Low Temperature Scanning Laser Microscopy

H. B. Wang,[1] S. Guénon,[2] J. Yuan,[1] A. Iishi,[1] S. Arisawa,[1] T. Hatano,[1] T. Yamashita,[1] D. Koelle,[2] and R. Kleiner[2]
[1]*National Institute for Materials Science, Tsukuba 3050047, Japan*
[2]*Physikalisches Institut – Experimentalphysik II and Center for Collective Quantum Phenomena, Universität Tübingen, Auf der Morgenstelle 14, D-72076 Tübingen, Germany*

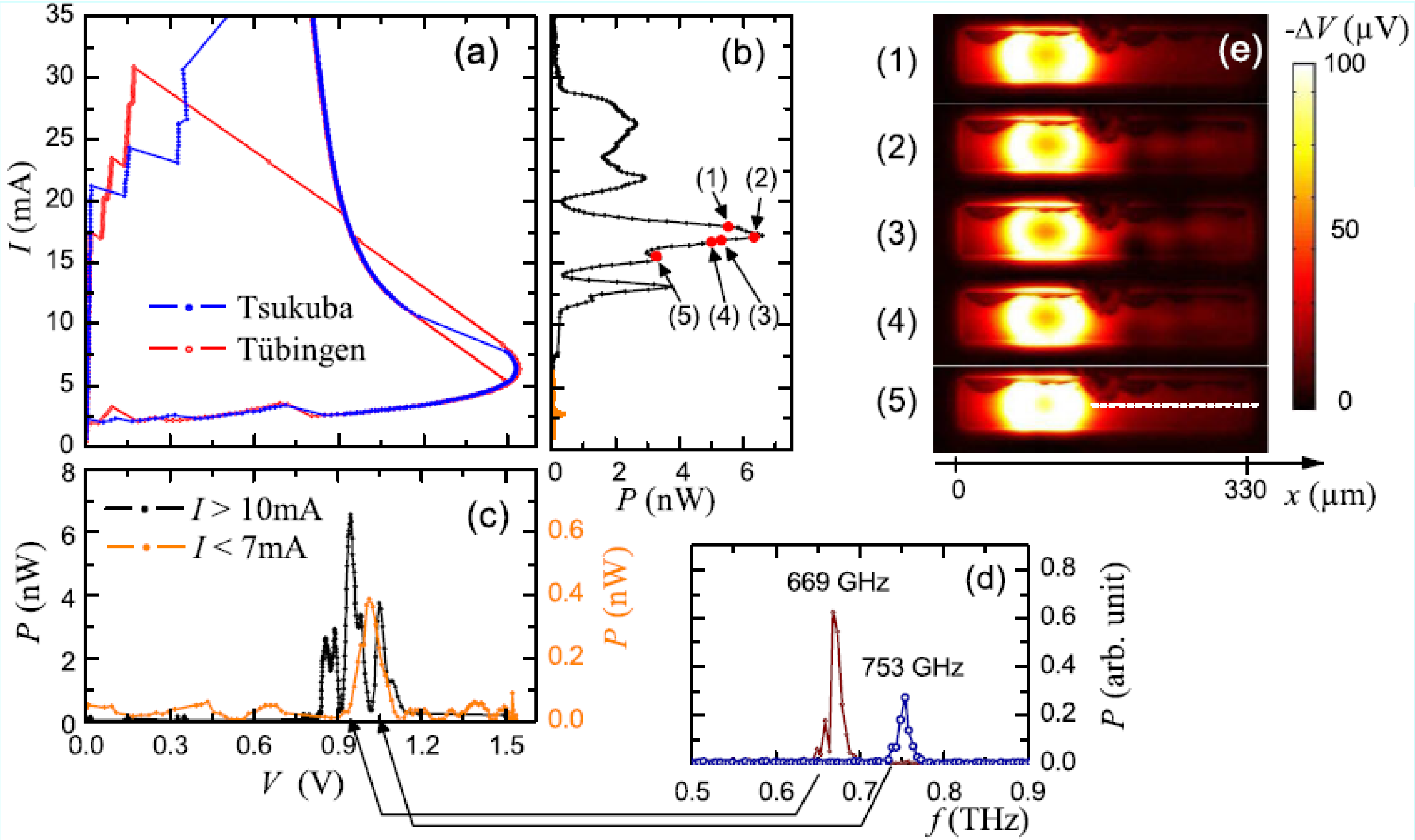


Emissions in low-bias and high-bias regimes

PRL **105,** 057002 (2010) PHYSICAL REVIEW LETTERS week ending
30 JULY 2010

Coherent Terahertz Emission of Intrinsic Josephson Junction Stacks in the Hot Spot Regime

H. B. Wang,[1] S. Guénon,[2] B. Gross,[2] J. Yuan,[1] Z. G. Jiang,[3] Y. Y. Zhong,[3] M. Grünzweig,[2] A. Iishi,[1] P. H. Wu,[3] T. Hatano,[1]
D. Koelle,[2] and R. Kleiner[2]



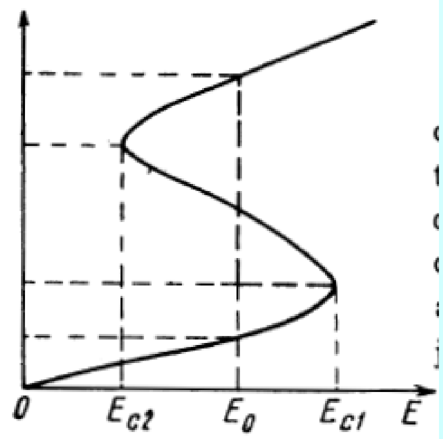
Hot-spot instability

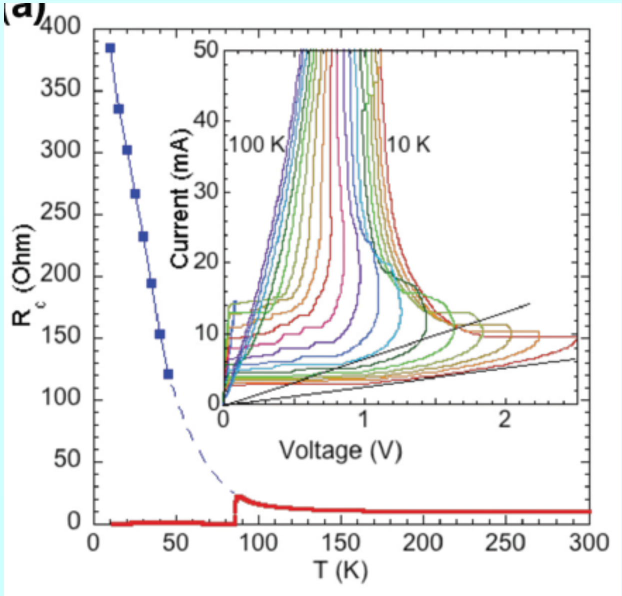
SOVIET PHYSICS JETP VOLUME 25, NUMBER 6 DECEMBER, 1967

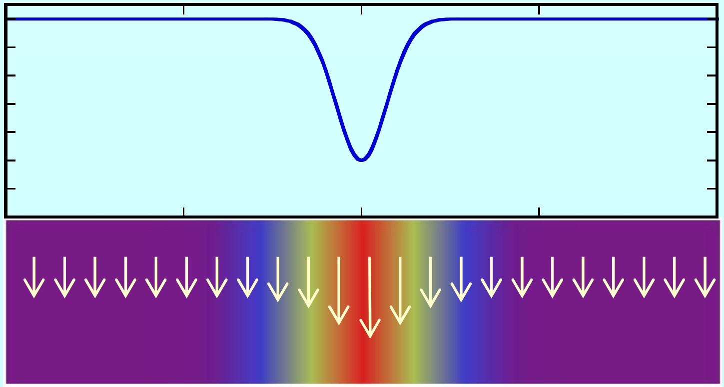
NONUNIFORM CURRENT DISTRIBUTION IN SEMICONDUCTORS WITH NEGATIVE DIFFERENTIAL CONDUCTIVITY

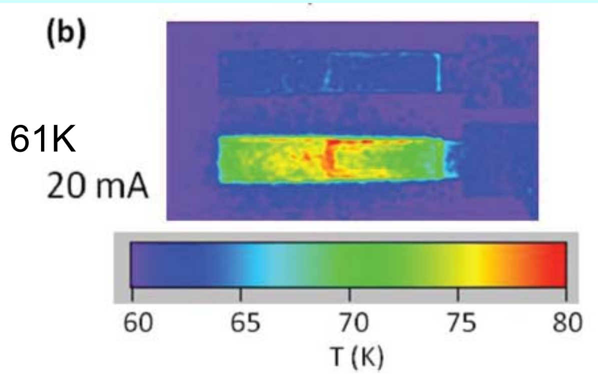
A. F. VOLKOV and Sh. M. KOGAN

Institute of Radio Engineering and Electronics, Academy of Sciences, U.S.S.R.



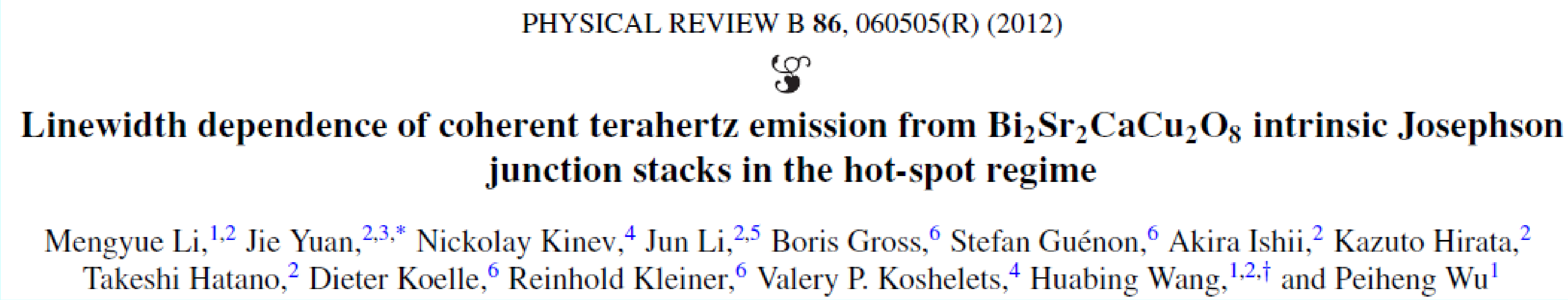






Benseman *et al.*,
J. of Appl. Phys. **113**, 133902 (2013)

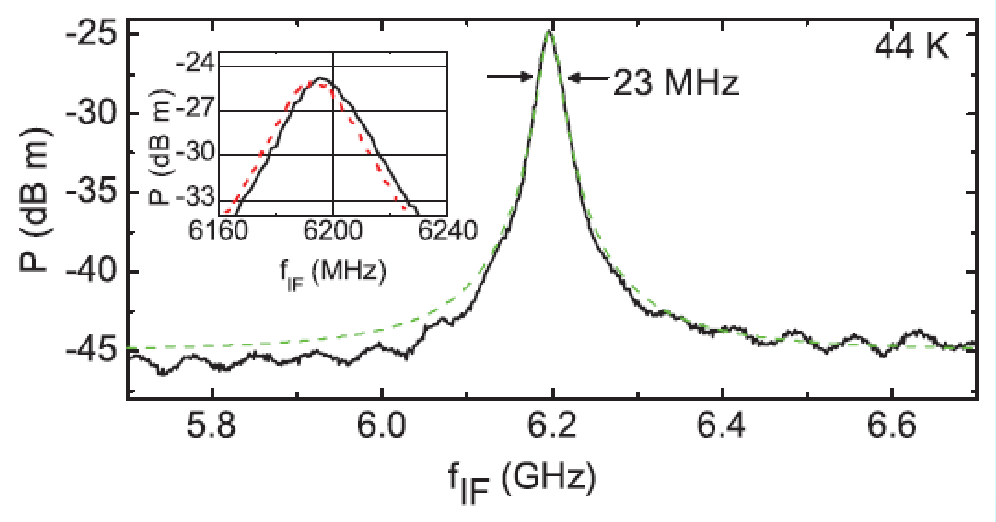
Measurements of line shape



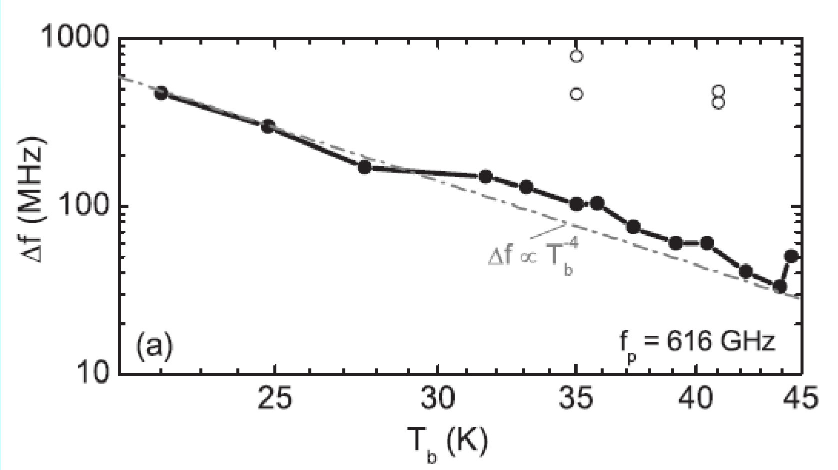
PHYSICAL REVIEW B **86**, 060505(R) (2012)

Linewidth dependence of coherent terahertz emission from $Bi_2Sr_2CaCu_2O_8$ intrinsic Josephson junction stacks in the hot-spot regime

Mengyue Li,[1,2] Jie Yuan,[2,3,*] Nickolay Kinev,[4] Jun Li,[2,5] Boris Gross,[6] Stefan Guénon,[6] Akira Ishii,[2] Kazuto Hirata,[2]
Takeshi Hatano,[2] Dieter Koelle,[6] Reinhold Kleiner,[6] Valery P. Koshelets,[4] Huabing Wang,[1,2,†] and Peiheng Wu[1]



Unexpected T dependence



Detector:
Nb/AlN/NbN integrated receiver
(Koshelets group)

Scientific issues

- Coupling to the resonance mode
- Structure and stability of coherent states
- Mechanisms of damping of cavity mode
- Limits of radiation power
- Mechanisms of line width

Excitation of in-phase cavity mode

Homogeneous state + External modulation, AEK and L. Bulaevskii, PR B **77**, 014530 (2008)

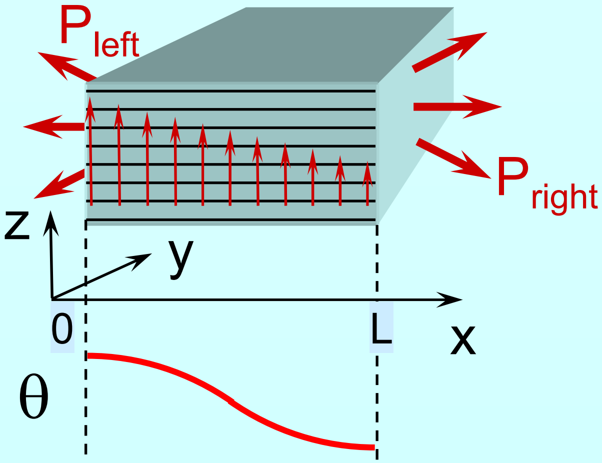
Lateral modulation of the Josephson critical current, $j_J(x)=j_J g(x)$

Equation for c-axis homogeneous phase (reduced form)

$$\frac{\partial^2\theta}{\partial t^2}+\nu_c\frac{\partial\theta}{\partial t}+g(x)\sin\theta-\frac{\partial^2\theta}{\partial x^2}=0$$

+ radiative boundary conditions

$1/\omega_p$ unit of time
λ_c unit of length



Resistive state near resonance $\omega_1 = c_0\pi/L$

$$\theta(x,t) = \omega t + Re[\psi \exp(-i\omega t)] \cos(\pi x/L)$$

Mode amplitude: $$\psi \approx \frac{ig_1}{\omega^2-\omega_1^2+i\nu\omega}$$

Coupling parameter

$$g_1=\frac{2}{L}\int_0^L \cos(\pi x/L)g(x)dx$$

Damping parameters: $\nu = \nu_c + \nu_r + \nu_b \ldots$

$$\nu_c=\frac{4\pi\sigma_c}{\varepsilon_c\omega_p}$$ quasiparticle ~ 10^{-2}-10^{-3}

radiation from edges

radiation to crystal

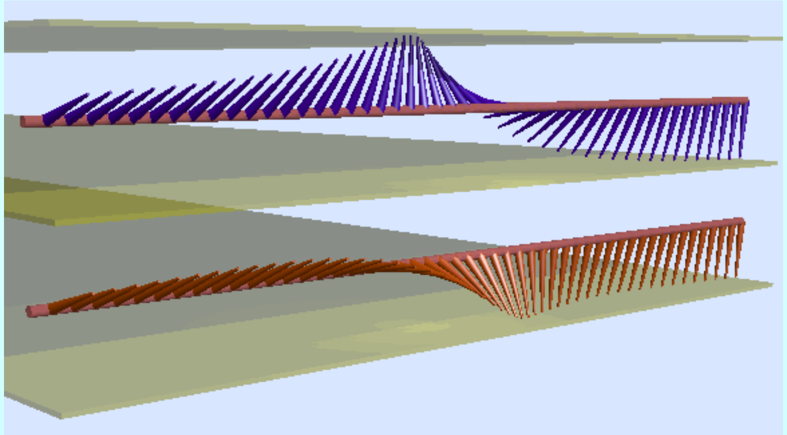
Cavity quality factor $$Q_c=\frac{\omega}{\nu\omega_p}$$

Alternating kink state

S.-Z. Lin and X. Hu, PRL **100**, 247006 (2008); AEK, Phys. Rev. B **78**, 174509 (2008)

$$\varphi_n(x,t) \approx \omega t + (-1)^n \varphi_{kink}(x) + \mathrm{Re}\left[\psi \exp(-i\omega t)\right]\cos(\pi x/L) + \ldots$$

Static soliton (kink) at *x* = *L/2*



$$\varphi_{kink}(0) \approx 0;\ \varphi_{kink}(L) \approx \pi$$

kink width $l_s \approx \left(\dfrac{L\lambda_J^2}{16}\dfrac{\omega_1^2-\omega^2}{\omega_p^2}\right)^{1/3} \ll L_x$

Effective modulation

$$g(x) = \cos\left[\varphi_{kink}(x)\right] \approx -sign(x - L/2)$$

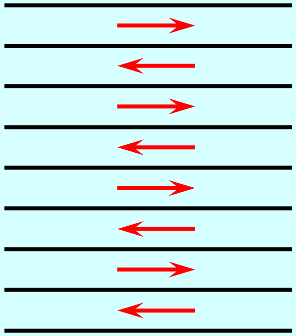
$$\psi = \frac{ig_1}{\omega^2 - \omega_1^2 + i\nu\omega} \qquad g_1 = \frac{2}{L}\int_0^L \cos(\pi x/L)\cos(\varphi_{kink})dx \approx 4/\pi$$

Maximum possible coupling

- exists without external modulations
- provides efficient pumping of energy into the cavity mode

Short-scale instability

A.E.K., Phys. Rev. B **82, 174512 2010**



Local plasma frequency $\omega_p(x) \propto \sqrt{g(x)\bar{C}(x)}$

$$\bar{C}(x) \equiv \left\langle \cos\left(\omega t + \theta(x,t)\right)\right\rangle_t \approx \frac{g_1(\omega_1^2 - \omega^2)}{(\omega_1^2 - \omega^2)^2 + \nu^2\omega^2} \frac{\cos(\pi x / L)}{2}$$

For decreasing positive g(x) $g(x)\bar{C}(x) < 0$ at x > L/2 → source of instability !!

Homogeneous state is only stable if g(x) = 0 at x > L/2

For alternating-kink state g(x) ≈ -sgn(x - L/2) and

$$g(x)\bar{C}(x) = \frac{(\omega_1^2 - \omega^2)}{(\omega_1^2 - \omega^2)^2 + \nu^2\omega^2} \frac{|\cos(\pi x / L)|}{2} \geq 0$$

→ stable

Radiative boundary conditions for oscillating phase

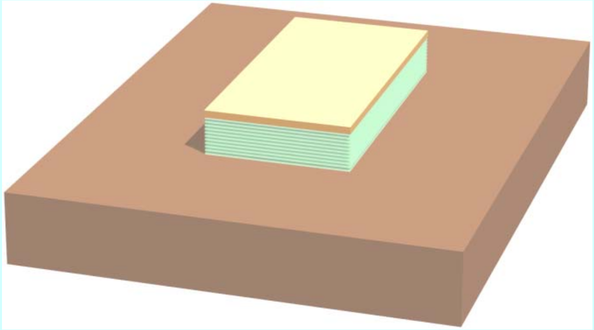
A.E.K. and Bulaevskii, Phys. Rev. B **77**, 014530 (2008)

Phase in resistive state

$$\varphi_n(x,t) \approx \omega t + Re[\theta_{n,\omega}(x) \exp(-i\omega t)]$$

Boundary conditions for homogeneous oscillating phase $\theta_\omega(x) = \left\langle \theta_{\omega,n}(x) \right\rangle_n$

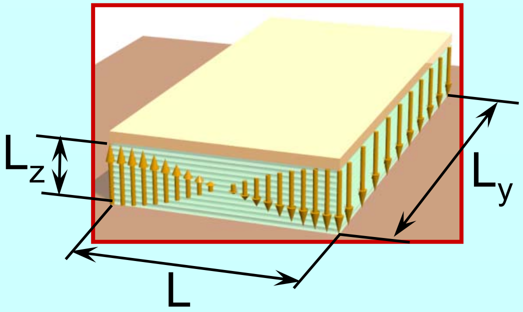
Long symmetric mesa



$$\frac{\partial \theta_\omega(L)}{\partial x} = i\zeta\theta_\omega(L) + i\tilde{\zeta}\theta_\omega(0)$$

$$\frac{\partial \theta_\omega(0)}{\partial x} = -i\zeta\theta_\omega(0) - i\tilde{\zeta}\theta_\omega(L)$$

For isolated mesa on metallic plate
with thin metallic contact on the top



$$\zeta \approx \frac{k_\omega^2 L_z}{2}\left[1 - \frac{2i}{\pi}\ln\frac{C}{k_\omega L_x}\right]$$

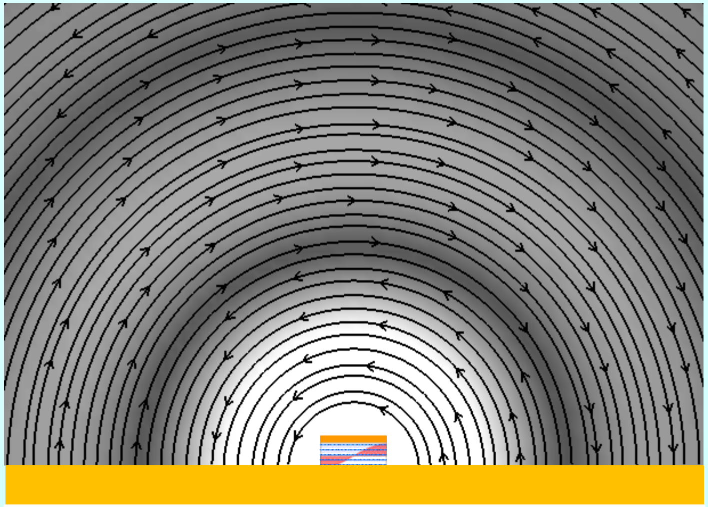
$k_\omega = \omega/c$

$$\tilde{\zeta} \approx -\frac{k_\omega^2 L_z}{2} H_0^{(1)}(k_\omega L)$$

Radiation dampings

1. Radiation into free space
(sensitive to mode and geometry)

$$\nu_r = \frac{4\omega_p \lambda_c^2}{\omega L} \mathrm{Re}\left[\zeta - \tilde{\xi}\right] = \frac{2\omega L_z}{\varepsilon_c \omega_p L}\left[1 + J_0(k_\omega L)\right] \propto \frac{L_z}{\lambda_\omega}$$



Stand alone mesa

Kashiwagi *et al. JJAP, 2012*

An *et al.* APL, 2013

2. Radiation into crystal

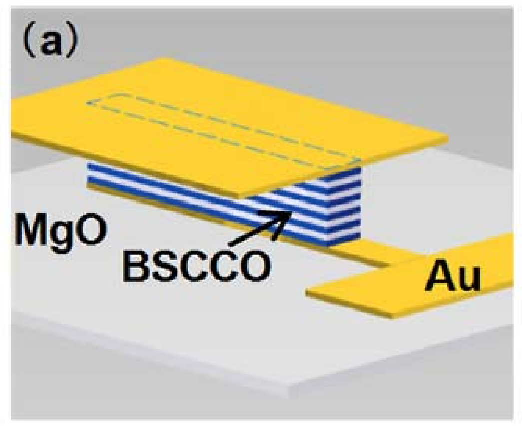
Journ. of Phys., **150**, 052124 (2009)

Power flow to the crystal:

$$P_{bottom} = \frac{C_{ab}\Phi_0^2 \lambda_{ab}\omega}{64\pi s^2 L}|\psi|^2$$

$$\nu_b = \frac{\lambda_{ab}\lambda_c}{L_x L_z} \square\ 0.5$$

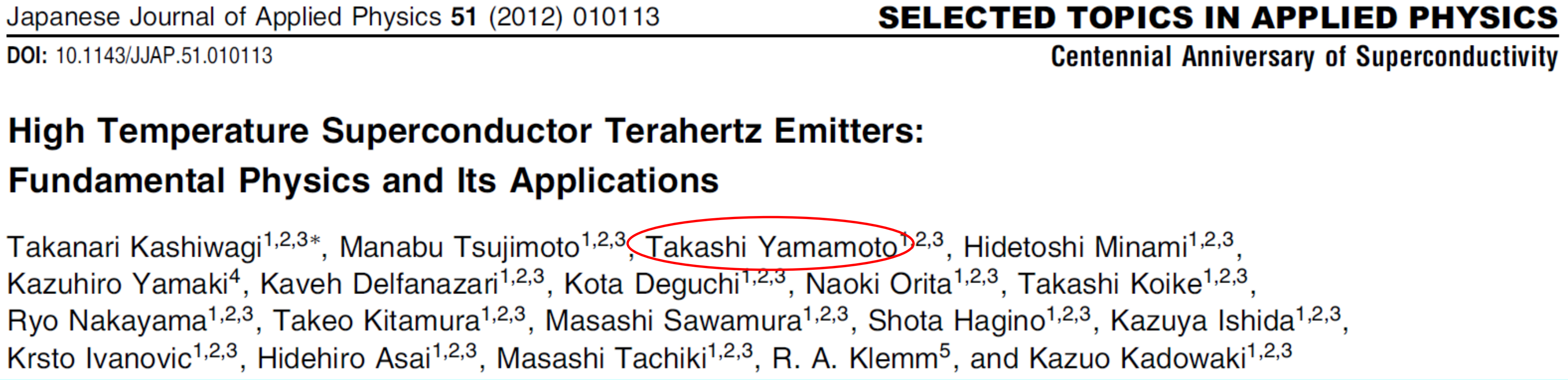
$$\frac{\nu_r}{\nu_b} \square\ \frac{L_z^2}{\varepsilon_c \lambda_{ab} L_x} \square\ 0.06$$



Dominating mechanism of damping!

Argonne
NATIONAL LABORATORY

Stand-alone mesa



Japanese Journal of Applied Physics **51** (2012) 010113

SELECTED TOPICS IN APPLIED PHYSICS

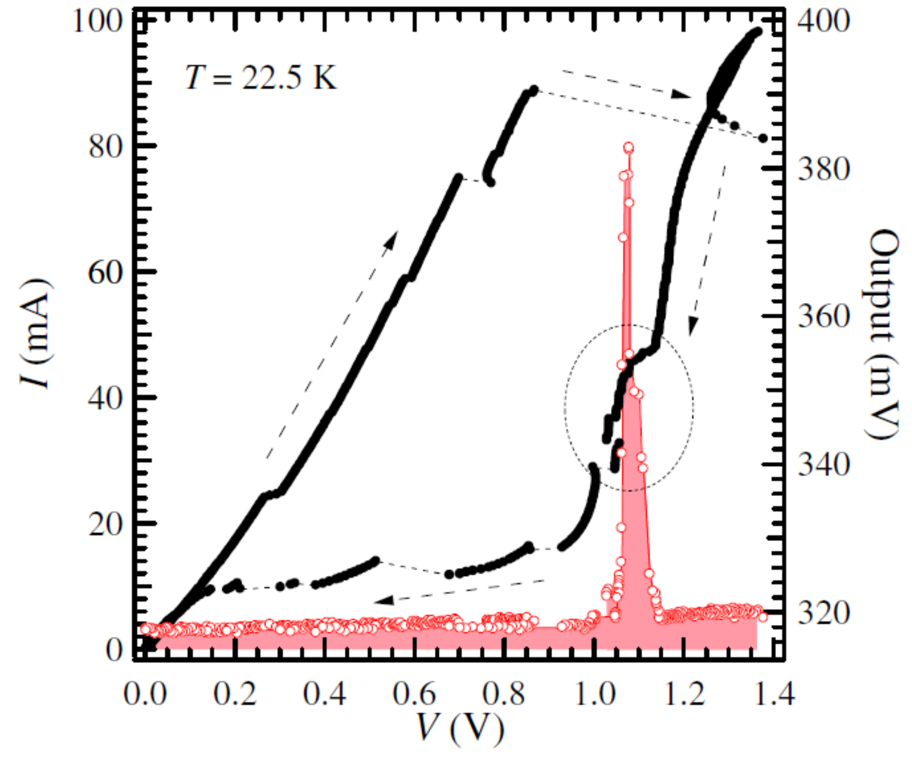
DOI: 10.1143/JJAP.51.010113

Centennial Anniversary of Superconductivity

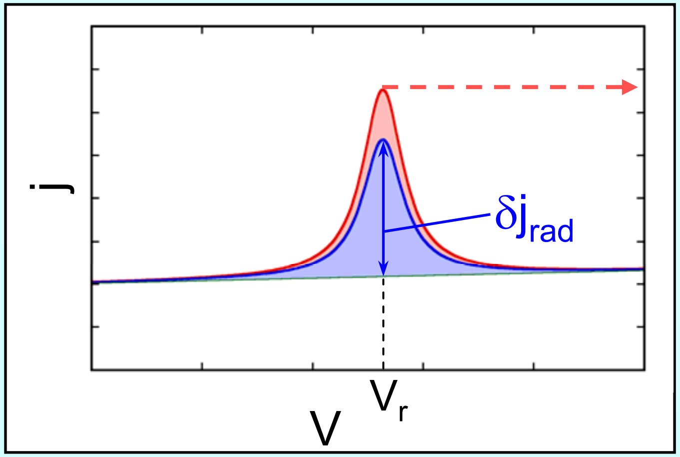
High Temperature Superconductor Terahertz Emitters: Fundamental Physics and Its Applications

Takanari Kashiwagi[1,2,3]*, Manabu Tsujimoto[1,2,3], Takashi Yamamoto[1,2,3], Hidetoshi Minami[1,2,3], Kazuhiro Yamaki[4], Kaveh Delfanazari[1,2,3], Kota Deguchi[1,2,3], Naoki Orita[1,2,3], Takashi Koike[1,2,3], Ryo Nakayama[1,2,3], Takeo Kitamura[1,2,3], Masashi Sawamura[1,2,3], Shota Hagino[1,2,3], Kazuya Ishida[1,2,3], Krsto Ivanovic[1,2,3], Hidehiro Asai[1,2,3], Masashi Tachiki[1,2,3], R. A. Klemm[5], and Kazuo Kadowaki[1,2,3]

$87 \times 383 \times 1.3\ \mu m^3$



Transport and radiation near resonance



Excess current (units of j_J)

$$\delta j = \frac{1}{4}\frac{g_1^2 \nu\omega}{[\omega^2 - \omega_1^2]^2 + \nu^2\omega^2}$$

Similar to: M. Russo and R. Vaglio, Phys. Rev. B 17, 2171 (1978)
(single junction, no radiation)

Energy balance

$$IV = V^2/R + \underbrace{\langle \tilde{E}^2 \rangle L_z^2/R + P}_{\delta I\, V}$$

Radiated power

$$P_{edge} = AL_y \frac{\Phi_0^2 \omega^3 N^2}{32\pi^3 c^2} |\psi|^2 \quad A \sim 1 \qquad P_{edge} = \frac{\nu_r}{\nu}\delta I V$$

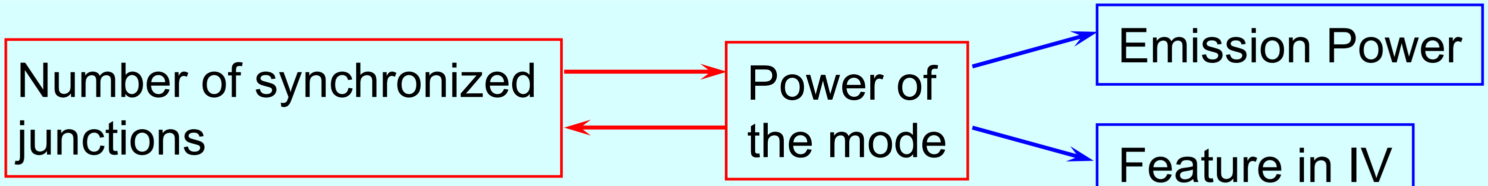
In resonance, for $\nu_r \ll \nu$ $\quad P_{edge} \propto N^2$

for $\nu \approx \nu_r$ $\quad P_{\max} \approx \frac{\pi L_y L^2 g_1^2 j_J^2}{2\omega}$ Does not depend on N!

$g_1=0.3,\ j_J=500\ A/cm^2,\ L_y=300\mu m$
$\omega/2\pi=1THz,\ L=43\ \mu m$ → P = 1.5 mW

achieved ~ 50 μW

Synchronization in inhomogeneous mesas



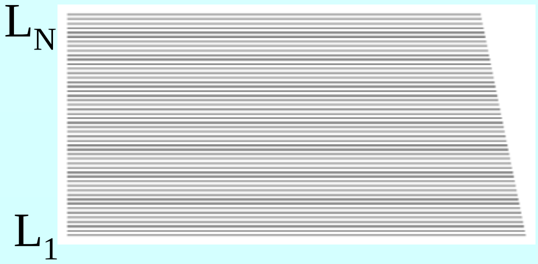
Number of synchronized junctions

Power of the mode

Emission Power

Feature in IV

Mesas with inhomogeneous cross section

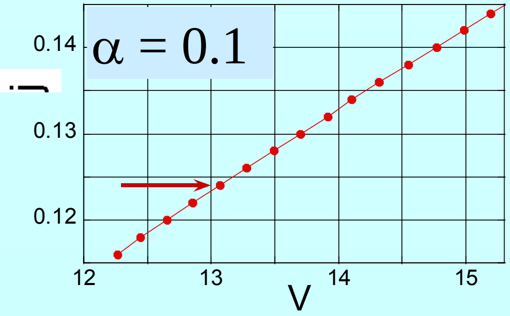


$$\alpha = (L_1 - L_N)/L_{N/2}$$

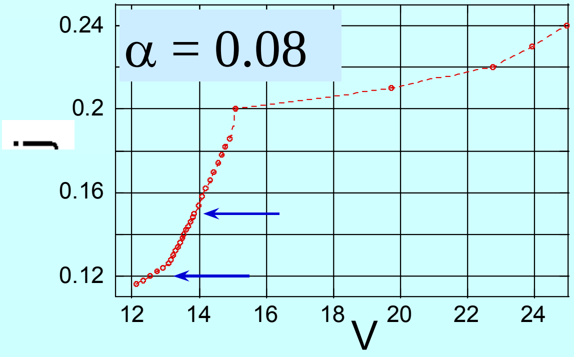
Natural frequencies:

$$\omega_n \propto V_n \propto j_n \propto 1/L_n$$

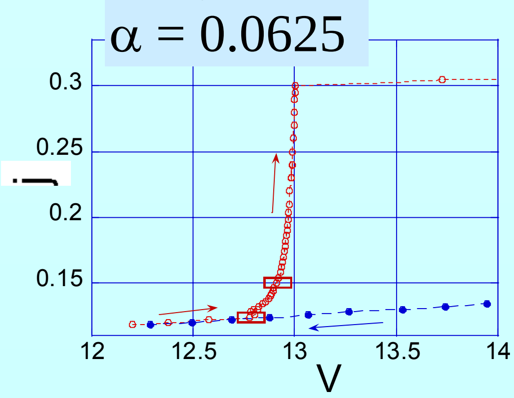
No synchronization



Partial synchronization



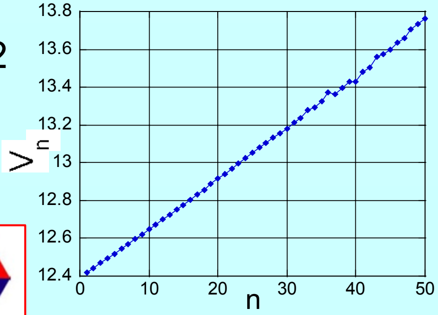
Full synchronization

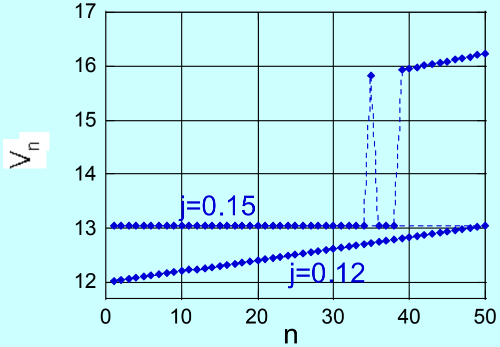


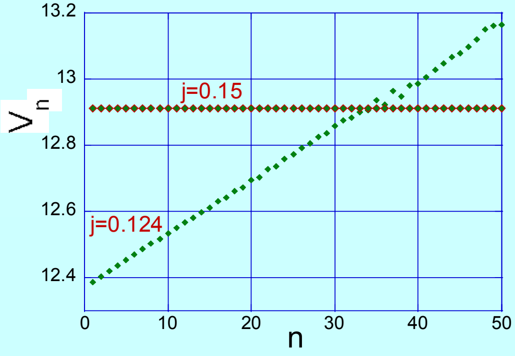
N=50

ν_c=0.01,ν_{ab}=0.2

l=50, L_1=12.5

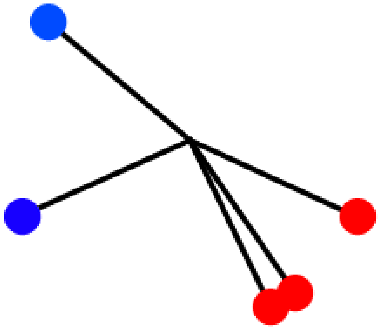


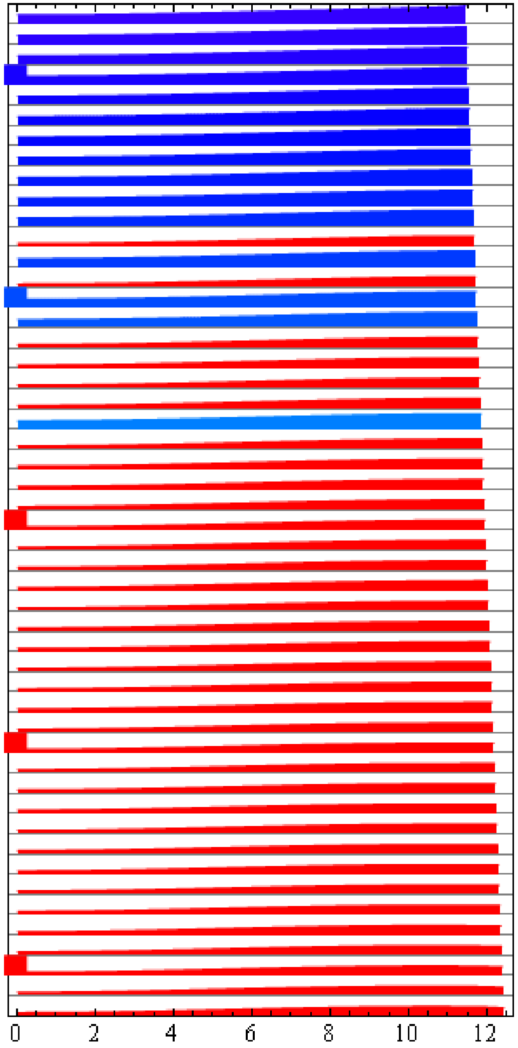


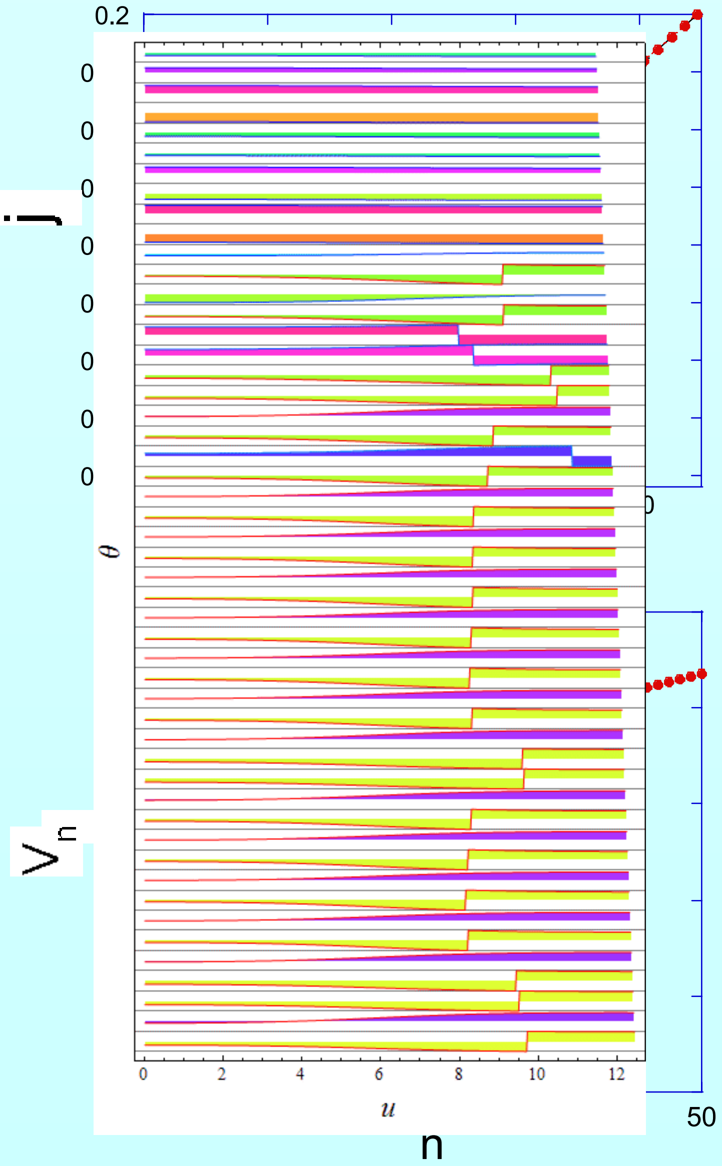


Visualization of electric field

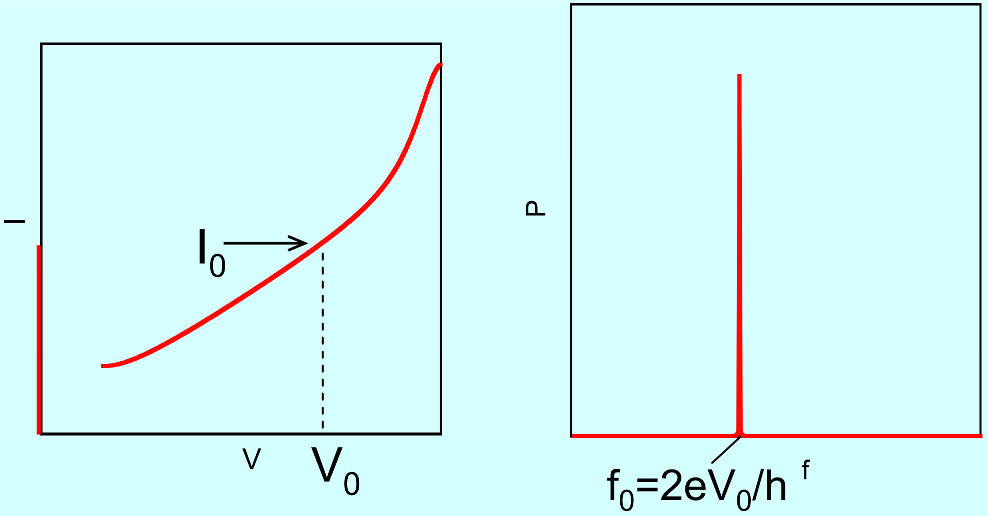
$\alpha = 0.08$





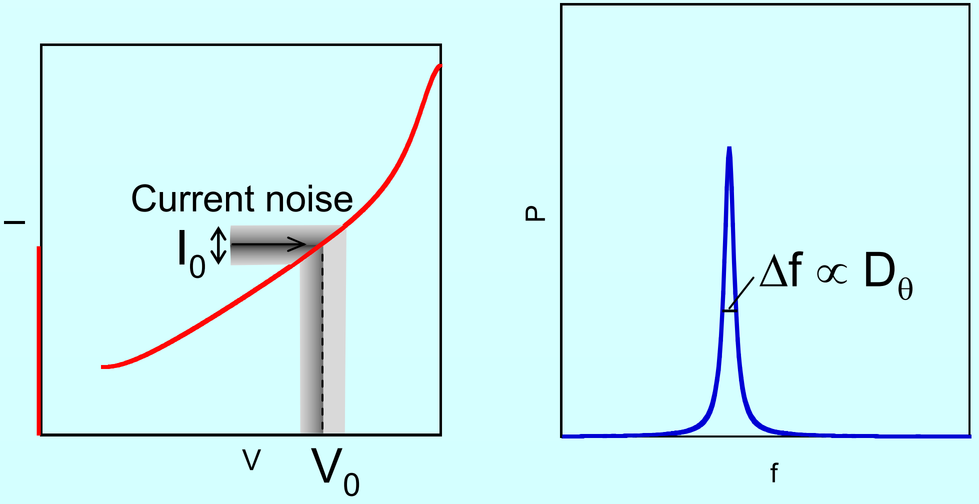


Line width of Josephson radiation



$J \propto \sin(\omega_0 t)$

Line width of Josephson radiation



$$J \propto \sin[\omega_0 t + \theta(t)] \qquad \theta^2(t) = D_\theta t$$

Single junction:

Line width due to quasiparticle-current fluctuations, $k_B T > \hbar\omega$

Larkin and Ovchinnikov, Zh. Eksp. Teor. Fiz., 1967
Dahm et al., Phys. Rev. Lett. , 1969

$$D_\theta \propto \frac{\langle j_f^2 \rangle}{\nu^2} \qquad \langle j_f^2 \rangle \propto \frac{k_B T}{R} \qquad R = V/I$$

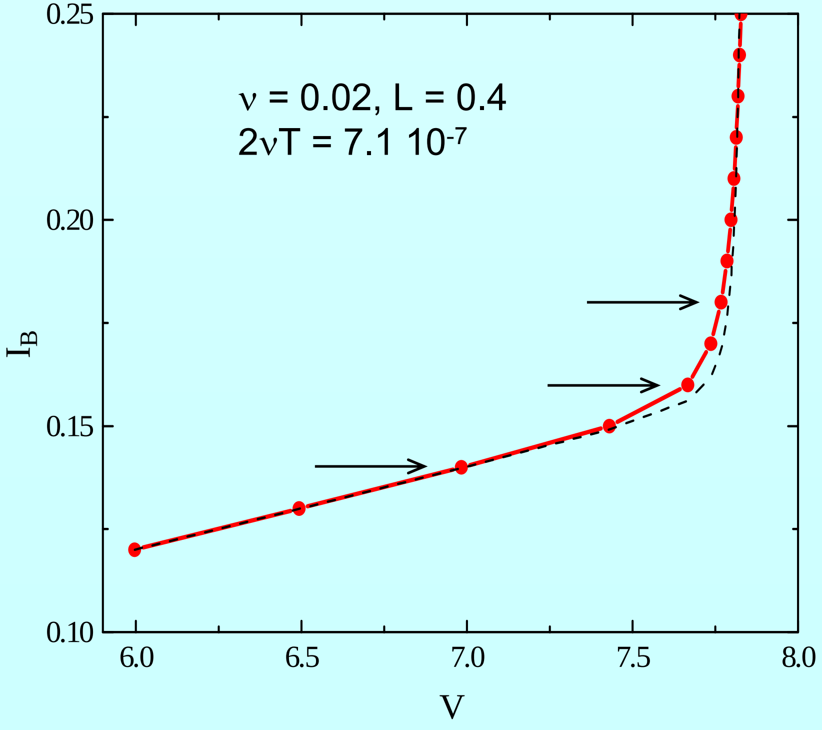
$$\nu \propto 1/R_d \qquad R_d = dV/dI$$

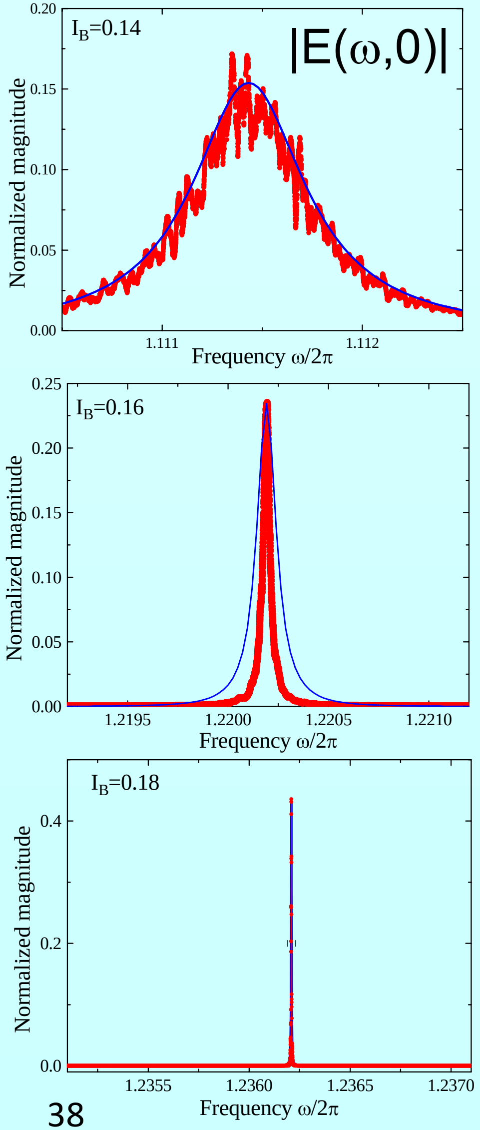
$$\Delta f = \frac{1}{\pi}\left(\frac{2e}{\hbar}\right)^2 \frac{R_d^2}{R} k_B T$$

Line shrinking near cavity resonance

Lin and AEK, Phys. Rev. B, 2013

$$\Delta f = \frac{1}{\pi}\left(\frac{2e}{\hbar}\right)^2 \frac{R_d^2}{R} k_B T$$





Line width of synchronized stack

Dominating quasiparticle current dissipation

Effective noise current

$$\tilde{J} = \frac{1}{N}\sum_{n=1}^{N}\tilde{j}_n \qquad \left\langle \left|\tilde{J}\right|^2 \right\rangle = \frac{\left\langle \left|\tilde{j}_n\right|^2 \right\rangle}{N}$$

→ 1/N narrowing

Line width $\Delta f_0 = \frac{T}{\pi N L_x L_y \nu_c \ell^2} \to \frac{4}{\pi}\frac{R}{N}\left(\frac{e}{\hbar}\right)^2 k_B T$ $\qquad R = \frac{s\rho_c}{L_x L_y}$

Estimate

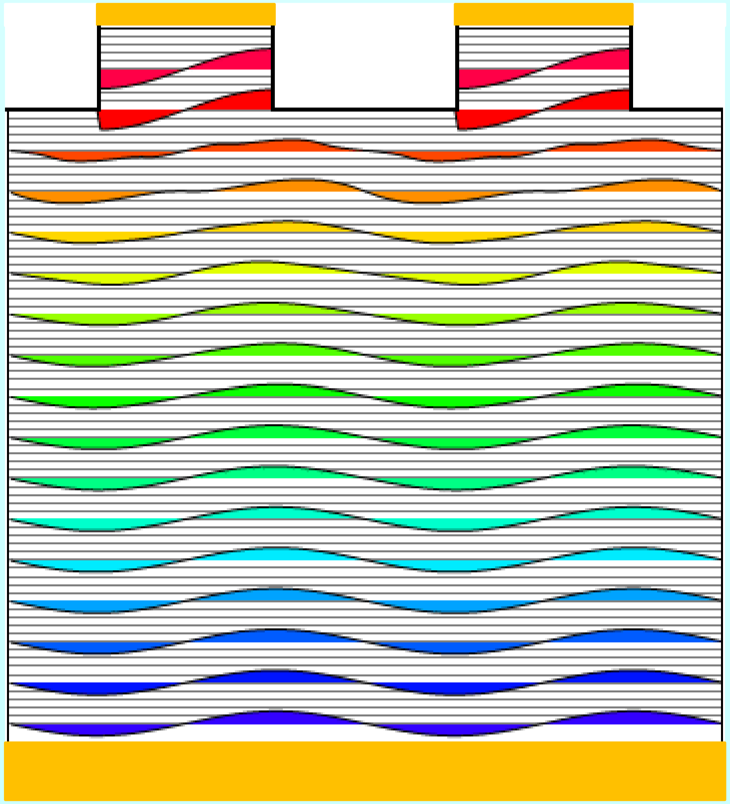
ρ_c = 50 ohm cm, L_x=80 μm, L_y=200 μm, N=600, T=60K → Δf_0 =0.2 MHz

Much smaller than best experimental observation (Δf_0 ~ 20 MHz)

Mesa arrays

Synchronized mesa arrays → Route to enhance power

Synchronization via base crystal



Radiation field:
Lin and AEK
Physica C **491** 24 (2013)

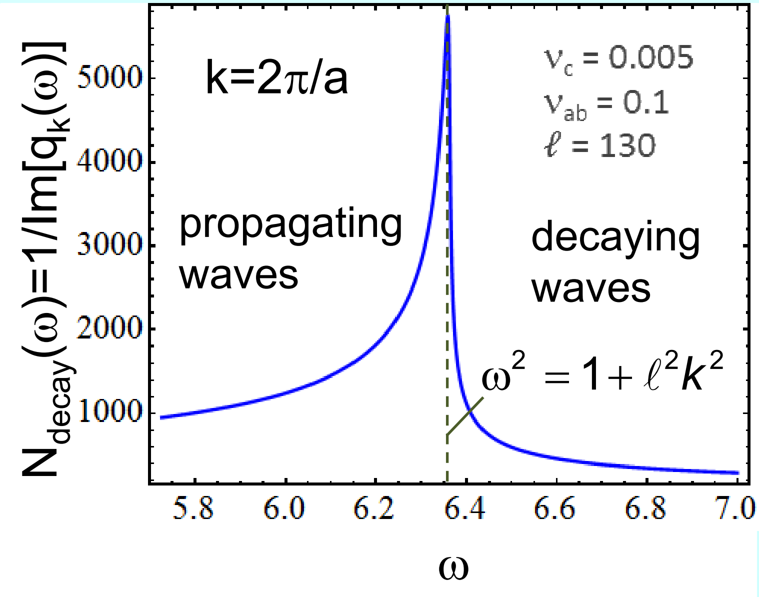
Two resonances

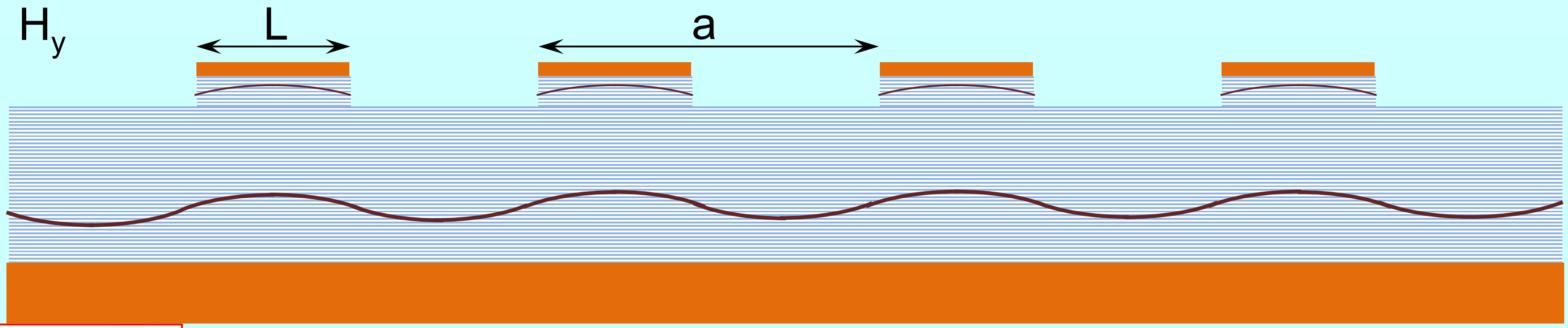
Uniform cavity mode in mesa → uniform standing wave in crystal

$$\omega^2 = c^2(\pi / L)^2 = \omega_p^2 + c^2(2\pi / a)^2$$

$$a = \frac{2L}{\sqrt{1 - \frac{\omega_p^2}{(c\pi / L)^2}}}$$

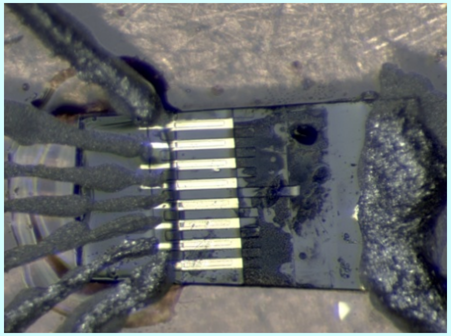
Strongest interaction between the mesas

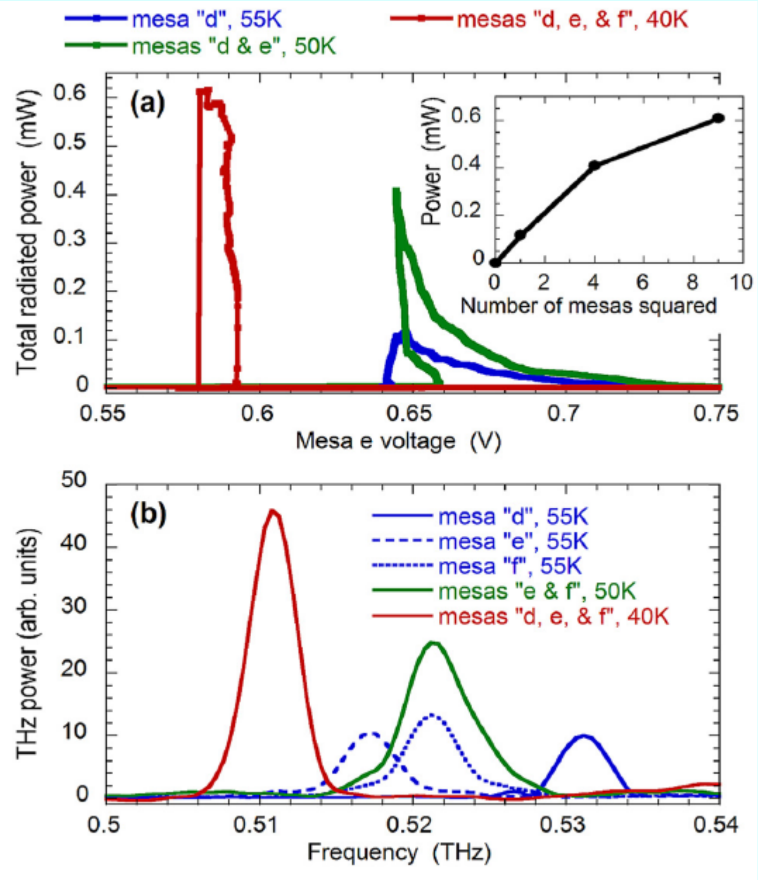




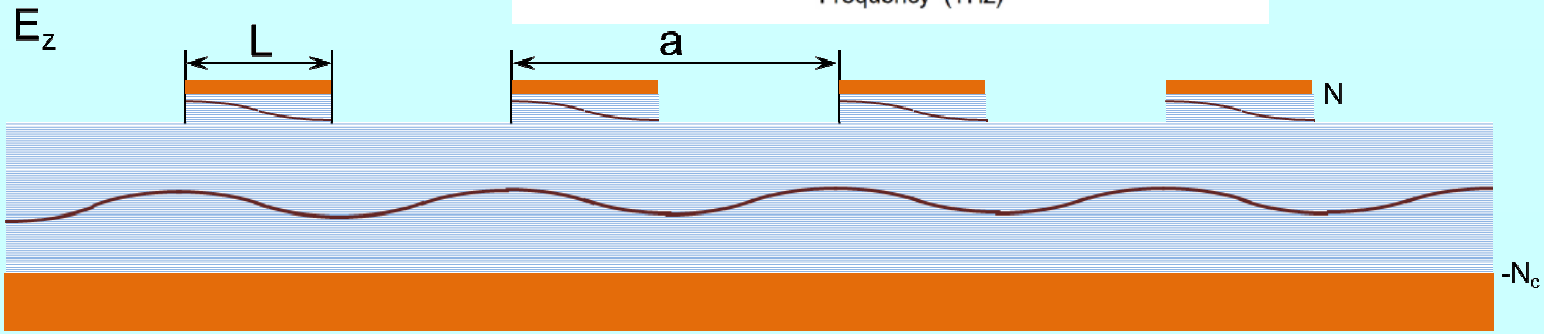
Mesa arrays, experiment

Tim Benseman *et al.*, Appl. Phys. Lett. 103, 022602 (2013): 3 mesas synchronized





Total power up to 610 μW



Summary

- Artificial and intrinsic Jos. junction arrays
- Resonant emission from BSCCO mesas
  - Frequency and polarization → Josephson origin
  - Frequency $\propto$ 1/width, from 0.4 to 0.85 THz, power up to 50 μW
  - Line width ~ 20 MHz
- Scientific issues and properties
  - radiation losses, radiation power in ideal case
  - external and self-generated modulation of Josephson current
    - alternating kink state
  - synchronization in inhomogeneous mesas
    - partial synchronization
  - intrinsic line width
- Mesa arrays

Potential for powerful, efficient and compact THz source!