Novel phase transitions in disordered quantum systems with long-range hops

Gora Shlyapnikov
LPTMS, Orsay, France
Russian Quantum Center, Moscow

- Introduction.
- Dipolar excitations in 3D
- Ergodic and non-ergodic extended states
- Quasicrystals with power-law hops
- Ergodic-non-ergodic transition
- Conclusions

Collaborations/discussions V. Kravtsov (ICTP), B. Altshuler (Col. Univ.)
X.Deng/L. Santos (Hannover Univ.)
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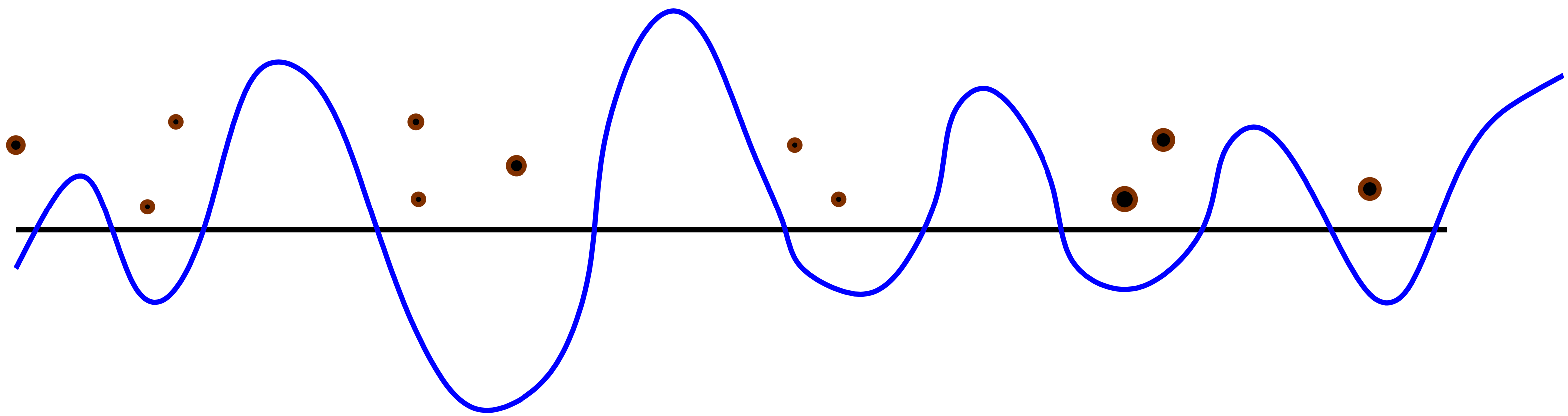
Many-body system in disorder



Many-particle system in disorder $\Rightarrow$ Transport and localization properties

Anderson localization (P.W. Anderson, 1958)

Destructive interference in the scattering of a particle from random defects

Introduction

Disordered quantum systems

What is known $\Rightarrow$ Anderson transition (localization-delocalization)

Already found: Anderson localization of

Light

Microwaves

Sound waves

Electrons in solids

Anderson localization of neutral atoms

All single-particle states in 1D and 2D are localized

for short-range disorder

Long-range hops

Anderson localization in disordered quantum systems

Long-range interactions $1/r^a$ are crucial

Regular lattice with on-site disorder and $1/r^a$ hops

$a > d \rightarrow$ localization; $a = d \rightarrow$ critical; $a < d \rightarrow$ extended (Levitov, 1997)

2D dipolar excitations $\rightarrow d = 2 = a$ (Aleiner et al (2011)

critical behavior for the T-inv. case

Ergodic and non-ergodic extended states

Disordered systems $\Rightarrow$ Ergodicity of extended states

The states at the mobility edge are non-ergodic (multifractal)

Wegner, 1981; Altshuler/Kravtsov/Lerner, 1986

Bethe lattice (Biroli et al, 2012; De Luca et al, 2014)

Random matrix models (Kravtsov et al, 2015) $\Rightarrow$

$\Rightarrow$ finite-width band of NEE states

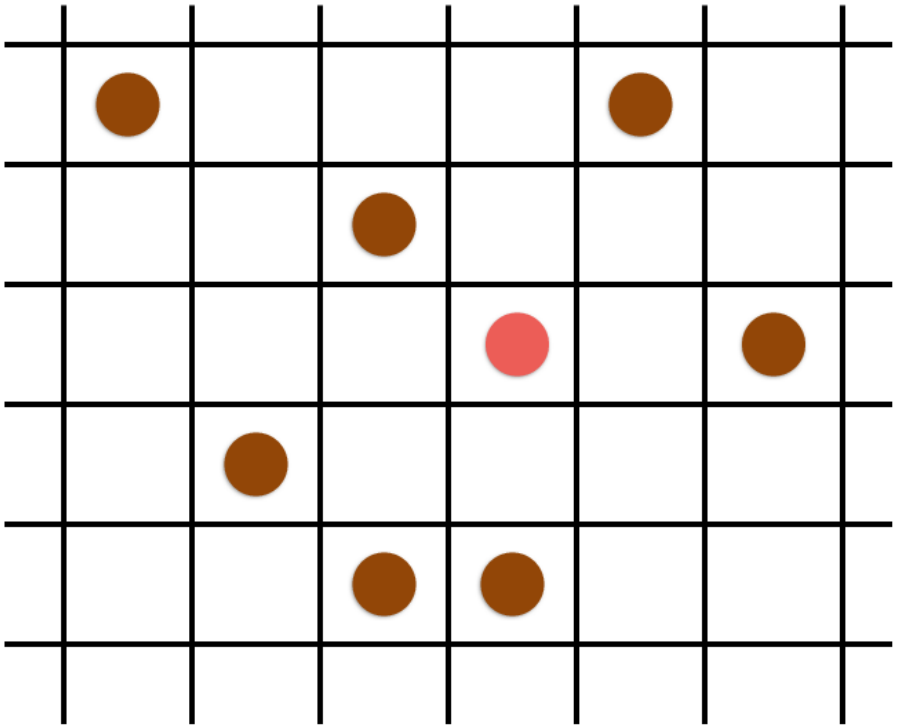
Great interest $\Rightarrow$ Systems with NEE $\leftrightarrow$ EE transition

Physical systems?

Systems with dipolar excitations

Polar molecules randomly spaced in an optical lattice (JILA)

● $\to J = 1,$ ● $\to J = 0$



Nuclear spins, Rydberg atoms, NV-centers, etc.

Long-range hops (Levy flights); Hopping amplitude $\propto 1/r^3$

Hamiltonian and methods

Deng, Altshuler, Santos, G.S. (2016)

Problem with off-diagonal disorder

$$\hat{H} = -\sum_{i,j} t_{ij} |i\rangle\langle j|$$

$$t_{ij} = -\frac{d^2}{a^3|\mathbf{r_i} - \mathbf{r_j}|^{\mathbf{3}}}(1 - 3\cos^2\theta_{ij})$$

Exact diagonalization. From 100 to 1000 realizations of disorder

$\rho = N/L^d$; N up to 80000. Extrapolation to $N \to \infty$

Eigenstate properties

$$\hat{H}|\psi_n\rangle = E_n|\psi_n\rangle; \quad \delta_n = E_{n+1} - E_n$$

$$r_n = \frac{\min\{\delta_n, \delta_{n-1}\}}{\max\{\delta_n, \delta_{n-1}\}}$$

Localized states $\Rightarrow$ Poissonian distribution of r_n with $\langle r \rangle \approx 0.386$

EE states $\rightarrow$ Wigner-Dyson distribution with $\langle r \rangle \approx 0.53$

Moments $I_q = \sum_i |\psi(i)|^{2q} \propto N^{-\tau(q)}$

EE states $\Rightarrow \tau = q - 1$

localized states $\Rightarrow \tau = 0$

Fractal dimensions $D_q = \dfrac{\tau(q)}{q-1}$

D_q depends on $q \rightarrow$ multifractal $\psi(i)$

Spectrum of fractal dimensions

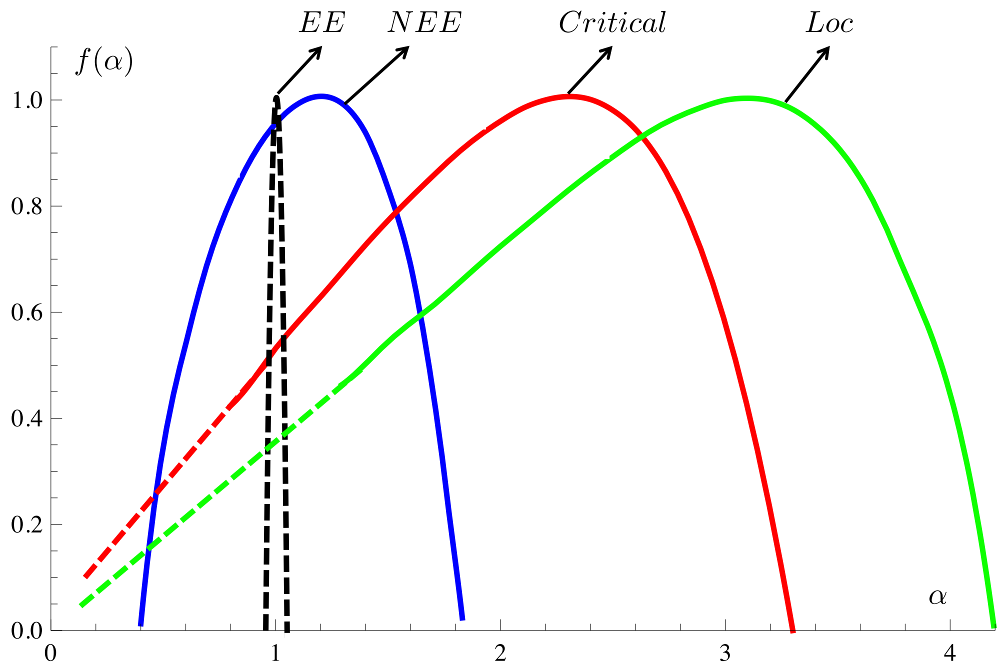
$$P(|\psi|^2) \propto \frac{1}{|\psi|^2} N^{f(\alpha)-1}; \quad \alpha = -\frac{\ln |\psi|^2}{\ln N}$$

$$\langle I_q \rangle = N \langle |\psi|^{2q} \rangle \propto \int d\alpha N^{f(\alpha)-q\alpha)}$$

$\tau(q) = q\alpha - f(\alpha); \;\; \alpha(q)$ is a solution of $f'(\alpha) = q$

$f(\alpha) \to$ fractal dimension of the set of those points $\mathbf{r}$ where $|\psi(\mathbf{r})|^2 \sim N^{-\alpha}$

Spectrum of fractal dimensions



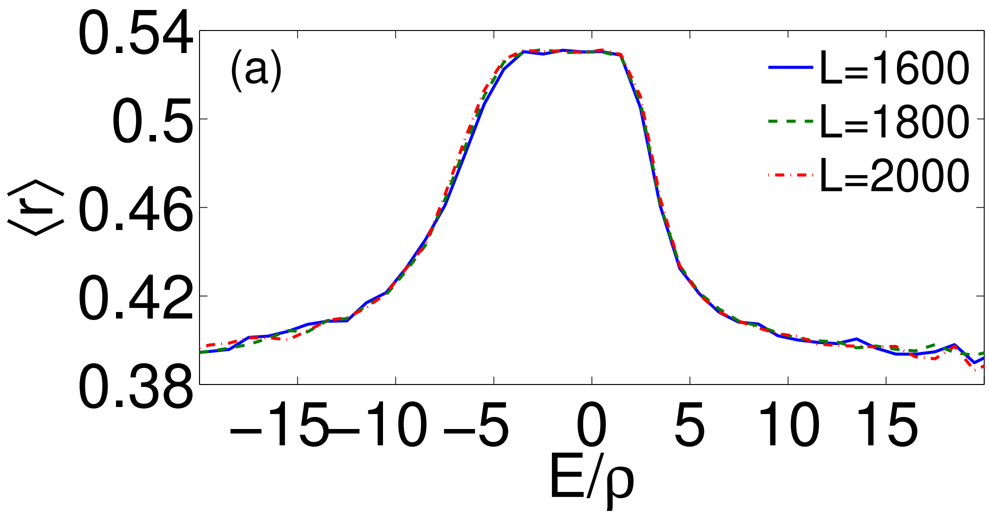
$$f(\alpha, N) = f(\alpha) + \frac{C}{\ln N}$$

Dipolar excitations in 3D. Dilute limit

All states extended

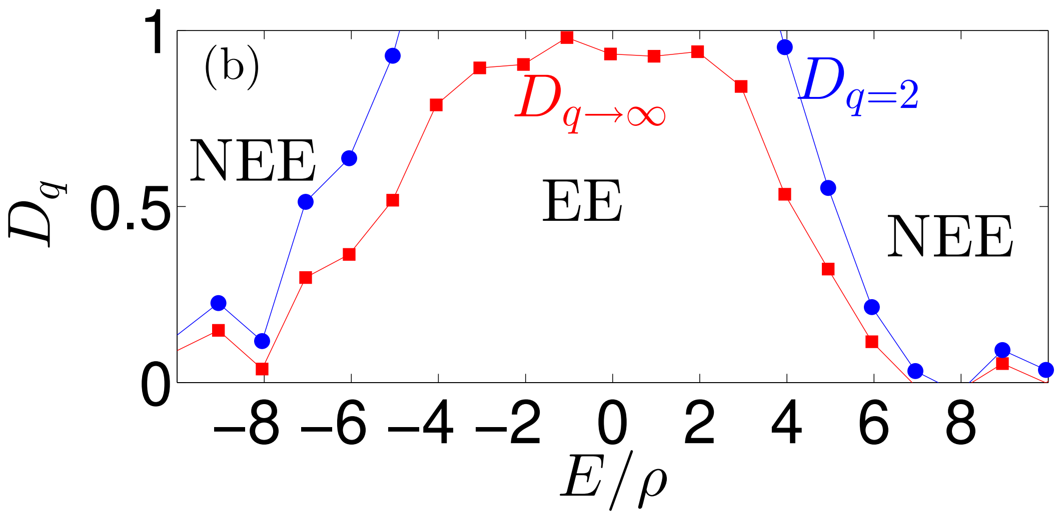
Energy scale ρ - hopping at mean interparticle distance

$|E|/\rho \lesssim 2 \Rightarrow$ EE states ($\langle r \rangle = 0.53$ and $D_q = 1$ for all q)



Wings $\Rightarrow$ NEE ($0.386 < \langle r \rangle < 0.53$)

Dipolar excitations in 3D



$0 < D_q < 1$ decays with growing q)

Dipolar excitations in 3D

NEE $\leftrightarrow$ EE transition at a given ρ by varying E

Central region of EE states grows with increasing ρ.

For $\rho > 0.5$ the EE region covers almost all the spectrum

NEE $\leftrightarrow$ EE transition at a given E by varying ρ

Why EE $\leftrightarrow$ NEE is a phase transition?

D_2 has at least a cusp at NEE-EE edges

Excitation dynamics

Experimentally the lattice with $\sim 10^6$ sites and ρ up to ~ 0.3 are possible

Create a dipolar excitation in a particular site

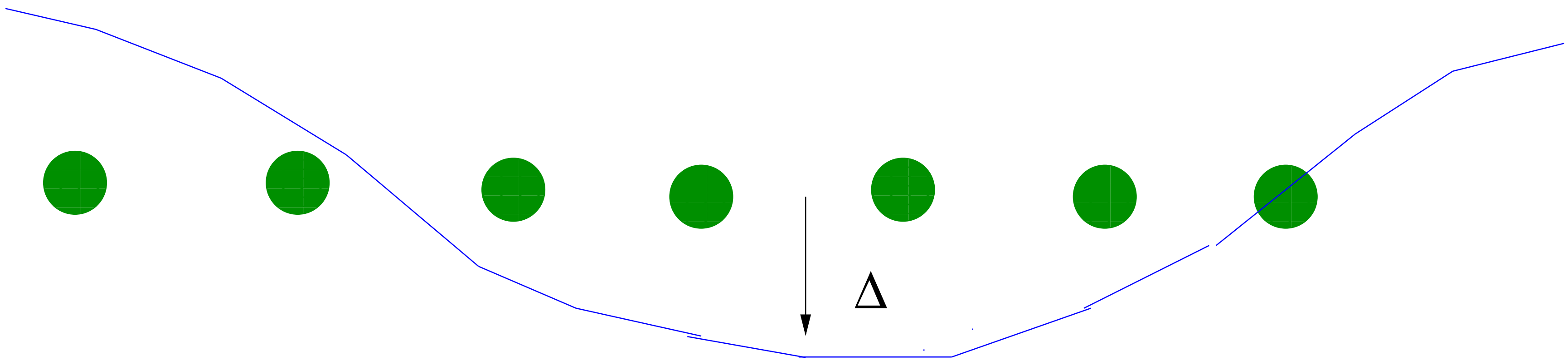
For the non-ergodic case the broadening of the wave packet is much slower: Chalker; Kravtsov et al; etc.

Measure the return probability $P(t)$ after time t

In the NEE case $P(t)$ decays with increasing t much slower

1D quasicrystals with power law hops

Superposition of 2 incommensurate lattices



Nearest neghbour hopping $J \rightarrow$ Aubrey-Azbel-Harper model

$\Delta < 2J \rightarrow$ ergodic extended states

$\Delta > 2J \rightarrow$ localized states

Power law hops $1/r^a$ $(a > 0)$

Many interesting regimes

1D crystals with powerlaw hops

$$\hat{H} = -J \sum_{i,j\neq i} \frac{1}{|i-j|^a} |i\rangle\langle j| + \Delta \sum_j \cos(\beta(2\pi j + \phi)) |j\rangle\langle j|$$

$a < 1 \rightarrow$ laser-driven interaction between trapped ions

All states are extended (NEE and EE)

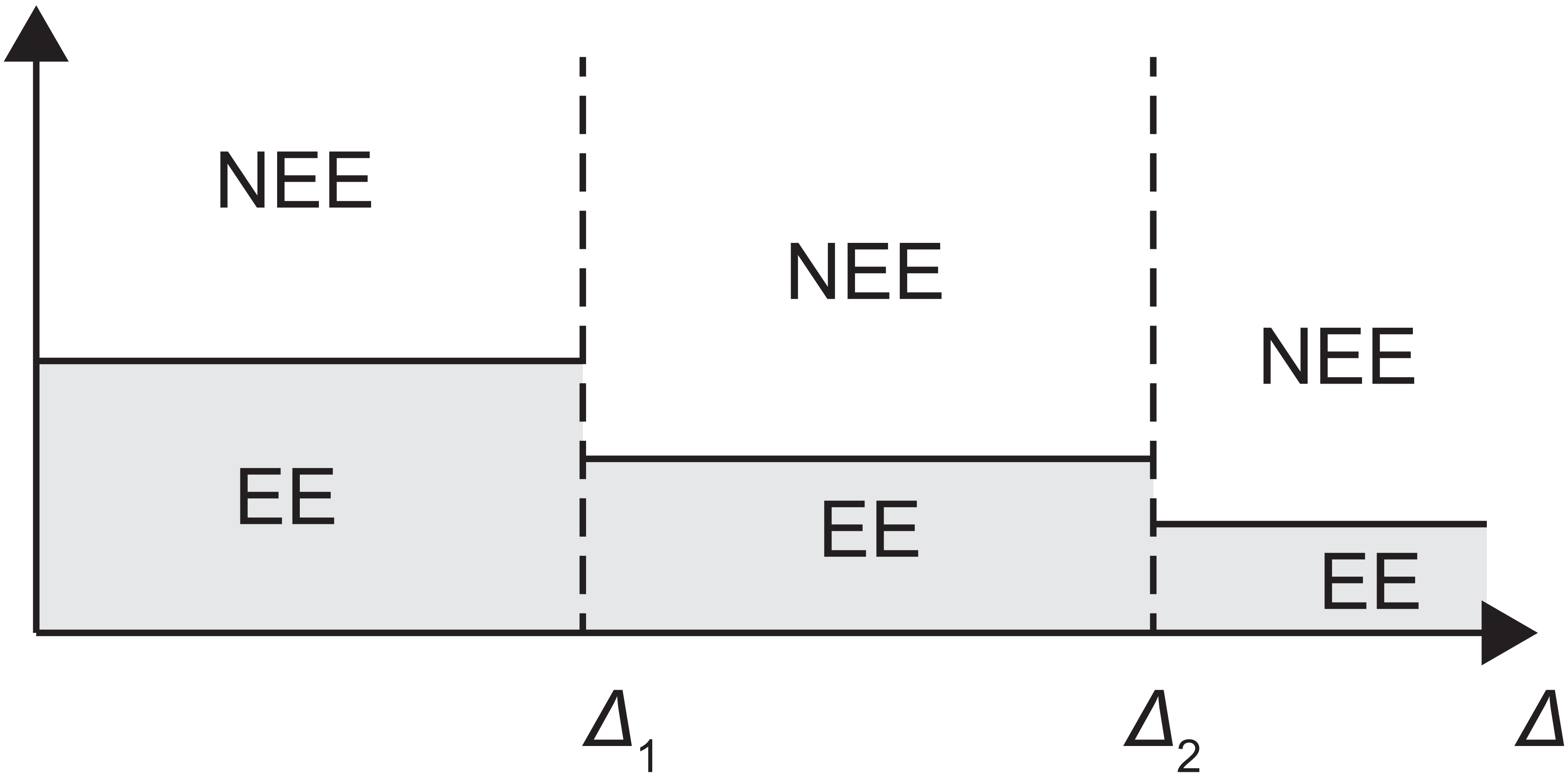
Deng, Ray, Sinha, Santos, GS

Ergodic and multifractal states

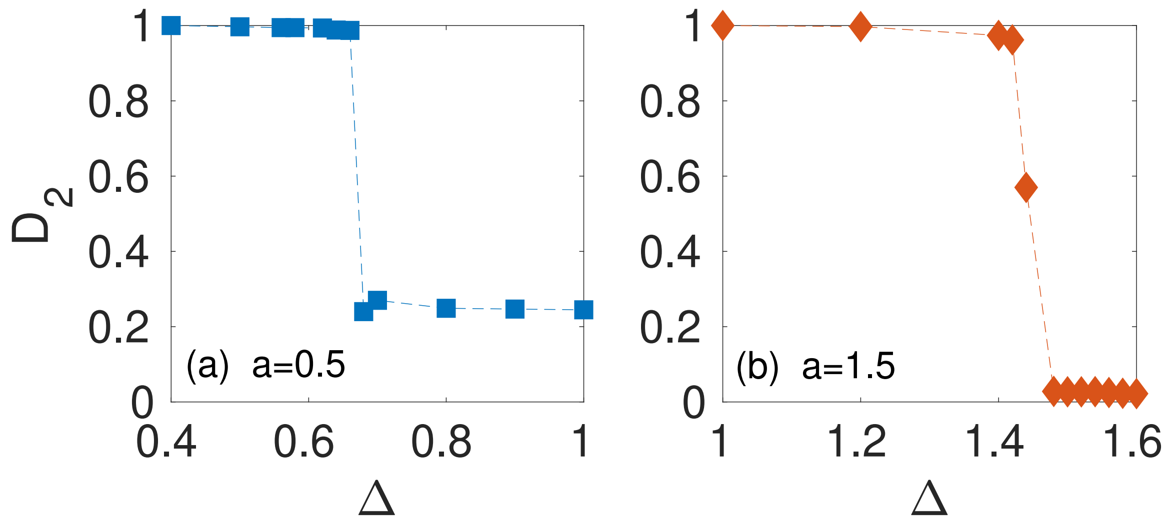
$$\beta = (\sqrt{5} - 1)/2$$

$\Delta < \Delta_1 \to$ lowest βL states EE, and the rest NEE

$\Delta_1 < \Delta < \Delta_2 \to$ lowest $\beta^2 L$ states EE, and the rest NEE



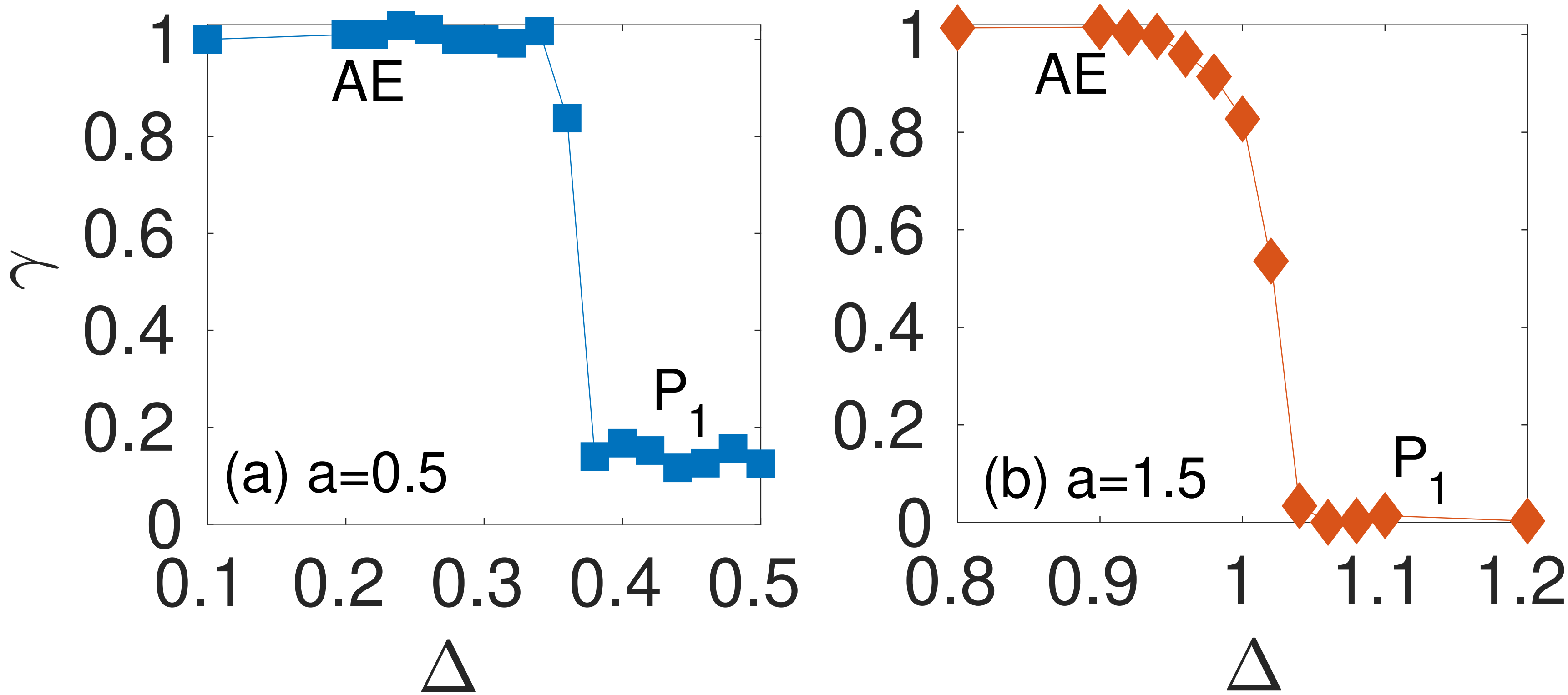
Nonergodic-ergodic transition



Phase transition

Excitation dynamics

Return probability of an initially localized excitation $P(t) \propto t^{-\gamma}$ for large t



$\gamma = 0$ (localized); $\gamma = 1$ (EE); $\gamma \simeq D_2/(2-a)$ (NEE)

Conclusions

Ergodic-nonergodic phase transition

Thank you for attention!