Многочастичная Локализация Андерсона

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Колумбийский Университет

Летняя школа Фонда Дмитрия Зимина "Династия"
"Актуальные проблемы теории конденсированного состояния"
4 – 14 июля 2010г.

Previous Lectures:

1. Anderson Localization as Metal-Insulator Transition
   Anderson model.
   Localized and extended states. Mobility edges.

2. Spectral Statistics and Localization.
   Poisson versus Wigner-Dyson.
   Anderson transition as a transition between different types of spectra.
   Thouless conductance

3 Quantum Chaos and Integrability and Localization.
   Integrable ⟺ Poisson; Chaotic ⟺ Wigner-Dyson

4. Anderson transition beyond real space
   Localization in the space of quantum numbers.
   KAM ⟺ Localized; Chaotic ⟺ Extended

Previous Lectures:

4. Anderson Localization and Many-Body Spectrum in finite systems. BA, Gefen, Kamenev & Levitov. PRL 1996

   Q: Why nuclear spectra are statistically the same as RM spectra – Wigner-Dyson?
   A: Delocalization in the Fock space.

   Q: What is relation of exact Many Body states and quasiparticles?
   A: Quasiparticles are "wave packets"

5. Anderson Model and Localization on the Cayley tree

   Ergodic and Nonergodic extended states
   Wigner – Dyson statistics requires ergodicity!

6. Phononless conductivity

Definition: We will call a quantum state $|\mu\rangle$ **ergodic** if it occupies the number of sites N_μ on the Anderson lattice, which is proportional to the total number of sites N:

$$\frac{N_\mu}{N} \xrightarrow[N\to\infty]{} 0$$

nonergodic

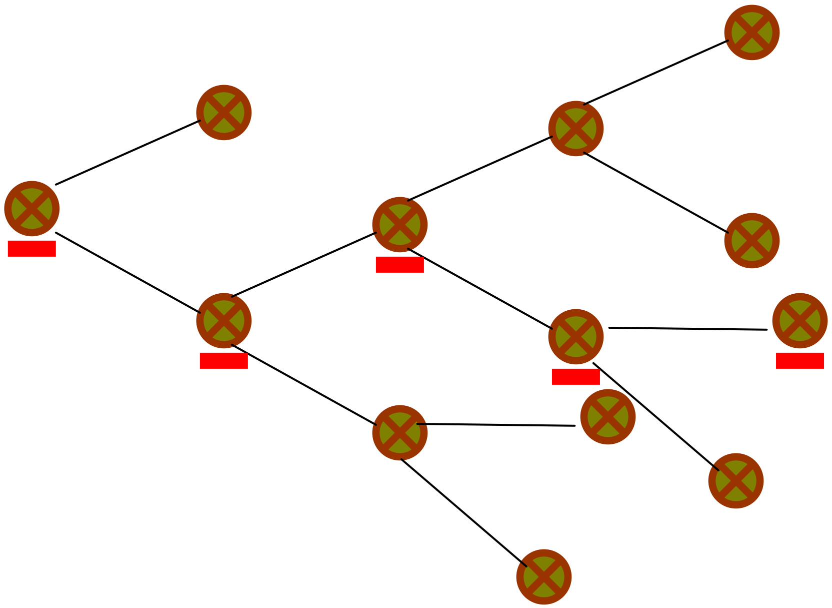
$$\frac{N_\mu}{N} \xrightarrow[N\to\infty]{} const > 0$$

ergodic

nonergodic states

Such a state occupies infinitely many sites of the Anderson model but still negligible fraction of the total number of sites

Example of nonergodicity: Anderson Model Cayley tree:



$$N_\mu \propto \ln N$$

transition

K– branching number

$$I_c = \frac{W}{K \ln K}$$

ergodicity

$$I_{erg} \sim W$$

crossover

$$I < W/(K \ln K)$$

Resonance is typically far $N_\mu = const$ localized

$$W/K > I > W/(K \ln K)$$

Resonance is typically far $N_\mu \sim \ln N$ nonergodic

$$W > I > W/K$$

Typically there is a resonance at every step $N_\mu \sim \ln N$ nonergodic

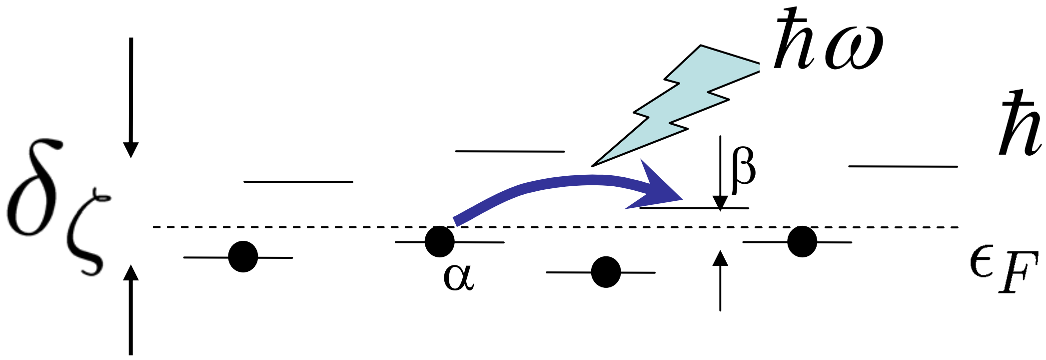
$$I > W$$

Typically each pair of nearest neighbors is at resonance $N_\mu \sim N$ ergodic

Lecture 3.

1. Many-Body localization

Phonon-assisted hopping



$$\hbar\omega = \varepsilon_\alpha - \varepsilon_\beta$$

Variable Range Hopping

N.F. Mott (1968)

$$\sigma(T) \propto T^\gamma \exp\left[-\left(\frac{\delta_\zeta}{T}\right)^{\frac{1}{d+1}}\right]$$

δ_ζ is mean localization energy spacing – typical energy separation between two localized states, which strongly overlap

Any bath with a continuous spectrum of delocalized excitations down to $\omega = 0$ will give the same exponential

In disordered metals phonons limit the conductivity, but at low temperatures one can evaluate ohmic conductivity without phonons, i.e. without appealing to any bath (Drude formula)!

A bath is needed only to stabilize the temperature of electrons.

Q1: Is the existence of a bath crucial even for ohmic conductivity? ?

Q2: Can a system of electrons left alone relax to the thermal equilibrium without any bath? ?

Main postulate of the Gibbs Statistical Mechanics – equipartition (microcanonical distribution):

In the equilibrium all states with the same energy are realized with the same probability.

Without interaction between particles the equilibrium would never be reached – each one-particle energy is conserved.

Common believe: Even weak interaction should drive the system to the equilibrium. Is it always true?

No external bath!

Many-Body Localization:

1. It is not localization in a real space!

2. There is **no relaxation** in the localized state in the same way as wave packets of localized wave functions do not spread.

Fermi Pasta Ulam **1955**

Q: Will a nonlinear system (system of interacting particles) completely isolated from the outside world evolve to a microcanonical distribution (reach equipartition). ?

Anderson **1958**

Q: Will a density fluctuation (a wave packet) in a system of quantum particles in the presence of disorder dissolve in the diffusive way. ?

Common belief: Anderson Insulator weak e-e interactions ⇒ Phonon assisted hopping transport

Can hopping conductivity exist **without phonons** ?

Given:
1. All one-electron states are localized
2. Electrons interact with each other
3. The system is closed (no phonons)
4. Temperature is low but finite

Find: DC conductivity $\sigma(T, \omega=0)$ (zero or finite?)

Q: Can e-h pairs lead to phonon-less variable range hopping in the same way as phonons do ?

A#1: Sure

A#2: No way (L. Fleishman. P.W. Anderson (1980))

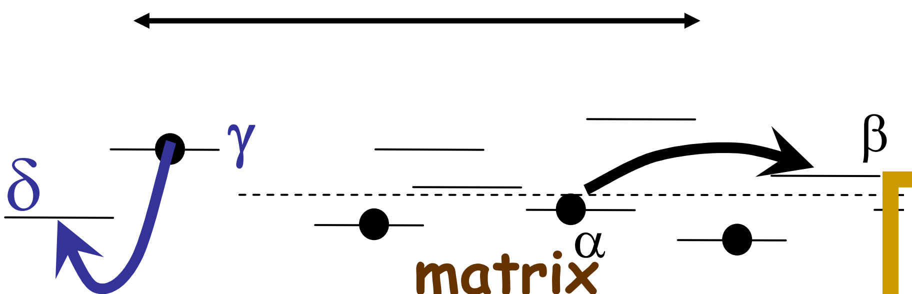
Except maybe Coulomb interaction in 3D

$$\sigma(\omega) \simeq \frac{e^2 \zeta_{loc}^{d-2}}{\hbar} \left(\frac{\hbar\omega}{\delta_\zeta}\right)^2 \ln^{d+1}\left|\frac{\delta_\zeta}{\hbar\omega}\right|$$

is contributed by rare resonances

$R \to \infty$

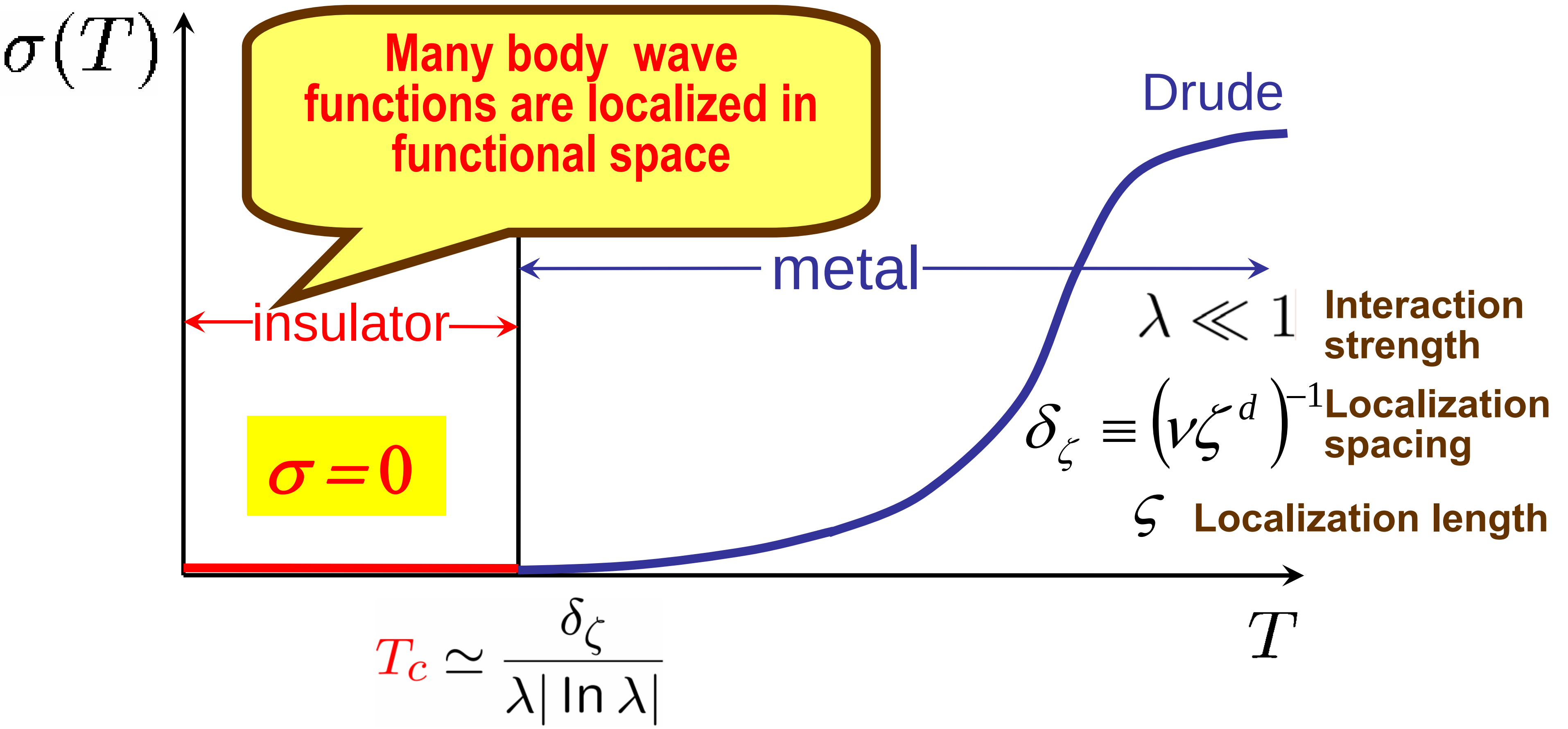
$$\omega = \xi_\beta - \xi_\alpha = \xi_\gamma - \xi_\delta$$



$R \to \infty \Rightarrow$ matrix element vanishes

$$\sigma(T) \propto 0 \times \exp\left[-\left(\frac{\delta_\zeta}{T}\right)^{\frac{1}{d+1}}\right]$$

Finite temperature Metal-Insulator Transition



D.M. Basko, I.L. Aleiner & BA, Annals of Phys. 321, 1126 (2006)

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Many-Body Localization:

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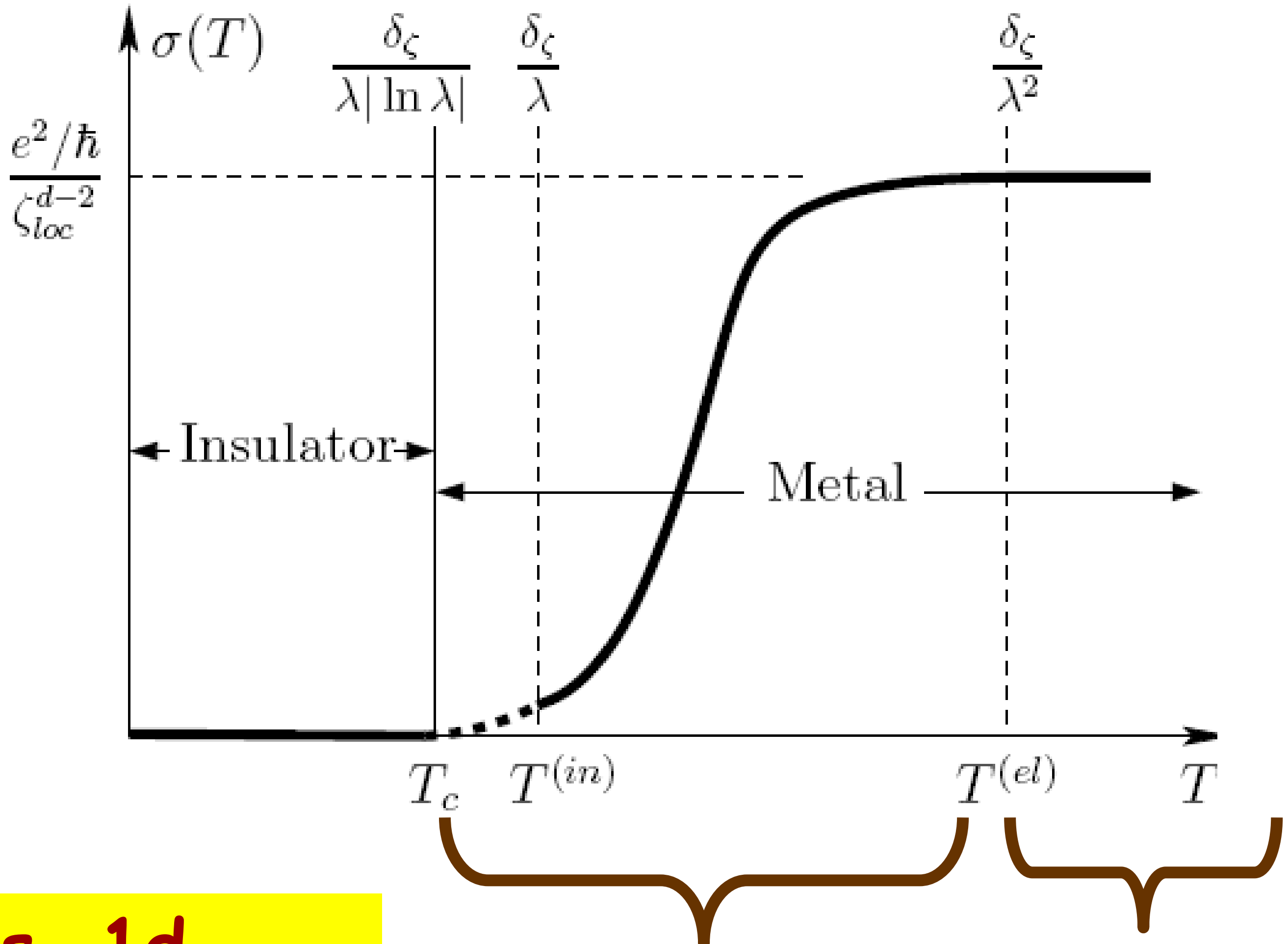
Finite temperature Metal-Insulator Transition



Bad metal

Good (Drude) metal

Includes, 1d case, although is not limited by it.

Dogma

There can be no phase transitions at a finite temperature in 1D

Van Howe, Landau

Reason

Thermal fluctuation destroy any long range correlations in 1D

$T \neq 0$ Normal fluid – Insulator Phase Transition:

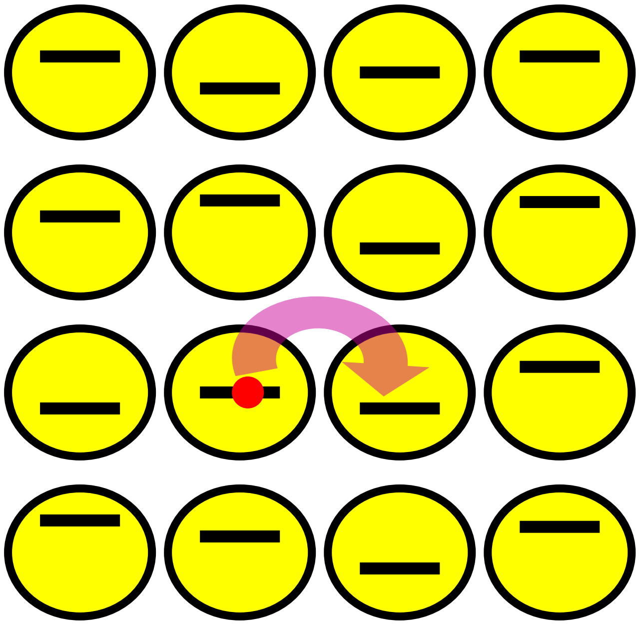
Neither normal fluids (metals) nor glasses (insulators) exhibit long range correlations

still

True phase transition: singularities in transport (rather than thermodynamic) properties

Conventional Anderson Model

- one particle,
- one level per site,
- onsite disorder
- nearest neighbor hoping

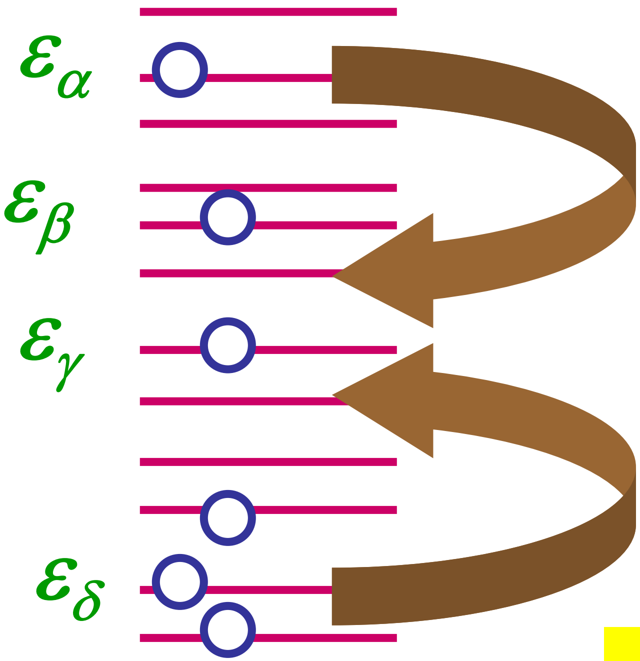


Basis: $\left| i \right\rangle$, $\quad i$ labels sites

Hamiltonian: $\hat{H} = \hat{H}_0 + \hat{V}$

$$\hat{H}_0 = \sum_i \varepsilon_i \left| i \right\rangle \left\langle i \right| \qquad \hat{V} = \sum_{i,j=n.n.} I \left| i \right\rangle \left\langle j \right|$$

0d system with interactions



Basis: $\left|\mu\right\rangle$ $\quad \mu = \left\{ n^\alpha \right\}$

$n^\alpha = 0,1$ occupation numbers $\quad \alpha$ labels levels

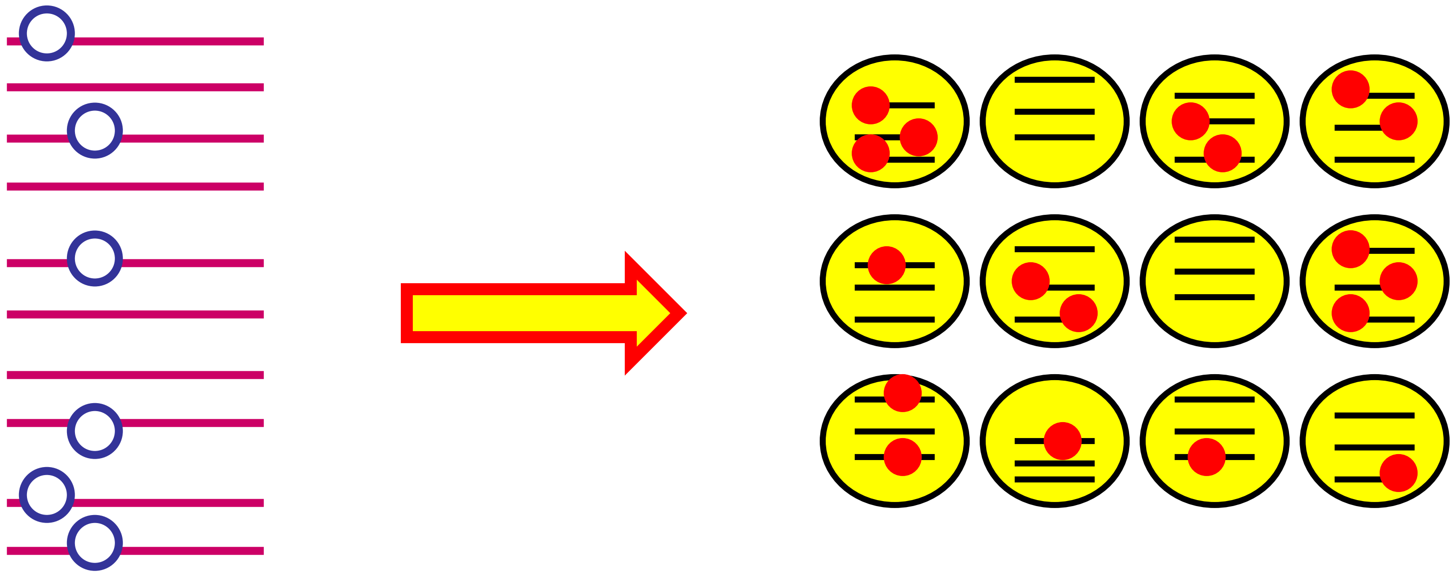
Hamiltonian: $\widehat{H} = \hat{H}_0 + \hat{V}$

$$\hat{H}_0 = \sum_\mu E_\mu \left|\mu\right\rangle\left\langle\mu\right|$$

$$\hat{V} = \sum_{\mu,\eta(\mu)} I \left|\mu\right\rangle\left\langle\nu(\mu)\right|$$

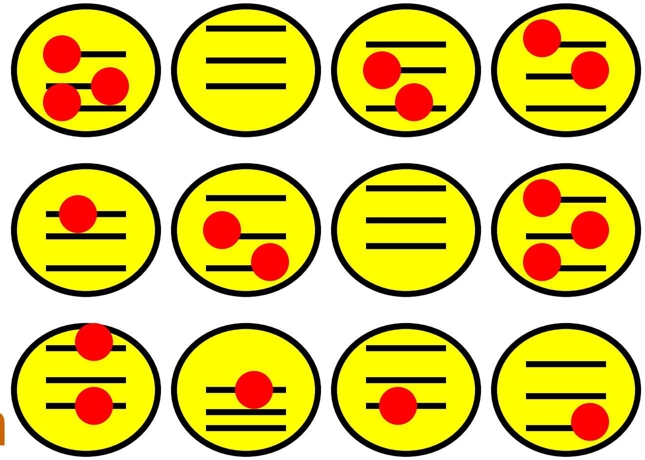
$$\left|\nu(\mu)\right\rangle = \left|..,n^\alpha - 1,..,n^\beta - 1,..,n^\gamma + 1,..,n^\delta + 1,..\right\rangle$$

Many body Anderson-like Model



Many body Anderson-like Model

- many particles,
- several levels per site, spacing δ_ς
- onsite disorder
- Local interaction



Basis: $|\mu\rangle$

$$\mu = \left\{ n_i^\alpha \right\}$$

i labels sites $\quad \alpha$ labels levels

$n_i^\alpha = 0,1$ occupation numbers

Hamiltonian:

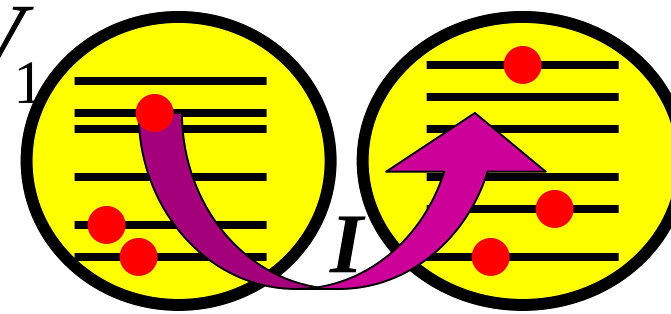
$$\widehat{H} = \hat{H}_0 + \hat{V}_1 + \hat{V}_2$$

$$\hat{H}_0 = \sum_\mu E_\mu |\mu\rangle\langle\mu|$$

$$\hat{V}_1 = \sum_{\mu,\nu(\mu)} I |\mu\rangle\langle\nu(\mu)|$$

$$|\nu(\mu)\rangle = |.., n_i^\alpha - 1, .., n_j^\beta + 1, ..\rangle, \quad i, j = n.n.$$

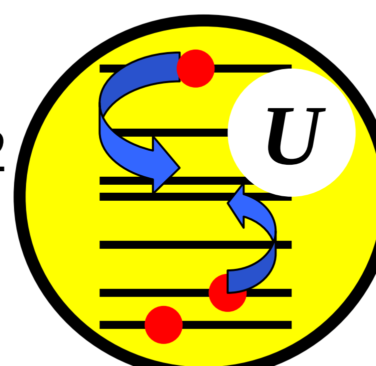
$\hat{V}_1$



$$\hat{V}_2 = \sum_{\mu,\eta(\mu)} U |\mu\rangle\langle\eta(\mu)|$$

$$|\eta(\mu)\rangle = |.., n_i^\alpha - 1, .., n_i^\beta - 1, .., n_i^\gamma + 1, .., n_i^\delta + 1, ..\rangle$$

$\hat{V}_2$



Conventional Anderson Model

Basis: $\left| i \right\rangle$

i labels sites

$$\hat{H} = \sum_i \varepsilon_i \left| i \right\rangle \left\langle i \right| + \sum_{i,j=n.n.} I \left| i \right\rangle \left\langle j \right|$$

Many body Anderson-like Model

Basis: $\left| \mu \right\rangle, \quad \mu = \left\{ n_i^{\alpha} \right\}$

i labels sites

α labels levels

$n_i^{\alpha} = 0,1$ occupation numbers

$$\hat{H} = \sum_{\mu} E_{\mu} \left| \mu \right\rangle \left\langle \mu \right| + \sum_{\mu,\nu(\mu)} I \left| \mu \right\rangle \left\langle \nu(\mu) \right| + \sum_{\mu,\eta(\mu)} U \left| \mu \right\rangle \left\langle \eta(\mu) \right|$$

N sites

M one-particle levels per site

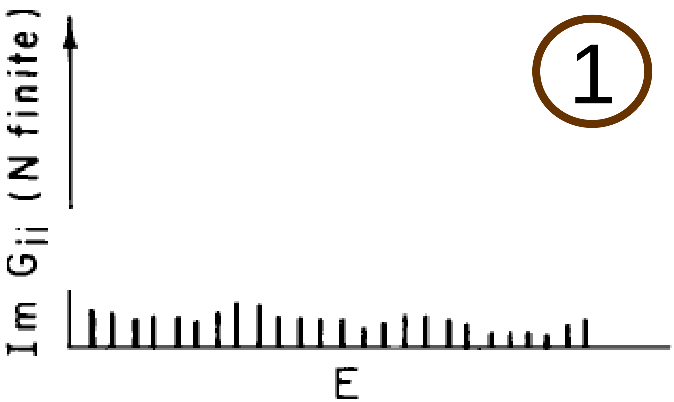
Two types of "nearest neighbors":

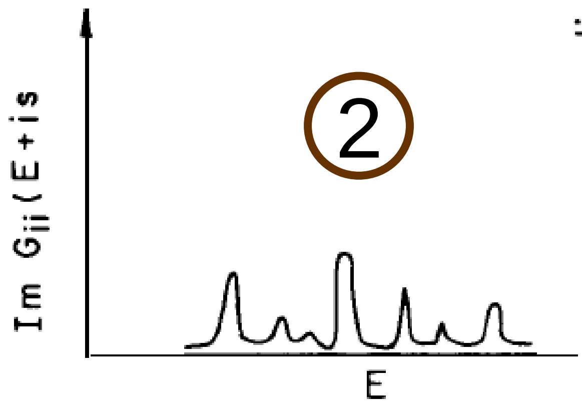
$$\left| \nu(\mu) \right\rangle = \left| .., n_i^{\alpha} - 1, .., n_j^{\beta} + 1, .. \right\rangle, \quad i, j = n.n.$$

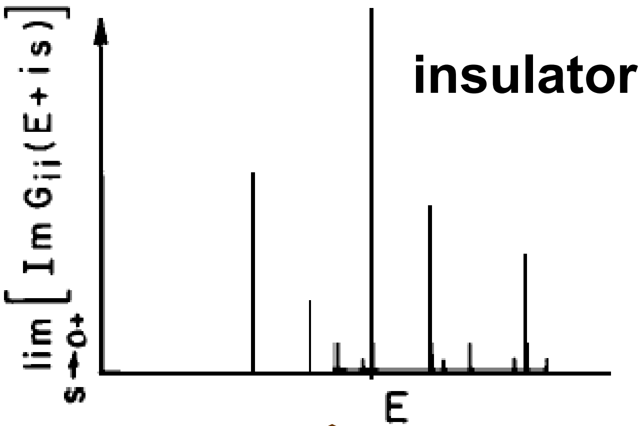
$$\left| \eta(\mu) \right\rangle = \left| .., n_i^{\alpha} - 1, .., n_i^{\beta} - 1, .., n_i^{\gamma} + 1, .., n_i^{\delta} + 1, .. \right\rangle$$

Anderson's recipe:

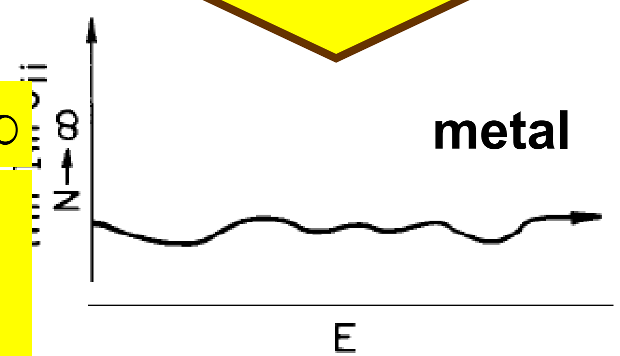
1. take discrete spectrum E_μ of H_0
2. Add an infinitesimal *Im* part *is* to E_μ
3. Evaluate $Im\Sigma_\mu$





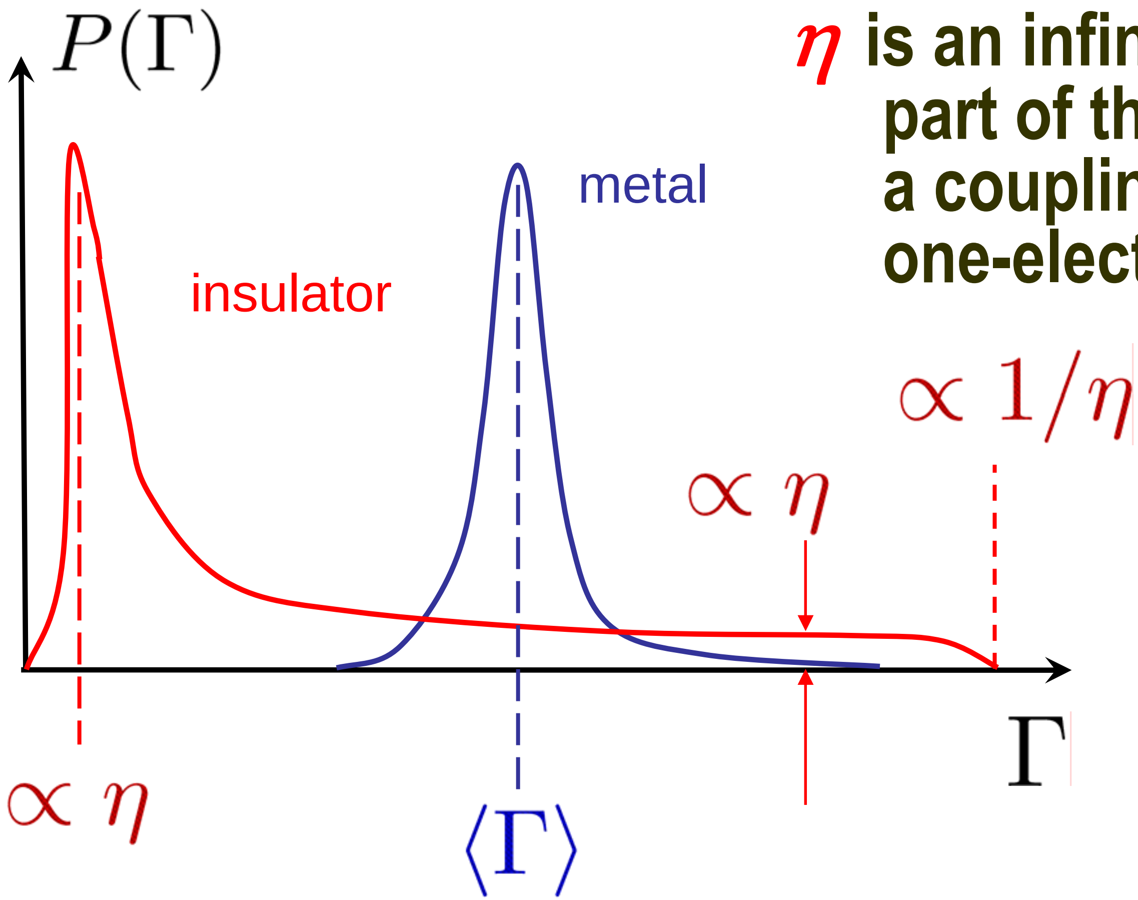


4 **limits** 1) $N \to \infty$ 2) $s \to 0$



4. take limit $s \to 0$ but only after $N \to \infty$
5. "What we really need to know is the *probability distribution* of $Im\Sigma$, *not* its average…" !

Probability Distribution of $\Gamma = Im\ \Sigma$



η is an infinitesimal width (*Im* part of the self-energy due to a coupling with a bath) of one-electron eigenstates

Look for:

$$\lim_{\eta \to +0} \lim_{V \to \infty} P(\Gamma > 0) = \begin{cases} > 0; & metal \\ 0; & insulator \end{cases}$$

Stability of the insulating phase: NO spontaneous generation of broadening

$$\Gamma_\alpha(\varepsilon) = 0$$

is always a solution

$$\varepsilon \to \varepsilon + i\eta$$

linear stability analysis

$$\frac{\Gamma}{(\varepsilon - \xi_\alpha)^2 + \Gamma^2} \to \pi\delta(\varepsilon - \xi_\alpha) + \frac{\Gamma}{(\varepsilon - \xi_\alpha)^2}$$

After ***n*** iterations of the equations of the Self Consistent Born Approximation

$$P_n(\Gamma) \propto \frac{\eta}{\Gamma^{3/2}} \left(const \frac{\lambda T}{\delta_\zeta} \ln \frac{1}{\lambda} \right)^n$$

first $n \to \infty$

then $\eta \to 0$

$(\ldots) < 1$ – insulator is stable !

Stability of the metallic phase: Finite broadening is self-consistent

- $$P(\Gamma) = \frac{1}{\sqrt{2\pi\langle\delta\Gamma^2\rangle}} \exp\left[-\frac{(\Gamma - \langle\Gamma\rangle)^2}{2\langle\delta\Gamma^2\rangle}\right]$$

  $\sqrt{\langle\delta\Gamma^2\rangle} \ll \langle\Gamma\rangle$ **as long as** $T \gg \frac{\delta_\zeta}{\lambda}$

- $\langle\Gamma\rangle \ll \delta_\zeta$ **(levels well resolved)**

- **quantum kinetic equation for transitions between localized states**

  $\sigma(T) \propto \lambda^2 T^\alpha$ **(model-dependent)**

$$T >> T^{(el)} = \frac{\delta_\zeta}{16\pi^2 d\lambda^2}$$

good metal

$$\sigma(T \gg \sqrt{\delta_\zeta T_{el}}) \approx \sigma_\infty \left(1 - \frac{2}{3}\frac{\delta_\zeta T_{el}}{T^2}\right);$$

$$\kappa(T \gg \sqrt{\delta_\zeta T_{el}}) \approx \kappa_\infty(T)\left[1 - \left(\frac{14}{5} - \frac{24}{\pi^2}\right)\frac{\delta_\zeta T_{el}}{T^2}\right]$$

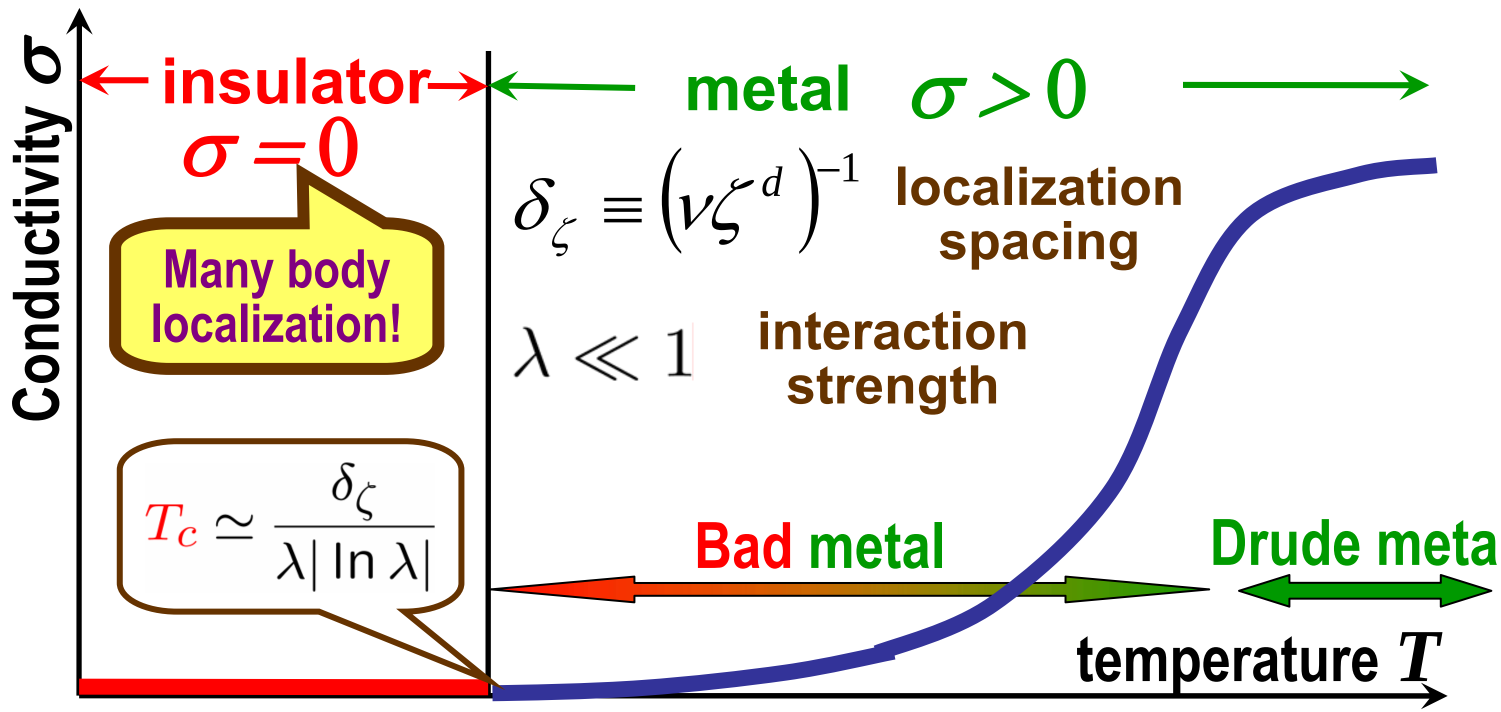
$$\sigma_\infty \equiv \frac{2\pi e^2 I^2 \zeta_{loc}^{2-d}}{\hbar}, \quad \kappa_\infty(T) \equiv \frac{2\pi^3 e^2 T I^2 \zeta_{loc}^{2-d}}{3\hbar}.$$

$$T^{el} >> T >> T^{(in)} = \frac{\delta_\zeta}{6\pi\lambda}$$

bad metal

$$\sigma(T \ll \sqrt{\delta_\zeta T_{el}}) = \sigma_\infty \frac{\pi}{4}\left(\frac{T^2}{\delta_\zeta T_{el}}\right),$$

$$\kappa(T \ll \sqrt{\delta_\zeta T_{el}}) = \kappa_\infty(T)\frac{48G^2}{\pi^3}\left(\frac{T^2}{\delta_\zeta T_{el}}\right)$$

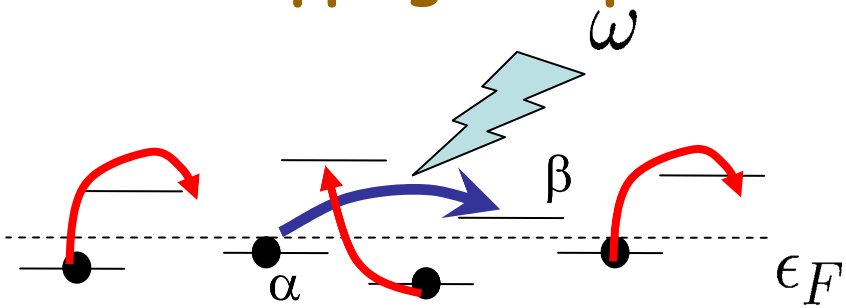


Q: Does "localization length" have any meaning for the Many-Body Localization ?

Physics of the transition: cascades

Conventional wisdom:

For phonon assisted hopping one phonon – one electron hop



It is maybe correct at low temperatures, but the higher the temperature the easier it becomes to create e-h pairs.

Therefore with increasing the temperature the typical number of pairs created n_c (i.e. the number of hops) increases. Thus phonons create cascades of hops.

Size of the cascade n_c ⟺ "localization length"

Physics of the transition: cascades

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It is maybe correct at low temperatures, but the higher the temperature the easier it becomes to create e-h pairs.

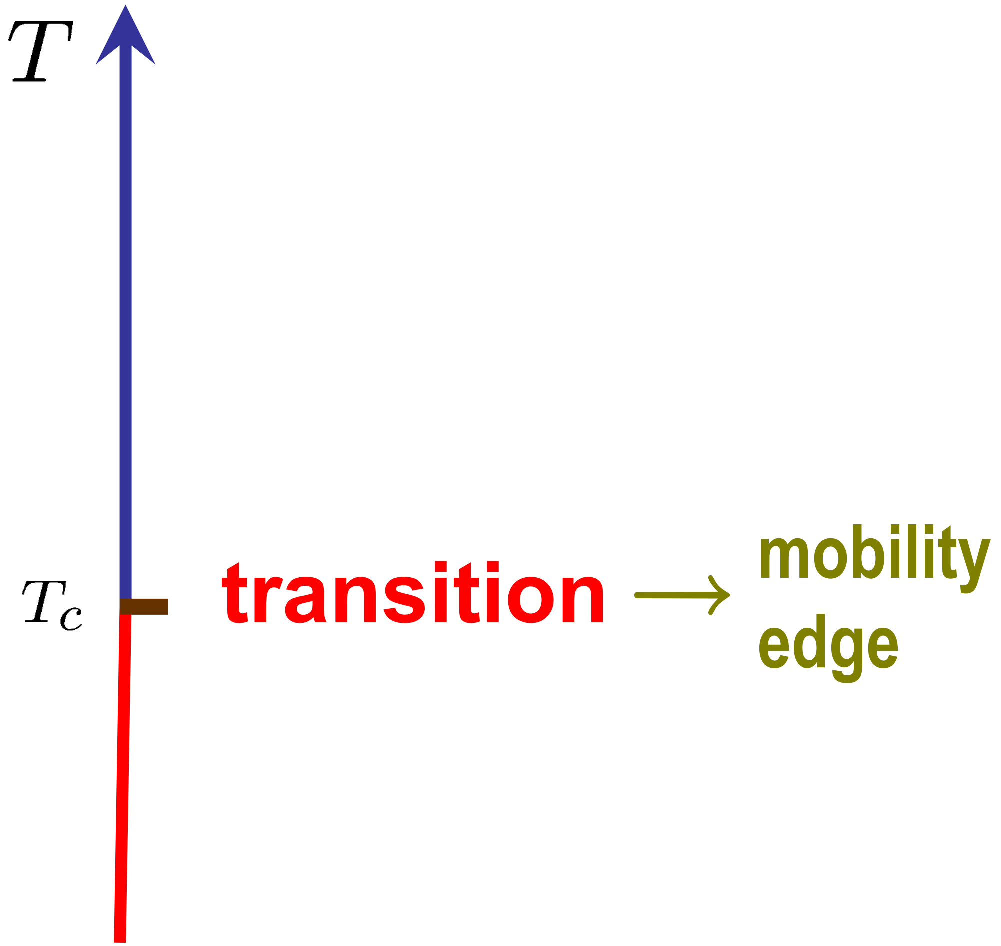
Therefore with increasing the temperature the typical number of pairs created n_c (i.e. the number of hops) increases. Thus phonons create cascades of hops.

At some temperature $T = T_c \quad n_c(T) \to \infty.$

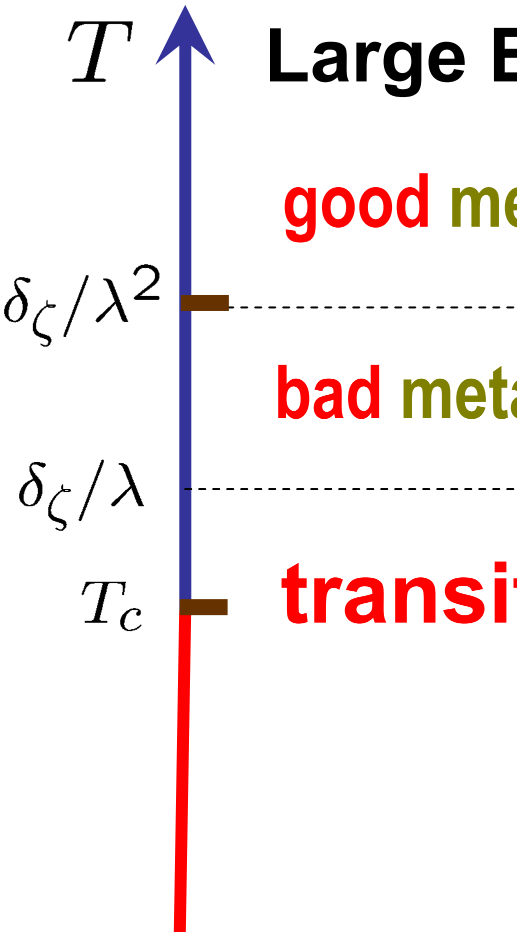
This is the critical temperature T_c .

Above T_c one phonon creates infinitely many pairs, i.e., the charge transport is sustainable without phonons.

Many-body mobility edge



Metallic States



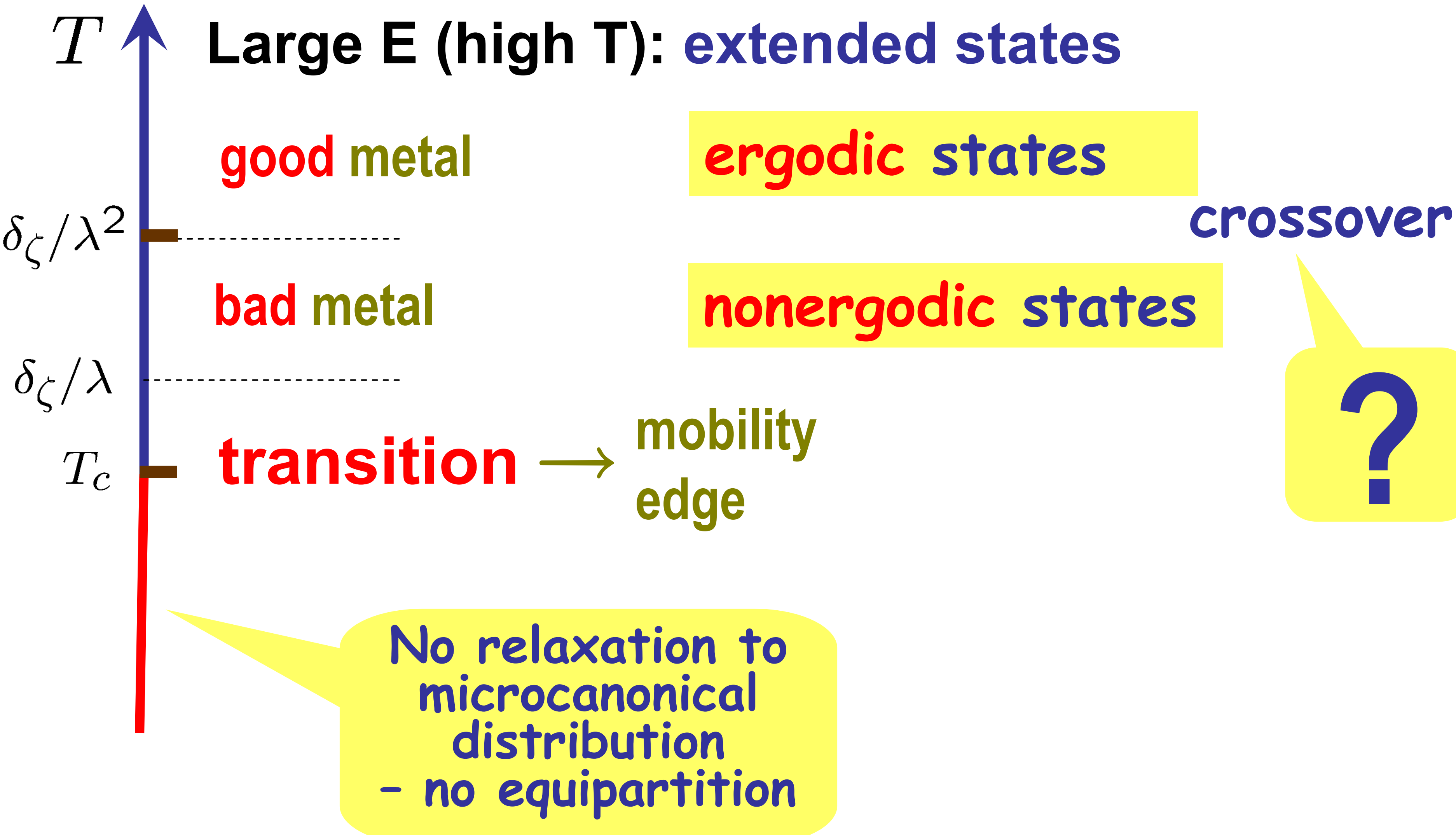
Large E (high T): extended states

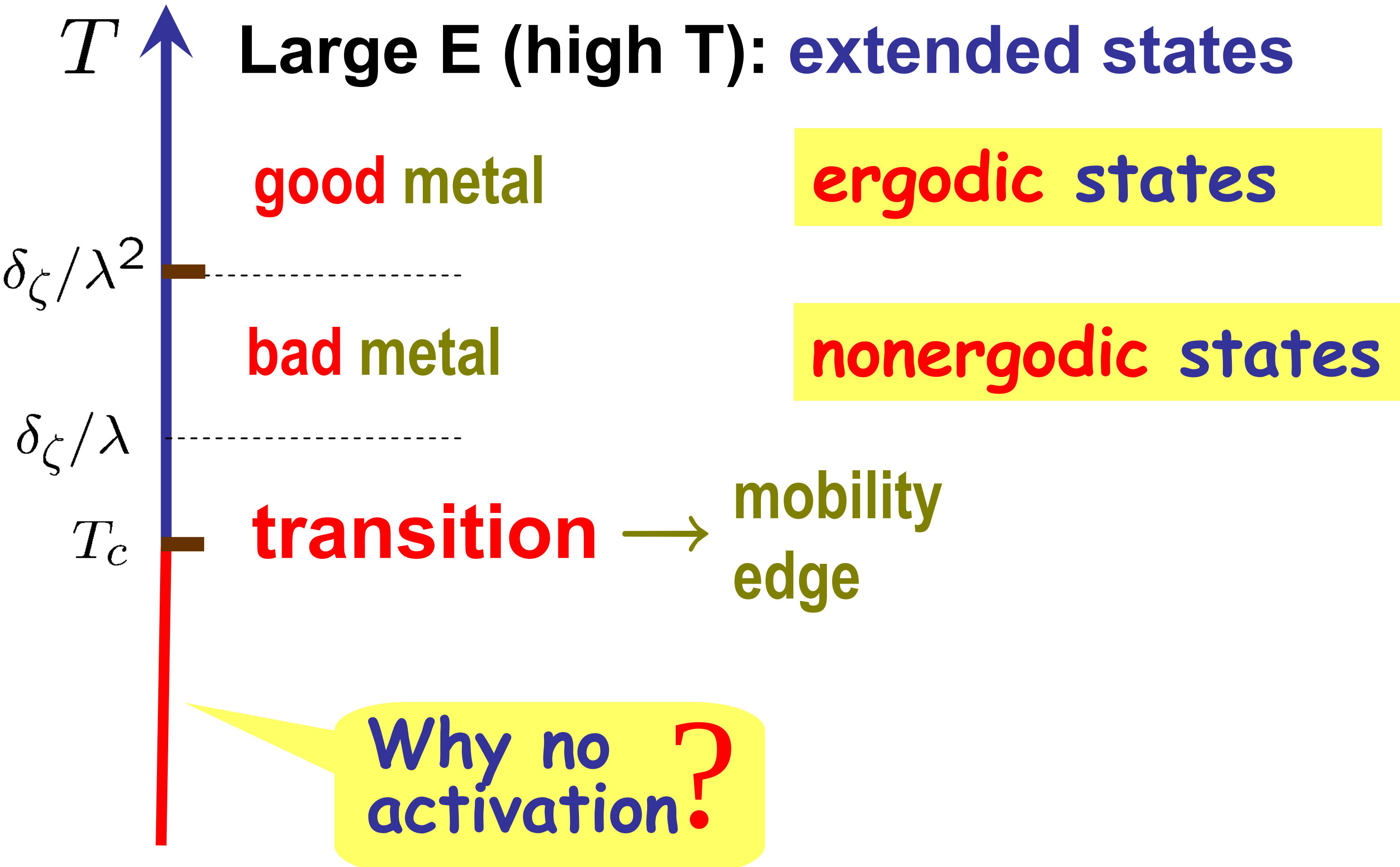
good metal — **ergodic states**

bad metal — **nonergodic states**

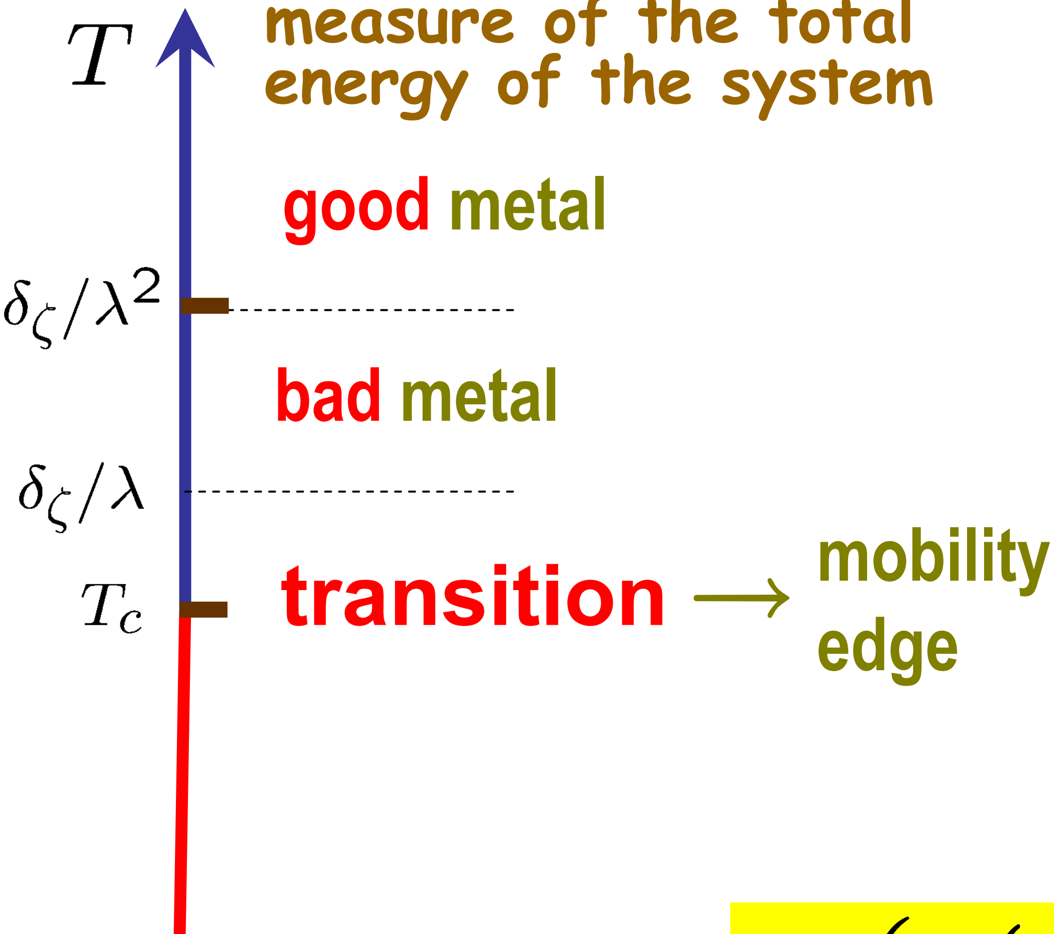
transition $\longrightarrow$ mobility edge

Such a state occupies infinitely many sites of the Anderson model but still negligible fraction of the total number of sites


T
Large E (high T): extended states
good metal
ergodic states
δ_ζ/λ^2
crossover
bad metal
nonergodic states
δ_ζ/λ
?
T_c
transition $\longrightarrow$ mobility edge
No relaxation to microcanonical distribution
- no equipartition


T
Large E (high T): extended states
good metal
ergodic states
δ_ζ/λ^2
bad metal
nonergodic states
δ_ζ/λ
T_c
transition
mobility
edge
Why no
activation?

Temperature is just a measure of the total energy of the system



$$E, E_c \propto volume$$

No activation:

$$E_c \propto \frac{T_c^{\,2}}{\delta_\zeta \zeta^d} \qquad E \propto \frac{T^2}{\delta_\zeta \zeta^d} \qquad \times\, volume$$

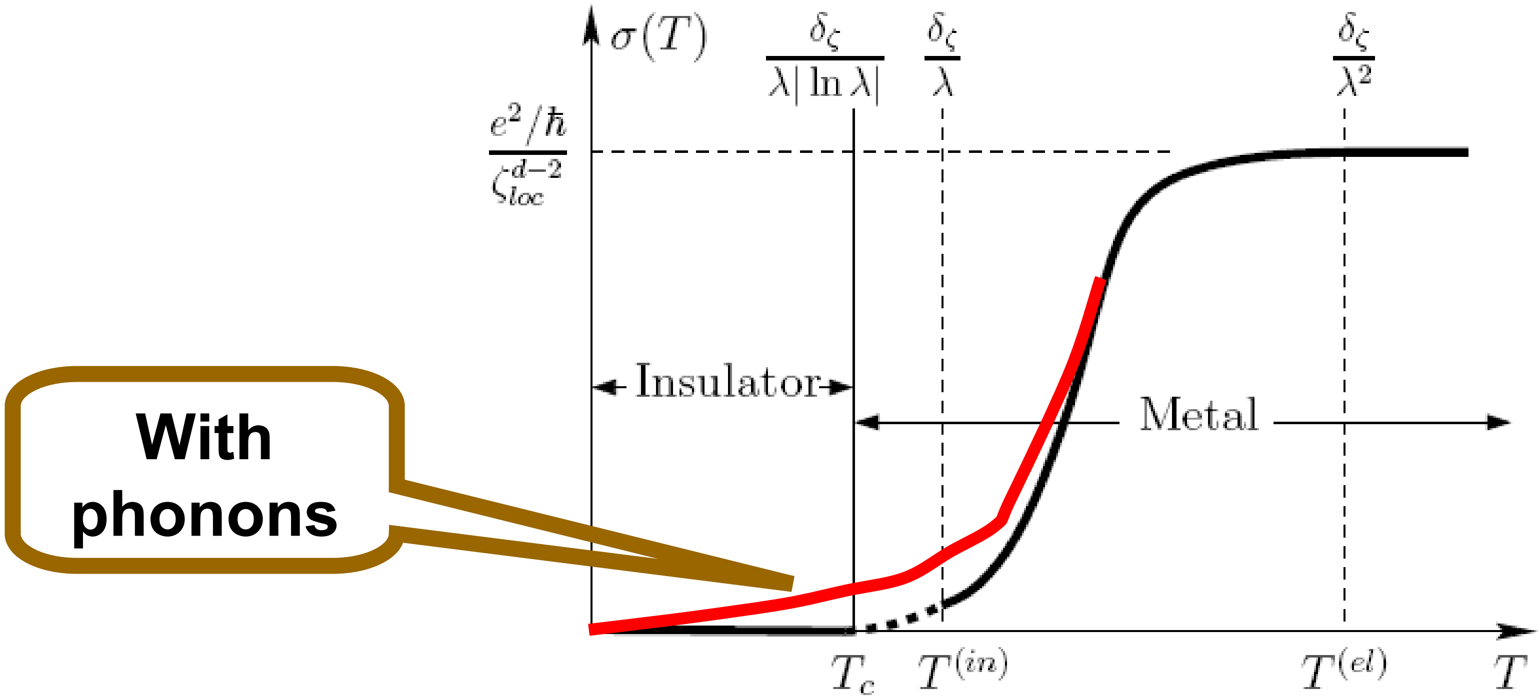
$$\exp\left(\frac{E(T)-E_c}{T}\right) \xrightarrow[volume \to \infty]{} 0$$

Lecture 3.

3. Experiment

What about experiment?

1. Problem: there are no solids without phonons



2. Cold gases look like ideal systems for studying this phenomenon.

F. Ladieu, M. Sanquer, and J. P. Bouchaud, Phys. Rev.B 53, 973 (1996)

YSi

G. Sambandamurthy, L. Engel, A. Johansson, E. Peled & D. Shahar, Phys. Rev. Lett. 94, 017003 (2005).

InO

M. Ovadia, B. Sacepe, and D. Shahar, PRL (2009).

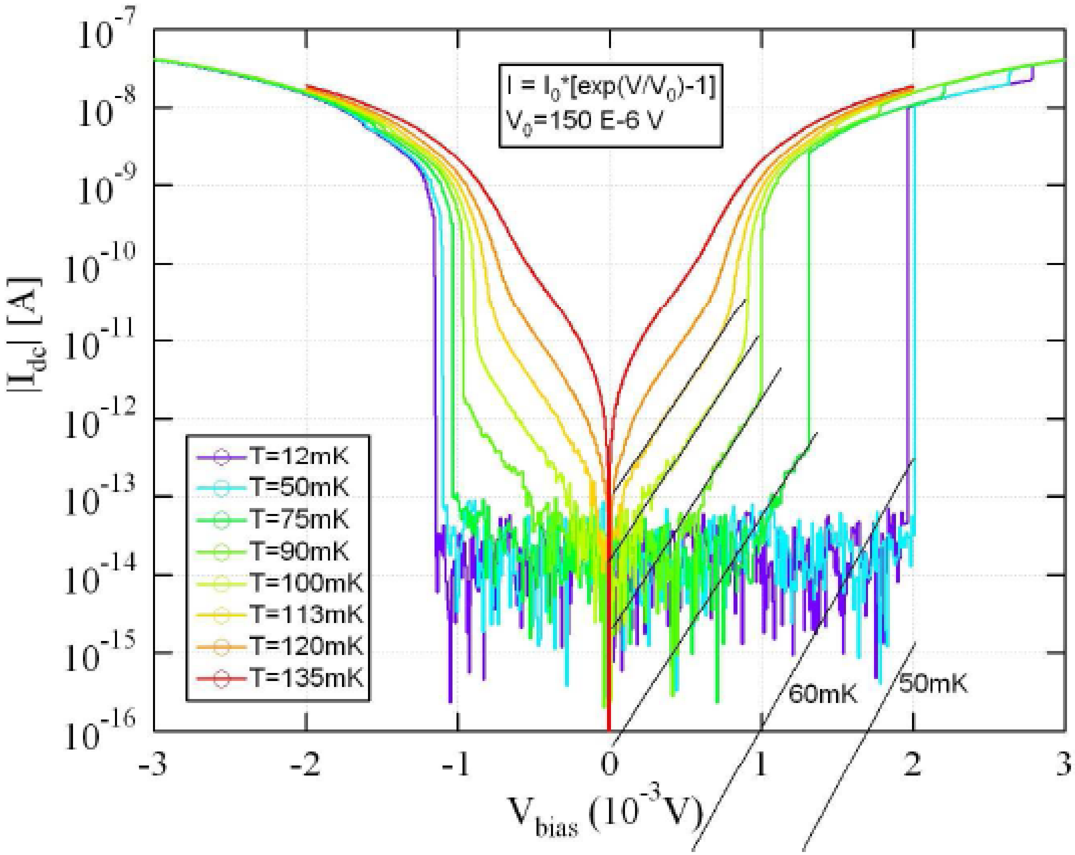
V. M. Vinokur, T. I. Baturina, M. V. Fistul, A. Y.Mironov, M. R. Baklanov, & C. Strunk, Nature 452, 613 (2008)

TiN

Superconductor – Insulator transition

S. Lee, A. Fursina, J.T. Mayo, C. T. Yavuz, V. L. Colvin, R. G. S. Sofin, I. V. Shvetz and D. Natelson, Nature Materials v 7 (2008)

FeO_4 magnetite



$$\frac{\partial I_{dc}}{\partial V} \propto \exp\left(-\frac{T_0}{T}\right)$$

Arrhenius law

M. Ovadia, B. Sacepe, and D. Shahar

Kravtsov, Lerner, Aleiner & BA:

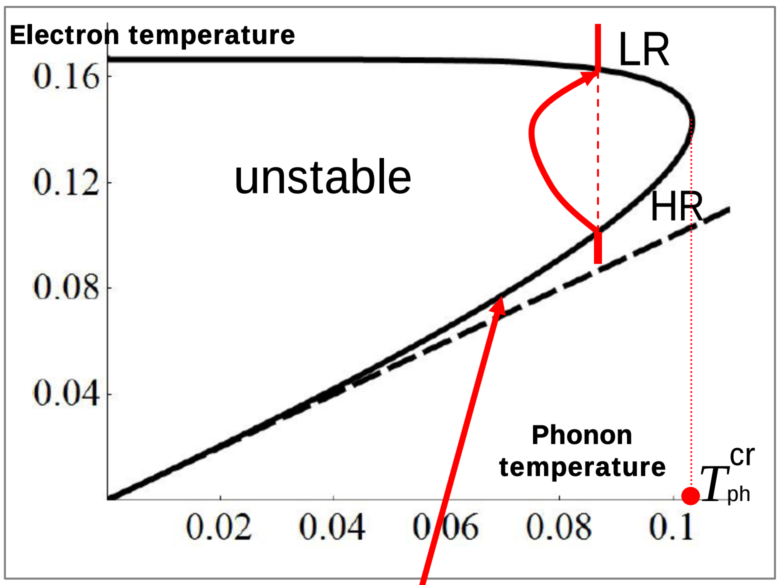
} PRL, 2009

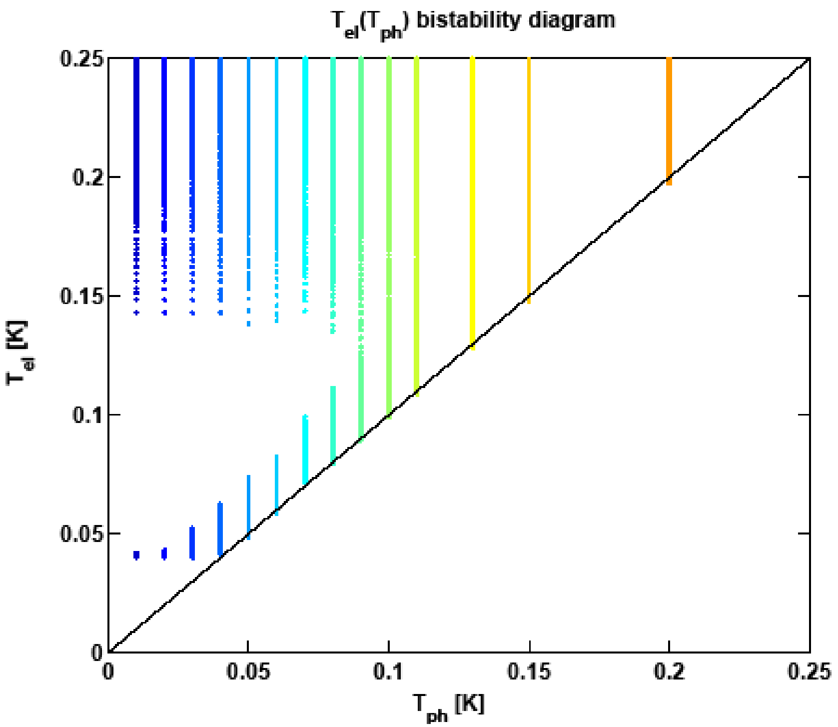
Switches ⇐ Bistability ⇐ Electrons are overheated:

Low resistance => high Joule heat => high el. temperature

High resistance => low Joule heat => low el. temperature

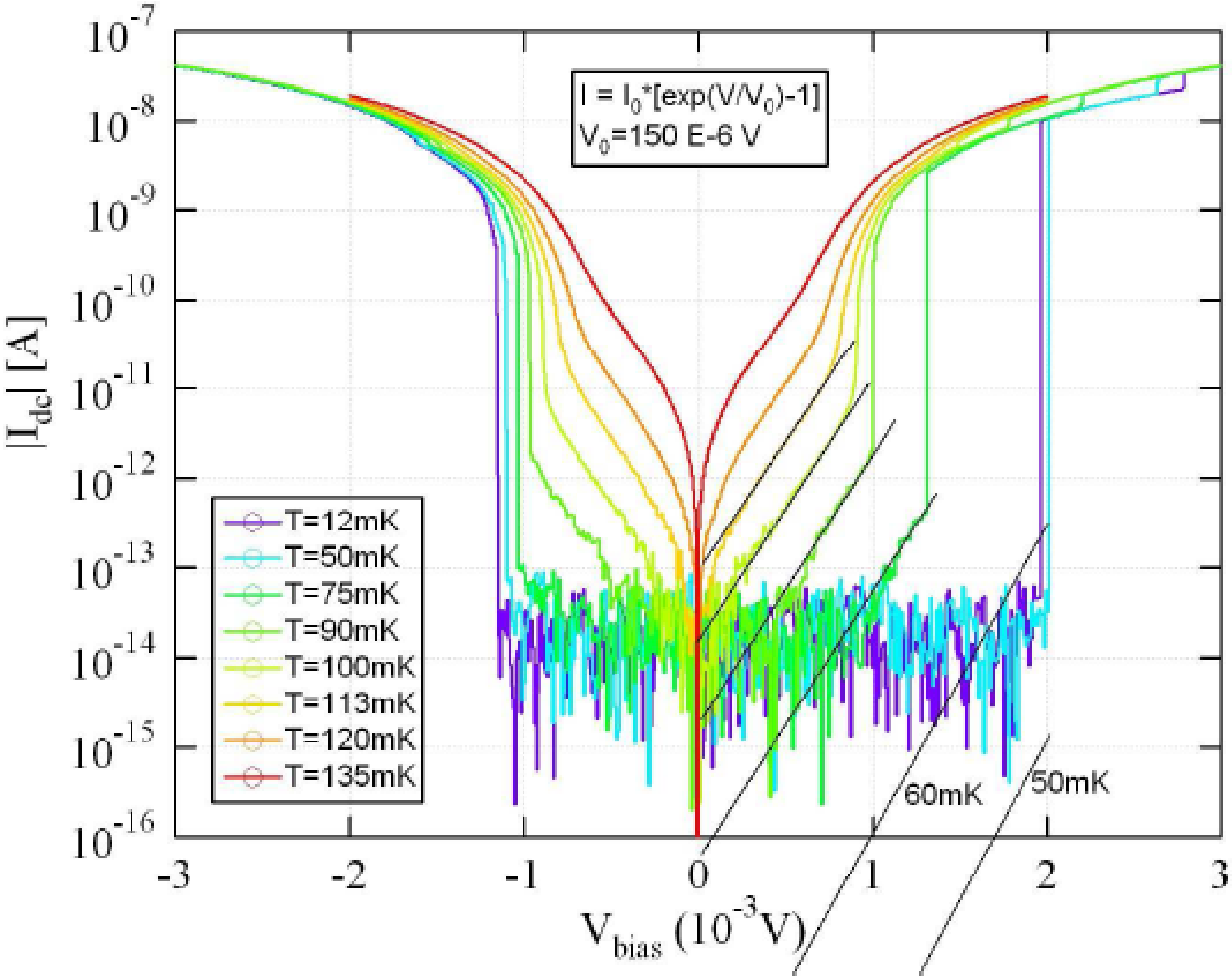
Electron temperature versus bath temperature





Arrhenius gap T_0~1K, which is measured independently is the only “free parameter”

Experimental bistability diagram (Ovadia, Sasepe, Shahar, 2008)



Common wisdom:

no heating in the insulating state

no heating for phonon-assisted hopping

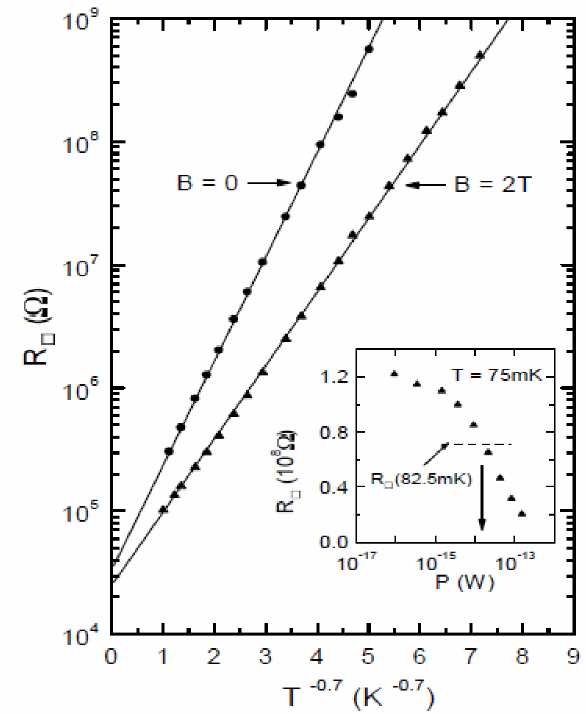
Kravtsov, Lerner, Aleiner & BA:

Switches ⇐ Bistability ⇐ Electrons are overheated:

Low resistance => high Joule heat => high el. temperature

High resistance => low Joule heat => low el. temperature

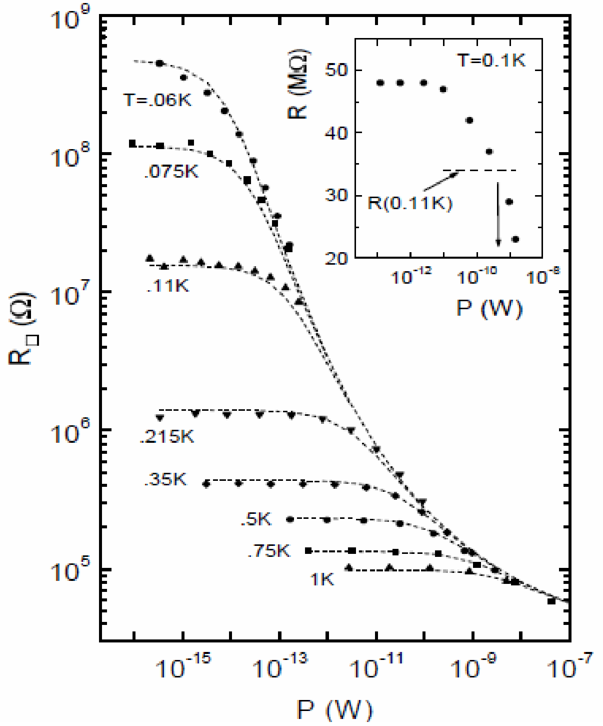
M. E. Gershenson, Yu. B. Khavin, D. Reuter, P. Schafmeister, and A. D. Wieck Phys. Rev. Lett. 85, 1718 (2000).



Si δ - doped GaAs structure

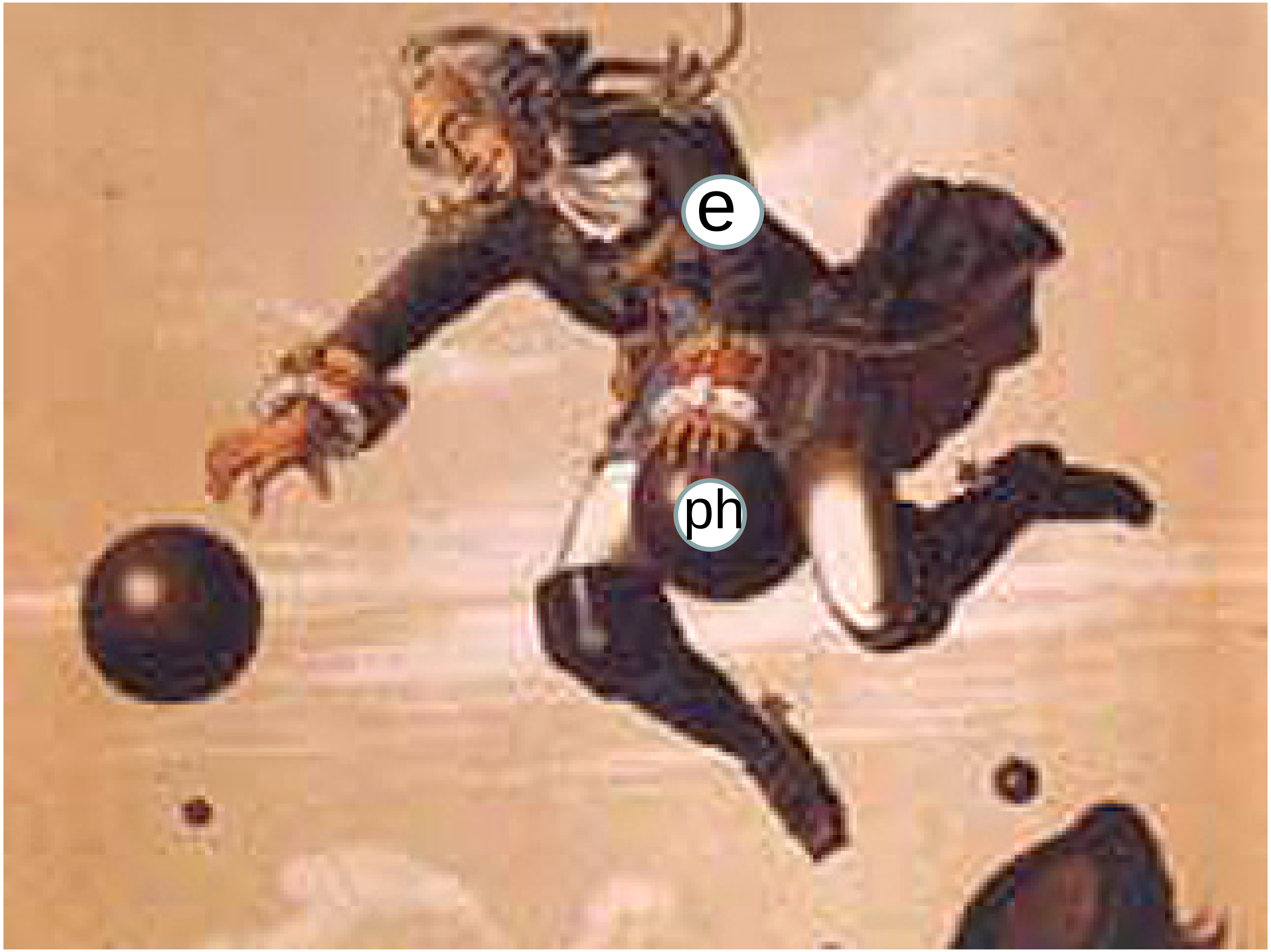
$$R_{\square} = R^{*} \exp\left[\left(\frac{T_o}{T}\right)^{0.7}\right]$$

	R^{*}	T_0
$B=0$	33 KΩ	2.6 °K
$B=2T$	25 KΩ	1.6 °K



Power: $P = 3.7\times10^{-9}\,[W]\left(T_e^{4.5} - T^{4.5}\right)$

Phonon-assisted variable range hopping

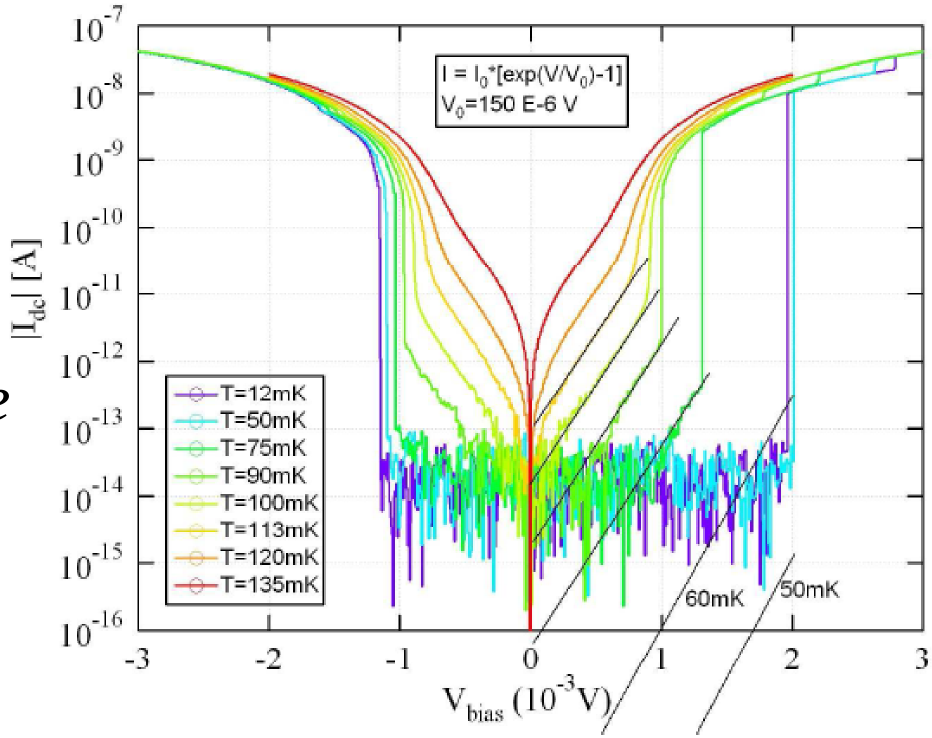


Low temperature anomalies

Voltage dependence of the conductance in the High Resistance phase

Theory : $G(V_{HL})/G(V \to 0) < e$

Experiment: this ratio can exceed *30*

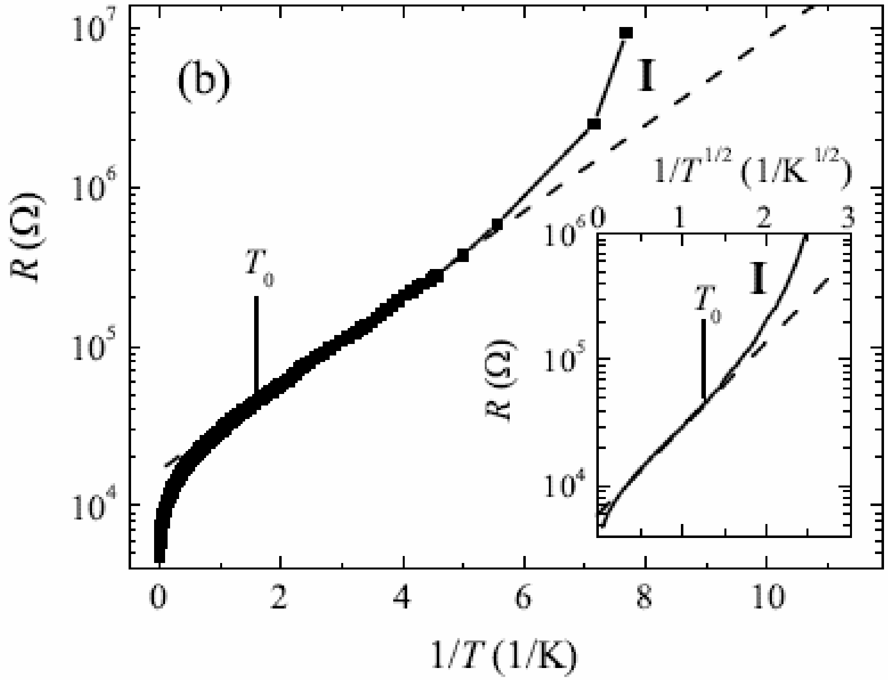


Many-Body Localization ?

Low temperature anomalies

1. Low T deviation from the Ahrenius law

"Hyperactivated resistance in TiN films on the insulating side of the disorder-driven superconductor-insulator transition"



T. I. Baturina, A.Yu. Mironov, V.M. Vinokur, M.R. Baklanov, and C. Strunk, 2009

Also:

- D. Shahar and Z. Ovadyahu, Phys. Rev. B (1992).
- V. F. Gantmakher, M.V. Golubkov, J.G. S. Lok, A.K. Geim,. JETP (1996)].
- G. Sambandamurthy, L.W. Engel, A. Johansson, and D.Shahar, Phys. Rev. Lett. (2004).

Lectute 3.

4. Many-Body Localization

1D bosons + disorder

1D Localization

Exactly solved:
all states are localized

Gertsenshtein & Vasil'ev, 1959

Conjectured:

Mott & Twose, 1961

-
-
-

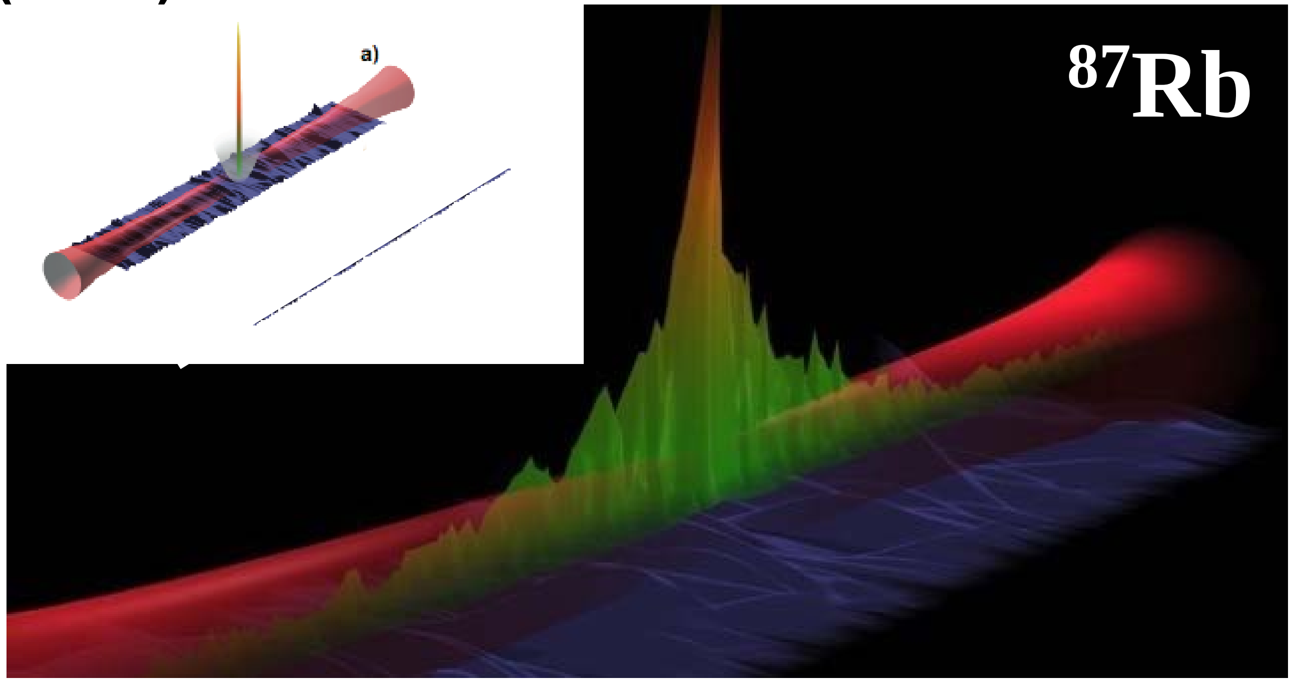
1-particle problem → correct for bosons as well as for fermions

Bosons without disorder

- Bose - Einstein condensation
- Bose–condensate even at weak enough repulsion
- Even in 1D case at T=0 – “algebraic superfluid”
- Finite temperature – Normal fluid

Localization of cold atoms

Billy et al. "Direct observation of Anderson localization of matter waves in a controlled disorder". Nature <u>453</u>, 891- 894 (2008).



Roati et al. "Anderson localization of a non-interacting Bose-Einstein condensate". Nature <u>453</u>, 895-898 (2008).

No interaction !

Thermodynamics of ideal Bose-gas in the presence of disorder is a pathological problem: all particles will occupy the localized state with the lowest energy

→

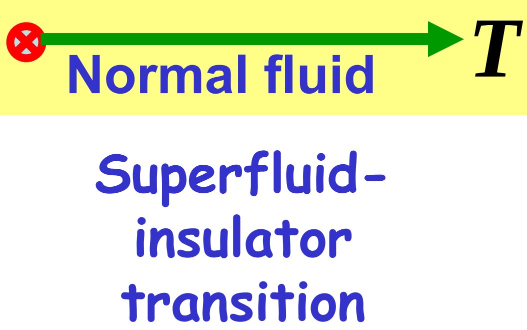
Need repulsion

Weakly interacting bosons

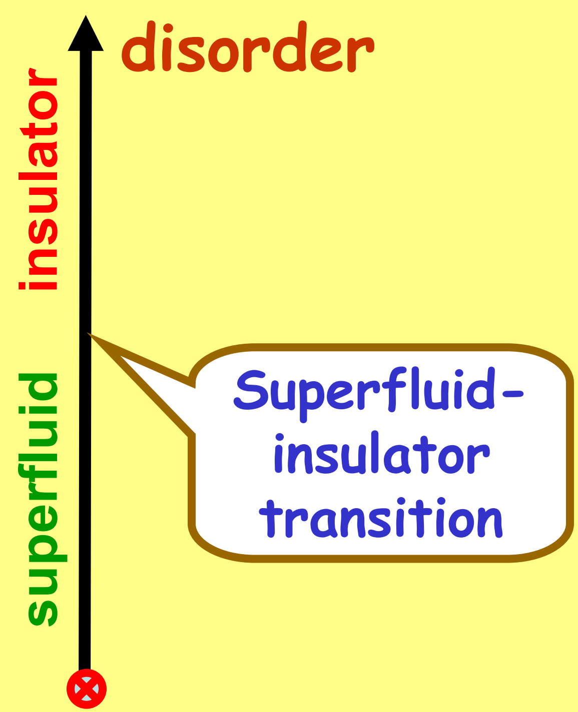
- Bose - Einstein condensation
- Bose-condensate even at weak enough repulsion
- Even in 1D case at T=0 – "algebraic superfluid"

1. No interaction

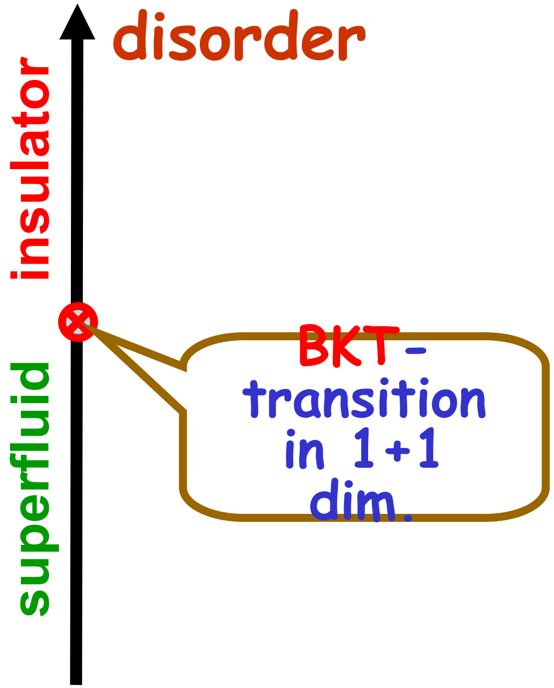
2. No disorder



3. Weak repulsion

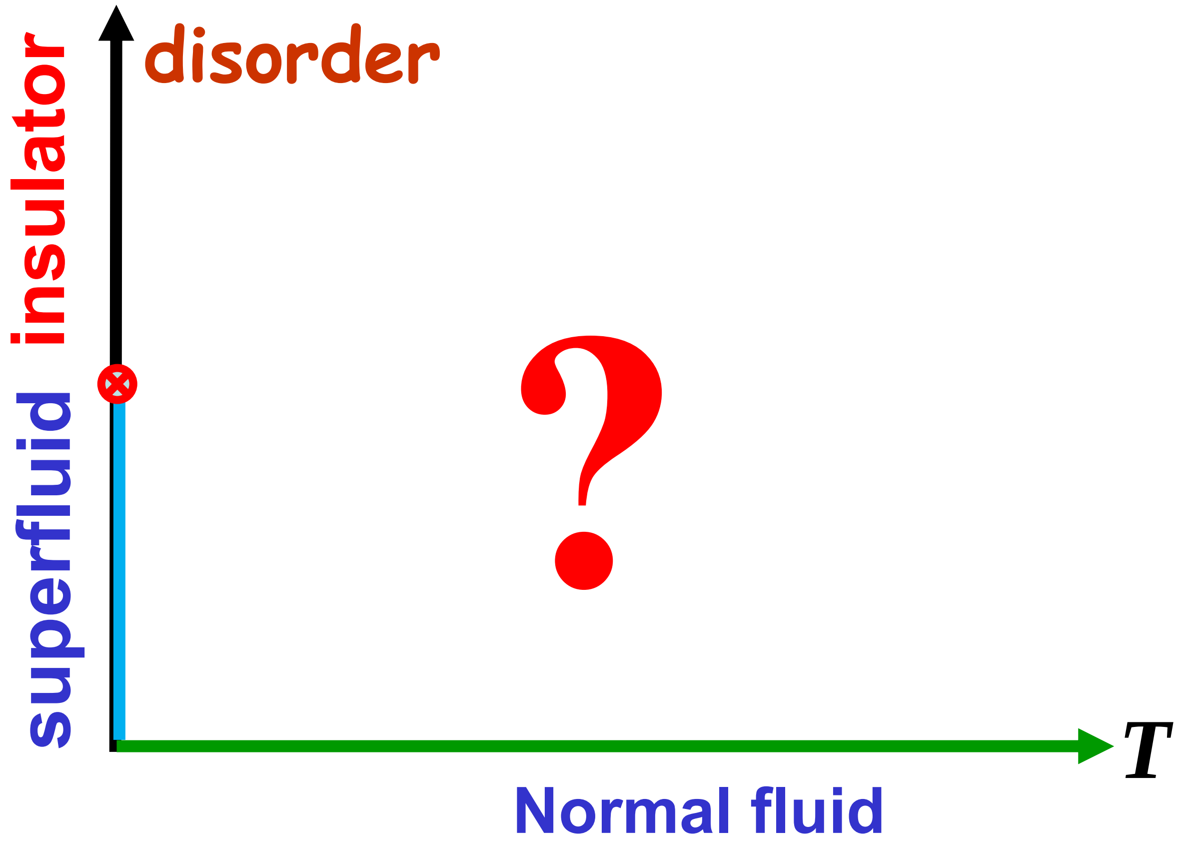


$T=0$ Superfluid - Insulator Quantum Phase Transition



E. Altman, Y. Kafri, A. Polkovnikov & G. Refael,
Phys. Rev. Lett., **100**, 170402 (2008).

G.M. Falco, T. Nattermann, & V.L. Pokrovsky,
Phys. Rev., **B80**, 104515 (2009).



Is it a normal fluid at any temperature?

What is insulator?

Perfect Insulator

Zero DC conductivity at finite temperatures

Possible if the system is decoupled from any outside bath

Normal metal (fluid)

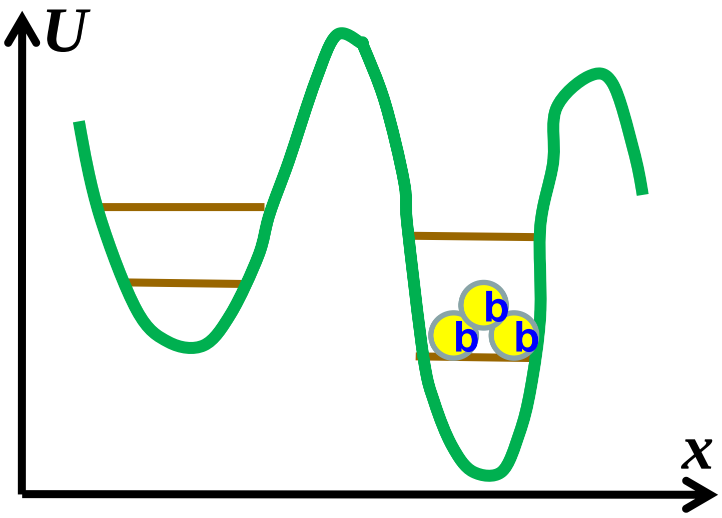
Finite (even if very small) DC conductivity at finite temperatures

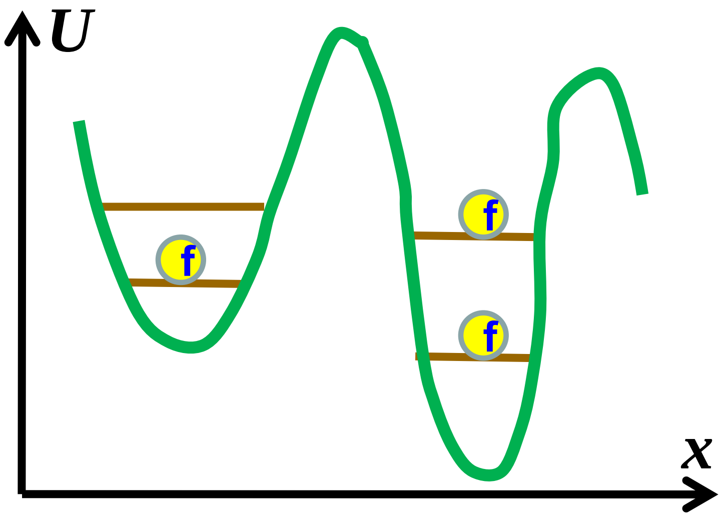
1D Luttinger liquid: bosons = fermions ?

Bosons with infinitely strong repulsion ≈ Free fermions

Free bosons ≈ Fermions with infinitely strong attraction

Weakly interacting bosons ≈ Fermions with strong attraction





As soon as the occupation numbers become large the analogy with fermions is not too useful

All one-particle states are localized in 1D – perfect insulator without interaction

This is correct for both fermions and bosons

Fermi-systems remain perfect insulators at low enough temperatures even in the presence of the interaction Basko, Aleiner & BA, 2005

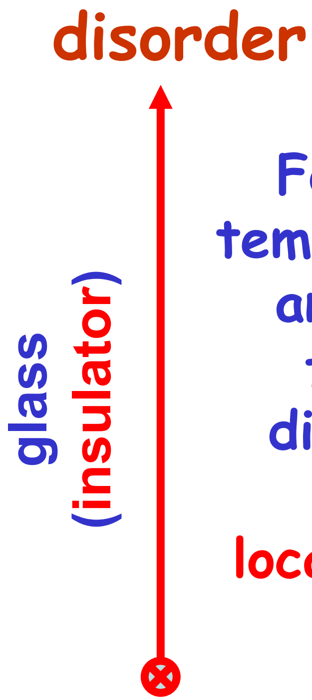
What about bose-systems ?

Difference: many bosons can occupy a given one-particle state. Interaction matrix elements increase with occupation numbers

1D Weakly Interacting Bosons + Disorder

Aleiner, BA & Shlyapnikov, 2010, Nature Physics, to be published
cond-mat 0910.4534

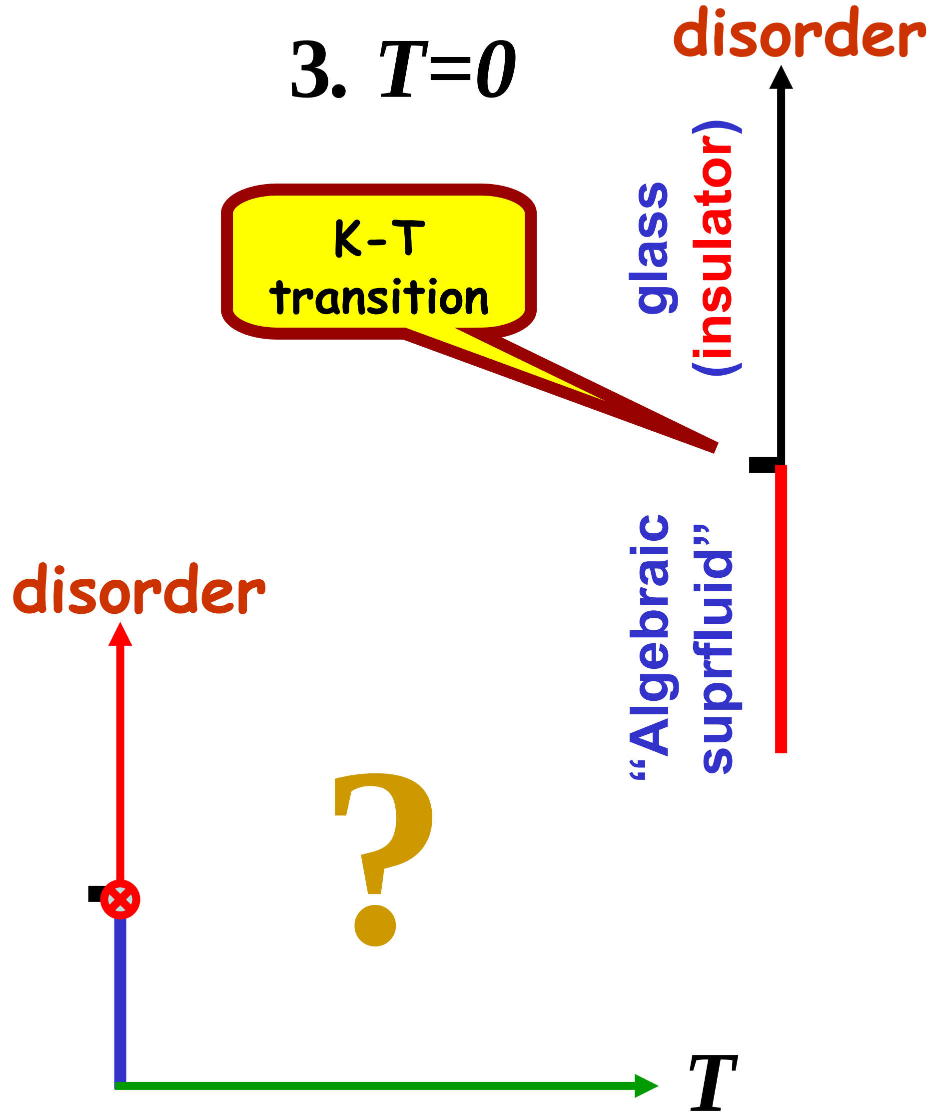
1. No interaction



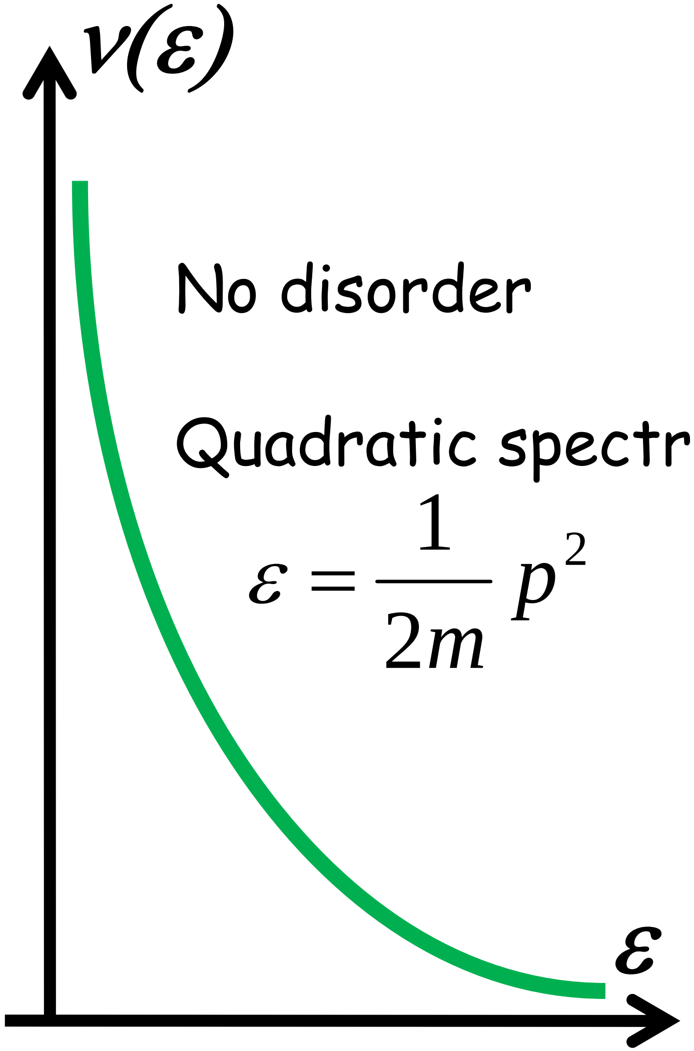
For any temperature and any finite disorder
1D localization

2. No disorder

3. *T*=0



Density of States $\nu(\varepsilon)$ in one dimension



No disorder

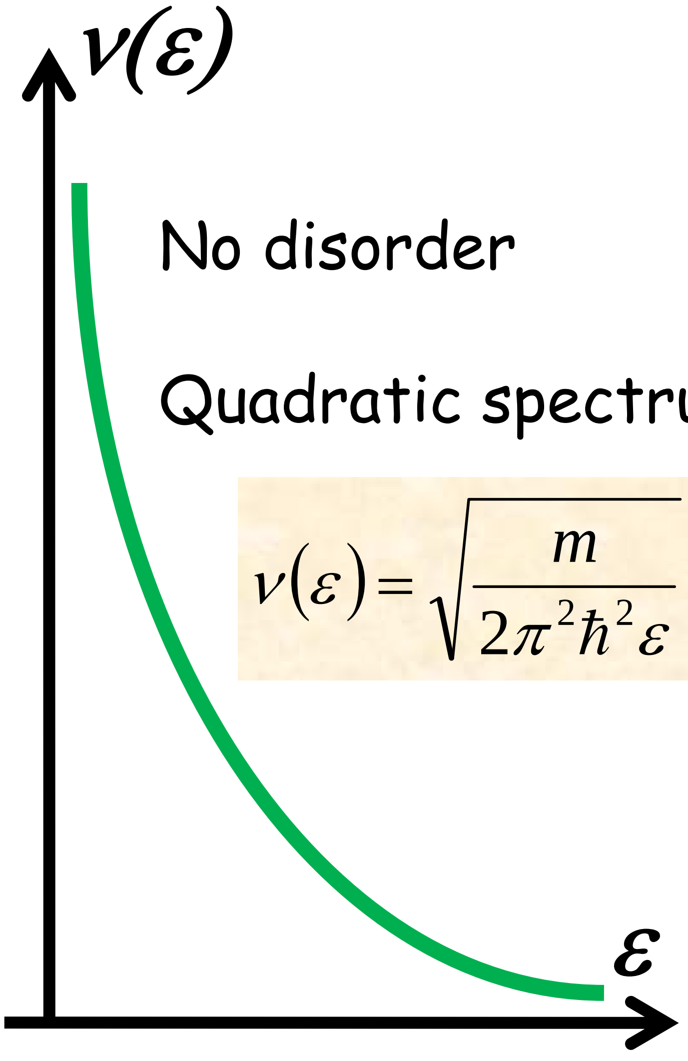
Quadratic spectrum

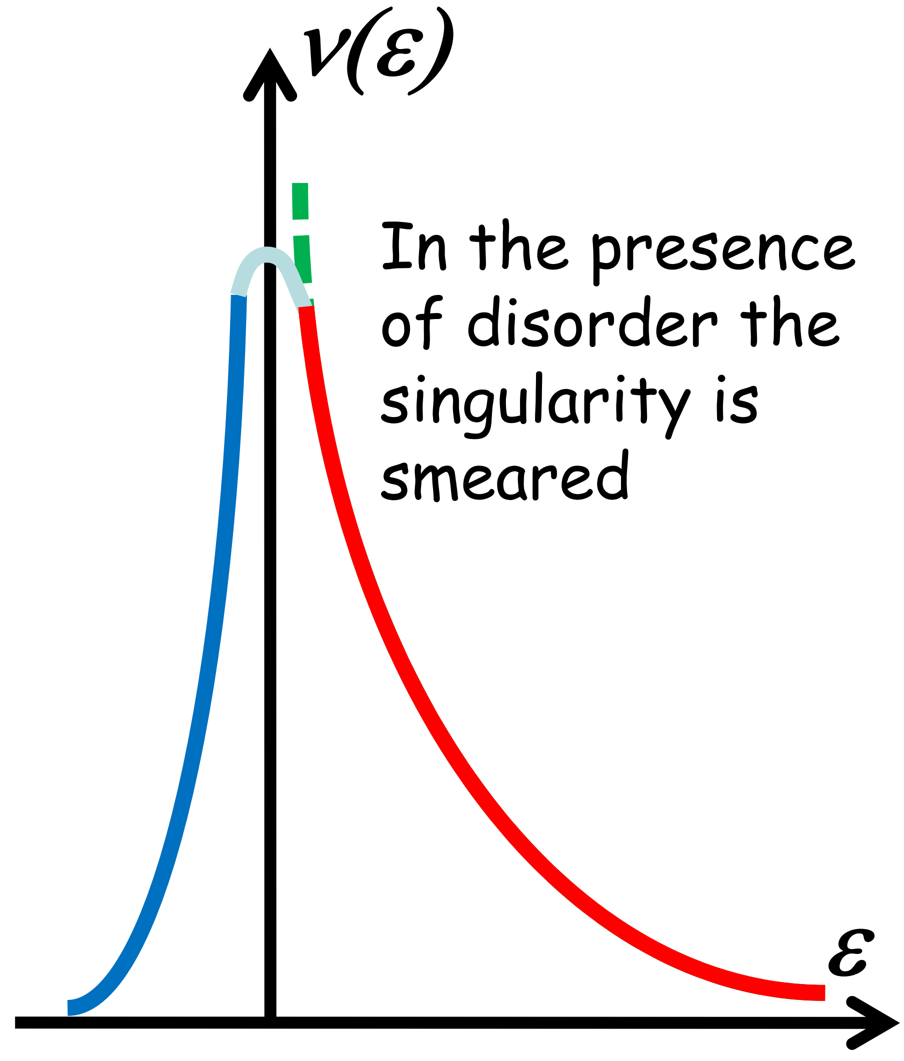
$$\varepsilon = \frac{1}{2m} p^2$$

$$\nu(\varepsilon) = \sqrt{\frac{m}{2\pi^2 \hbar^2 \varepsilon}}$$

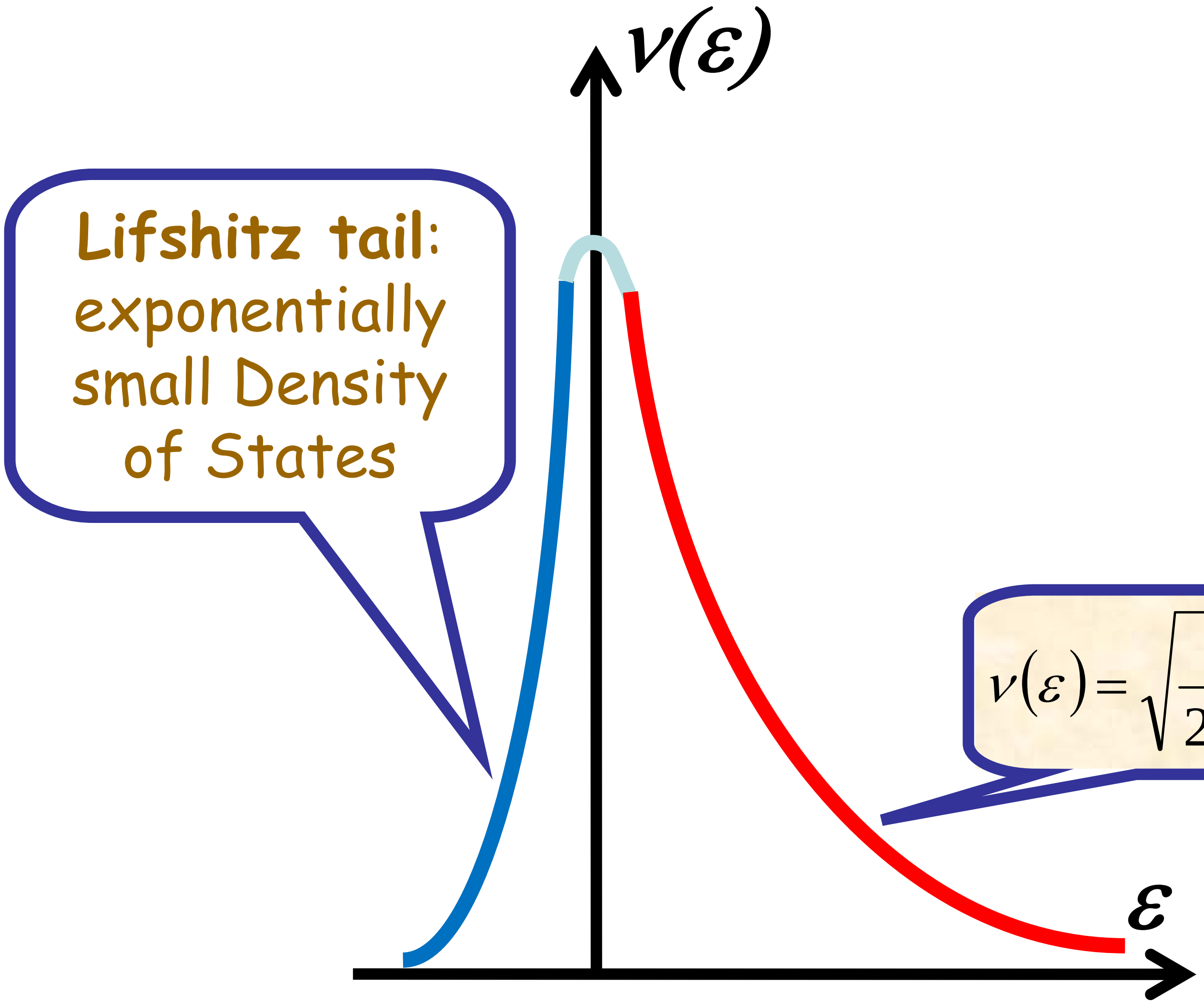
$\sqrt{\ }$ - singularity

Density of States $\nu(\varepsilon)$ in one dimension

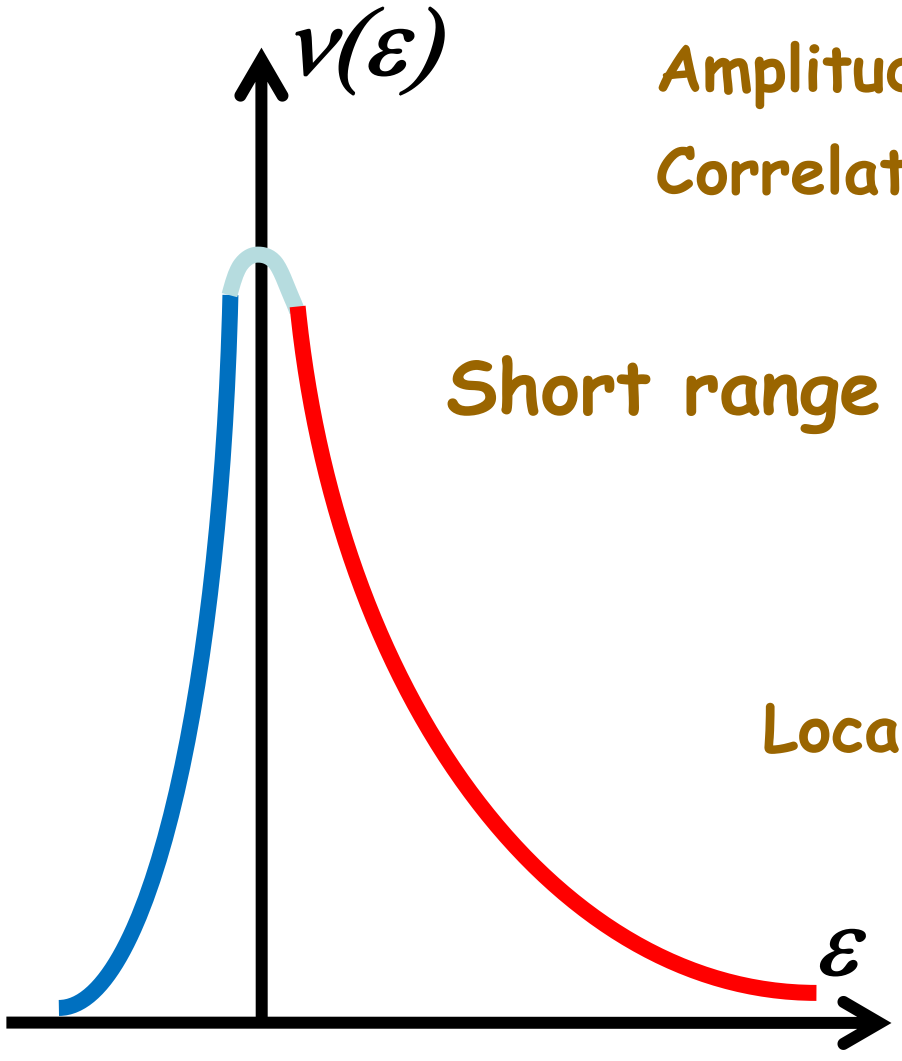




Density of States $\nu(\varepsilon)$ in one dimension



Weak disorder – random potential $U(x)$



Random potential $U(x)$:

Amplitude U_0

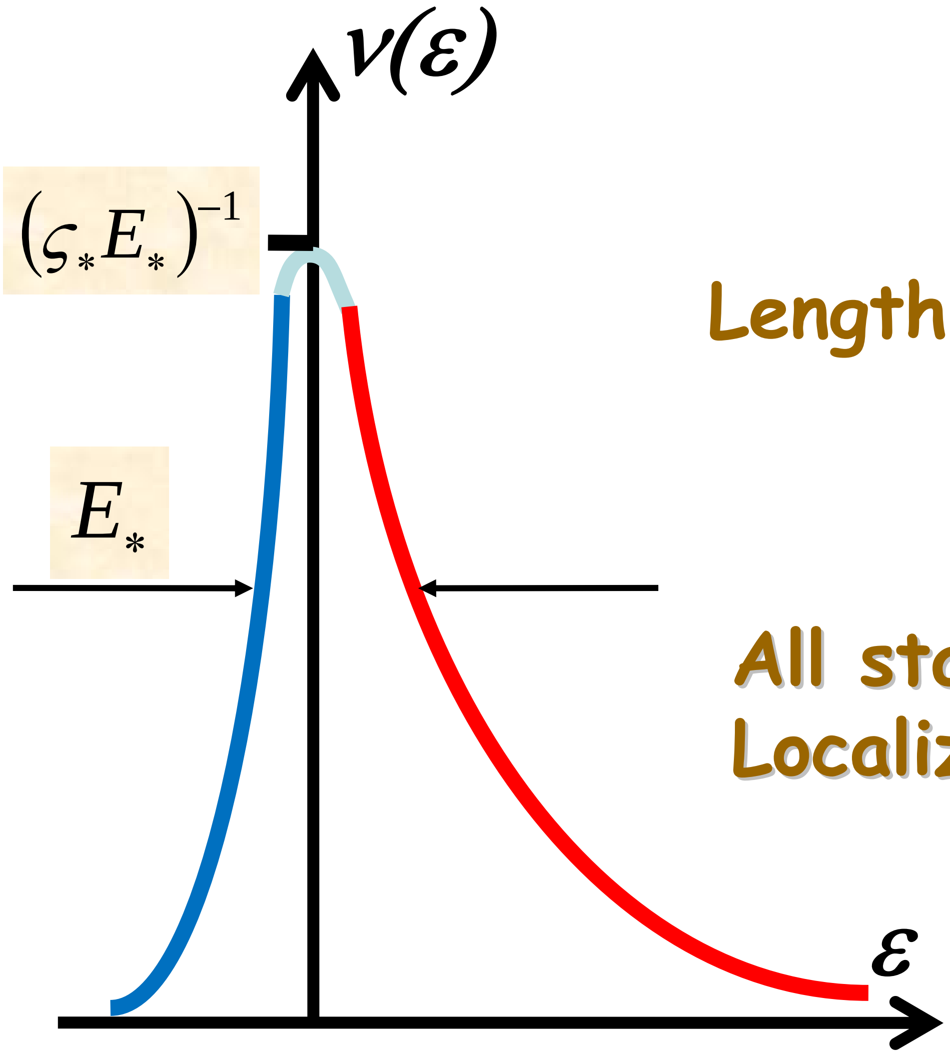
Correlation length σ

Short range disorder: $U_0 << \frac{\hbar^2}{m\sigma^2}$

⇓

Localization length $\zeta >> \sigma$

Characteristic scales:



Energy

$$E_* \equiv \left(\frac{U_0^4 \sigma^2 m}{\hbar^2}\right)^{1/3}$$

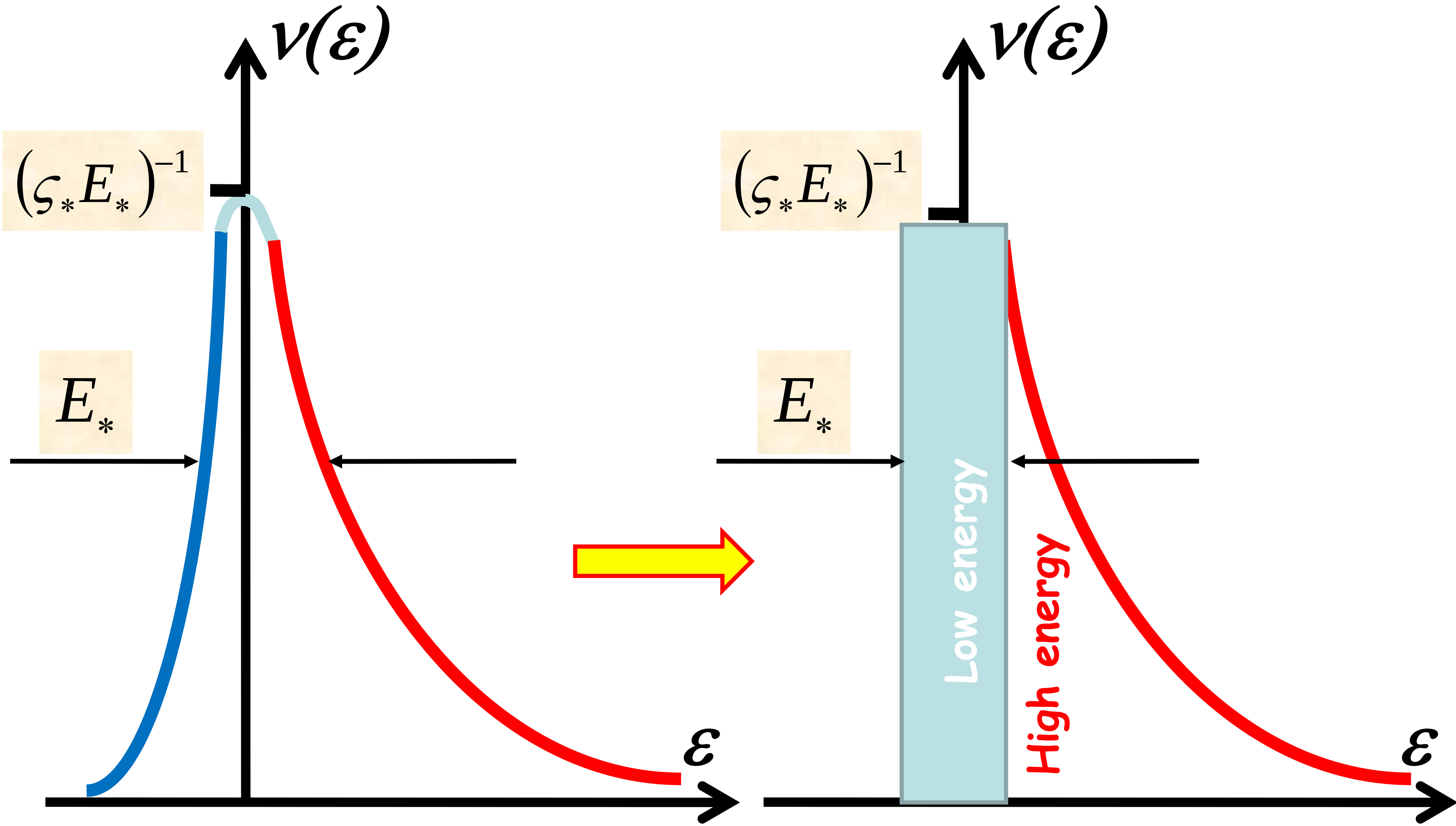
Length

$$\varsigma_* \equiv \left(\frac{\hbar^4}{U_0^2 \sigma m}\right)^{1/3} >> \sigma$$

All states are localized

Localization length:

$$\varsigma(\varepsilon) \sim \begin{cases} \varsigma_* & \varepsilon \sim E_* \\ \varsigma_* \dfrac{\varepsilon}{E_*} & \varepsilon >> E_* \end{cases}$$



$\nu(\varepsilon)$
$(\varsigma_* E_*)^{-1}$
E_*
ε
$\nu(\varepsilon)$
$(\varsigma_* E_*)^{-1}$
E_*
Low energy
High energy
ε

Finite density Bose-gas with repulsion

Density n

Two more energy scales

Temperature of quantum degeneracy $T_d \equiv \frac{\hbar^2 n^2}{m}$

Interaction energy per particle ng

Two dimensionless parameters

$\kappa \equiv E_* / ng$ Characterizes the strength of disorder

$\gamma \equiv ng / T_d$ Characterizes the interaction strength

Strong disorder $\kappa >> 1$

Weak interaction $\gamma << 1$

Dimensionless temperature $t = T/ng$

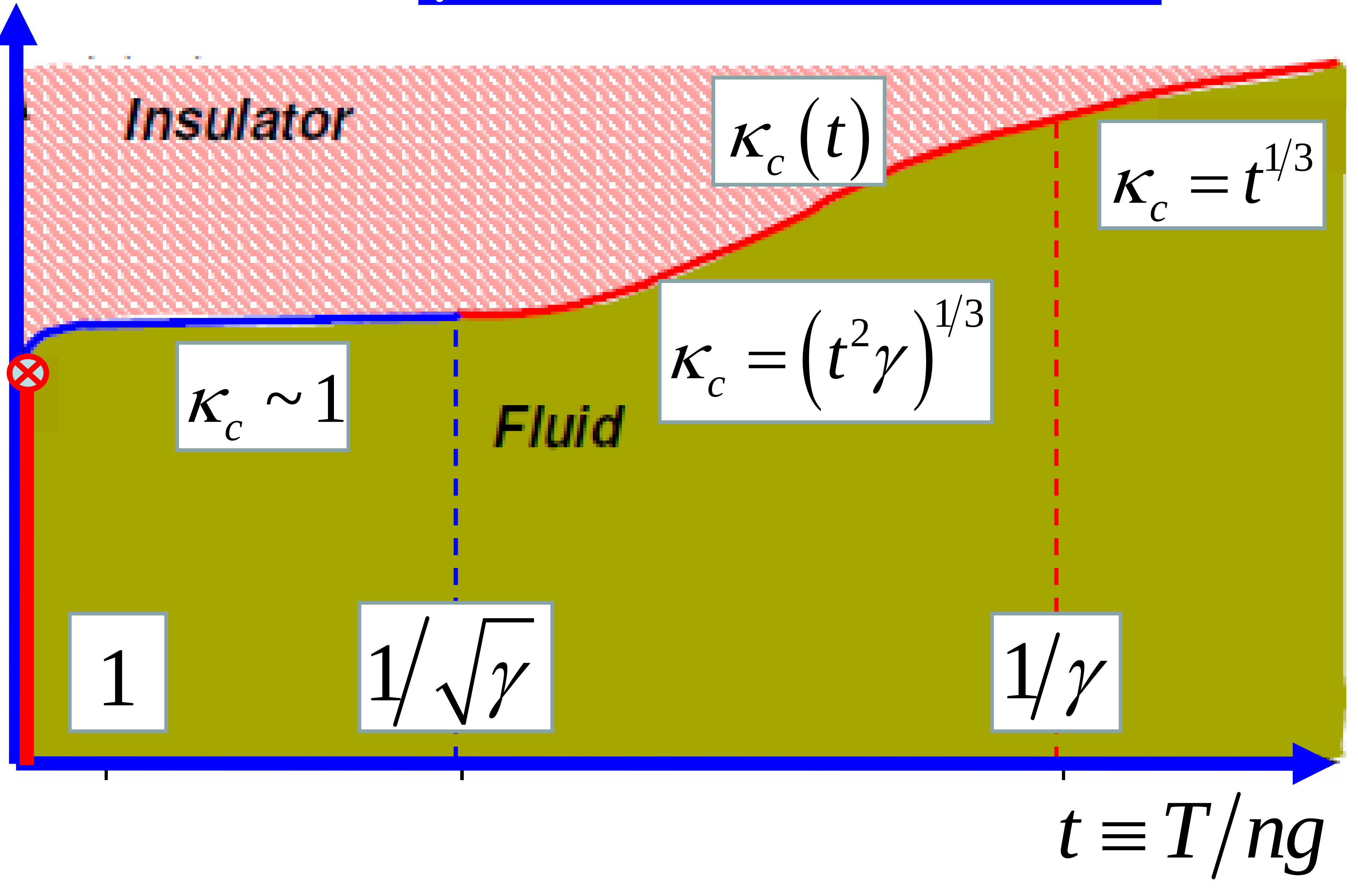
Critical temperature T_c $t_c = t_c(\kappa, \gamma)$

Critical disorder $\kappa_c = \kappa_c(t, \gamma)$

Phase transition line on the t, κ - plane

Finite temperature phase transition in 1D

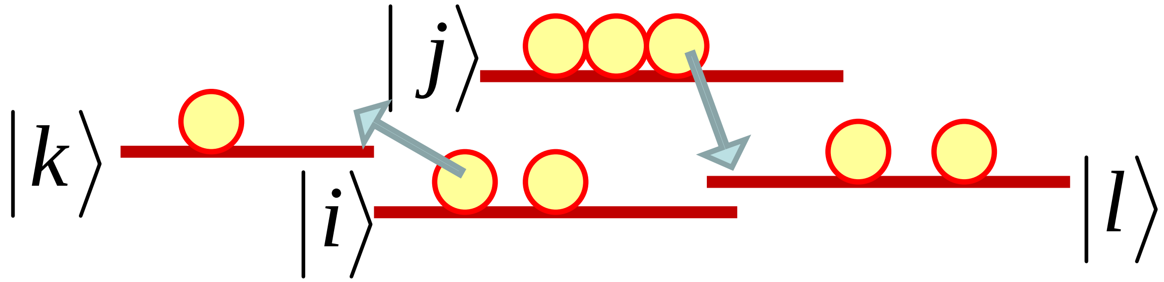
$\gamma << 1$



Transition temperature: $T_c \equiv t_c(ng)$

$|i\rangle, |j\rangle \Rightarrow |k\rangle, |l\rangle$

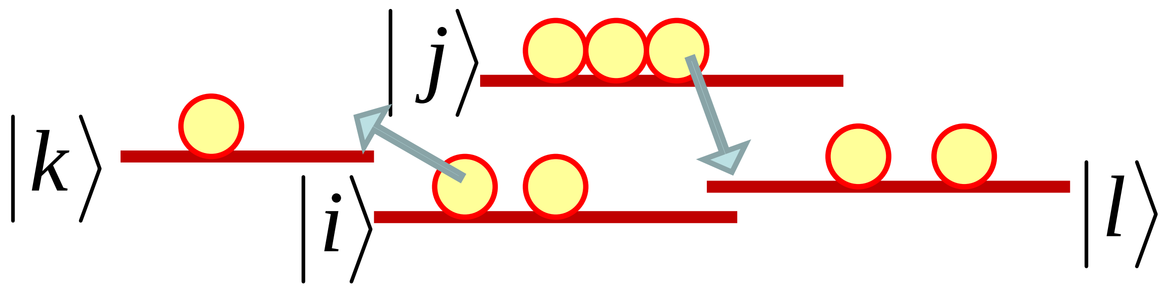
transition



Transition temperature: $T_c \equiv t_c(ng)$

$|i\rangle, |j\rangle \Rightarrow |k\rangle, |l\rangle$

transition



$\Delta_{ij,kl} \equiv \varepsilon_i + \varepsilon_j - \varepsilon_k - \varepsilon_l$ energy mismatch

$M_{ij,kl}$ matrix element

Decay of a state $|i\rangle$

Δ typical mismatch

N_1 typical # of channels

M typical matrix element

Transition:

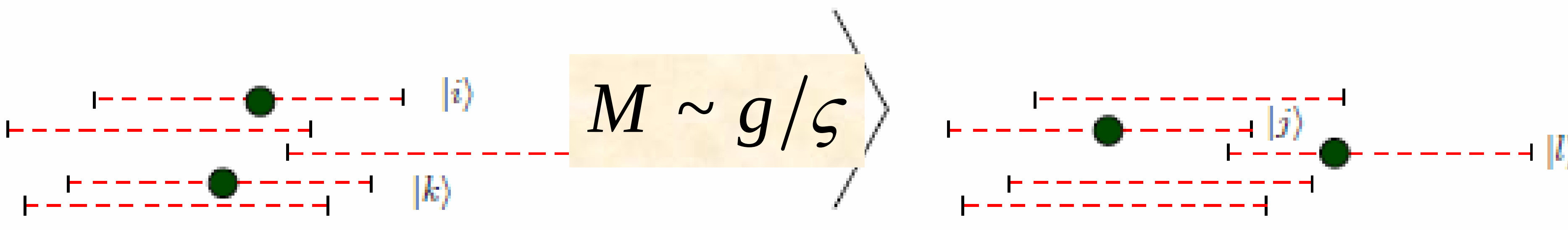
$$M(T) \begin{matrix} \gg \Delta(T)/N_1(T) \\ \ll \Delta(T)/N_1(T) \end{matrix} \quad \begin{matrix} \text{extended} \\ \text{localized} \end{matrix}$$

analog of

$$I \begin{matrix} \gg W \\ \ll W \end{matrix}$$

High temperatures: $T \gg T_d$ ⟺ $t \gg \gamma^{-1}$

Bose-gas is not degenerated;
occupation numbers either **0** or **1**



Matrix element of the transition

$$M \sim g/\varsigma(\varepsilon = T) \sim (gE_*)/(\varsigma_* T)$$

should be compared with the minimal energy mismatch

$$(\nu\varsigma)^{-1}/(n\varsigma) \sim \left(\nu n \varsigma_*^{\,2} T^2\right)^{-1} E_*^2$$

Localization spacing δ_ς

Number of channels

$$\kappa_c(t) \propto t^{1/3} \qquad t\gamma \gg 1$$

Intermediate temperatures: $\gamma^{-1/2} << t << \gamma^{-1}$

1. $T << T_d$ ⟺ $t\gamma << 1$

2. Bose-gas is degenerated; occupation numbers either >>1.

3. Typical energies $|\mu| = T^2/T_d$, μ is the chemical potential. Correct as long as

$|\mu| >> ng, E_*$ ⟺ $t\sqrt{\gamma} >> 1$

multiple occupation $N(\varepsilon) \sim \frac{T}{\varepsilon}$

4. Characteristic energies $\varepsilon \sim |\mu|$ $<< T$, $>> ng, E_*$

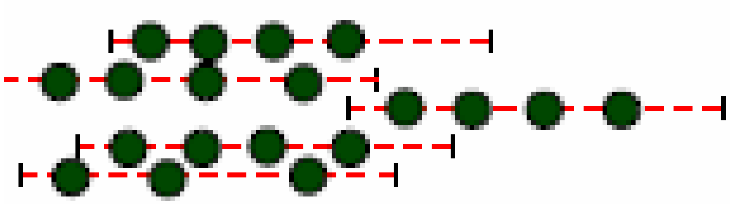
We are still dealing with the high energy states

Intermediate temperatures: $\gamma^{-1/2} << t << \gamma^{-1}$

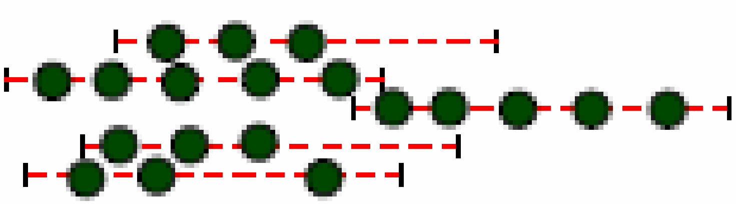
$$|\mu| = T^2 / T_d >> ng, E_*$$

$$T << T_d$$

Bose-gas is degenerated; typical energies ~ $|\mu| >> T$ ⟹ occupation numbers $>> 1$ ⟹ matrix elements are enhanced



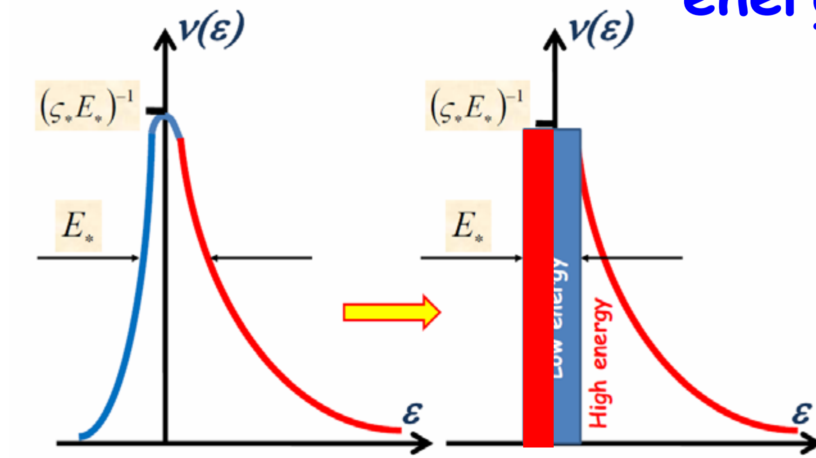
$$MN_1 \sim \frac{g}{\varsigma(\varepsilon)} \frac{T}{\varepsilon}$$



$$\kappa_c(t) \propto t^{2/3} \gamma^{1/3} \qquad \sqrt{\gamma} << t\gamma << 1$$

Low temperatures: $t << \gamma^{-1/2}$

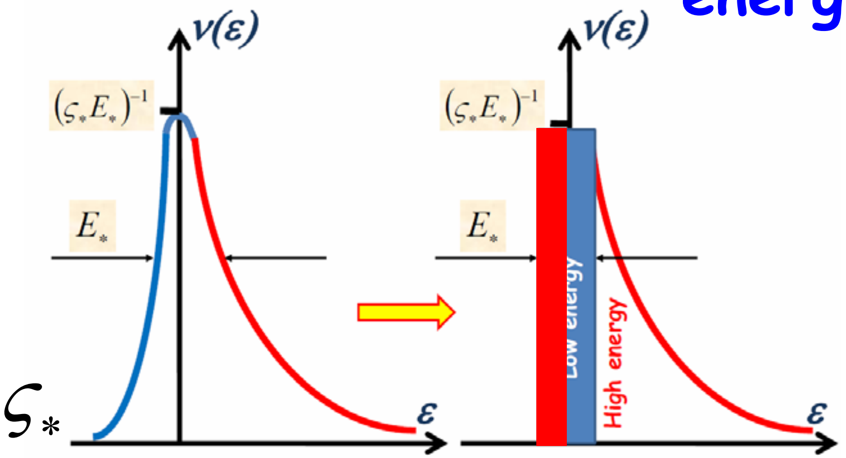
Suppose $\kappa \equiv E_* / ng >> 1$ ⟹ $|\mu| << E_*$ ⟹ **Bosons occupy only small fraction of low energy states** $\varepsilon_i < \mu$



Low temperatures: $t << \gamma^{-1/2}$

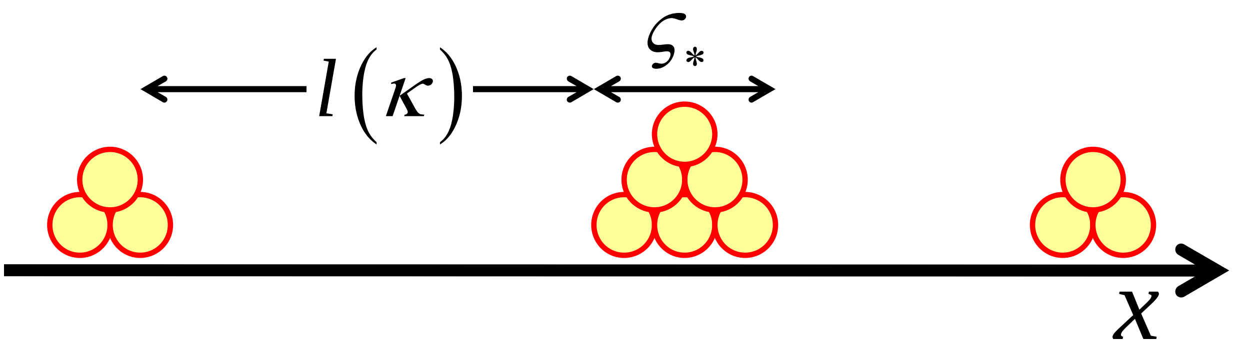
Suppose $\kappa \equiv E_* / ng >> 1$ ⟹ $|\mu| << E_*$ ⟹ **Bosons occupy only small fraction of low energy states** $\varepsilon_i < \mu$

Localization length ς_*



Occupation #: $(\mu - \varepsilon_i)\varsigma_* / g$

DoS: $\nu(\varepsilon) = (E_* \varsigma_*)^{-1}$ ⟹ $n = \dfrac{\mu^2}{2gE_*}$ ⟹ $\mu = E_* \sqrt{\kappa}$



$l(\kappa) = \varsigma_* \sqrt{\kappa} >> \varsigma_*$

Occupation

$nl(\kappa)/\varsigma_* = \gamma^{-1/2} >> 1$

Low temperatures: $t << \gamma^{-1/2}$

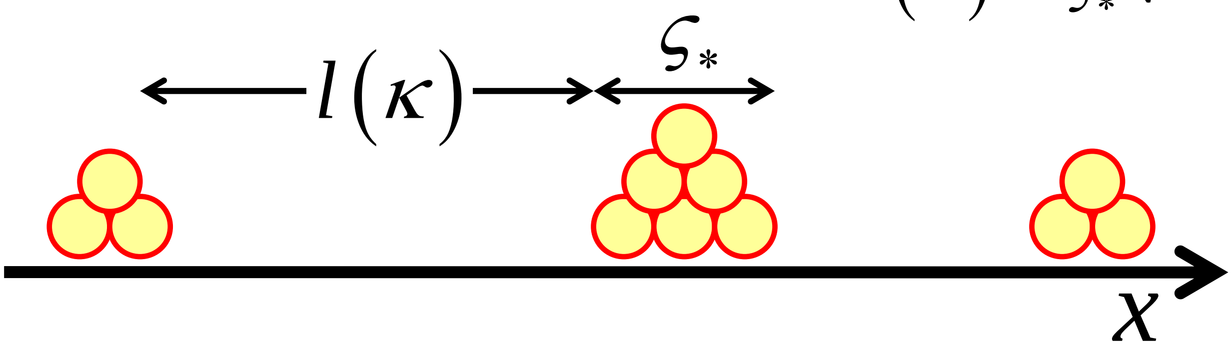
$\kappa \equiv E_* / ng >> 1$ ⟹ "lakes"

Occupation

$nl(\kappa)/\varsigma_* = \gamma^{-1/2} >> 1$

Distance

$l(\kappa) = \varsigma_* \sqrt{\kappa} >> \varsigma_*$



$l(\kappa) >> \varsigma_*$ ⟹ **Strong insulator**

$\kappa \to \kappa_c$

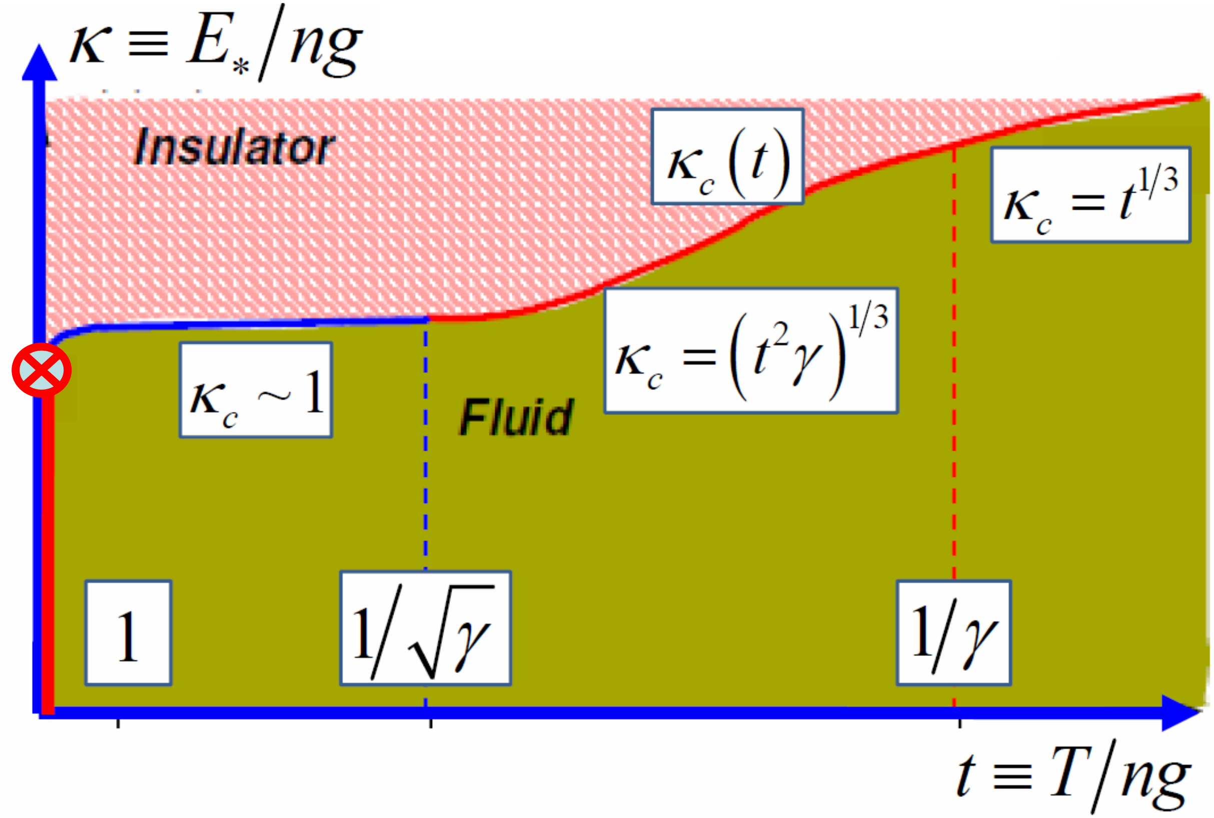
$l(\kappa) << \varsigma_*$ ⟹ **Insulator – Superfluid transition in a chain of "Josephson junctions"**

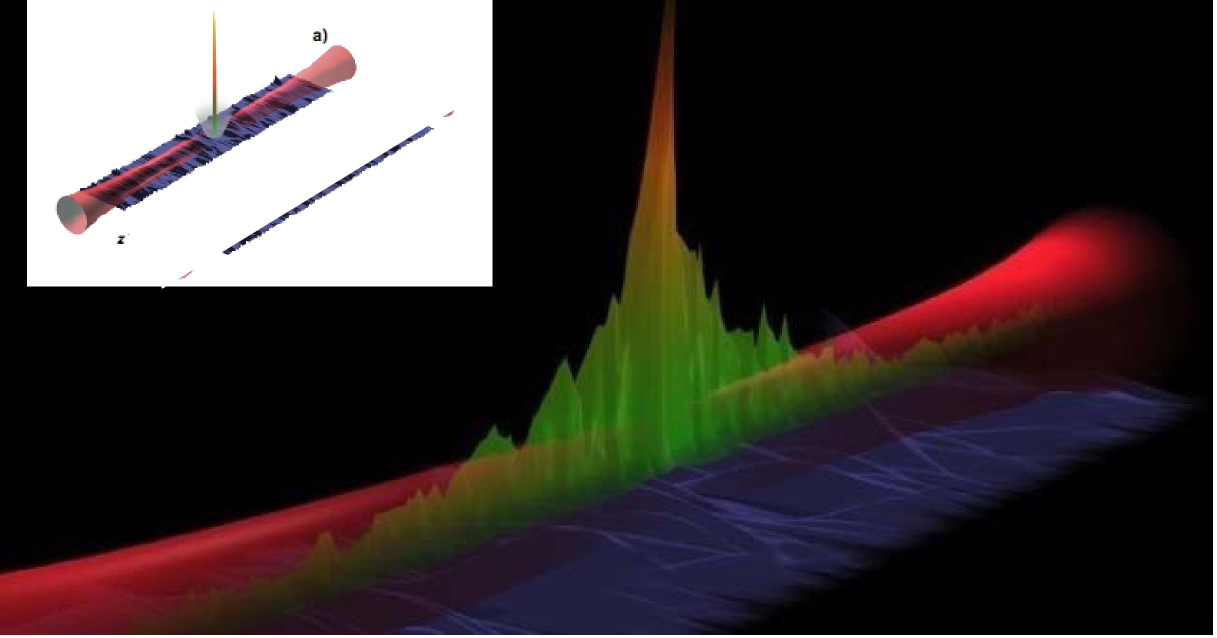
Low temperatures: $t << \gamma^{-1/2}$

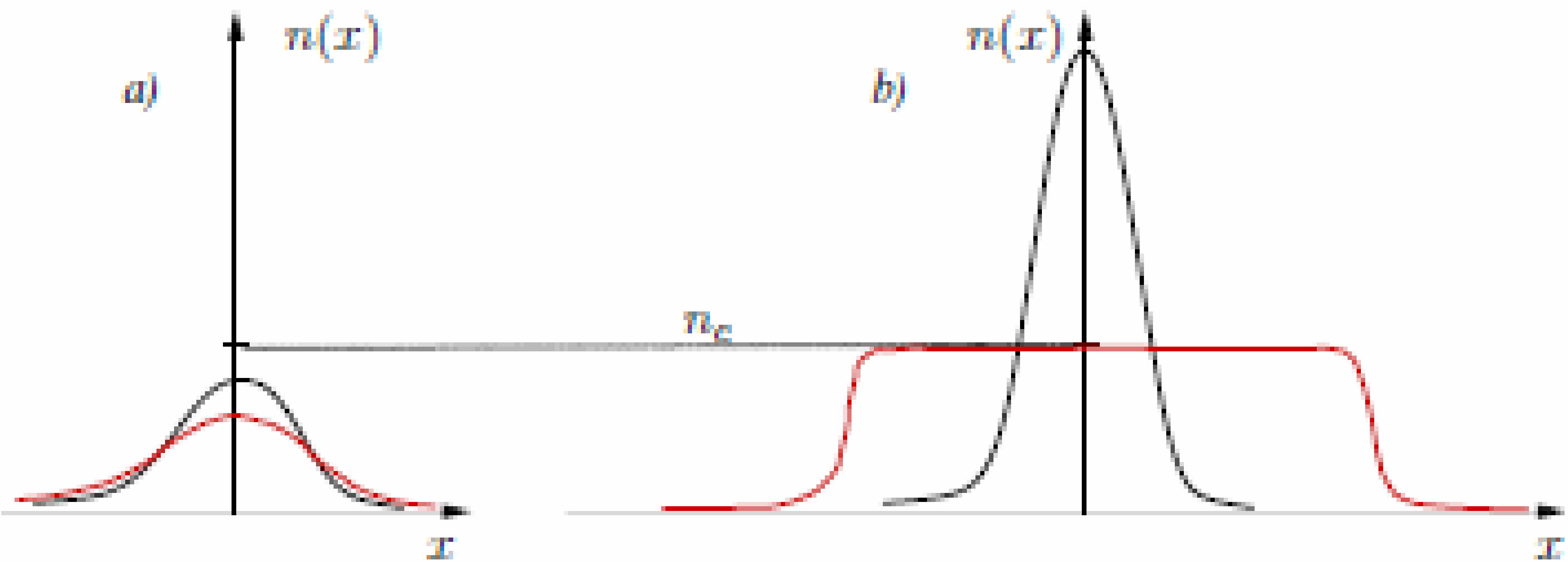
$\kappa \equiv E_* / ng >> 1$ → **Strong insulator**

$T = 0$ **transition** $\kappa_c \sim 1$

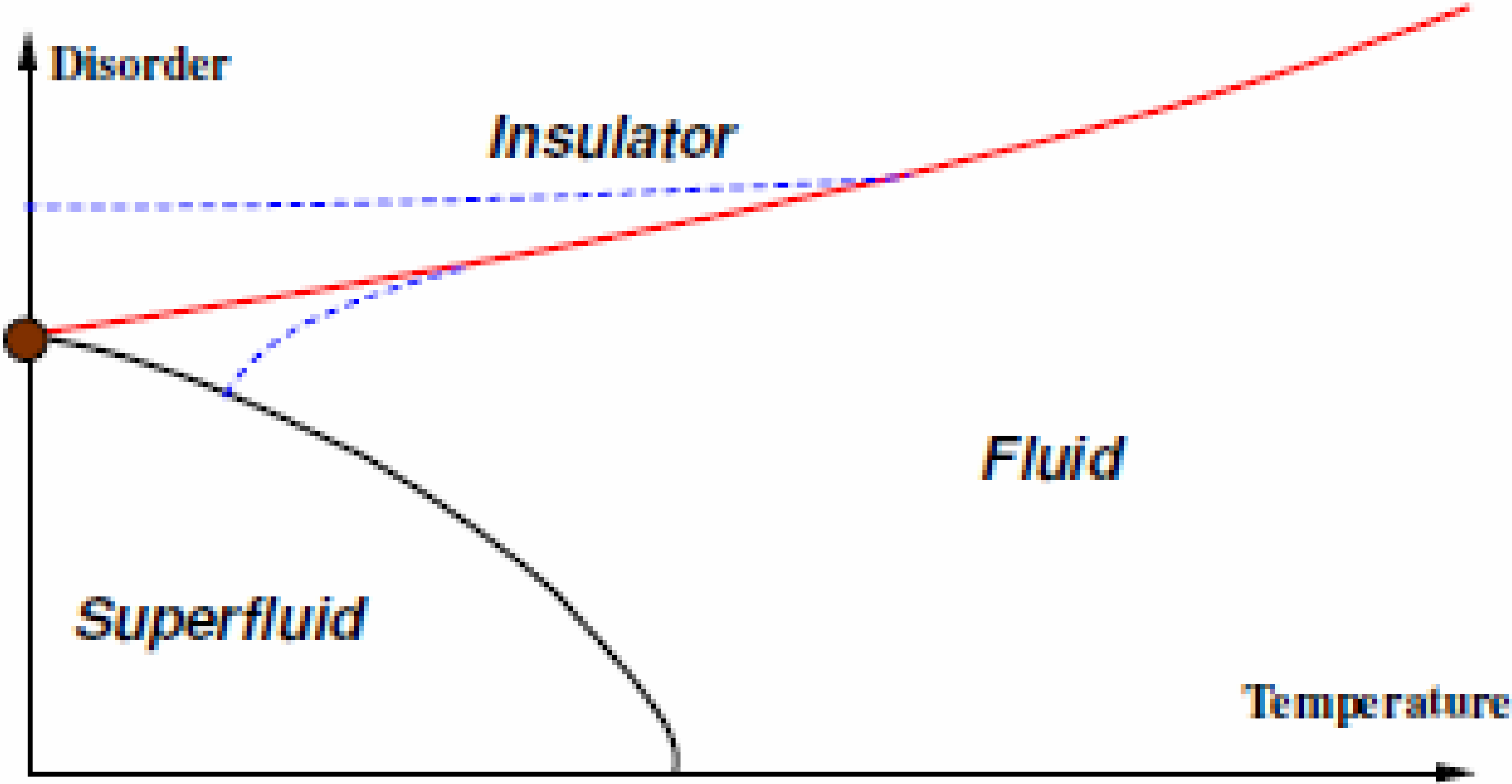
⇒ $\kappa_c \sim 1$ **for** $t << \gamma^{-1/2}$



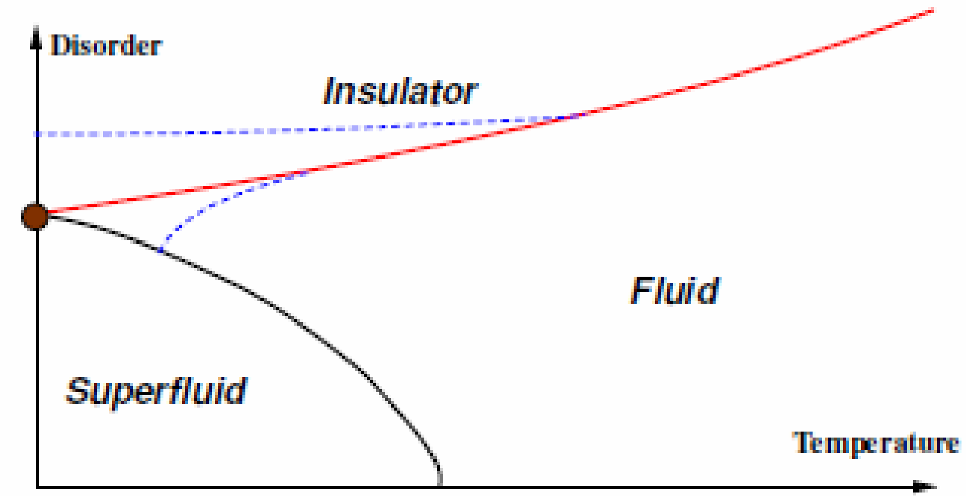

a)


n(x)
a)
n(x)
b)
n_c
x
x

Disordered interacting bosons in two dimensions



Disordered interacting bosons in two dimensions



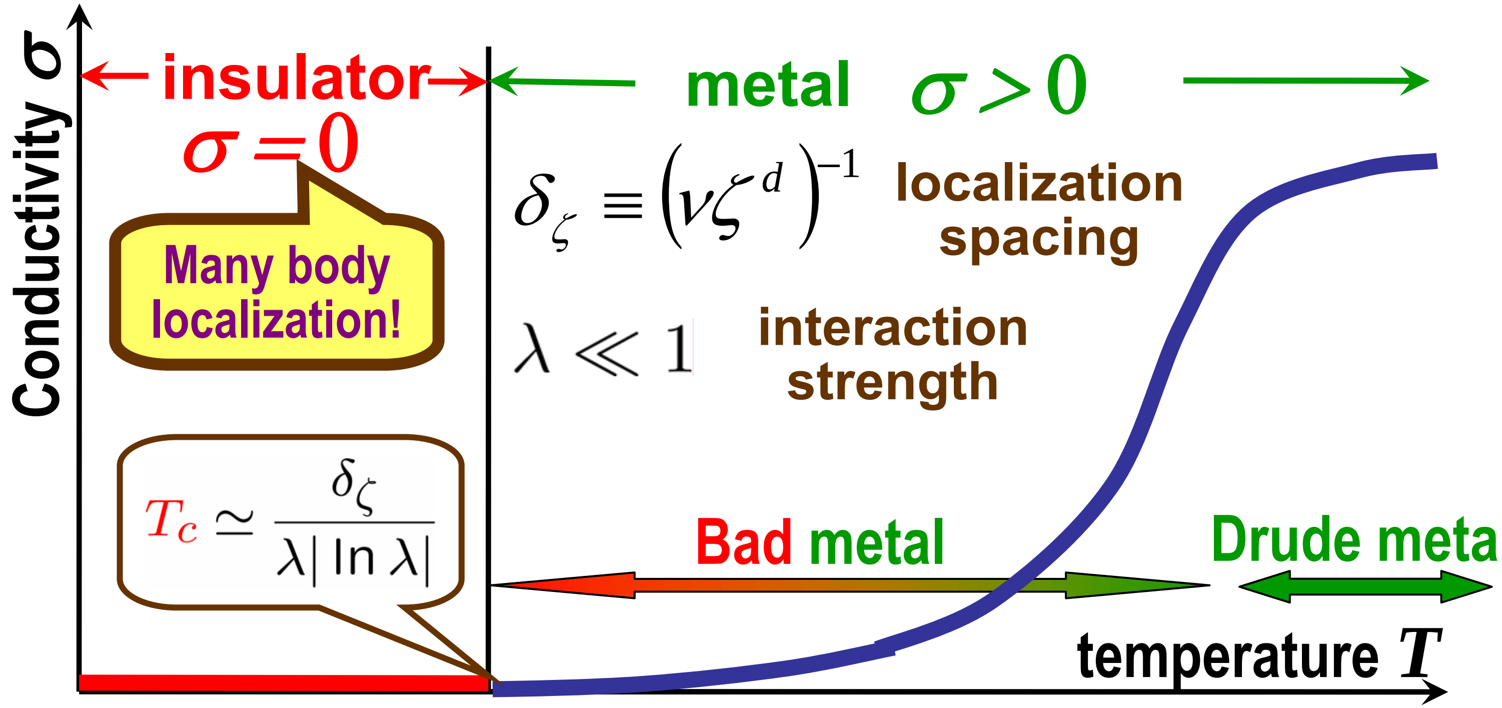
Justification:

1. At $T=0$ normal state is unstable with respect to either insulator or superfluid.

2. At finite temperature in the vicinity of the critical disorder the insulator can be thought of as a collection of "lakes", which are disconnected from each other. The typical size of such a "lake" diverges. This means that the excitations in the insulator state are localized but the localization length can be arbitrary large. Accordingly the many –body delocalization is unavoidable at an arbitrary low but finite T.

Lecture 3.

4. Speculations



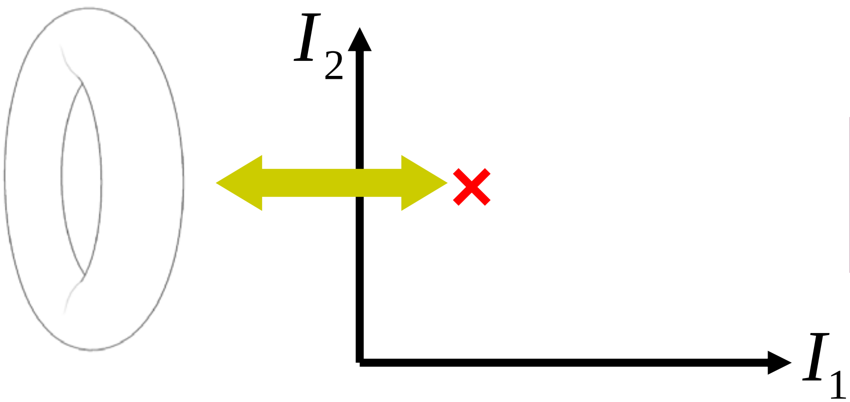
Q: What happens in the classical limit $\hbar \to 0$?

Speculations: 1. No transition $T_c \to 0$

2. Bad metal still exists

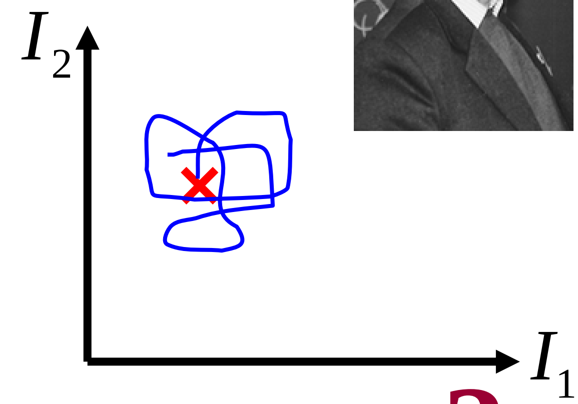
Reason: Arnold diffusion

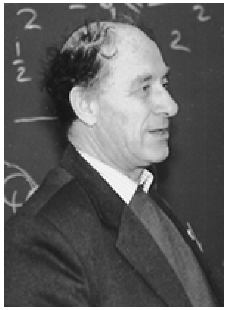
Arnold diffusion



Each point in the space of the **integrals of motion** corresponds to a torus and vice versa

$$\hat{V} \neq 0$$





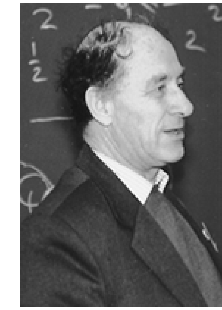
Finite motion?

$d = 2$ All classical trajectories correspond to a finite motion

$d > 2$ Most of the trajectories correspond to a finite motion

However small fraction of the trajectories goes infinitely far

Arnold diffusion



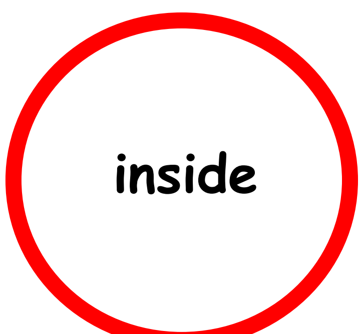
1. Most of the tori survive – KAM

2. Classical trajectories do not cross each other

space	# of dimensions
real space	d
phase space	$2d$
energy shell	$2d\text{-}1$
tori	d

$$d = 2 \quad \Rightarrow \quad d_{en.shell} - d_{tori} = 1$$

Each torus has "inside" and "outside"



$$d = 2 \quad \Rightarrow \quad d_{en.shell} - d_{tori} = 1$$

A torus does not have "inside" and "outside" as a ring in >2 dimensions

Speculations:

1. Arnold diffusion ⟺ Nonergodic (bad) metal
2. Appearance of the transition (finite T_c) – quantum localization of the Arnold diffusion

Conclusions

Anderson Localization provides a relevant language for description of a wide class of physical phenomena – far beyond conventional Metal to Insulator transitions.

Transition between integrability and chaos in quantum systems

Interacting quantum particles + strong disorder. Three types of behavior:

ordinary ergodic metal
"bad" nonergodic metal
"true" insulator

A closed system without a bath can relaxation to a microcanonical distribution only if it is an ergodic metal

Thank you